Lecture - 05

Analysis and Design of Two-way Slab System without Beams (Flat Plates and Flat Slabs)

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- **Session – I:** Flexural Analysis of Two-Way Slab System without Beams (Direct Design Method)

- **Session – II:** Shear Design for Two-Way Slab System without Beams (Flat Plates and Flat Slabs)

Session – I

Flexural Analysis and Design of Two-Way Slab System without Beams (Direct Design Method)

Topics Addressed

- Background
- Introduction to Direct Design Method (DDM)
- Frame Analysis steps using Direct Design Method (DDM)
- Example 1
- Two-Way Slabs (Other Requirements of ACI)
- Example 2
- Example 3
- Summary of Direct Design Method (DDM)

Objectives

At the end of this lecture, students will be able to

- Analyze flat slabs and flat plates for flexure using Direct Design Method (DDM)
- Analyze and design flat slabs and flat plates for shear

Background

- In previous lectures, a 90′ × 60′ Hall was analyzed as a one-way slab system as shown below.

Option 01

Option 02

Background

- Also, in previous lecture, the slab of 100′ × 60′ commercial building was analyzed as slab with beams using Moment coefficient method.

Background

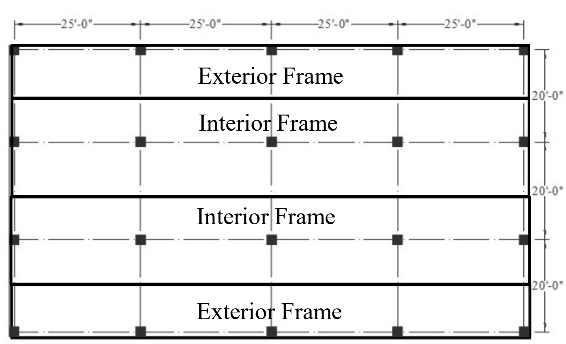
- In the same 100′ × 60′ commercial building, if there are shallow or no beams, Moment Coefficient Method cannot be used.
- Direct Design Method (DDM) can be used for such cases.

Introduction to DDM

- Direct Design Method (DDM) is one of the methods for flexural analysis of two-way slabs with or without beams.
- DDM is relatively difficult for slabs with beams or walls therefore we will use it only for slabs without beams.

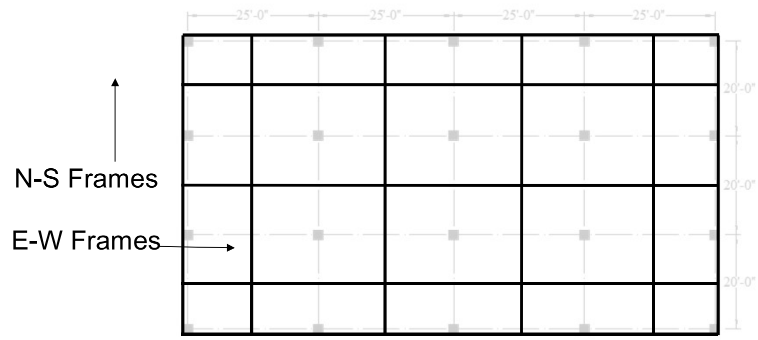
Introduction to DDM

- In DDM, frames rather than panels are analyzed.



Introduction to DDM

- For complete analysis of slab system, frames are analyzed in E-W and N-S directions.

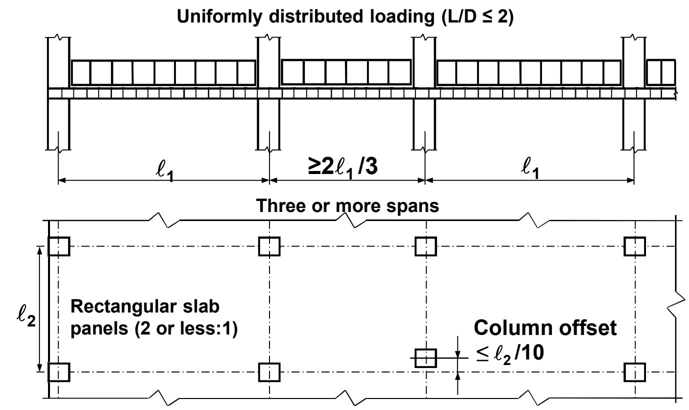


Introduction to DDM

- **Limitations**
  - Though DDM is useful for analysis of slabs, specially without beams, the method is applicable with some limitations as discussed next.

Introduction to DDM

- **Limitations (ACI 8.10.2):**



Steps In DDM

- **Step 01: Sizes**
  - ACI table 8.3.1.1 is used for finding the slab thickness.

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (in.)[1]

	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, psi[2]	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

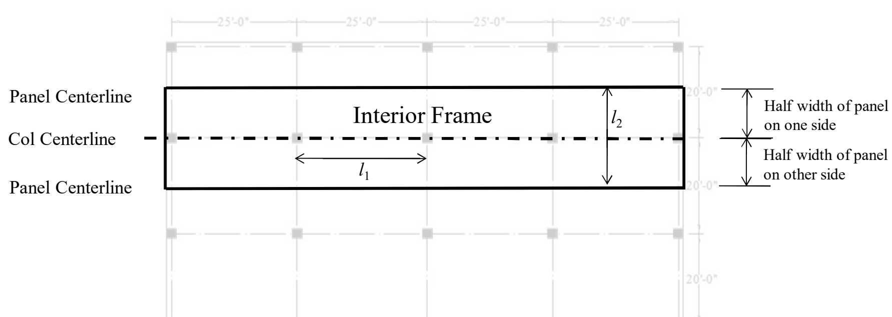
- h_{min} = 5 inches (slabs without drop panels)
- h_{min} = 4 inches (slabs with drop panels)

Steps In DDM

- **Step 02: Loads**
  - The slab load is calculated in usual manner.
    - Factored Load (w_u) = 1.2D.L + 1.6L.L

Steps In DDM

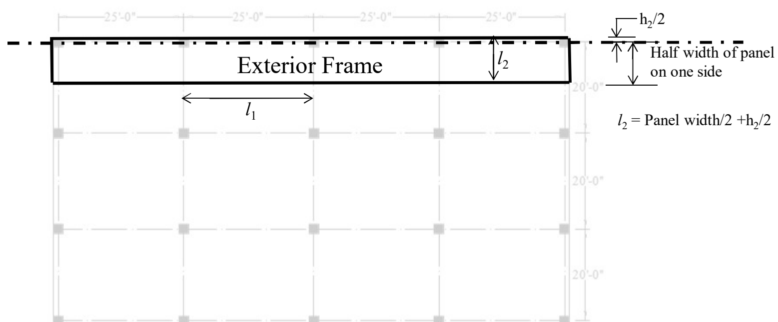
- **Step 03: Analysis**
  - **Step I:** a) Marking E-W Frame (Interior Frame)



According to ACI 8.10.3.2.2: Where the transverse span of panels on either side of the centerline of supports varies, l_2 shall be taken as the average of adjacent transverse spans.

Steps In DDM

- **Step 03: Analysis**
  - **Step I:** b) Marking E-W Frame (Exterior Frame)



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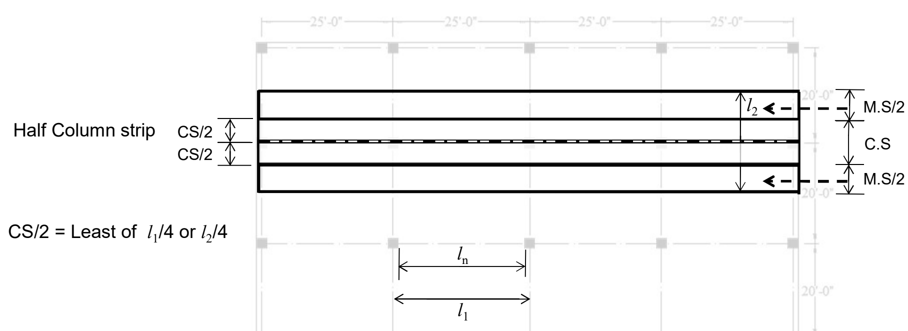
Steps In DDM

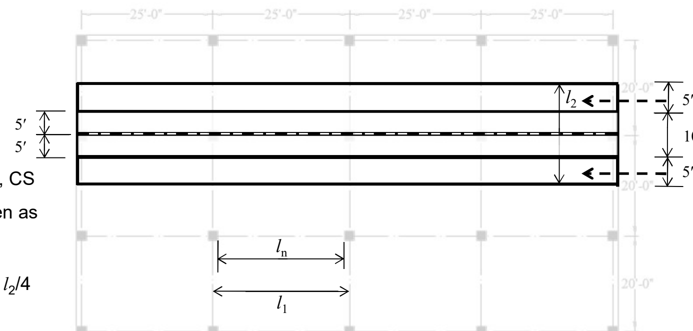
- **Step 03: Analysis**
  - **Step II:** **a)** Marking Column Strip (For Interior Frame)
    b) Marking Middle Strip (For Interior Frame)



Steps In DDM

- **Step 03: Analysis**
  - **Step II:** **a)** Marking Column Strip (For Interior Frame)
    b) Marking Middle Strip (For Interior Frame)

Half Column Strip



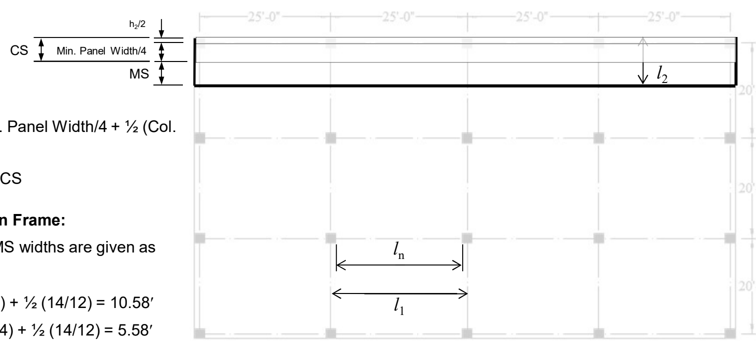
For l_1 = 25′ and l_2 = 20′, CS and MS widths are given as follows:

CS/2 = Least of $l_1/4$ or $l_2/4$

$l_2/4$ = 20/4 = 5′

Steps In DDM

- **Step 03: Analysis**
  - **Step II:** **c)** Marking Column Strip (For Exterior Frame)
    d) Marking Middle Strip (For Exterior Frame)



CS = Min. Panel Width/4 + ½ (Col. Size)

MS = l_2 - CS

For Given Frame:

CS and MS widths are given as follows:

l_2 = ½ (20) + ½ (14/12) = 10.58′

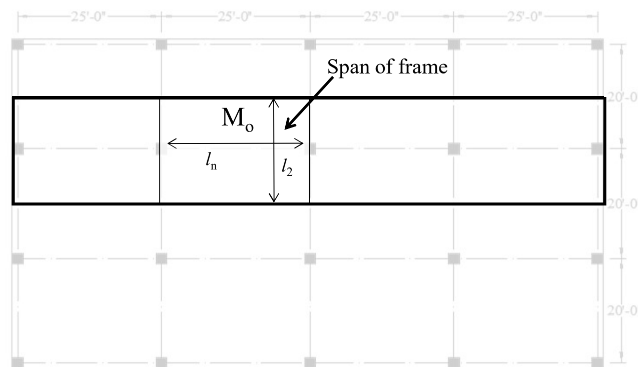
CS = (20/4) + ½ (14/12) = 5.58′

MS = 10.58 – 5.58 = 5′

Steps In DDM

- **Step 03: Analysis**
  - **Step III:** Calculate Static Moment (M_o) for interior span of frame.

