Lecture-07

Idealized Structural Modeling of RC Structures

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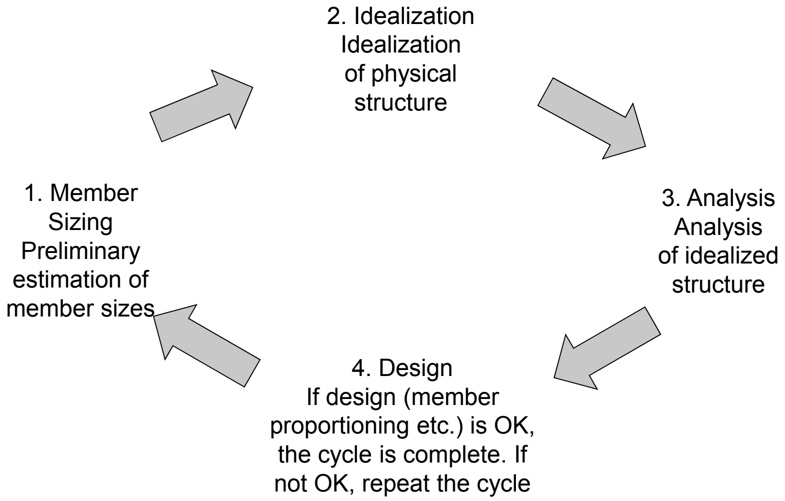
Topics Addressed

- Design Cycle
- Member Sizes Estimation
- Idealization of Physical Structure
- Stiffness Properties Calculation
- Transformation from physical structure to idealized model
- Loads Idealization and Placement
- Closing Note

Design Cycle



2. Idealization
Idealization
of physical
structure

3. Analysis
Analysis
of idealized
structure

4. Design
If design (member
proportioning etc.) is OK,
the cycle is complete. If
not OK, repeat the cycle

1. Member
Sizing
Preliminary
estimation of
member sizes

Member Sizes Estimation

- **General**
  - The experienced designer can estimate member sizes with surprising accuracy.
  - Those with little or no experience must rely on trial calculations.
  - Architecture requirements, availability of formwork and other similar field issues are also important aspects and must be considered before the sizes of various RC members are finalized.

Member Sizes Estimation

- **Slab Thickness**
  - Controlled by either:
    - Deflection requirements,
    - Negative moments at the faces of the supporting beams.
  - A practical minimum thickness of 5 inches is often used, except for joist construction.

Member Sizes Estimation

- **Beam Sizes**
  - Usually governed by:
    - Negative moments and Shears at the supports.
    - Deflection at mid span.
  - Many designers prefer following estimate:
    - Depth = 3/4 inch per foot of span
    - Width = ½ depth.

Member Sizes Estimation

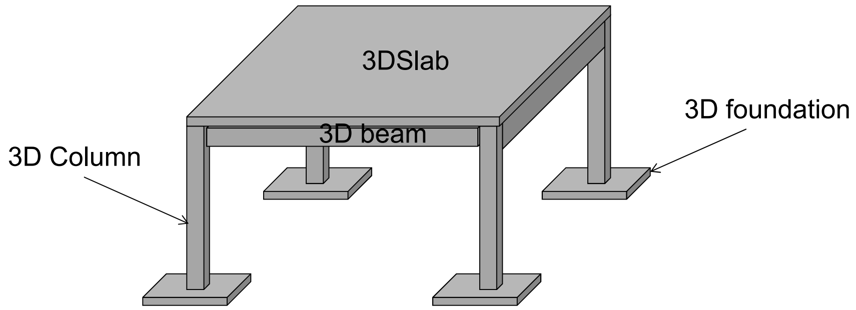
- **Column Sizes**
  - Column sizes are governed primarily by axial loads which can be estimated quickly.
  - Development length of beam reinforcement in column may also be a deciding factor for selecting column dimension parallel to beam longitudinal axis.

Member Sizes Estimation

- **Column Sizes**
  - For minimum forming costs, it is highly desirable to use the same column dimensions throughout the height of a building.
  - This can be accomplished by using higher-strength concrete on the lower stories and reducing concrete strength in upper stories, as appropriate.

Idealization of Physical Structure

- **Physical Structure**
  - Every structure in the universe is basically a 3D structure and is composed of 3D members.



Idealization of Physical Structure

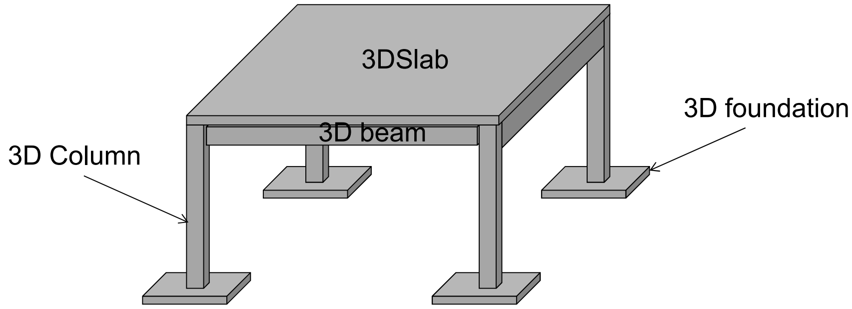
- **Idealized Structure**
  - For analysis purpose, a physical structure is represented as an idealized structure.
  - While transforming a physical structure to its equivalent idealized form, following characteristics of the idealized structure shall be selected as close as possible to the actual structure:
    - Stiffness,
    - Span length,
    - Supports,
    - Loads etc.

Idealization of Physical Structure

- **Idealization (Steps)**
  - **Step 1:** Calculation of Stiffness Properties such as Area and Moment of Inertia of the Members based on Estimated Sizes.
  - **Step 2:** Transformation of Physical Structure into Idealized Model.
    - i. Idealization of Structural Members
    - ii. Idealization of Joints Connecting RC Members
    - iii. Idealization of Restraints (Supports) to provide proper Boundary Conditions to Idealized Structure
    - iv. Idealization of Walls
  - **Step 3:** Load idealization and Load Placement.

Step 1: Stiffness Properties Calculation

- Given a 3D physical structure, its member stiffness can be calculated easily. The most important property is moment of inertia "*I*" of members.

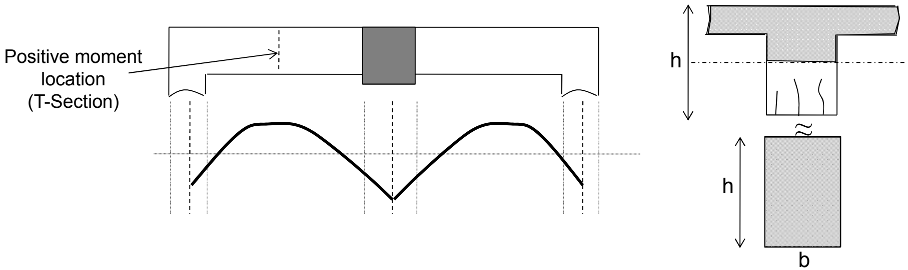


Step 1: Stiffness Properties Calculation

- Effects of Cracking on Moment of Inertia of Members
  - We know that RC members in a physical structure crack well below the ultimate load. Therefore using full moment of inertia of the member in the idealized model is not justified.
  - Though the use of full moment of inertia of members will not affect the total value of bending (positive plus negative), the distribution between the members may change considerably.
  - It is not the absolute moment of inertia that matters, rather it is the relative stiffness of members that must be considered.

