

Lab # 10

Transfer function representation and the Laplace Transform, the inverse Laplace.

Objective:

Transfer Function Representation:

Transfer function of a system is defined as the ratio of output to input in frequency domain. The transfer functions are defined in MATLAB by storing the coefficients of numerator and denominator in vector form.

Generally a transfer function is represented as

$$H(s) = B(s) / A(s)$$

Where B and A are defined as

$$B = b_m S^m + b_{m-1} S^{m-1} + b_{m-2} S^{m-2} + \dots$$
$$A = a_m S^m + a_{m-1} S^{m-1} + a_{m-2} S^{m-2} + \dots$$

Coefficients of A and B are stored in vectors and then used forward.

Example: Given the following transfer function,

$$H(S) = \frac{2S^2 + 3}{S^3 + 4S^2 + 5}$$

We implement the transfer function in Matlab as

```
num=[2 0 3];           %Coefficients of the numerator  
den=[1 4 0 5];        %Coefficients of the denominator
```

To convert the vectors in transfer function we use:

```
h=tf(num,den);         %Transfer function
```

Output:

```
num =  
      2      0      3  
  
den =  
      1      4      0      5  
  
h =  
  
      2 s^2 + 3  
-----  
s^3 + 4 s^2 + 5
```

Zero, poles and gain can be found using the following

```
[z,p,k]=tf2zp(num,den);
```

```
z =
```

```
0 + 1.2247i
```

```
0 - 1.2247i
```

```
p =
```

```
-4.2737
```

```
0.1369 + 1.0729i
```

```
0.1369 - 1.0729i
```

```
k =
```

```
2
```

Transfer function can be obtained back from zero poles and gain.

```

[num,den]=zp2tf(z,p,k);
num =

    0  2.0000    0  3.0000

den =

    1.0000  4.0000    0  5.0000

tf(num,den)

Transfer function:
  2 s^2 + 3
-----
s^3 + 4 s^2 + 5

```

Now suppose we want to check the response of the system against a range of frequencies, we can this response using following

```

w=pi:pi/1000:2*pi*f;
[h,k]=freqz(num,den,w);

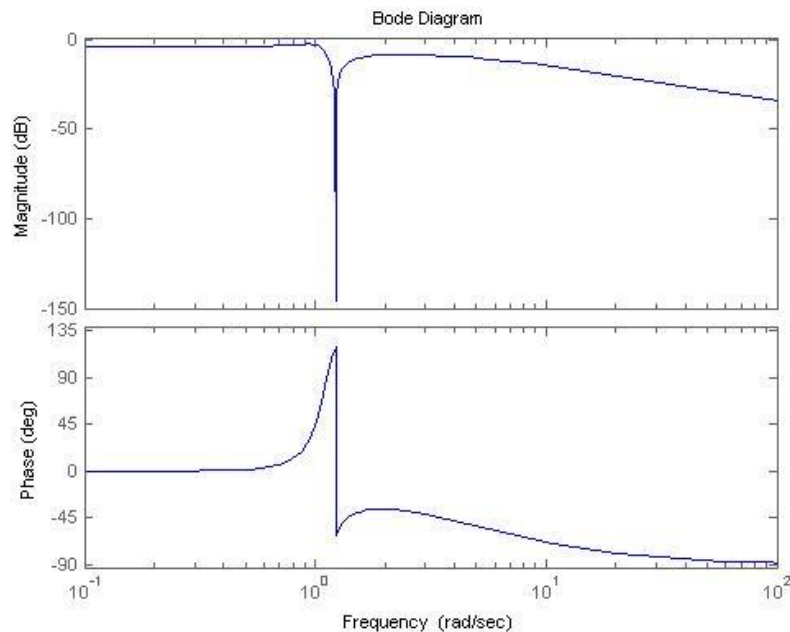
```

To draw the poles, zeros, gain and phase of the transfer function, we draw bode plot using following function

```

Bode(num,den)

```



Laplace Transform

Laplace transform is used to transform a function/signal from time domain to s domain (complex domain). System can be analyzed and performed different operations in S domain and then can be taken back into t domain. Laplace transform converts the function expression into simple algebraic polynomial. Following expression is used to calculate the Laplace transform of a time domain function f(t).

$$F(S) = \int_{-\infty}^{\infty} f(t)e^{-st} dt$$

In Matlab we solve the Laplace transform using symbolic math toolbox. Function used for finding the Laplace is Laplace().