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8}$$



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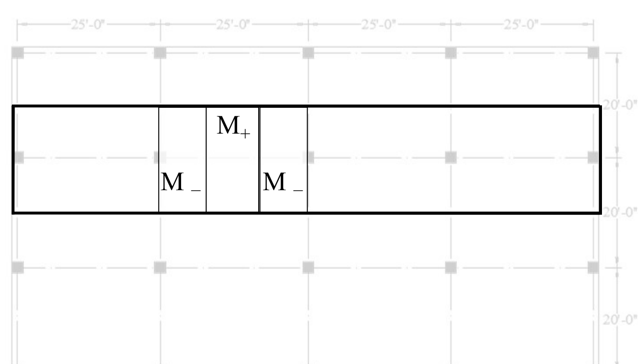
Steps In DDM

- **Step 03: Analysis**
  - **Step IV:** Longitudinal Distribution of Static Moment (M_o).

$M_- = 0.65M_o$

$M_+ = 0.35M_o$



Steps In DDM

- **Step 03: Analysis**
  - **Step V:** Lateral Distribution to column and middle strips.

$M_{-} = 0.65M_o$

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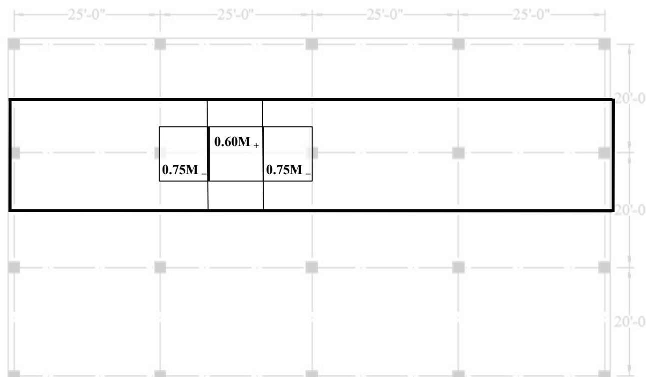
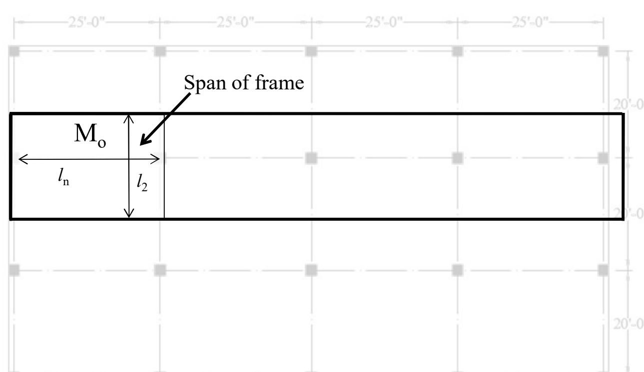


Steps In DDM

- **Step 03: Analysis**
  - **Step III:** Calculate Static Moment (M_o) for exterior span of frame.

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8}$$



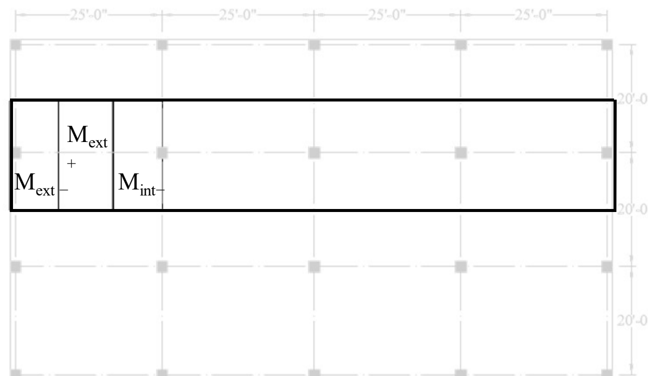
Steps In DDM

- **Step 03: Analysis**
  - **Step IV:** Longitudinal distribution of static moment (M_o).

M_{ext-} = $0.26M_o$

M_{ext+} = $0.52M_o$

M_{int-} = $0.70M_o$



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Steps In DDM

- **Step 03: Analysis**
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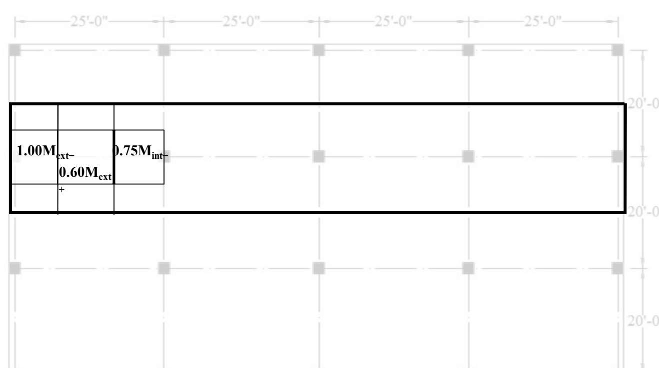

Steps In DDM

- **Step 03: Analysis**
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M_{ext-} = 0.26M_o

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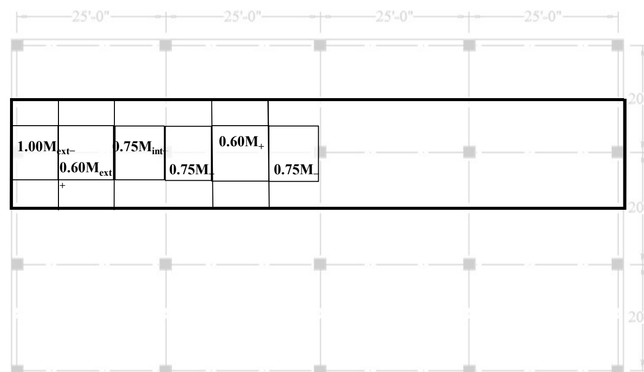


Example 1

- **Example 1:**
  - Analyze the flat plate shown below using DDM. The slab supports a live load of 144 psf. All columns are 14″ square. Take f_c' = 4 ksi and f_y = 60 ksi.

Example 1

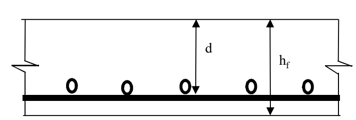
- **Step 01: Sizes**
  - ACI table 8.3.1.1 is used for finding flat plate and flat slab thickness.

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (in.)[1]

f_y, psi[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
40,000	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
75,000	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

Example 1

- **Step 01: Sizes**
  - Exterior panel governs. Therefore,
  - $h_f = l_n/30 = [\{25 - (2 \times 14/2)/12\}/30] \times 12 = 9.53''$ (ACI minimum requirement)
  - Take $h_f = 10''$
  - For #6 bars: $d = h_f - 0.75 - 0.75 = 8.5''$



Slab

Example 1

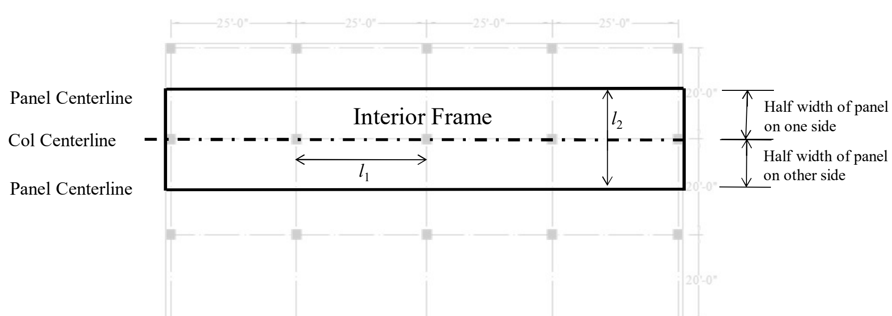
- **Step 02: Loads**
  - Service Dead Load (D.L) = $\gamma_{slab} h_f$

    = 0.15 × (10/12) = 0.125 ksf
  - Superimposed Dead Load (SDL) = Nil
  - Service Live Load (L.L) = 144 psf or 0.144 ksf
  - Factored Load (w_u) = 1.2D.L + 1.6L.L

    = 1.2 × 0.125 + 1.6 × 0.144 = 0.3804 ksf

Example 1

- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step I:** Marking E-W Interior Frame.



Example 1

- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step II:** Marking column and middle strips.

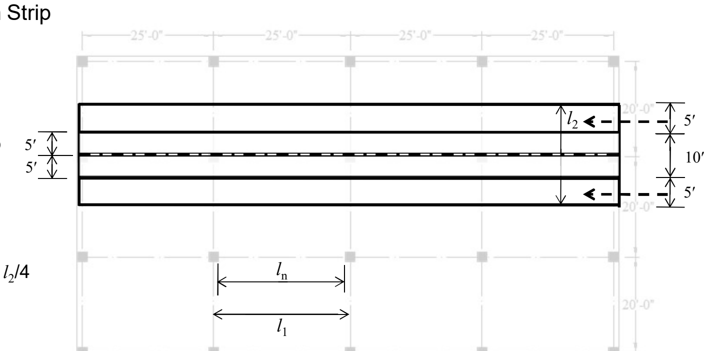
a) Marking Column Strip

b) Middle Strip

Half Column strip

CS/2 = Least of $l_1/4$ or $l_2/4$

$l_2/4 = 20/4 = 5'$

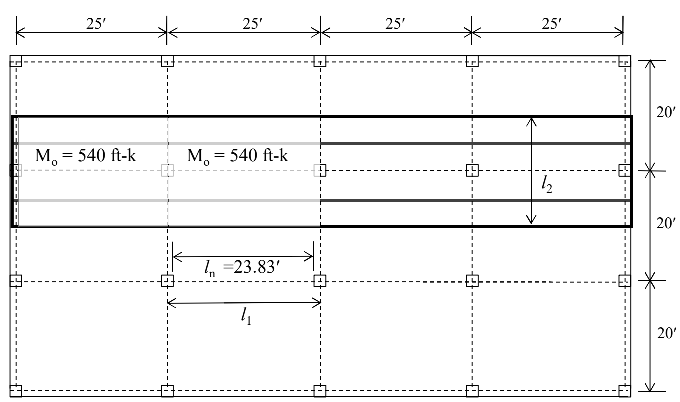


Example 1

- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step III:** Static Moment (M_o) calculation.

$M_o = w_u l_2 l_n^2/8$

$= 540$ ft-kip

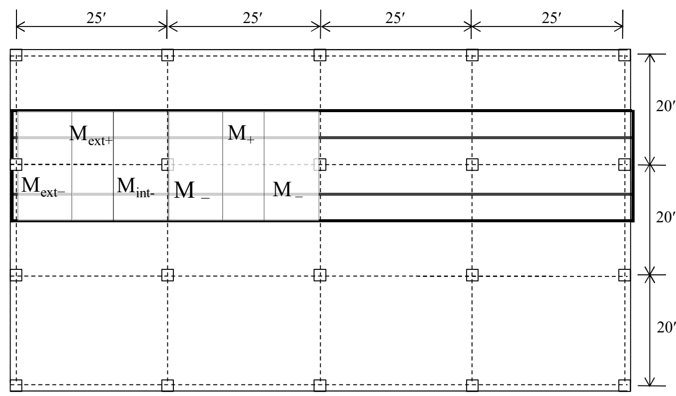


Example 1

- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step IV:** Longitudinal distribution of Static Moment (M_o).

$M_{ext-} = 0.26M_o = 140$
$M_{ext+} = 0.52M_o = 281$
$M_{int-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$



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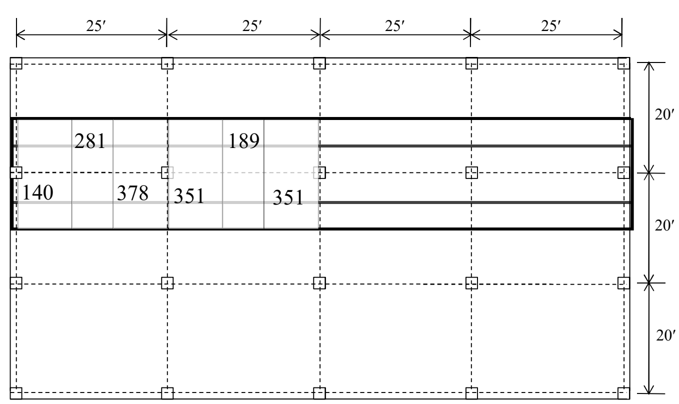

Example 1

- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step V:** Lateral distribution to column and middle strips.