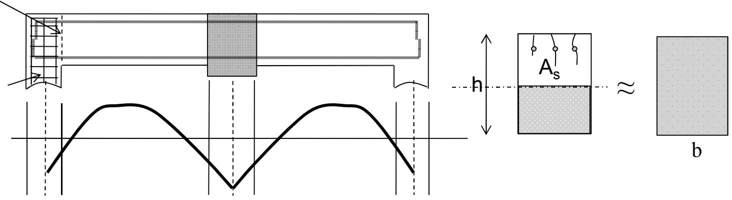
Step 1: Stiffness Properties Calculation

- Effects of Cracking on Moment of Inertia of Members
  - Beam Section:
    - At positive moment locations where T-section is effective, the cracked T-section moment of inertia is almost equal to a rectangular section with the same dimensions.



Step 1: Stiffness Properties Calculation

- Effects of Cracking on Moment of Inertia of Members
  - Beam Section:
    - At negative moment locations where rectangular section is effective, the cracked rectangular section moment of inertia is almost equal to rectangular section with same dimensions due to additional stiffness provided by beam and column reinforcements.



Negative moment location (Rectangular Section)

Column reinforcement adds to section stiffness

Step 1: Stiffness Properties Calculation

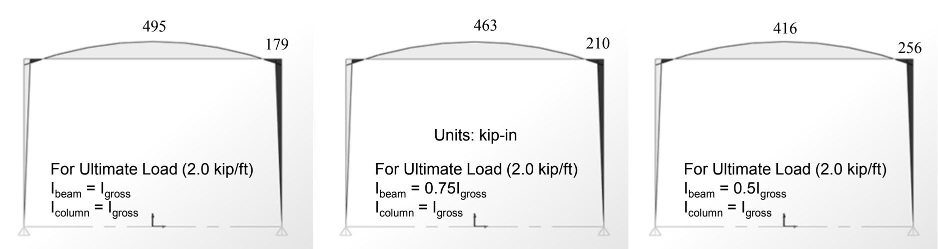
- Effects of Cracking on Moment of Inertia of Members
  - Column Section:
    - The effect of cracks on variation of moment of inertia of columns is relatively less due to axial load on column.

Step 1: Stiffness Properties Calculation

- Case Studies
  - In the next 2 slides, case studies regarding the effects of variation in moment of inertia of frame members on the bending moment values are presented.

Step 1: Stiffness Properties Calculation

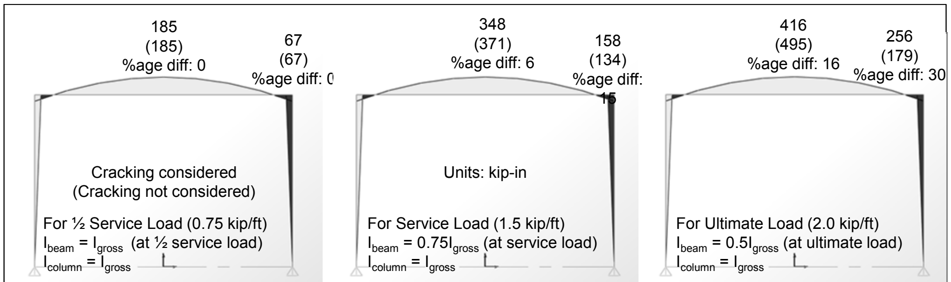
- Case Study 1: Distribution of bending moments in beam and column at same loads and different I.



Beam: 12″ × 18″; Column: 12″ × 12″; DL = 1 kip/ft; LL = 0.5 kip/ft

Step 1: Stiffness Properties Calculation

- Case Study 2: Distribution of bending moments in beam and column at different loads and different I.



Beam: 12″ × 18″; Column: 12″ × 12″; DL = 1 kip/ft; LL = 0.5 kip/ft

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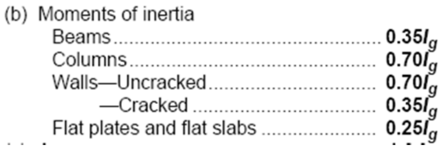
Step 1: Stiffness Properties Calculation

- ACI Code on Stiffness
  - **ACI 8.6.1** states that use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.
  - **ACI R8.6.1** states that relative values of stiffness are important. Common assumptions are:
    - Gross EI values for all members
    - Half the gross EI of the beam stem for beams and the gross EI for the columns.

Step 1: Stiffness Properties Calculation

- ACI Code on Stiffness
  - Additional guidance is given in ACI Code 10.11.1, which specifies the section properties to be used for frames subject to side-sway.



(b) Moments of inertia

Beams	$0.35I_g$
Columns	$0.70I_g$
Walls—Uncracked	$0.70I_g$
—Cracked	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

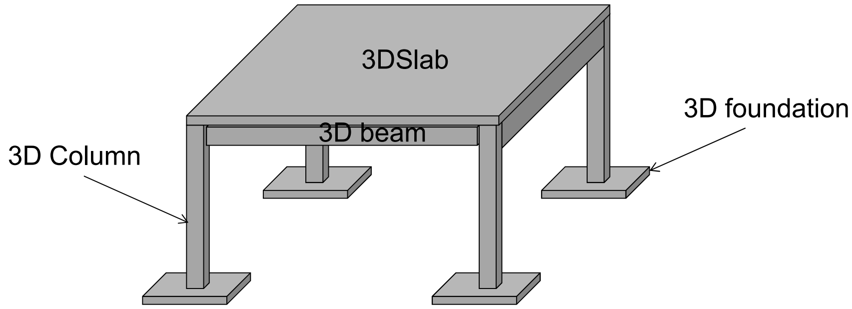
  - *Note: Effective moment of inertia shall be used for calculation of deflection at a particular load stage*.

Step 1: Stiffness Properties Calculation

- Stiffness Assignment in the FEA Software
  - In reality, all beams in a monolithic RC structure are either T or L beams. However in software like SAP2000, the beams are usually modeled as rectangular sections. This is justified mathematically as below:
    - Generally, $I_T = 2I_{rect}$ (I_T and I_{rect} are moment of inertia of T and rectangular section)
    - Now as per ACI 10.11.1 (sway case), to model cracked beam, I of beam = 0.35I
    - Therefore, $I_{T, cracked} = 0.35 \times 2I_{rect} = 0.7I_{rect}$; Similarly $I_{col} = 0.70\ I_g$
    - Therefore, finally all columns and rectangular beam section's stiffnesses shall be reduced by a factor of 0.7 in the software to model stiffnesses as per ACI 10.11.1.

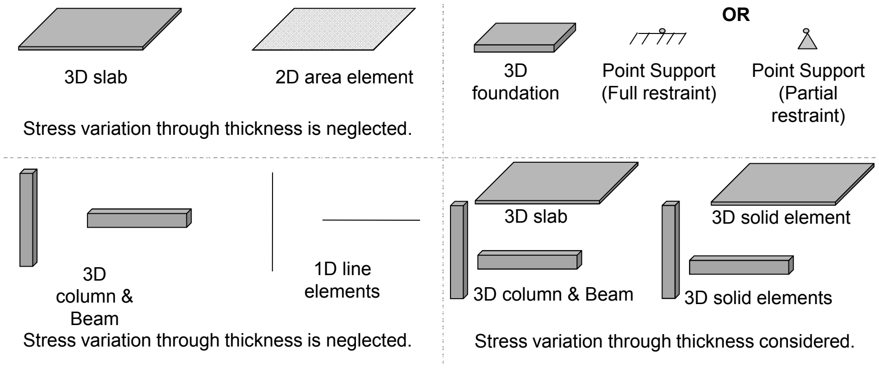
Step 1: Stiffness Properties Calculation

- Once the stiffnesses are in hand, physical structure can be converted into an idealized model using structural model elements.



