

## Lab # 11

### Fourier Transform, CTFT, DTFT and their properties.

#### Objective:

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#### Fourier Transform

Fourier Transformation transforms a composite time domain signal to frequency domain (complex) which shows the frequency components present in the system. The system can easily be analyzed through this transform. Fourier Transform falls in two types

1. Continuous time Fourier transform
2. Discrete Time Fourier transform

We will discuss them one by one

#### 1. Continuous time Fourier transform

CTFT is defined by the following relation.

$$F(e^{j\omega}) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

Inverse Fourier transform transforms the signal back into time domain and give mathematically as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(e^{j\omega})e^{j\omega t} d\omega$$

In Matlab we implement CTFT using symbolic math toolbox. Fourier and Fourier are the two functions used for Fourier and inverse furrier transform.

## **Frequency Response:**

The frequency response is a representation of the system's response to sinusoidal inputs at varying frequencies. The output of a linear system to a sinusoidal input is a sinusoid of the same frequency but with a different magnitude and phase. Any linear system can be *completely* described by how it changes the amplitude and phase of cosine waves passing through it. This information is called the system's frequency response. Since both the impulse response and the frequency response contain complete information about the system, there must be a one-to-one correspondence between the two.

## **Discrete Time Fourier Transform:**

The discrete-time Fourier transform (DTFT)  $X(e^{j\omega})$  of a sequence  $x[n]$  is defined

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

In general  $X(e^{j\omega})$  is a complex function of the real variable  $\omega$  and can be written as

$$X(e^{j\omega}) = X_{re}(e^{j\omega}) + jX_{im}(e^{j\omega}),$$

where  $X_{re}(e^{j\omega})$  and  $X_{im}(e^{j\omega})$  are, respectively, the real and imaginary parts of  $X(e^{j\omega})$ , and are real functions of  $\omega$ .  $X(e^{j\omega})$  can alternately be expressed in the form

$$X(e^{j\omega}) = |X(e^{j\omega})|e^{j\theta(\omega)},$$

$$\theta(\omega) = \arg\{X(e^{j\omega})\}.$$

The quantity  $|X(e^{j\omega})|$  is called the magnitude function and the quantity  $\theta(\omega)$  is called the phase function

In many applications, the Fourier transform is called the Fourier spectrum and, likewise,  $|X(e^{j\omega})|$  and  $\theta(\omega)$  are referred to as the magnitude spectrum and phase spectrum, respectively.

The DTFT  $X(e^{j\omega})$  is a periodic continuous function in  $\omega$  with a period  $2\pi$ . The DTFT satisfies a number of useful properties that are often utilized in a number of applications.

### **Matlab Commands:**

For complex  $Z$ , the magnitude  $R$  and phase angle  $\theta$  are given by:

$$\mathbf{R = abs (Z)}$$

$$\mathbf{\Theta = angle (Z)}$$

$Y = \text{fft}(X)$  returns the discrete Fourier transform of vector  $X$ , computed with a fast Fourier

transform (FFT) algorithm.

$$\mathbf{Y = fft(X)}$$

$Y = \text{fft}(X,n)$  returns the  $n$ -point FFT.

$$\mathbf{Y = fft(X, n)}$$

### **Example 1:**

Compute the discrete Fourier transform of the following function analytically and Then plot the magnitude and phase:

$$x(n) = 3(0.9)^n u(n)$$

Its DTFT is given as:

$$\frac{3}{1 - 0.9 e^{-j\omega}}$$

### **Matlab Code:**

```
w = [0:500]*pi/500;
```

```
z = exp(-j*w);
```

```
x = 3*(1-0.9*z).^(-1);
```

```
a = abs(x);
```

```
b = angle(x)*180/pi;
```

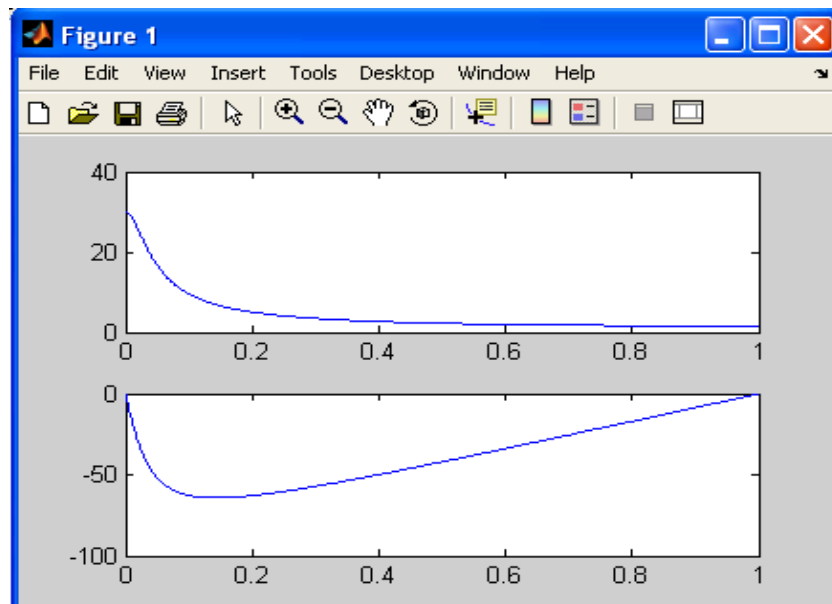
```
subplot(2,1,1);
```

```
plot(w/pi,a);
```

```
subplot(2,1,2);
```

```
plot(w/pi,b);
```