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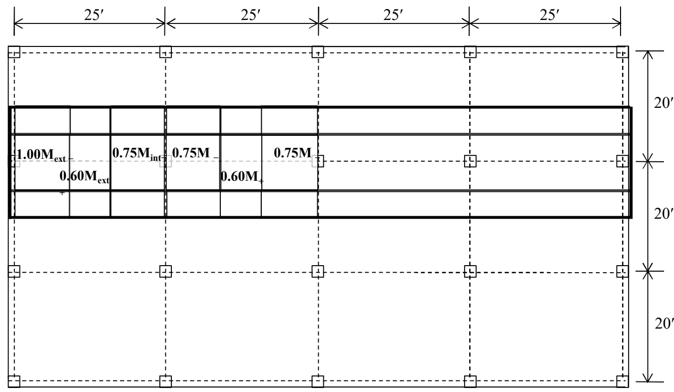

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Example 1

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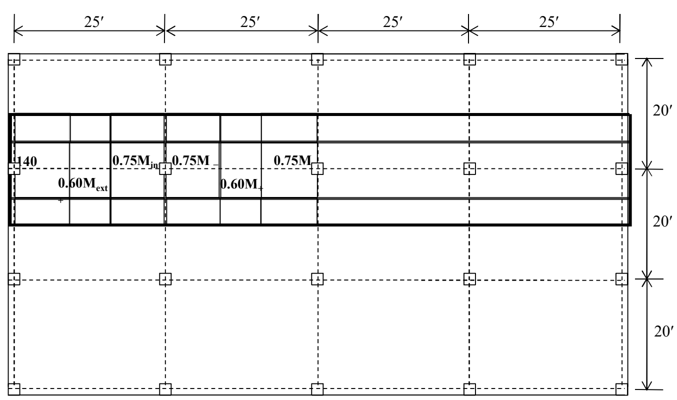
100 % of M_{ext-} goes to column strip and remaining to middle strip



Example 1

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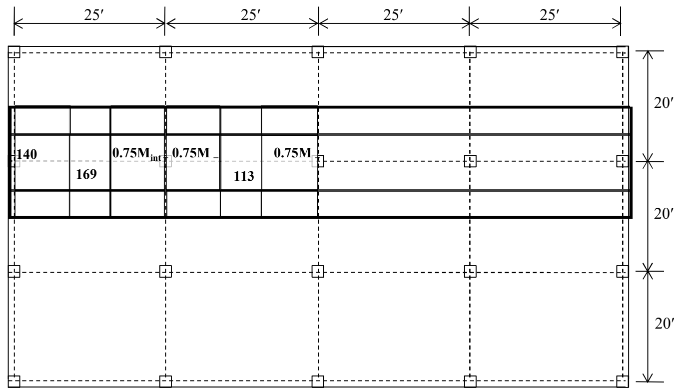
$M_{int\,-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$

60 % of M_{ext+} & M_{+} goes to column strip and remaining to middle strip



Example 1

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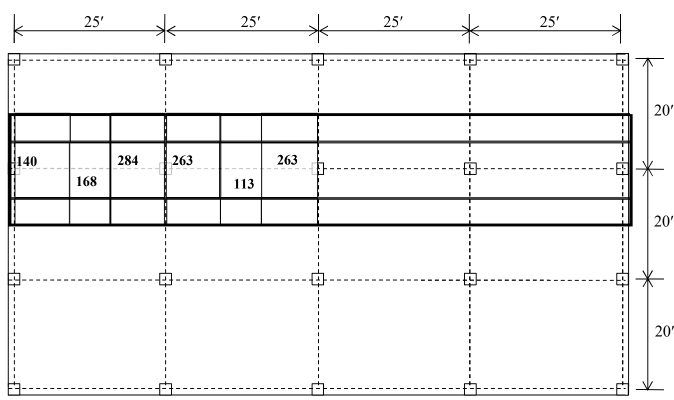
$M_{int\,-} = 0.70M_o = 378$

$M_{-} = 0.65M_o = 351$
$M_{+} = 0.35M_o = 189$

75 % of M_{int-} goes to column strip and remaining to middle strip



Example 1

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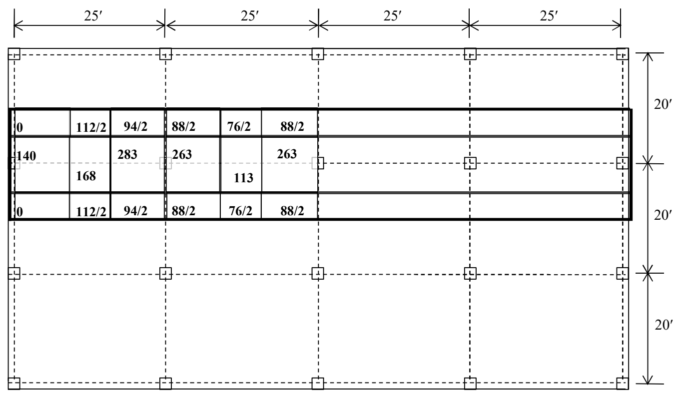
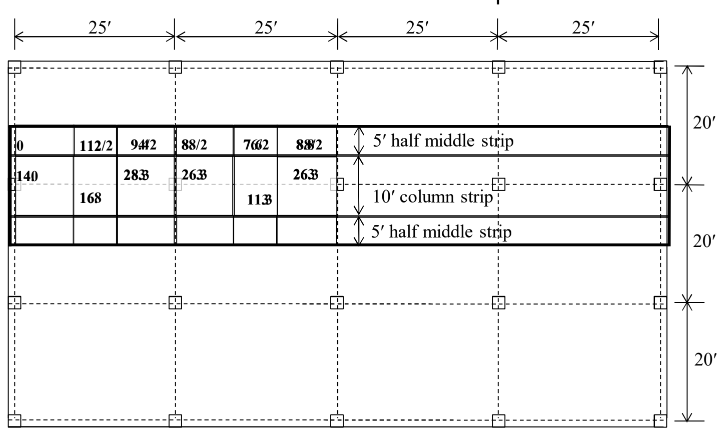


Example 1

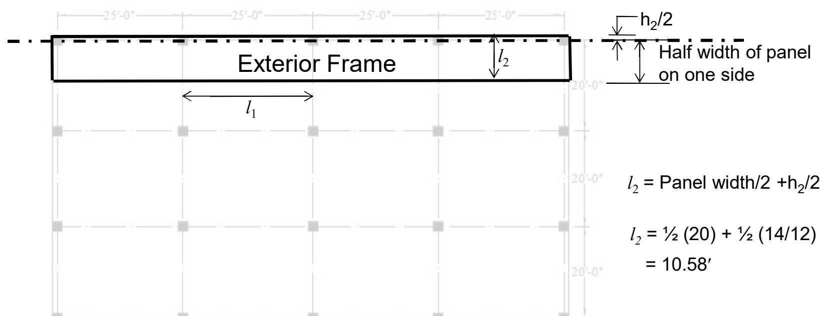
- **Step 03: Frame Analysis (E-W Interior Frame)**
  - **Step V:** Lateral distribution to column and middle strips.

M_u (per foot width)
= M / strip width



Example 1

- **Step 03: Frame Analysis (E-W Exterior Frame)**
  - **Step I:** Marking E-W Exterior Frame



l_2 = Panel width/2 +$h_2/2$

l_2 = ½ (20) + ½ (14/12)
= 10.58′

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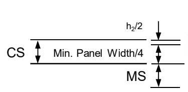
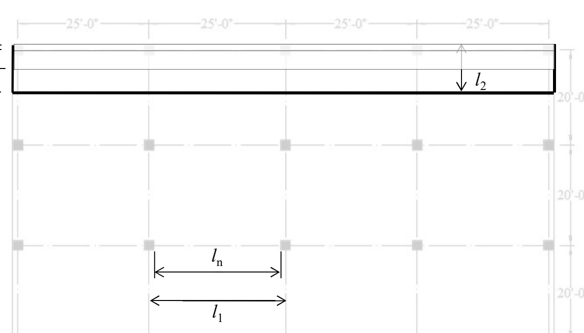
Example 1

- **Step 03: Frame Analysis (E-W Exterior Frame)**
  - **Step II:** Marking Column Strip and Middle Strips





CS = Min. Panel Width/4 + ½ (Col. Size)

MS = l_2 - CS

CS and MS widths are given as follows:

l_2 = ½ (20) + ½ (14/12) = 10.58′

CS = (20/4) + ½ (14/12) = 5.58′

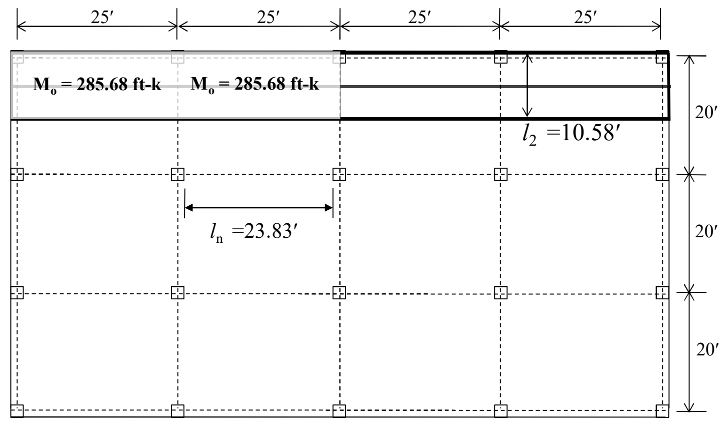
MS = 10.58 – 5.58 = 5′

Example 1

- **Step 03: Frame Analysis (E-W Exterior Frame)**
  - **Step III:** Calculation of static moment, M_o

$M_o = w_u l_2 l_n^2/8$

$= 285.68$ ft-kip



Example 1

- **Step 03: Frame Analysis (E-W Exterior Frame)**
  - **Step IV:** Longitudinal distribution to column and middle strips.

$M_o = 285.68$ ft-kip

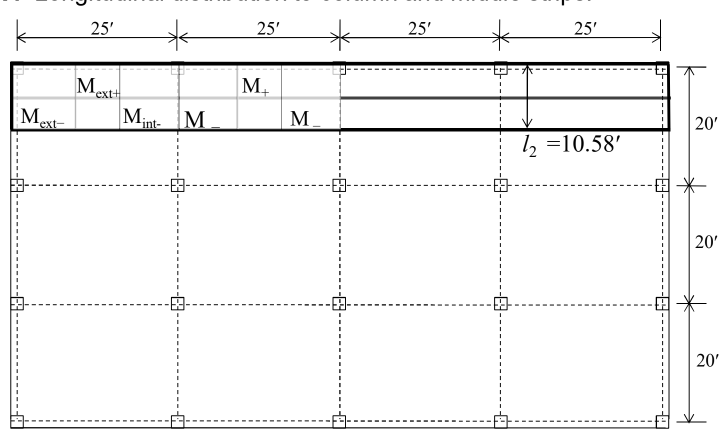
$M_{ext-} = 0.26M_o = 74$

$M_{ext+} = 0.52M_o = 148$

$M_{int-} = 0.70M_o = 200$

$M_{-} = 0.65M_o = 186$

$M_{+} = 0.35M_o = 100$



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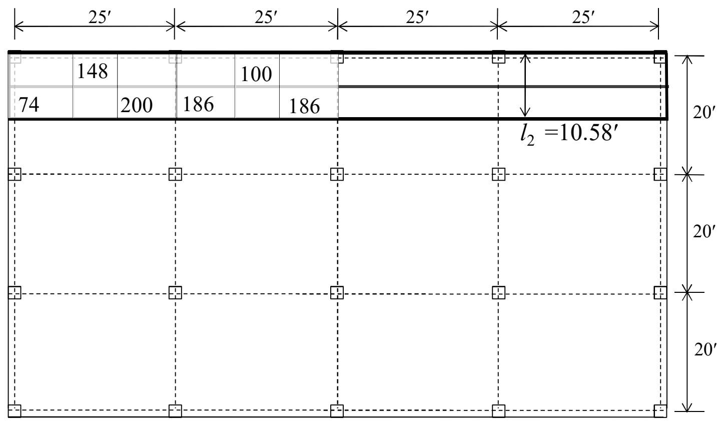