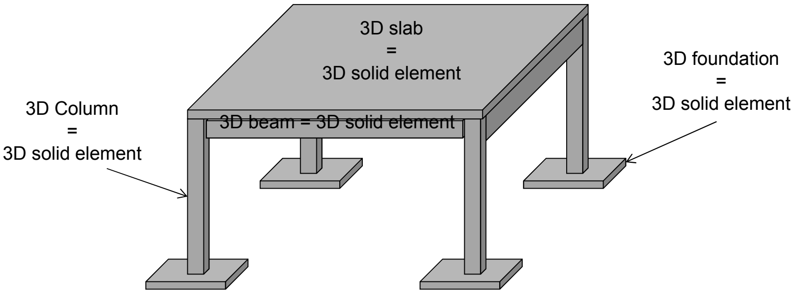
Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements



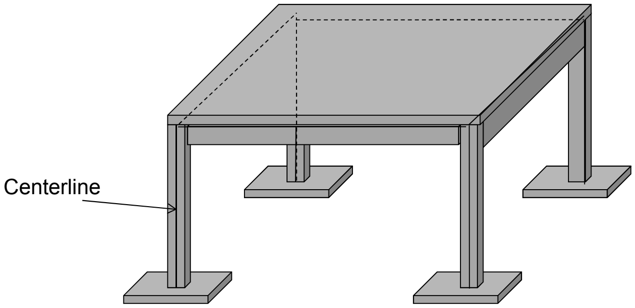
Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Using element models on the previous slide, several options are possible. Such as a full 3D structural model of the physical structure can be made using 3D solid elements.



Step 2: Transformation from physical structure to idealized model

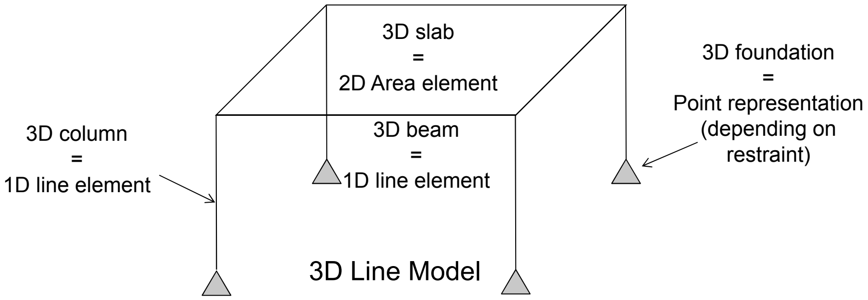
- (i). Idealization of Structural Model Elements
  - However, generally 3D idealized line models are used in engineering design practice to minimize computational effort. Such a model must be made on centerlines of 3D members.



In line modeling, the beam and column lengths are taken more than they actually are.

Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - 3D idealized line model of a physical structure. Analysis of such models can be done relatively easily.

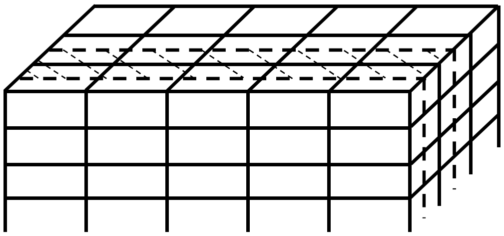


Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Conversion from 3D to 2D
    - Due to various constraints (software availability, software efficiency, time consumption, high skills required etc.), one would further like to transform a 3D model into a 2D model.
    - Most commonly used method for this purpose is Equivalent Frame Method.

Step 2: Transformation from physical structure to idealized model

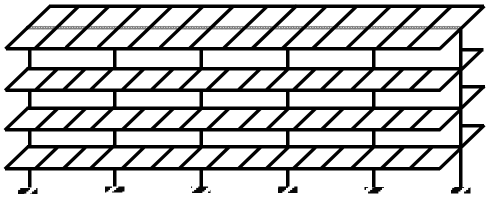
- (i). Idealization of Structural Model Elements
  - Conversion from 3D to 2D
    - In Equivalent frame analysis an equivalent 2D system of a 3D structure is extracted by considering stiffnesses of relevant members e.g., slab and beams.



1. 3D line model

Step 2: Transformation from physical structure to idealized model

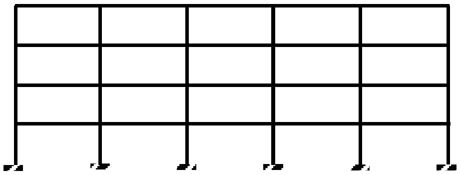
- (i). Idealization of Structural Model Elements
  - Conversion from 3D to 2D
    - 3D frame extracted from structure



2. 3D frame extracted from structure

Step 2: Transformation from physical structure to idealized model

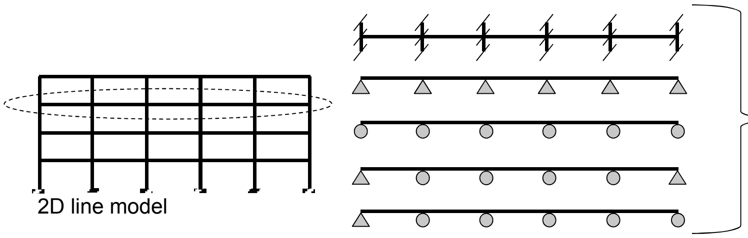
- (i). Idealization of Structural Model Elements
  - Conversion from 3D to 2D
    - Converted 2D line model
      - Once the 2D system is extracted, any method can be used for analysis.



3. Converted 2D line model

Step 2: Transformation from physical structure to idealized model

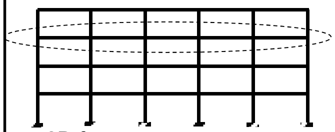
- (i). Idealization of Structural Model Elements
  - Conversion from 2D to 1D
    - 2D structures can be idealized into further simpler 1D line models.
    - End conditions must be selected with good structural engineering judgment for correct structural representation.



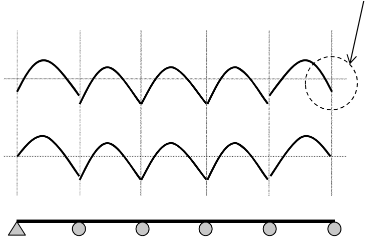
Which one is correct?

Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Conversion from 2D to 1D



2D frame



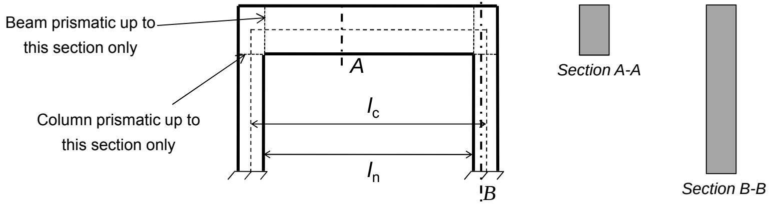
Must be considered
ACI Coefficient = 1/16 (for column support)

Actual BMD

BMD from selected support conditions

Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of prismatic instead of non-prismatic members:
      - The usual assumption in frame analysis is that the members are prismatic, with constant moment of inertia between centerlines. This is not strictly correct as shown below.