Example:

Let f(t)=t and we have to find the Laplace transform of the function. Matlab code for this is given below.

```
Syms t;
F=t;
```

F=laplace(t)

Output returned is

$$F=1/S^2$$

Example:

Laplace transform of e^{-at} is calculated as:

Syms a t s

f=exp(-a*t);

F=laplace(f);

Output comes out to be

$$F=1/s+a$$

Inverse Laplace Transform

Inverse Laplace transform is used to transform s domain(complex domain) to time domain. Following is the relation for the inverse laplace transform.

$$f(t) = \frac{1}{2\pi i} \lim_{T \rightarrow \infty} \int_{\gamma-iT}^{\gamma+iT} F(S)e^{st} ds$$

In Matlab we find out the inverse laplace transform using symbolic math toolbox. ilaplace is function for finding the inverse laplace.

Example:

Given below the transfer function

$$H(S) = \frac{(S+3)}{(S+1)(S+2)}$$

Find the inverse laplace.

Matlab Code is given below

Syms s

H=(s+3)/((s+1)*(s+2));

F=ilaplace(H);

Output comes out as:

$$F=2*\exp(-t)-\exp(-2*t)$$

In finding the inverse Laplace often we encounter improper fractions having numerator power greater than denominator. For such fractions we first divide the polynomials (numerator and denominator) and then make partial fractions before taking inverse Laplace.

Example:

Find the inverse Laplace of the following transfer function

$$H(s) = \frac{S^3 + 3S^2 + 3}{S^3 + 8S^2 + S + 5}$$

In Matlab we solve it as:

```
num=[1 3 0 3]; % Co-efficients of numerator
den=[1 8 15]; % co-efficients of denominator
[Q,R]=deconv(num,den);
% Divisions of two polynomials to make improper fraction a proper fraction
[r,p,k]=residue(num,den);
%Partial fraction of the transfer function

%By residue we get the following parameters
% r =

    23.5000
     1.5000

p =

    -5
    -3

k =

     1     -5 %
```

From values above we write Matlab expression for h(s) as

```
syms s
h=(s-5)+(23.5/(s+5))+(1.5/(s+3));
f=ilaplace(h);
```

Output comes out to be

```
f = dirac(1,t)-5*dirac(t)+47/2*exp(-5*t)+3/2*exp(-3*t)
```

Note:

1. Deconv returns the output as quotient (Q) and Remainder ®
2. Residue converts the fraction to partial fraction and returns three vectors r (constants of numerators), p (poles) and K (direct terms).
3. Dirac is the built-in function for unit impulse.

Post Lab Questions

a) What is Laplace transform?

b) Which command is used to find Zero, poles and gain?

c) What is residue and deconv used for?

d) How do we find laplace transform of improper fraction?

Lab Tasks

Task 1

a) Find the Laplace Transform of the following:

i) $f(t) = 3e^{-8t}$

ii) $f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}$

b) Find the inverse laplace transform of the following:

(i) $\frac{1}{s^2(s+1)}$ (ii) $\frac{1}{(s-1)(s-2)}$ (iii) $\frac{1}{(s^2+1)^2}$

Task 2

a) Given:

$$Y(s) = \frac{4s^2 + 4s + 4}{s^2(s^2 + 3s + 2)}$$

- i) Find the partial fraction of the above given transfer function.
- ii) Find the quotient and remainder of Y(s).

b) Plot the poles, zeros, gain and phase of the transfer function:

$$G(s) = \frac{s^2 - 3s - 4}{s^2 + 5s + 6}$$