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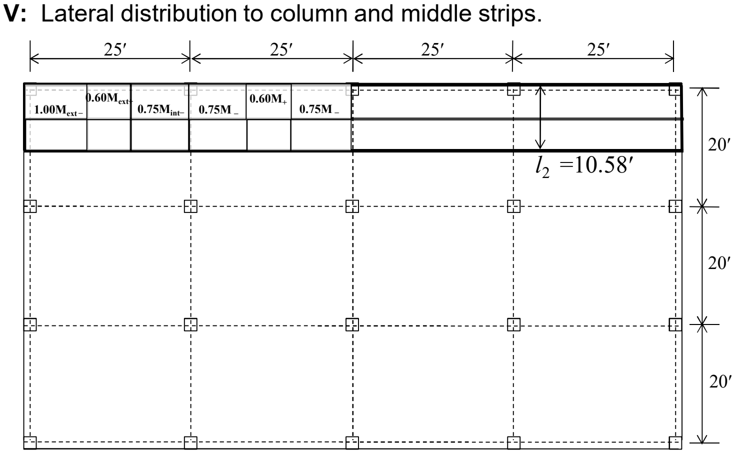

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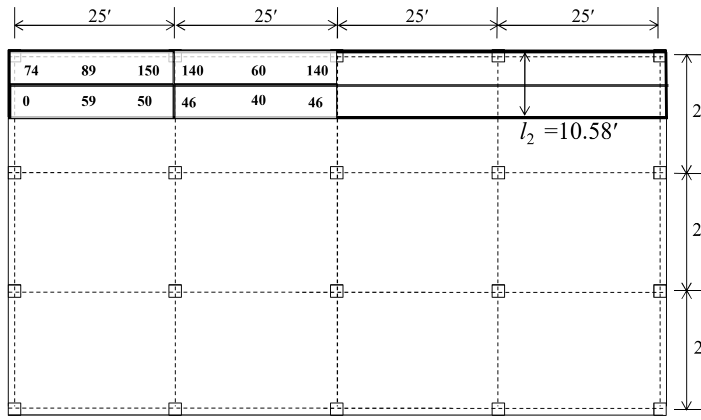
M_- = 0.65M_o = 186

M_+ = 0.35M_o = 100



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Example 1

- **Step 03: Frame Analysis (E-W Exterior Frame)**
  - **Step V:** Lateral distribution to column and middle strips (per strip width)

M_o = 285.68 ft-kip

M_{ext-} = 0.26M_o = 74

M_{ext+} = 0.52M_o = 148

M_{int-} = 0.70M_o = 200

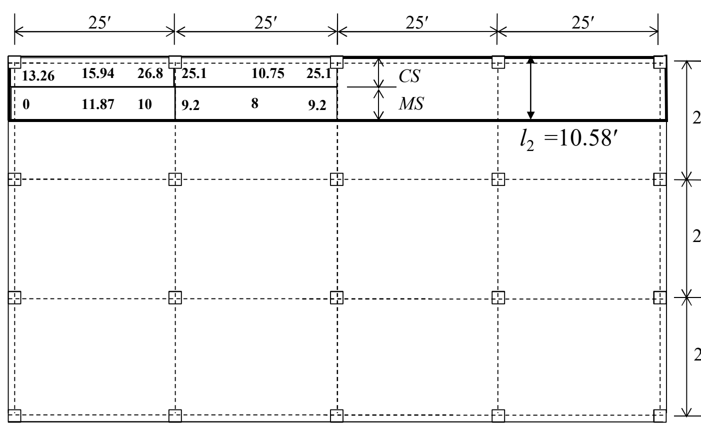
M_- = 0.65M_o = 186

M_+ = 0.35M_o = 100

l_2 = ½ (20) + ½ (14/12) = 10.58′

CS = (20/4) + ½ (14/12) = 5.58′

MS = 10.58 – 5.58 = 5′



Example 1

- **Step 03: Frame Analysis (N-S Interior Frame)**
  - **Column Strip and Middle strip Moments**

M_o = 421.5 ft-kip

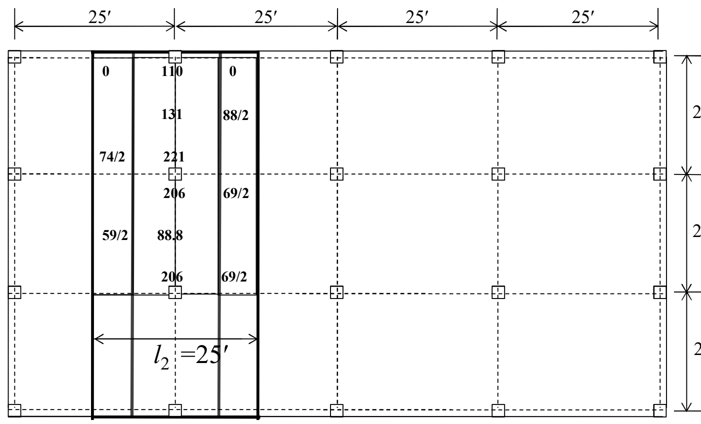
M_{ext-} = 0.26M_o = 110

M_{ext+} = 0.52M_o = 219

M_{int-} = 0.70M_o = 295

M_- = 0.65M_o = 274

M_+ = 0.35M_o = 148



Example 1

- **Step 03: Frame Analysis (N-S Interior Frame)**
  - **Column Strip and Middle strip Moments (Per Strip Width)**

M_o = 421.5 ft-kip

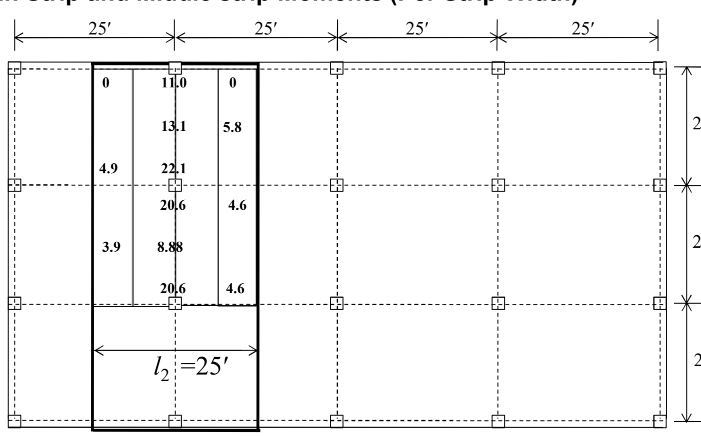
M_{ext-} = 0.26M_o = 110

M_{ext+} = 0.52M_o = 219

M_{int-} = 0.70M_o = 295

M_- = 0.65M_o = 274

M_+ = 0.35M_o = 148



Example 1

- **Step 03: Frame Analysis (N-S Exterior Frame)**
  - **Column Strip and Middle strip Moments**

M_o = 220.5 ft-kip

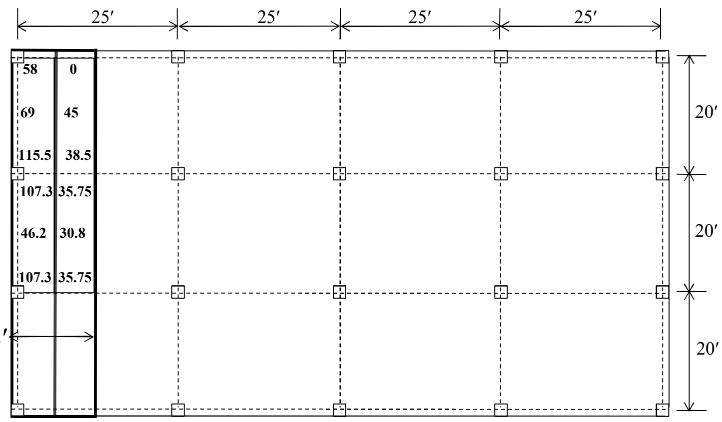
$M_{ext-} = 0.26M_o = 58$

$M_{ext+} = 0.52M_o = 114$

$M_{int-} = 0.70M_o = 154$

$M_- = 0.65M_o = 143$

$M_+ = 0.35M_o = 77$



l_2 =13.08′

l_2 = ½ (25) + ½ (14/12)
= 13.08′

Example 1

- **Step 03: Frame Analysis (N-S Exterior Frame)**
  - **Column Strip and Middle strip Moments (Per Strip Width)**

M_o = 220.5 ft-kip

$M_{ext-} = 0.26M_o = 58$

$M_{ext+} = 0.50M_o = 110$

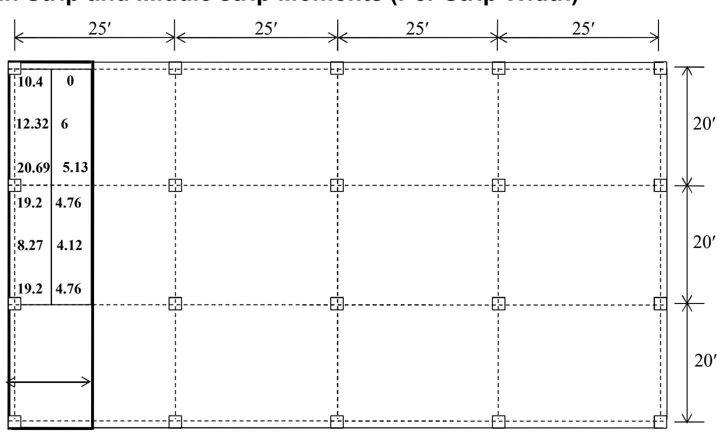
$M_{int-} = 0.70M_o = 154$

$M_- = 0.65M_o = 143$

$M_+ = 0.35M_o = 77$

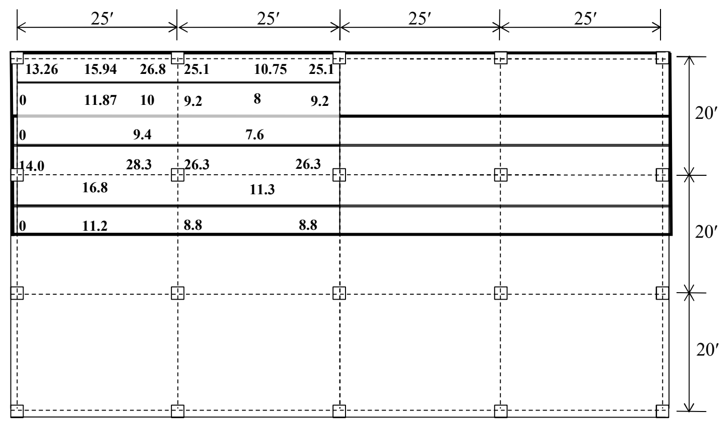
Column Strip Width
= (20/4) + ½ (14/12)
= 5.58′
MS = 13.08 – 5.58
= 7.5′

l_2 =13.08′



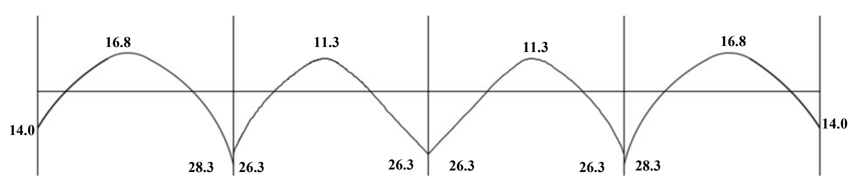
Example 1

- **Step 03: Frame Analysis (E-W Direction Moments)**

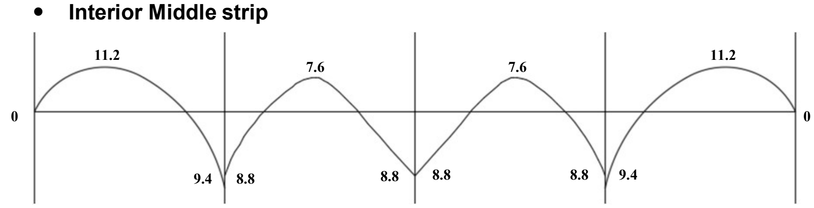


Example 1

- **Step 03: Frame Analysis (E-W Direction Moments)**
  - **Interior Column strip**