### **OUTPUT:**



### **Example 2:**

Evaluate the DTFT of the given coefficients.

**num=[2 1]**

**den=[1 -0.6]**

- Plot real and imaginary parts of Fourier spectrum.
- Also plot the magnitude and phase spectrum.

### **Matlab Code:**

```
% Evaluation of the DTFT
```

```
clc;
```

```
% Compute the frequency samples of the DTFT
```

```
w = -4*pi:8*pi/511:4*pi;
```

```
num = [2 1];
```

```
den = [1 -0.6];
```

```
h = freqz(num, den, w);
```

```
% Plot the DTFT
```

```
subplot(2,2,1)
```

```
plot(w/pi,real(h));
```

```
grid on;
```

```
title('Real part of  $H(e^{j\omega})$ ')
```

```
xlabel('\omega \pi');
```

```
ylabel('Amplitude');
```

```
subplot(2,2,2)

plot(w/pi,imag(h));

grid on;

title('Imaginary part of H(e^{j\omega})')

xlabel('\omega \pi');

ylabel('Amplitude');

subplot(2,2,3)

plot(w/pi,abs(h));

grid on;

title('Magnitude Spectrum |H(e^{j\omega})|')

xlabel('\omega \pi');

ylabel('Amplitude');

subplot(2,2,4)

plot(w/pi,angle(h));

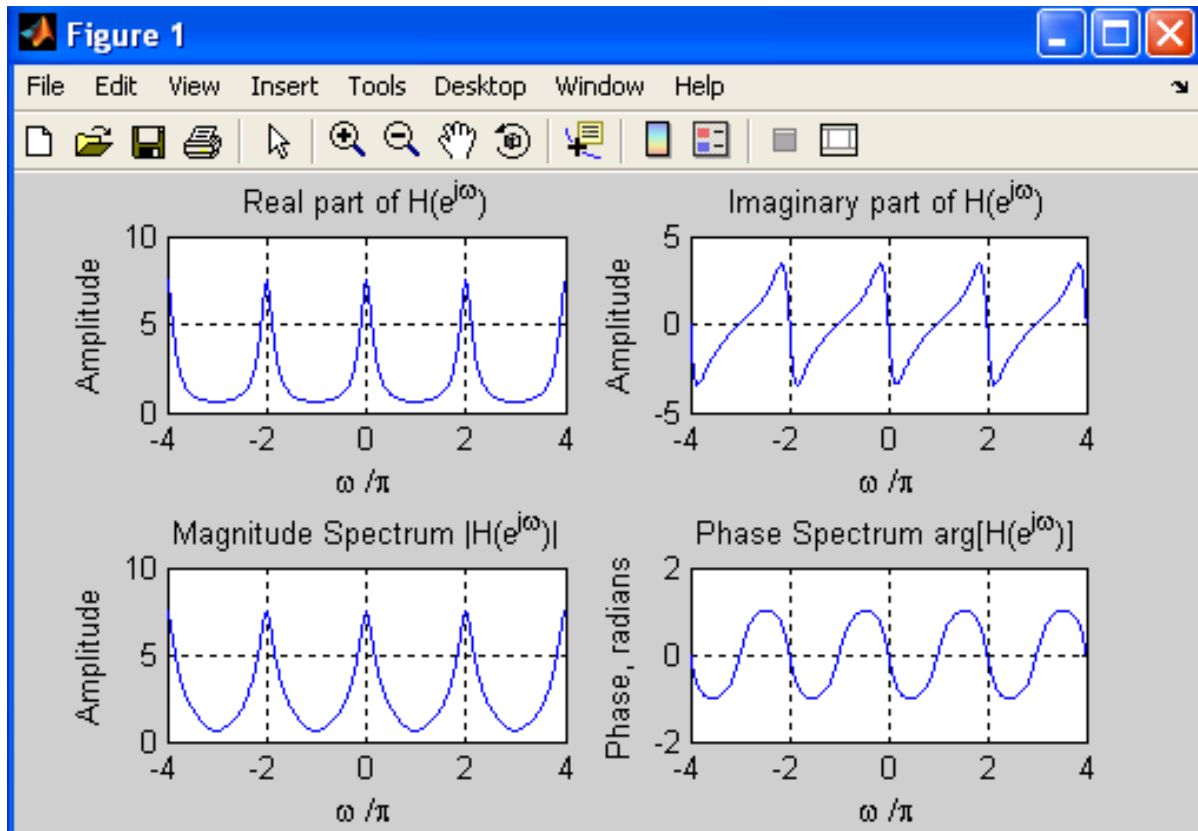
grid on;

title('Phase Spectrum arg[H(e^{j\omega})]')

xlabel('\omega \pi');

ylabel('Phase, radians');
```