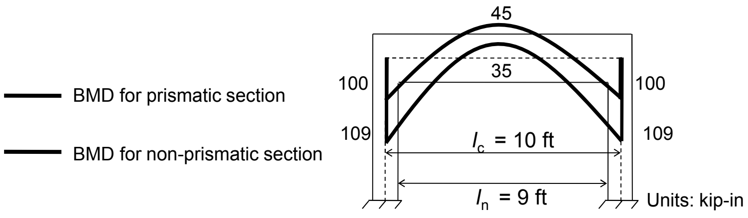


Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of prismatic instead of non-prismatic members:
      - This problem can be solved either with solid 3D member modeling or using varying moment of inertia along the length of member. If such a correct modeling is done, this will result in increase beam support moments somewhat and decrease span moments.

Step 2: Transformation from physical structure to idealized model

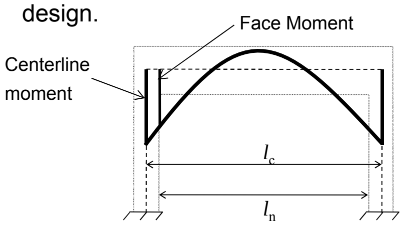
- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of prismatic instead of non-prismatic members:
      - As sectional dimensions of the members as compared to their length are very small, the variation in bending moment resulting from such deficient modeling will normally be not significant.



Beam: 12″ × 18″
Column: 12″ × 12″
Load = 1.0 kip/ft
Offset = 6″

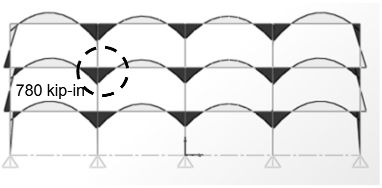
Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of centerline length instead of clear length.
      - Effect on Bending Moment in Beams: When centerline moment are used in design, unnecessarily large section would result. Therefore, it is better to use face moments in design.

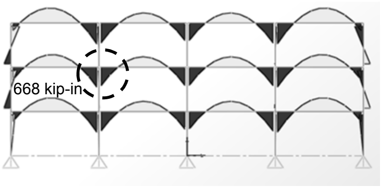


Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of centerline length instead of clear length
      - Effect on Bending Moment in Beams
        - Centerline and offset moments in a beam for the given case.



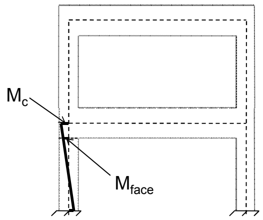
Centerline Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″



Offset Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″

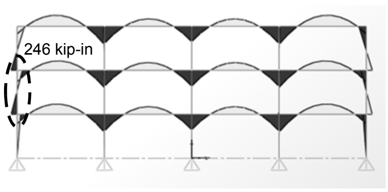
Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of centerline length instead of clear length.
      - Effect on Bending Moment in Columns: Because moment gradient is not very steep, the difference between M_{face} and M_c is small.

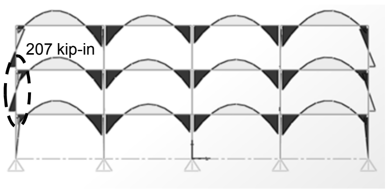


Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of centerline length instead of clear length
      - Effect on Bending Moment in Columns:
        - Centerline and offset moments in a column for the given case



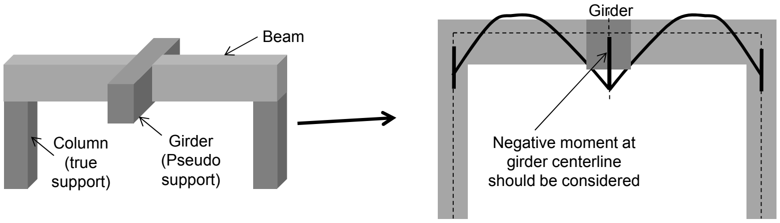
Centerline Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″



Offset Moment:
Beam: 12″ × 18″; Column size: 12″ × 12″

Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - Use of centerline length instead of clear length.
      - Negative Moments in Beam due to Girder: The negative moment in the beam at the centerline of the girder should be used to design the negative reinforcing steel.

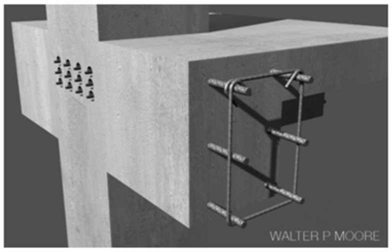


Step 2: Transformation from physical structure to idealized model

- (i). Idealization of Structural Model Elements
  - Deficiencies in Idealization Process due to Line Modeling
    - ACI Code on Span Length Selection:
      - ACI 8.7.2 allows the use of center to center distance for doing analysis of frames.
      - For beam design, ACI 8.7.3 permits the use of bending moment at face of the support.

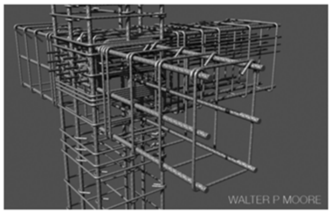
Step 2: Transformation from physical structure to idealized model

- (ii). Idealization of Joints Connecting RC Members
  - Joints in a cast in place RC structure are those locations where two or more members are connected monolithically.



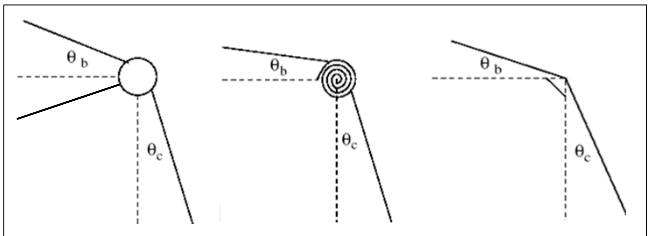
Step 2: Transformation from physical structure to idealized model

- (ii). Idealization of Joints Connecting RC Members
  - One of the important characteristics of RC structures is their capability of transferring load effects from one member to another through joints. Joints must be rigid and properly designed in order to transfer these actions.
  - The structures which exhibit continuity at joints are called monolithic structures.



Step 2: Transformation from physical structure to idealized model

- (ii). Idealization of Joints Connecting RC Members
  - Type of Joints



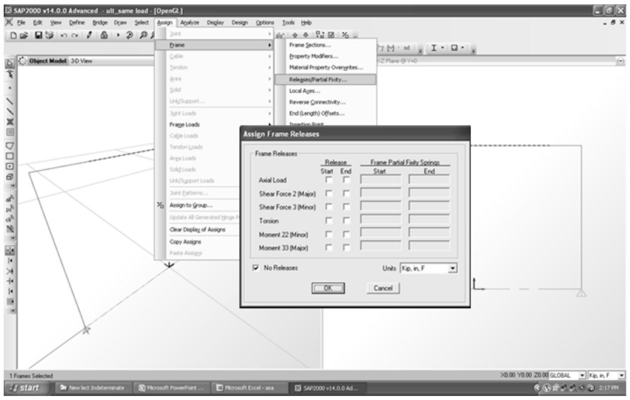
$\theta_{beam} \neq \theta_{column}$
Flexible
(Frictionless pin)

$\theta_{beam} \neq \theta_{column}$
Semi-Flexible
(Spring)

$\theta_{beam} = \theta_{column}$
Rigid

Step 2: Transformation from physical structure to idealized model

- (ii). Idealization of Joints Connecting RC Members
  - End release option in SAP2000 can create several type of joints



Assign Frame Releases

Frame Releases

	Release		Frame Partial Fixity Springs	
	Start	End	Start	End
Axial Load	☐	☐		
Shear Force 2 (Major)	☐	☐		
Shear Force 3 (Minor)	☐	☐		
Torsion	☐	☐		
Moment 22 (Minor)	☐	☐		
Moment 33 (Major)	☐	☐		

☑ No Releases

Units Kip, in, F

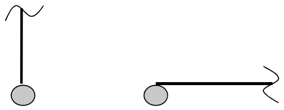
Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Supports may be idealized as point elements considering their overall behavior under the given demand.
  - Three types of idealized support representations are given next.