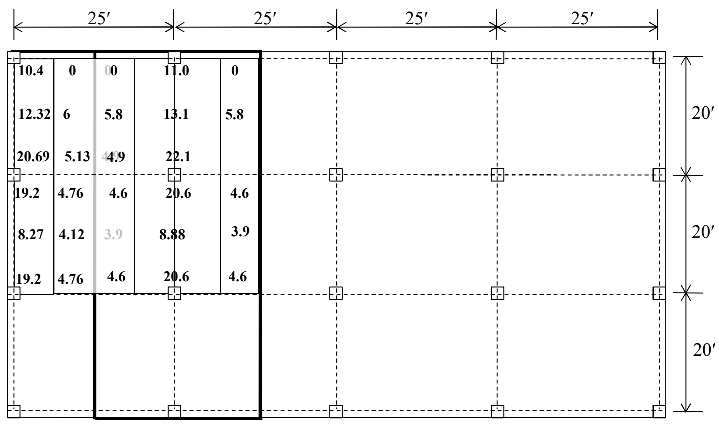
  - **Interior Middle strip**



- **All moments are in ft-kip unit.**

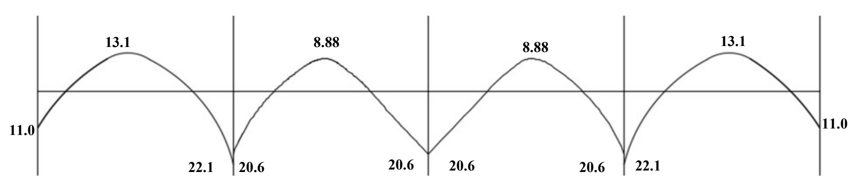
Example 1

- **Step 03: Frame Analysis (N-S Direction Moments)**

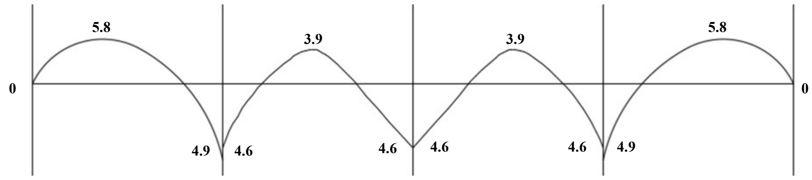


Example 1

- **Step 03: Frame Analysis (N-S Direction Moments)**
  - **Interior Column strip**



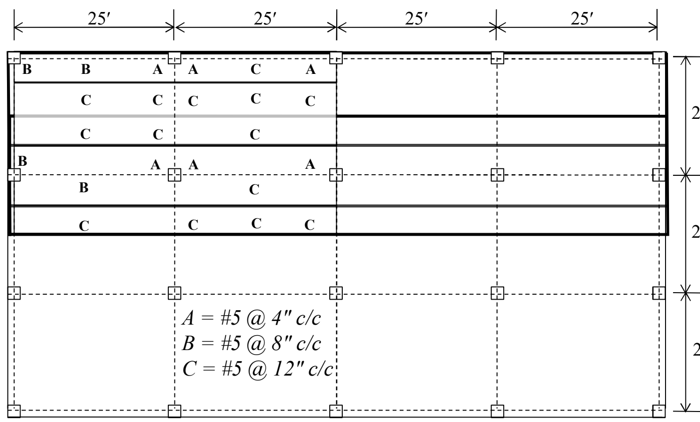
  - **Interior Middle strip**



- All moments are in ft-kip unit.

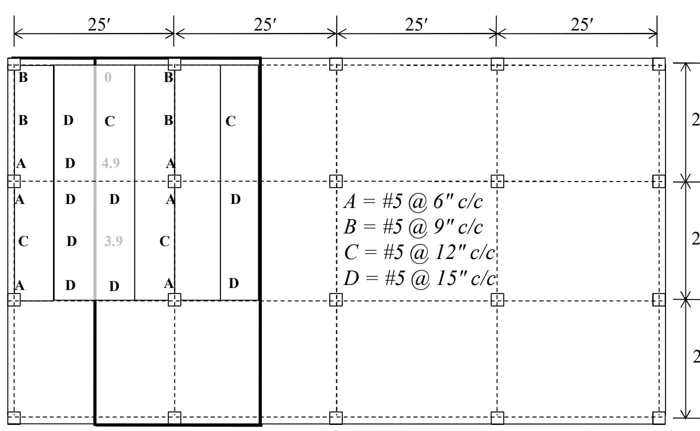
Example 1

- **Step 04: Design (E-W Direction)**



Example 1

- **Step 04: Design (N-S Direction)**

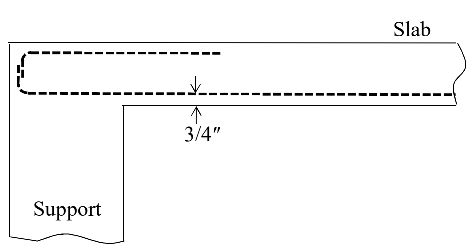


(General Requirements of ACI Code for two way slab)

- Maximum spacing and minimum reinforcement requirement:
  - **Maximum spacing (ACI 8.7.2.2):**
    - $s_{max} = 2h_f$ in each direction.
  - **Minimum Reinforcement (ACI 24.4.3.2):**
    - $A_{smin} = 0.0018\ bh_f$ for grade 60.
    - $A_{smin} = 0.002\ bh_f$ for grade 40 and 50.

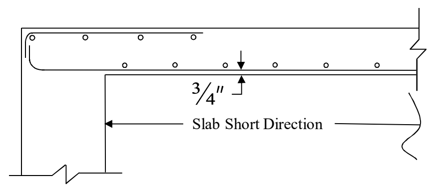
Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - At least 3/4" cover for fire or corrosion protection.



Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - Reinforcement placement: In case of two way slabs supported on beams, short-direction bars are normally placed closer to the top or bottom surface of the slab, with the larger effective depth because of greater moment in short direction.



Slab with stiff beams

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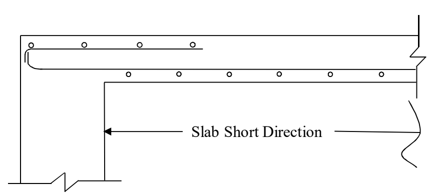
Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - Reinforcement placement: However in the case of flat plates/slabs, the long-direction negative and positive bars, in both middle and column strips, are placed closer to the top or bottom surface of the slab, respectively, with the larger effective depth because of greater moment in long direction.



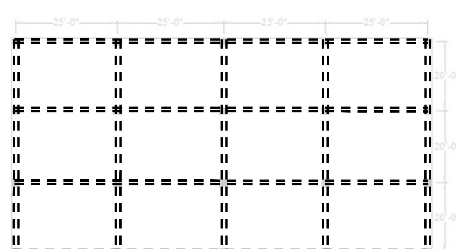
Flat Plate

Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - Splicing: ACI 8.7.4.2.1 requires that all bottom bars within the column strip in each direction be continuous or spliced with length equal to 1.0 l_d, (For development length see ACI 25.4.2.3 or Nelson 13th Ed, page 172 chapter 5 or mechanical or welded splices)
  - For #7 or larger no. of bars of f_y = 60 ksi, and normal weight, uncoated concrete of f_c' = 4 ksi:
    - $l_d = 47\ d_b$

Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - Continuity of bars: ACI 8.7.4.2.2 requires that at least two of the column strip bottom bars in each direction must pass within the column core and must be anchored at exterior supports.

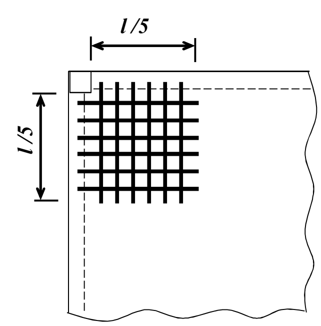


Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - Standard Bar Cut off Points (Practical Recommendation) for column and middle strips both.

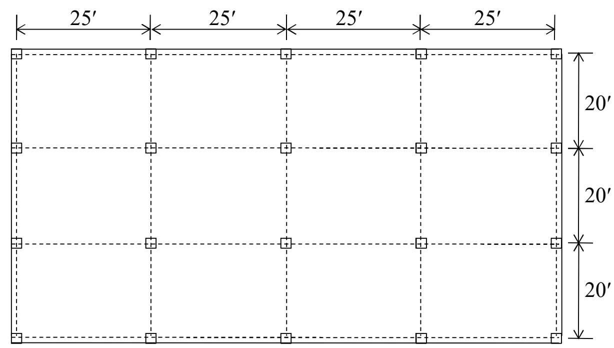
Other Requirements of ACI Code for Flat Slab

- Detailing of flexural reinforcement for column supported two-way slabs:
  - <u>Reinforcement at Exterior Corners:</u>
    - Reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the longer span of the corner panel as shown in figure.
    - The positive and negative reinforcement should be of size and spacing equivalent to that required for maximum positive moments (per foot of width) in the panel. (*ACI 8.7.3)*



Example 2

- **Homework:** Analysis results of the slab shown below using DDM are presented next. The slab supports a live load of 60 psf. Superimposed dead load is equal to 40 psf. All columns are 14″ square. Take f_c' = 3 ksi and f_y = 40 ksi.

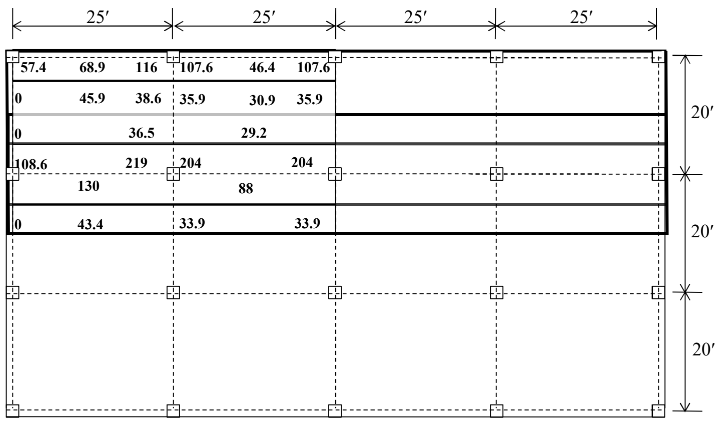


Example 2

- **Calculation summary**
  - Slab thickness h_f = 10″
  - Factored load (w_u) = 0.294 ksf
  - Column strip width = 5′

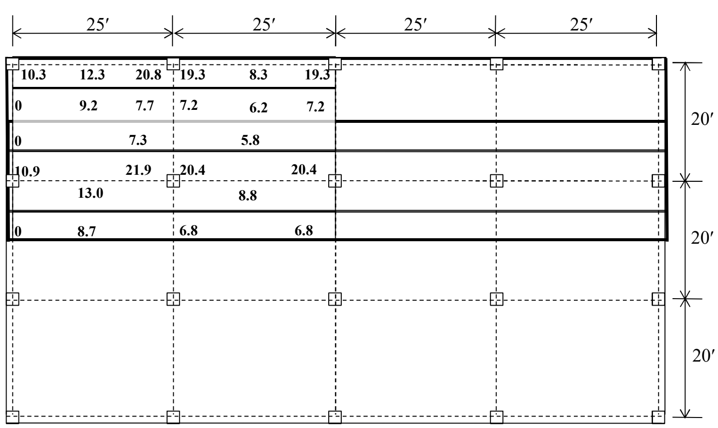
Example 2

- **E-W Direction Moments** (units: kip-ft)