**Output:**



### Fast Fourier Transform:

The Fast Fourier Transform (FFT) is just a computationally fast way to calculate the DFT.

### Example 3:

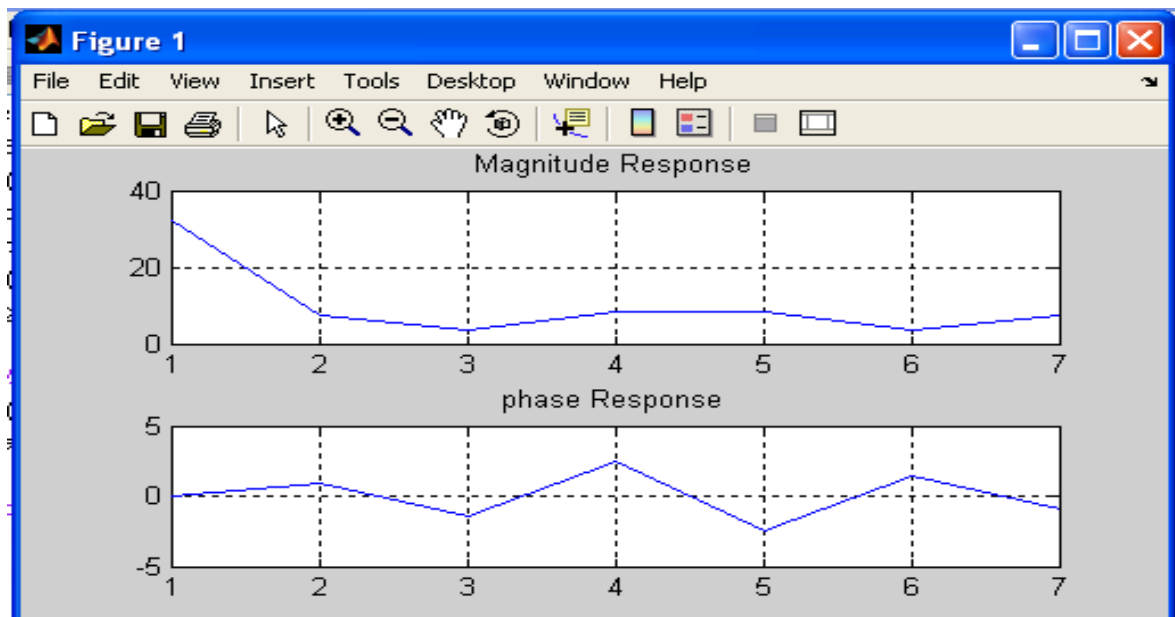
Determine the Fourier transform of the following sequence. Use the FFT (Fast Fourier Transform) function.

$$\mathbf{x(n) = \{4\ 6\ 2\ 1\ 7\ 4\ 8\}}$$

### Matlab Code:

```
n = 0:6;
x = [4 6 2 1 7 4 8];
a = fft(x);
mag = abs(a);
pha = angle(a);
subplot(2,1,1);
plot(mag);
grid on
title('Magnitude Response');
subplot(2,1,2);
plot(pha);
grid on
title('phase Response');
```

### **OUTPUT:**





## Post Lab Questions

a) What is Fourier Transform?

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b) Which command is used to find magnitude and phase angle of a complex Z?

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c) What is fast fourier transform and which command is used for it?

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## Lab Tasks

### Task 1

- a) Determine the Fourier transform of the following sequence. Use the FFT (Fast Fourier Transform) function,

$$x(n) = \{4 \ 3 \ 2 \ 1 \ 1 \ 2 \ 3 \ 4\}$$

- b) Evaluate the DTFT of the given coefficients.

$$\text{num}=[2 \ 1]$$

$$\text{den}=[1 \ -0.6]$$

- i. Plot real and imaginary parts of Fourier spectrum.
- ii. Also plot the magnitude and phase spectrum.

### Task 2

- a) Compute the discrete Fourier transform of the following function analytically and Then plot the magnitude and phase:

$$x(n)= 4(0.9)^n u(n)$$

Its DTFT is given as:

$$\frac{4}{1 - 0.9e^{-jw}}$$