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Roller
    - A support that allows horizontal movement and rotation but prevents movement vertically, up or down e.g., beam resting on a masonry wall with a small friction at the contact surface so that if there is any lateral movement, the beam can move laterally up to some extent.

Example:
Beam over masonry

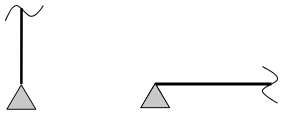


Roller Support Representation

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Hinge
    - A support that allows rotation but prevents translation e.g., beam resting on a masonry wall with a large friction at the contact surface so that lateral movement is prevented.

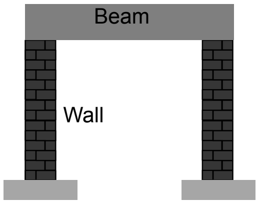
Example:
Beam over masonry

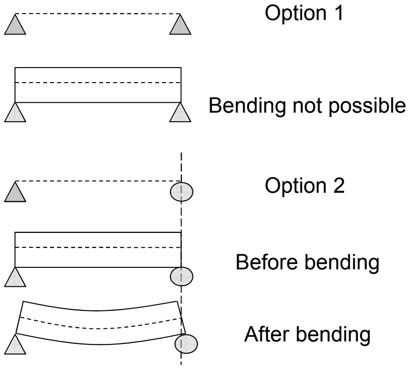


Hinge Support Representation

Step 2: Transformation from physical structure to idealized model

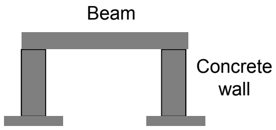
- (iii). Idealization of Restraints (Supports)
  - Idealization of Support for Beams over Masonry Wall



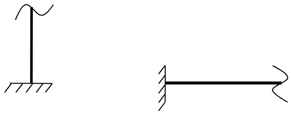


Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Fixed
    - A support in which both translation and rotation are prevented.



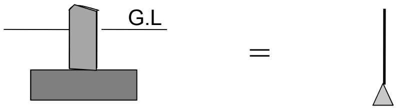
Example:
Concrete monolithic frame



Fixed Support Representation

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Support for Foundation
    - A shallow foundation with relatively smaller dimensions can be idealized as a hinge support.

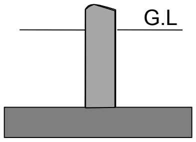


Shallow foundation with relatively smaller dimensions

Hinge

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Support for Foundation
    - A relatively deep foundation with relatively larger dimensions (e.g., continuous and raft footing) can be idealized as a fixed support.



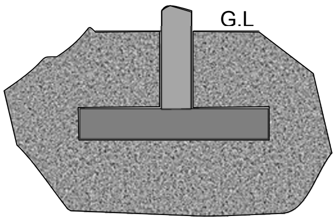
Relatively deep foundation with large dimensions resting on solid stratum

=

Fixed

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Support for Foundation
    - However a foundation may be more truly represented by modeling it as an area or solid element with soil represented as spring elements etc.



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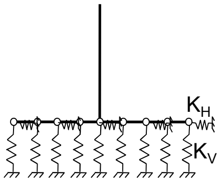
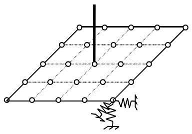


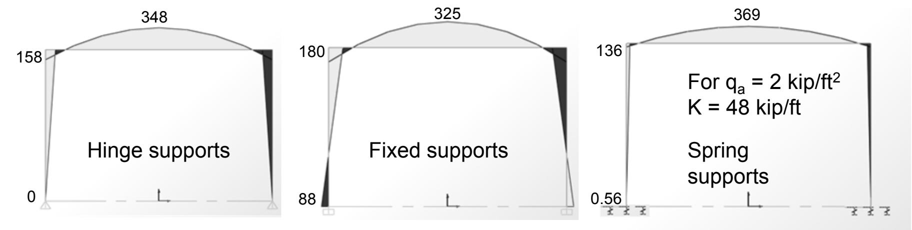
Elastic Spring Support
(Elevation view)



3D view

Step 2: Transformation from physical structure to idealized model

- (iii). Idealization of Restraints (Supports)
  - Case Study: Effect on bending moment in members due to change in type of restraints



Beam: 12″ × 18″; Column 12″ × 12″; Load: 1.5 kip/ft; Units: kip-in

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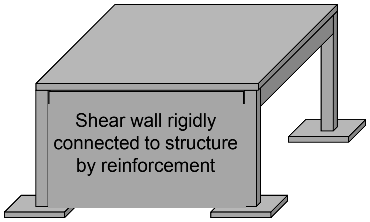
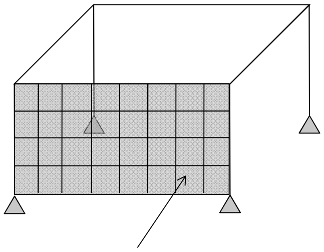
Step 2: Transformation from physical structure to idealized model

- (iv). Idealization of Walls
  - RC Shear Wall



3D structure

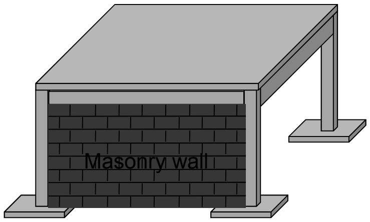
3D Line Model



- FE model of shear wall
- Membrane (in-plane resistance) element OR shell (in and out plane resistance) element
- Properly meshed to model rigid connection

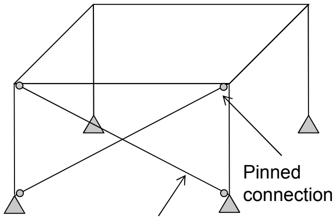
Step 2: Transformation from physical structure to idealized model

- (iv). Idealization of Walls
  - Masonry Wall



3D structure

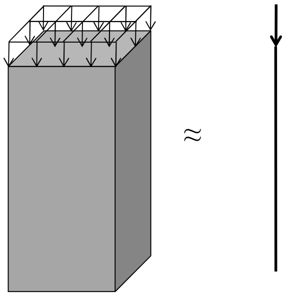
3D Line Model



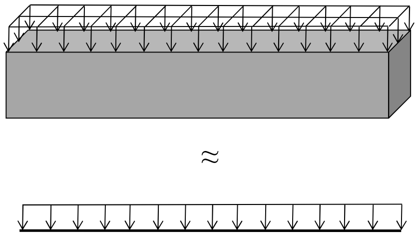
- Masonry wall modeled as equivalent diagonal bracing .
- Brace member is rectangular and has thickness equal to thickness of wall.
- Brace has depth equal to 0.25 times length of diagonal length.
- Brace is connected to structure through pinned connection
- Note : Masonry weight shall be separately applied on structure.

Step 3: Loads Idealization and Placement

- Loads Idealization



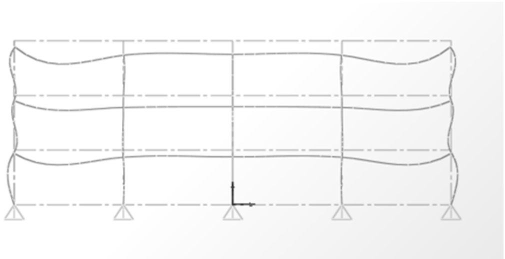
Column



Beam

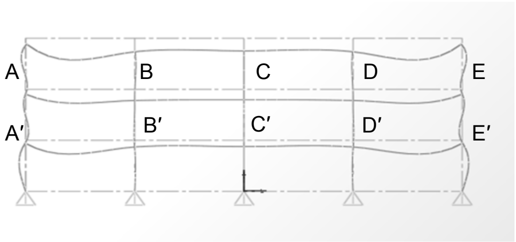
Step 3: Loads Idealization and Placement

- Placement of Loads
  - Consider a 2D frame with rigid joints.
  - The frame subjected to dead load only will have deflected shape as shown below.