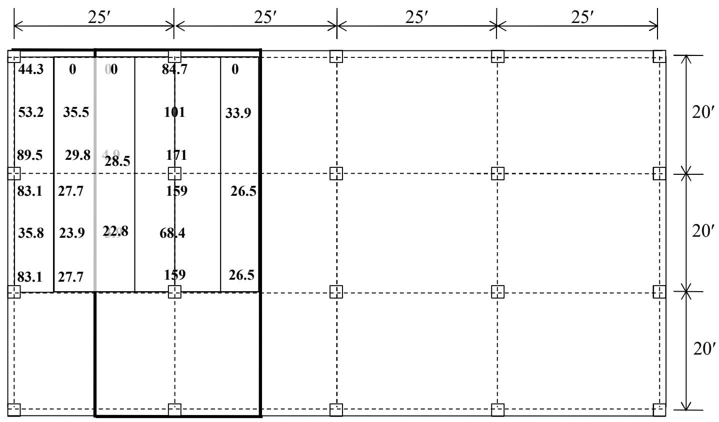
Example 2

- **E-W Direction Moments** (units: kip-ft/ft)
  - Moments per strip width



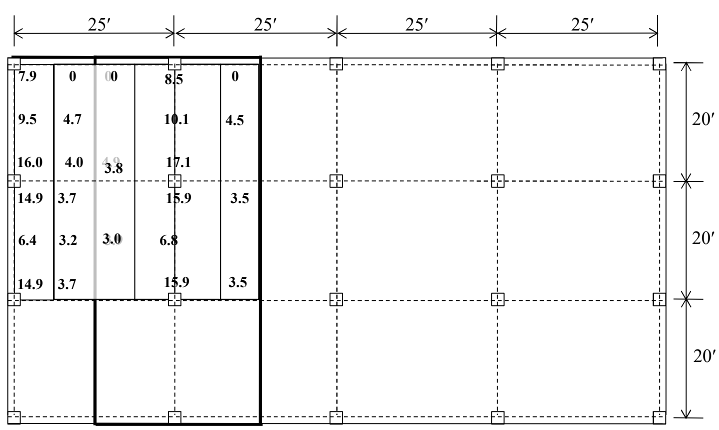
Example 2

- **N-S Direction moments** (units: kip-ft)



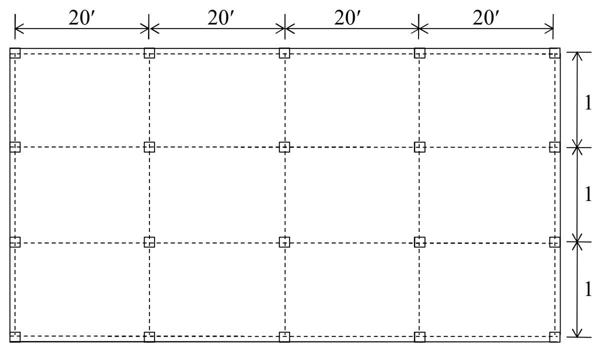
Example 2

- **N-S Direction moments** (units: kip-ft/ft)
  - Moments per strip width



Example 3

- **Homework:** Analysis results of the slab shown below using DDM are presented next. The slab supports a live load of 60 psf. Superimposed dead load is equal to 40 psf. All columns are 12″ square. Take f_c' = 3 ksi and f_y = 40 ksi.

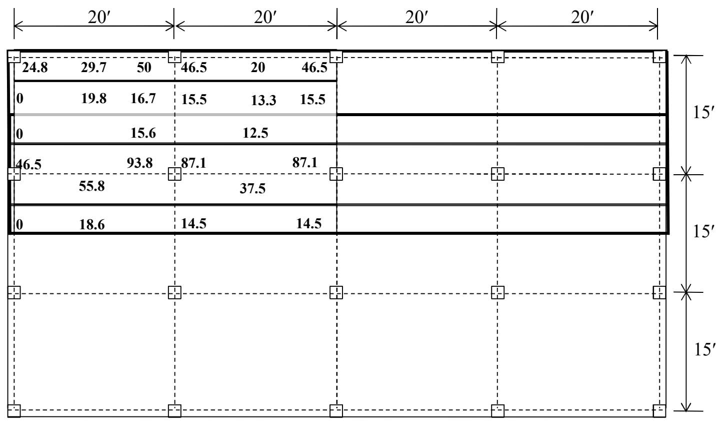


Example 3

- **Calculation summary**
  - Slab thickness h_f = 8″
  - Factored load (w_u) = 0.264 ksf
  - Column strip width = 3.75′

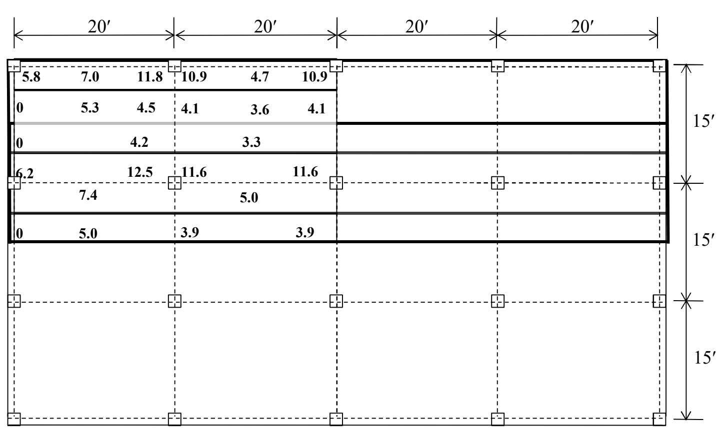
Example 3

- **E-W Direction Moments** (units: kip-ft)



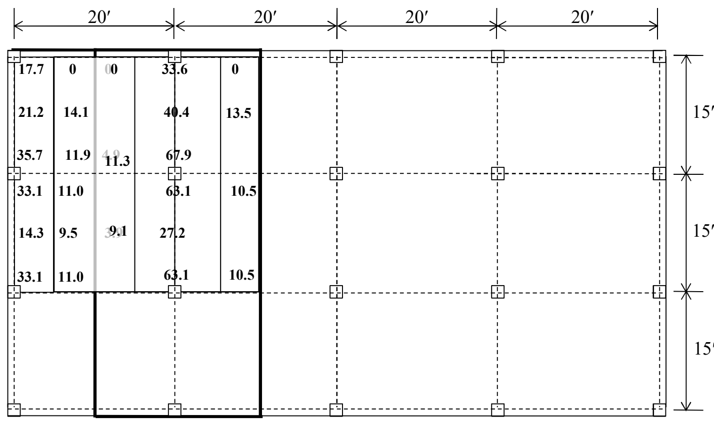
Example 3

- **E-W Direction Moments** (units: kip-ft/ft)
  - Moments per strip width



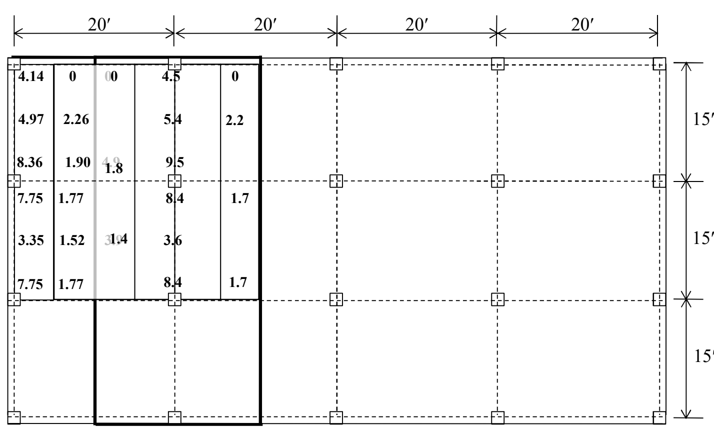
Example 3

- **N-S Direction moments** (units: kip-ft)



Example 3

- **N-S Direction moments** (units: kip-ft/ft)
  - Moments per strip width



Summary of Direct Design Method

- Select sizes of slab and columns. The slab depth can be calculated from ACI table 8.3.1.1.
- Find Load on slab (w_u = 1.2DL + 1.6LL)
- On given column plan of building, mark location and dimensions of all frames (exterior and interior)
- For a particular span of frame, find static moment ($M_o = w_u l_2 l_n^2/8$).

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Summary of Direct Design Method

- Longitudinal distribution of static moment:
  - Exterior span (M_{ext-} = 0.26M_o; M_{ext+} = 0.52M_o; M_{int-} = 0.70M_o)
  - Interior span (M_{int-} = 0.65M_o; M_{int+} = 0.35M_o)
- Lateral Distribution of each longitudinal moment:
  - 100 % of M_{ext-} goes to column strip
  - 60 % of M_{ext+} and M_{int+} goes to column strip
  - 75 % of M_{int-} goes to column strip
- The remaining moments goes to middle strips
- Design and apply reinforcement requirements ($s_{max} = 2h_f$)

Session – II

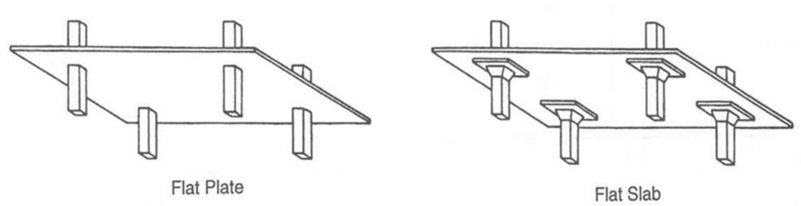
Shear Design for Two-Way Slab System without Beams (Flat Plates and Flat Slabs)

Topics Addressed

- Introduction
- Various Design Options
- Design Example

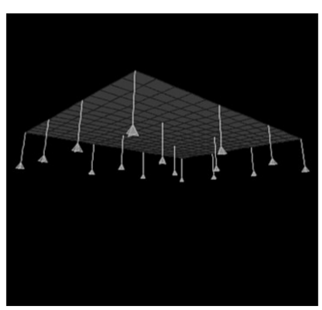
Introduction

- Flat Plates and Flat Slabs
  - These are the most common types of two-way slab system, which are commonly used in multi-story construction.
  - They render low story heights and have easy construction and formwork.



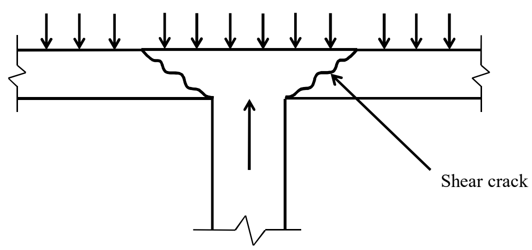
Introduction

- Behavior
  - When two-way slabs are directly supported by columns, shear near the column (punching shear) is of critical importance.
  - Therefore, in addition to flexure, flat plates shall also be designed for two way shear (punch out shear) stresses.



Introduction

- Punching Shear in Flat Plates
  - Punching shear occurs at column support points in flat plates and flat slabs.



Punch Out

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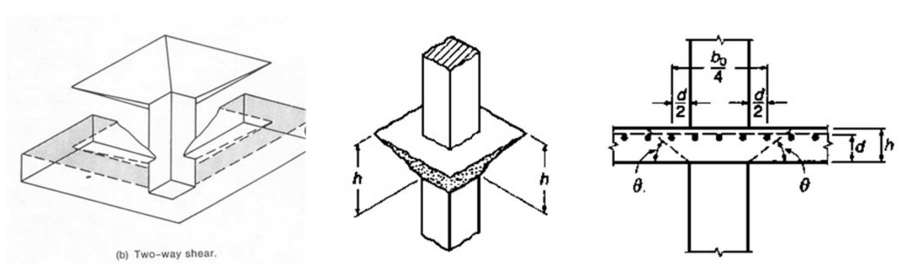
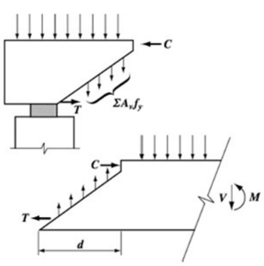
Introduction

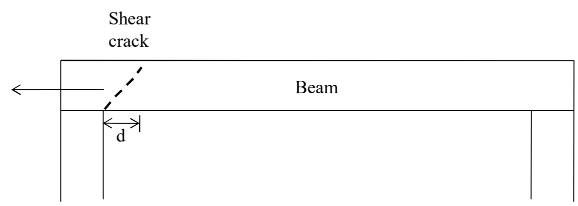
- Critical Section for Punching Shear



Introduction

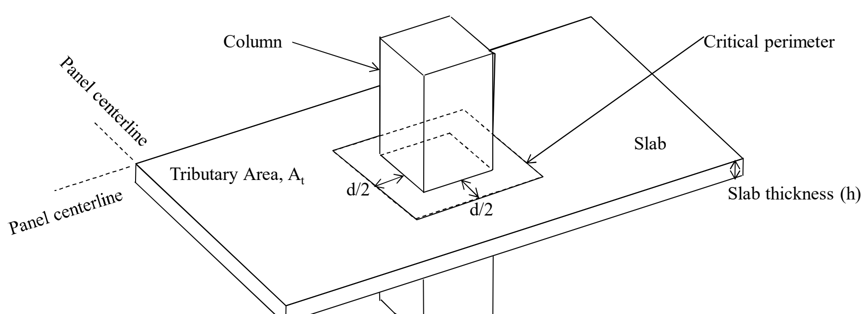
- Critical Section for Shear Design
  - In shear design of beams, the critical section is taken at a distance "d" from the face of the support.





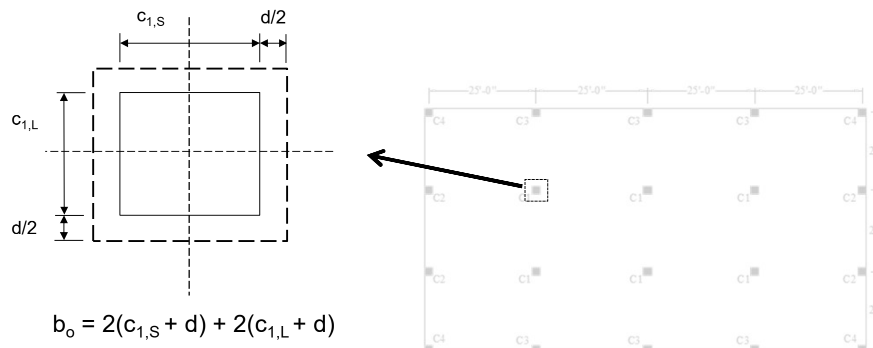
Introduction

- Critical Section for Shear Design
  - In shear design of flat plates, the critical section is an area taken at a distance "d/2" from all face of the support.



Introduction

- Punching Shear: Critical Perimeter, b_o



$b_o = 2(c_{1,S} + d) + 2(c_{1,L} + d)$

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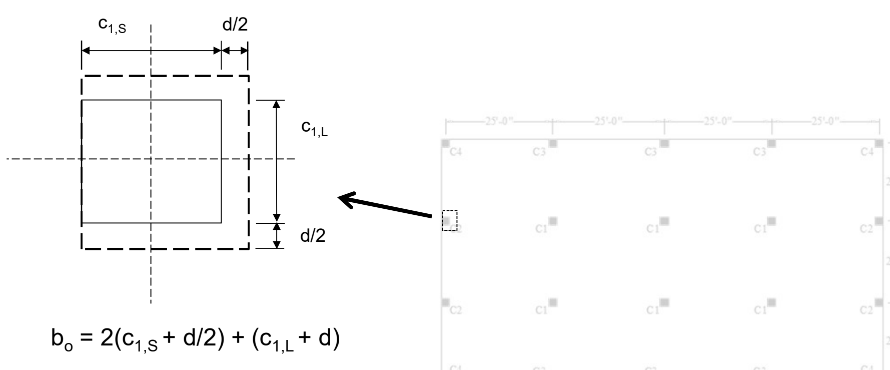
Introduction

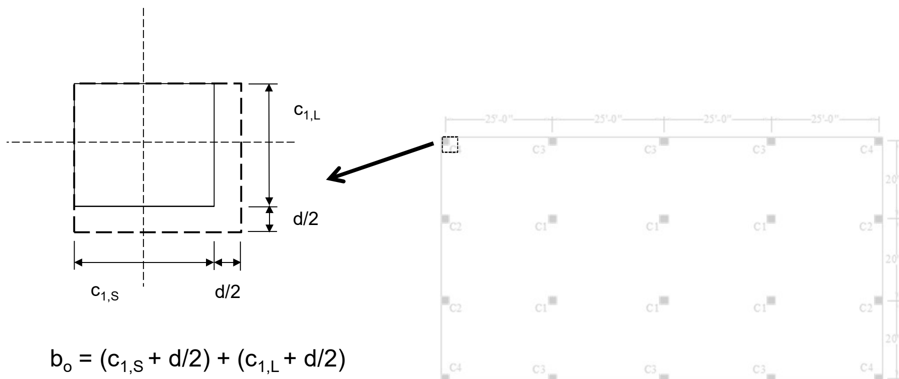
- Punching Shear: Critical Perimeter, b_o



$b_o = 2(c_{1,S} + d/2) + (c_{1,L} + d)$

Introduction

- Punching Shear: Critical Perimeter, b_o



$b_o = (c_{1,S} + d/2) + (c_{1,L} + d/2)$

Introduction

- Punching Shear Demand (V_u): For Square Column

Critical Perimeter, b_o:

$b_o = 4(c + d)$

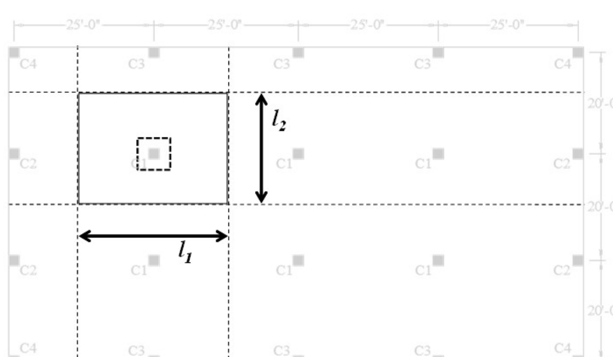
Area Contributing to Load (Excluding Area of b_o), A_t :

$A_t = (l_1 \times l_2) – (c + d)^2 / 144$

[l_1 & l_2 are in ft. units and c & d are in inches]

Punching Shear Demand:

$V_u = w_u \times A_t$



Introduction

- Capacity of Slab in Punching Shear:
  - $\Phi V_n = \Phi V_c + \Phi V_s$
  - ΦV_c is least of:
    - $\Phi 4\sqrt{(f_c')}\, b_o d$
    - $\Phi(2 + 4/\beta_c)\sqrt{(f_c')}\, b_o d$
    - $\Phi\{(\alpha_s d/b_o + 2\}\sqrt{(f_c')}\, b_o d$

    β_c = longer side of column/shorter side of column; for square column β_c=1

    α_s = 40 for interior column, 30 for edge column, 20 for corner columns

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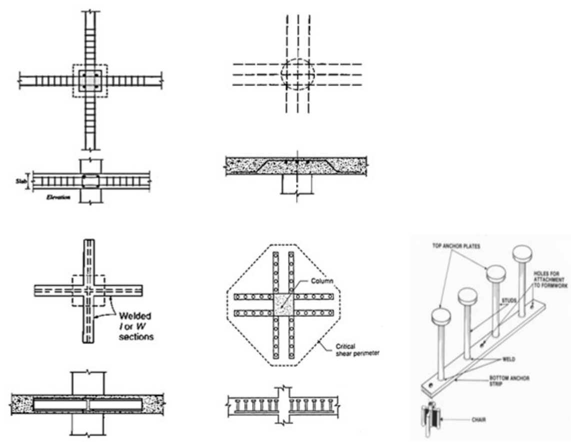
Various Design Options

- When $\Phi V_c \geq V_u$ (Φ = 0.75): Nothing is required.
- When $\Phi V_c < V_u$, it can be increased by either of the following ways:
  i. Increasing d depth of slab: This can be done by increasing the slab depth as a whole or in the vicinity of column (Drop Panel)
  ii. Increasing b_o critical shear perimeter: This can be done by increasing column size as a whole or by increasing size of column head (Column capital)
  iii. Increasing f_c' (high Strength Concrete)
- If it is not possible, provide shear reinforcement.

Various Design Options

- Shear reinforcement (ΦV_s) can be provided in the form of:
  - Integral beams
  - Bent Bars
  - Shear heads
  - Shear studs



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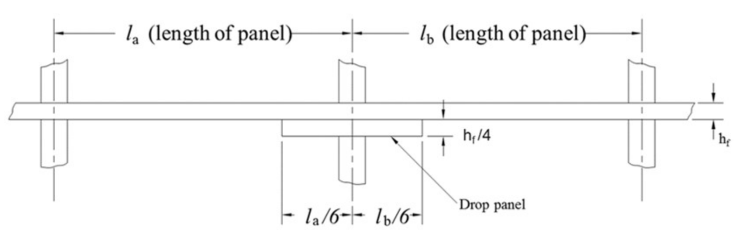
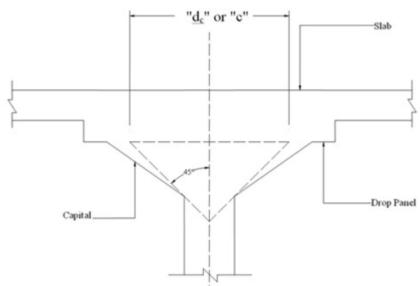
Various Design Options

- Drop Panels (ACI 8.3.1.1 and 8.2.4):



Various Design Options

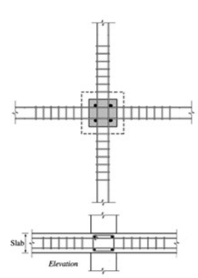
- Column Capital:



- ACI 8.4.1.4 requires the column capital should be oriented not greater than 45^0 to the axis of the column.
- ACI 26.5.7 (a) requires that the capital concrete be placed at the same time as the slab concrete. As a result, the floor forming becomes considerably more complicated and expensive.
- The increased perimeter can be computed by equating V_u to ΦV_c and simplifying the resulting equation for b_0

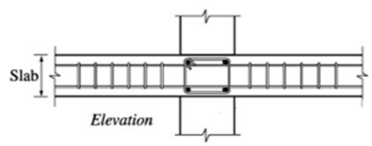
Various Design Options

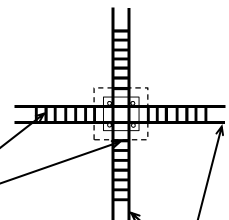
- Integral Beam and Bent Bars:
  - In case of integral beam or bent bar reinforcement following must be satisfied.
    - ACI 22.6.7.1 requires the slab effective depth d to be at least 6 in., but not less than 16 times the diameter of the shear reinforcement.
    - When bent bars and integral beams are to be used, reduce ΦV_c by 2



Various Design Options

- Integral Beams
  - Integral Beams require the design of two main components:
    1. Vertical stirrups
    2. Horizontal bars radiating outward from column faces.



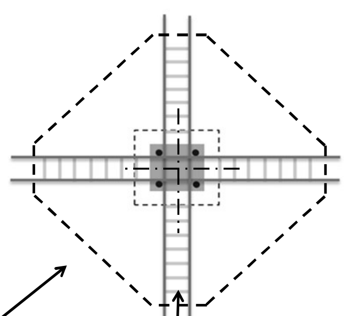


Various Design Options

- Integral Beams (Vertical Stirrups)

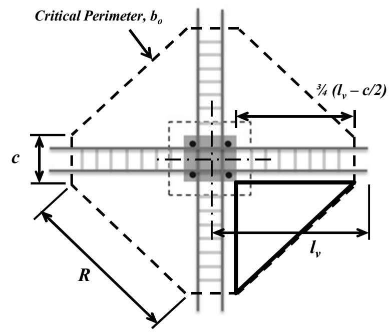
Vertical stirrups are used in conjunction with supplementary horizontal bars radiating outward in two perpendicular directions from the support to form what are termed *integral beams* contained entirely within the slab thickness.