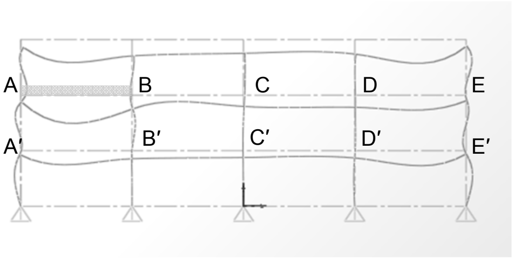
Step 3: Loads Idealization and Placement

- Placement of Loads
  - Live load may or may not occupy the complete floor of the structure at the same time. It is possible that only span AB or any other span is fully loaded at a time.
  - Several arrangements of live load are possible.



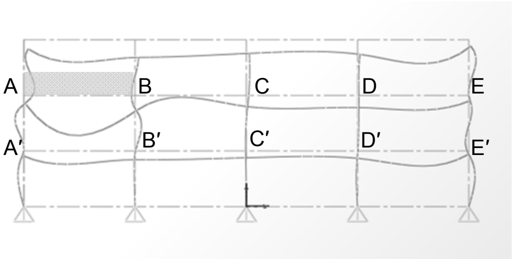
Step 3: Loads Idealization and Placement

- Placement of Loads
  - Consider the frame below. Due to application of uniform live load on span AB, the deflection of beam and adjacent columns will increase.



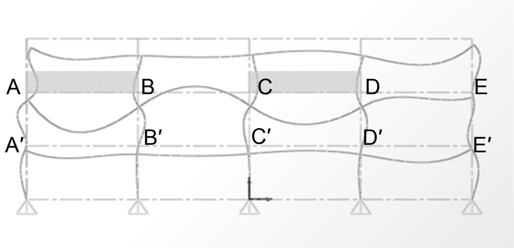
Step 3: Loads Idealization and Placement

- Placement of Loads
  - On further increase in live load, the span BC curvature can reverse, bringing tension in upper fibers.
  - Similarly the column AA′ converts from double to single curvature .



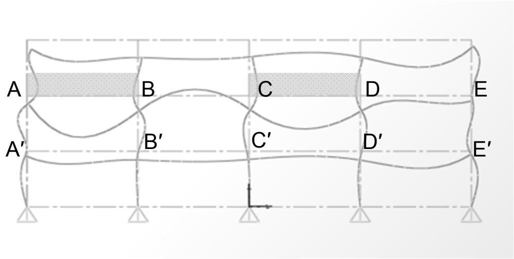
Step 3: Loads Idealization and Placement

- Placement of Loads
  - If span CD is loaded as well with live load, the deflections will further increase. Also span BC will reach reverse curvature earlier.
  - However if all spans AB, BC, and CD are loaded, the deflection in span AB decreases.



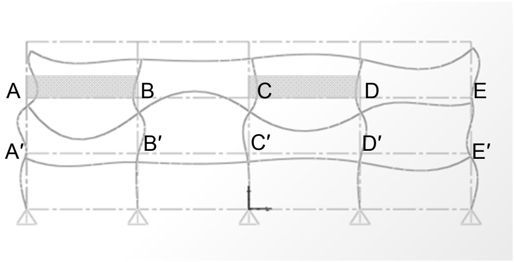
Step 3: Loads Idealization and Placement

- Placement of Loads
  - So it is not the full live load on floor ABCD that causes more bending in span AB, rather it is the arrangement or pattern shown below that causes more bending in span AB.



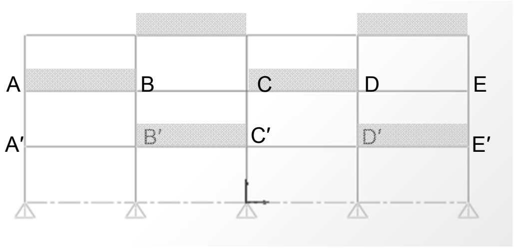
Step 3: Loads Idealization and Placement

- Placement of Loads
  - One pattern may give maximum mid span bending while other might give maximum support rotation.



Step 3: Loads Idealization and Placement

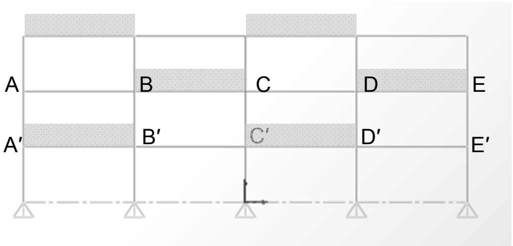
- Placement of Loads
  - Pattern for Maximum Mid Span Moment
    - With pattern 1, max. positive moments in spans AB and CD are obtained. Maximum mid span negative moment in BC is also obtained.



Pattern 1

Step 3: Loads Idealization and Placement

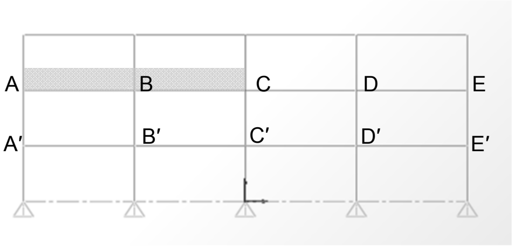
- Placement of Loads
  - Pattern for Maximum Mid Span Moment
    - With pattern 2, maximum positive moments in spans BC.
    - Maximum mid span negative moment in AB and CD is also obtained.



Pattern 2

Step 3: Loads Idealization and Placement

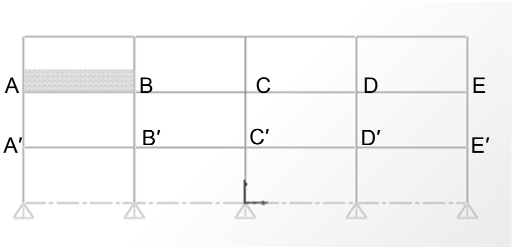
- Placement of Loads
  - Pattern for Maximum Support Moment
    - Maximum support moment (at support B for example) is obtained for the given frame from pattern 3.



Pattern 3

Step 3: Loads Idealization and Placement

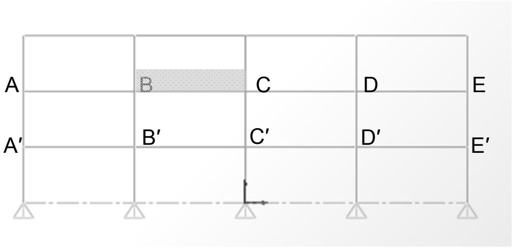
- Placement of Loads
  - Pattern for Maximum Column Moment
    - Maximum column moments (for column AA′ for example) is obtained for the given frame from pattern 4.