In such a way, critical perimeter is increased



Various Design Options

- Integral Beams (Horizontal Bars)



- How much should be the length of the horizontal bars, l_v
- l_v can be determined using the critical perimeter b_o
- Distance from the face of column to the boundary of critical perimeter = **¾ (l_v – c/2)**

X = ¾ (l_v – c/2)

X = ¾ (l_v – c/2)

R

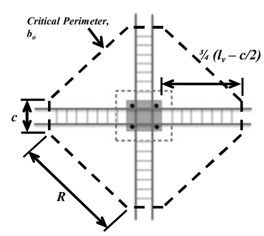
$b_o = 4R + 4c$

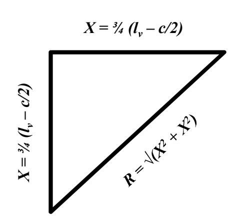
Various Design Options

- **Various Design Options**
  - Integral Beams
    - For Square Column of Size "c":
      - b_o = 4R + 4c .... (1)
      - R = $\sqrt{(X^2 + X^2)} = \sqrt{(2)}X$
      - Eq (1) => b_o = $4\sqrt{(2)}X$ + 4c

        putting value of X: $b_o = 4\sqrt{(2)}\{(3/4)(l_v – c_1/2)\} + 4c$

        after simplification, we get: $b_o = 4.24\, l_v + 1.88c$
      - The above equation can be used for determining the length up to which the horizontal bars should be extended beyond the face of column.





Design Example

- Step 01: Calculate Punching Shear Demand, V_u

- Critical Perimeter, b_o :

  $b_o = 4(c + d)$

  $= 4(14 + 8.5) = 90''$

- Tributary Area (Excluding Area of b_o), A_t :

  $A_t = (25 \times 20) - (14 + 8.5)^2/144$

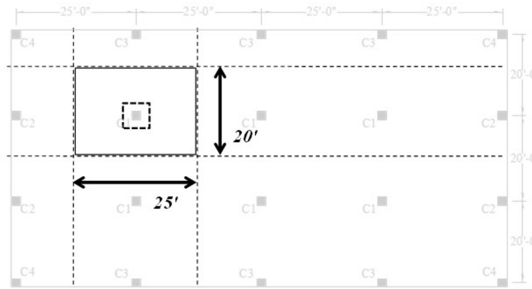
  $= 496.5$ ft²

- Punching Shear Demand, V_u :

  $w_u = 0.3804$ ksf

  $V_u = w_u \times A_t$

  $= 0.3804 \times 496.5 = 189$ kips



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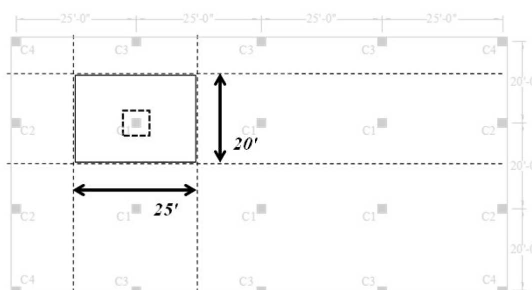
Design Example

- Calculation of Punching shear capacity (ΦV_c):

- Checking the punching shear capacity of concrete:

$\sqrt{f_c'}\, b_o d = \sqrt{4000} \times 90 \times 8.5 / 1000 = 48$ k

- ΦV_c is least of:
  - $\Phi 4 \sqrt{f_c'}\, b_o d = 145$ k
  - $\Phi\, (2 + 4/\beta_c) \sqrt{f_c'}\, b_o d = 216$ k
  - $\Phi\{(\alpha_s d/b_o + 2\} \sqrt{f_c'}\, b_o d = 208$ k



Therefore,

- $\Phi V_c = 145$ k < V_u (189 k)
- Shear design is required.

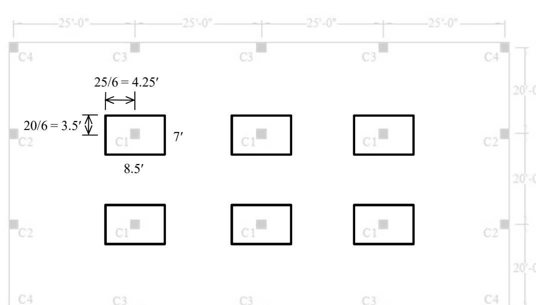
Design Example

- <u>Design for shear (Option 01):</u> Drop panels
  - In drop panels, the slab thickness in the vicinity of the columns is increased to increase the shear capacity ($\Phi V_c = \Phi 4 \sqrt{(f_c')} b_o d$) of concrete.
  - The increased thickness can be computed by equating V_u to ΦV_c and simplifying the resulting equation for "d" to calculate "h_{Total}".

    ($h_{Total} = h_{Slab} + h_{Drop\ Panle} = h_f + h_{DP}$)

Design Example

- <u>Design for shear (Option 01):</u> Drop panels



- <u>Equate V_u to ΦV_c:</u>

  $V_u = \Phi V_c$

  $189 = 0.75 \times 4 \sqrt{(4000)} \times 90 \times d$

  $d = 11.07''$

  Therefore, $h_{Total} = d + 1.5 \approx 12.6''$

  This gives 2.6″ drop panel.
- According to ACI, minimum thickness of drop panel = $h_f/4$ = $10/4 = 2.5''$, So using 2.6″ thick drop panel.
- <u>Drop Panel dimensions:</u>

  $25/6 \approx 4.25'$; $20/6 \approx 3.5'$

Design Example

- Design for shear (Option 02): Column Capitals

- Equate V_u to ΦV_c:

  $V_u = \Phi V_c$

  $189 = 0.75 \times 4 \sqrt{(4000)} \times b_o \times 8.5$

  $b_o = 117.20''$

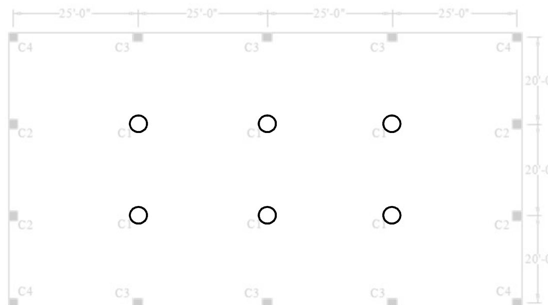
  Now,

  $b_o = 4 (c + d)$

  $117.20 = 4(c + 8.5)$

  Simplification gives,

  $c = 20.8 \approx 21''$



Design Example

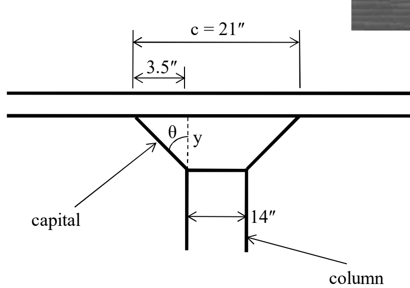
- Design for shear (Option 02): Column Capitals

- According to ACI code, $\theta < 45^o$

  $y = 3.5/ \tan\theta$

  Let $\theta = 30^o$, then $y \approx 6.06''$

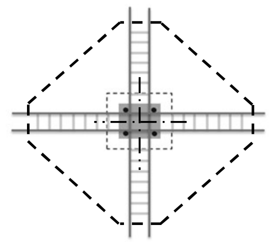
  For $\theta = 20^o$, $y \approx 9.62''$



Design Example

- Design for shear (Option 03): Integral Beams (Vertical Stirrups)
  - ΦV_c = 145 kips
  - When integral beams are to be used, reduces ΦV_c by 2. Therefore ΦV_c = 145/2 = 72.5 kips
  - Using 1/2″ Φ, 2 legged (0.4 in^2), 4 (side) = 4 × 0.4 = 1.6 in^2
  - Spacing (s) = $\Phi A_v f_y d/ (V_u - \Phi V_c)$

    s = 0.75 × 1.6 × 60 × 8.5/ (189 – 72.5) = 5.25 ≈ 5″
  - Maximum spacing allowed d/2 = 8.5/2 = 4.25″
    - Provide 4″ c/c
    - Using #4, 2 legged stirrups @ 4″ c/c



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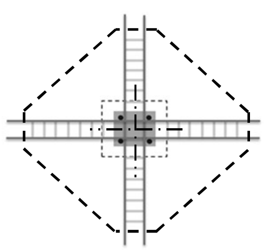
Design Example

- Design for shear (Option 03): Integral Beams (Horizontal Bars)
  - Four #5 bars are to be provided in each direction to hold the stirrups. We know minimum b_o = 117.20″
  - Using the equation $b_o = 4.24 l_v + 1.88c$, we get:

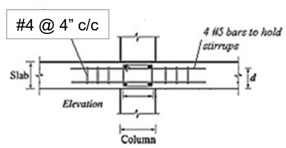
    l_v ≈ 21″
  - Hence the horizontal bars need to be extended beyond the center of the column up to at least 21″ (We will use 24″)

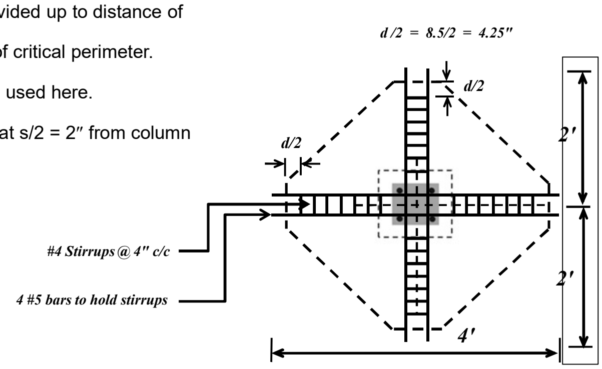


  - First stirrup should be provided at a distance not more than d/2 from the face of column up to distance of d/2 from boundary of critical perimeter.
  - It must be noted that integral beams along with stirrups will shift the critical perimeter. Beams alone cannot shift the critical perimeter.

Design Example

- <u>Design for shear (Option 03):</u> Integral Beams details.
  - Total length of horizontal bars is 4′ in both directions.
    4 #5 bars are used here to hold the vertical stirrups.
  - Vertical stirrups are provided up to distance of d/2 from the boundary of critical perimeter.
    #4 stirrups @ 4″ c/c are used here.
    First stirrup is provided at s/2 = 2″ from column face.





Summary

- For a particular span of frame, find static moment ($M_o = w_u l_2 l_n^2/8$).
- Find longitudinal distribution of static moment:
  - Exterior span ($M_{ext\,-} = 0.26M_o$; $M_{ext\,+} = 0.52M_o$; $M_{int\,-} = 0.70M_o$)
  - Interior span ($M_{int\,-} = 0.65M_o$; $M_{int\,+} = 0.35M_o$)
- Find lateral Distribution of each longitudinal moment (column strip):
  - 100 % of $M_{ext\,-}$ goes to column strip
  - 60 % of $M_{ext\,+}$ and M_{int+} goes to column strip
  - 75 % of $M_{int\,-}$ goes to column strip
- Max spacing requirements ($s_{max} = 2h_f$)

Summary

- Find Critical Perimeter, b_o:
  - $b_o = 4(c + d)$
- Area Contributing to Load (Excluding Area of b_o), A_t :
  - $A_t = (L1 \times L2) – (c + d)^2 / 144$; L1 & L2 are in ft. units and c & d are in inches.
- Punching Shear Demand:
  - $Vu = w_u \times A_t$
- Find ΦV_c is least of:
  - $\Phi 4\sqrt{(f_c')}\, b_o d$
  - $\Phi(2 + 4/\beta_c)\sqrt{(f_c')}\, b_o d$: β_c = longer side of column/shorter side of column
  - $\Phi\{(\alpha_s d/b_o + 2\}\sqrt{(f_c')}\, b_o d$: α_s = 40 for interior column, 30 for edge column, 20 for corner columns

References

- ACI 318-14
- Design of Concrete Structures by Nilson, Darwin and Dolan
- ACI Design Handbook

The End