Pattern 4

Step 3: Loads Idealization and Placement

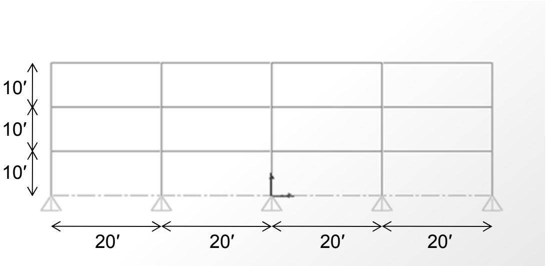
- Placement of Loads
  - Pattern for Maximum Column Moment
    - Maximum column moments (for column CC′ for example) is obtained for the given frame from pattern 5.



Pattern 5

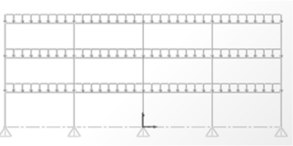
Step 3: Loads Idealization and Placement

- Case Study: Effect of Pattern Load on bending moment
  - Factored DL = 1.8 kip/ft (for 6" thick slab and 40 psf SDL)
  - Factored LL = 1.152 (for 60 psf LL on 12′ width)

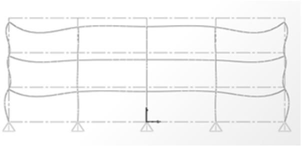


Step 3: Loads Idealization and Placement

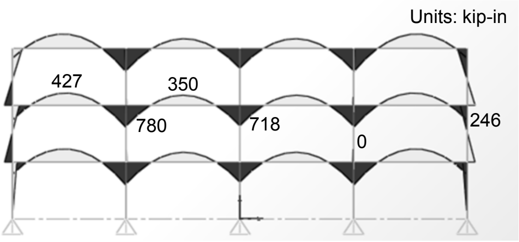
- Case Study: Effect of Pattern Load on bending moment



Load: Full dead only (1.80 kip/ft)

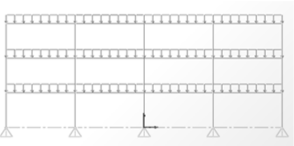


Deflected shape

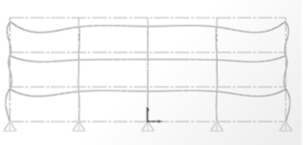


Step 3: Loads Idealization and Placement

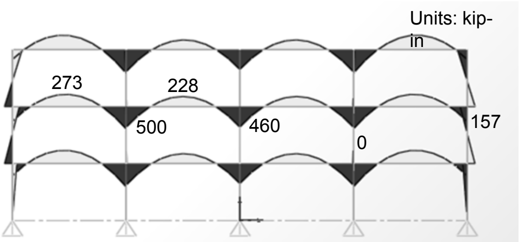
- Case Study: Effect of Pattern Load on bending moment



Load: Full Live only (1.152 kip/ft)



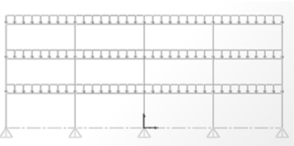
Deflected shape



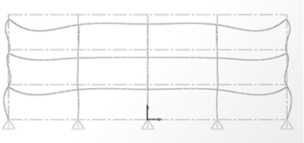
Units: kip-in

Step 3: Loads Idealization and Placement

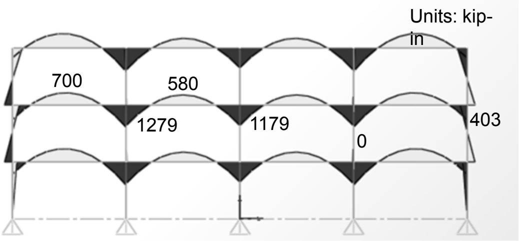
- Case Study: Effect of Pattern Load on bending moment



Load: Full dead + live {1.80 +1.152} kip/ft



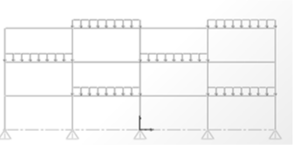
Deflected shape



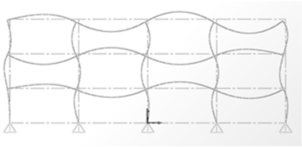
Units: kip-in

Step 3: Loads Idealization and Placement

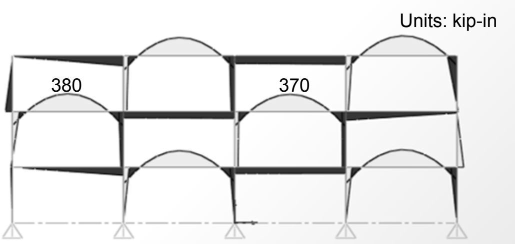
- Case Study: Effect of Pattern Load on bending moment



Load: Pattern Live Only (1.152 kip/ft) for $M_{mid+, max}$

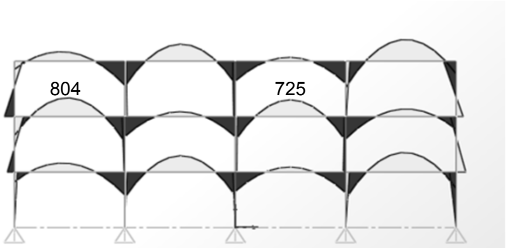


Deflected shape



Step 3: Loads Idealization and Placement

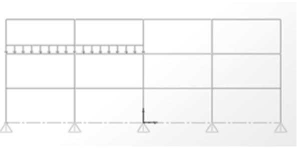
- Case Study: Effect of Pattern Load on bending moment
  - Dead + Pattern live for maximum positive mid span moment



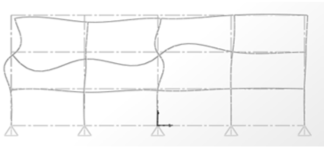
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{mid+,max}$

Step 3: Loads Idealization and Placement

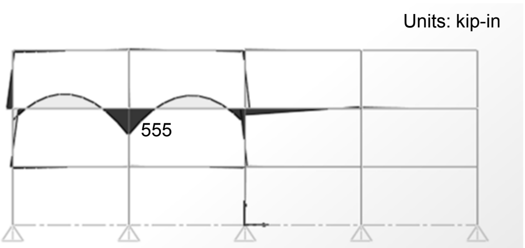
- Case Study: Effect of Pattern Load on bending moment



Load: Pattern Live Only (1.152 kip/ft) for $M_{1st\ int\ supp}$ -



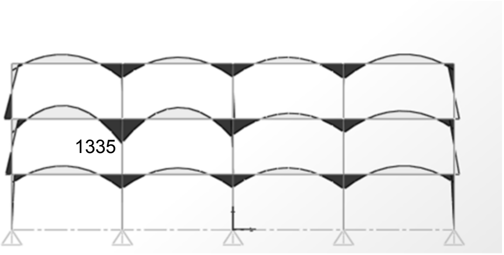
Deflected shape



Units: kip-in

Step 3: Loads Idealization and Placement

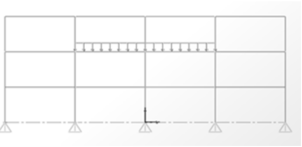
- Case Study: Effect of Pattern Load on bending moment
  - Dead + Pattern live for 1st interior support negative moment



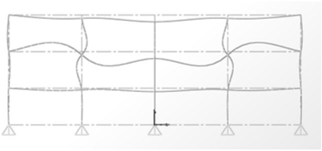
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{1st\ int\ supp\ -}$

Step 3: Loads Idealization and Placement

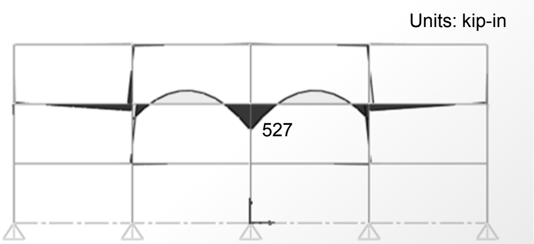
- Case Study: Effect of Pattern Load on bending moment



Load: Pattern Live Only (1.152 kip/ft) for $M_{int\ supp\ -}$

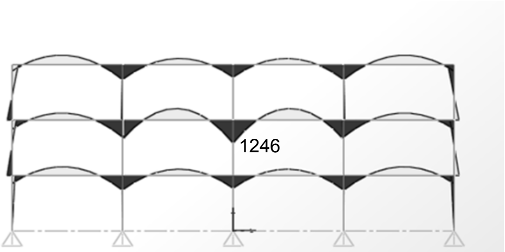


Deflected shape



Step 3: Loads Idealization and Placement

- Case Study: Effect of Pattern Load on bending moment
  - Dead + Pattern live for interior support negative moment

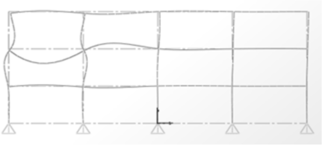


Load: Dead + Pattern Live (1.152 kip/ft) for $M_{int\ supp\ -}$

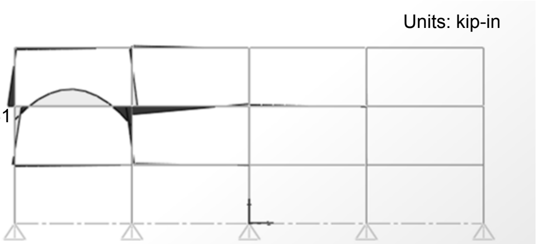
Step 3: Loads Idealization and Placement

- Case Study: Effect of Pattern Load on bending moment

Load: Pattern Live Only (1.152 kip/ft) for $M_{ext\ col}$

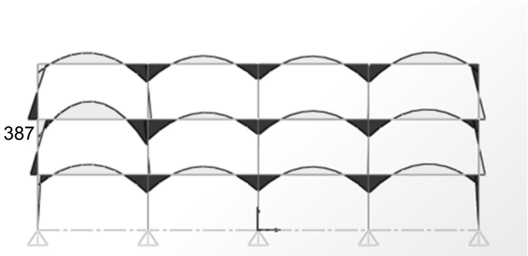


Deflected shape



Step 3: Loads Idealization and Placement

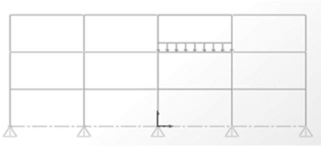
- Case Study: Effect of Pattern Load on bending moment
  - Dead + Pattern live for exterior column moment



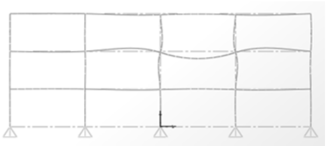
Load: Dead + Pattern Live (1.152 kip/ft) for $M_{ext\ col}$

Step 3: Loads Idealization and Placement

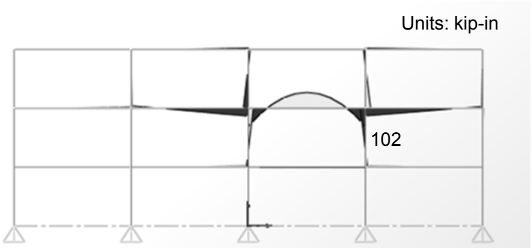
- Case Study: Effect of Pattern Load on bending moment



Load: Pattern Live Only (1.152 kip/ft) for $M_{\text{int col}}$

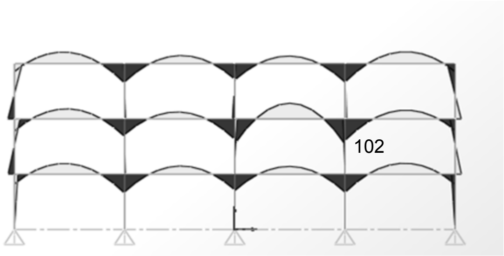


Deflected shape



Step 3: Loads Idealization and Placement

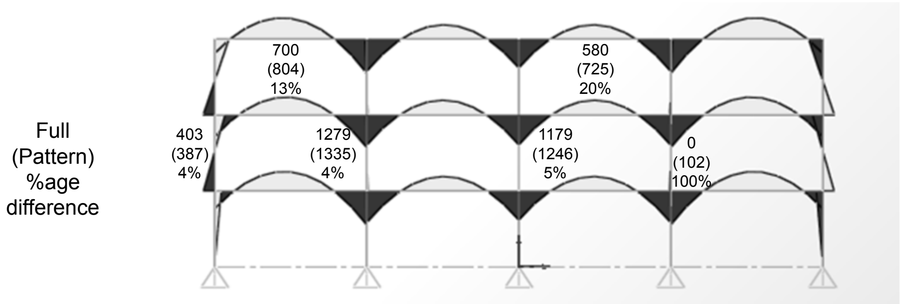
- Case Study: Effect of Pattern Load on bending moment
  - Dead + Pattern live for interior column moment



Load: Dead + Pattern Live (1.152 kip/ft) for $M_{\text{int col}}$

Step 3: Loads Idealization and Placement

- Case Study: Effect of Pattern Load on bending moment
  - All maximum load effects can be combined on a single diagram giving envelop. Comparison with full dead and live load is also shown.

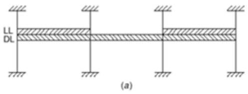


Step 3: Loads Idealization and Placement

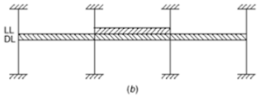
- Conclusions on Placement of Loads
  - To calculate the maximum possible moments at all critical points of a frame, live load must be placed in a great variety of different schemes.
  - In most practical cases, however, consideration of the relative magnitude of effects will permit limitation of analysis to a small number of significant cases.

Step 3: Loads Idealization and Placement

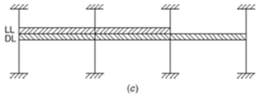
- Arrangement of Live Loads (ACI 8.9)
  - 8.9.2 — It shall be permitted to assume that the arrangement of live load is limited to combinations of:
    - (a) Factored dead load on all spans with full factored live load on two adjacent spans;
    - (b) Factored dead load on all spans with full factored live load on alternate spans.



Alternate spans loaded



Alternate spans loaded



Adjacent spans loaded

Closing Note

- The characteristics of the idealized structural model should be as close as possible to the physical structure.
- The engineer must not only accept the uncertainties of load placement, magnitude, and duration typical of any structural analysis, but must also cope with other complications that are unique to reinforced concrete such as effect of cracking, creep and shrinkage.

Closing Note

- The student may well despair of accurate calculation of the internal forces for which the members of a reinforced concrete frame must be designed.
- However mainly because of plastic flow, a concrete structure tries with admirable ductility to adapt itself to our calculations.

Closing Note

- As very correctly said by Halvard Birkeland
- ". . . the structure, in many instances, will accept our rash assumptions and our imperfect mathematical models to such an extent that the structure will exhaust all means of standing before it decides to fall.

Closing Note

- However too great a deviation from the actual distribution of internal forces can result in serviceability problems associated with cracking and deflection, and can even result in premature failure.
- But it is reassuring to know that, if good judgment is used in assigning internal forces to critical sections, the *wisdom of the structure* will prevail.

References

- Design of Concrete Structures 13th Ed. by Nilson, Darwin and Dolan.
- ACI 318.
- SAP2000 FEM based Software.
- Case Studies by Dr. Qaisar and Engr. Umer

The End