First Order Logic

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Why Not Propositional Logic?

- Consider the following statements:
  - All monitor are ready
  - X12 is a monitor
- Propositions are either true or false
- Consider
  - All monitors are ready
  - X12 is a monitor
  - Therefore X12 is ready
- Let p be all monitors ready
- Let q be…
- Sense of statement is lost

First Order Logic

- Term
  - Object that denotes some object other than true or false
  - Simplest kind of term is constant which stand for any kind of object
  - Another term is variable which stands for anything in a set of objects
    - E.g N for Natural numbers
- Definition: A constant of type T is a name that denotes some particular object in the set T

- Definition: A variable of type T is a name that can denote any value in the set T

First Order Logic

- Functions
  - The idea of functional terms in logic is similar to programming
  - Function symbol is associated with a natural number called arity. This is number of arguments it takes
  - Has return type
  - E.g. plus(2, 3)
  - But plus(-1, 0) isn’t

- Using functions, constant and variables we can build up expressions
- (x+3) * sin 90

Predicates

- In addition to hahving terms, FOL has relational operators, which capture relationships between objects.
- FOL contains stock of predicate symbol

- Gt(4, 3) is a predicate, which evaluates to true
- Gt(3, 4) is a predicate, which evaluates to false.
- Predicate symbols.
  - ><≤≥=
- Once again, the fact that we are normally write x>y instead of gt(x,y) is just convention.

FOL

- So a predicate just expresses a relationship between some values
- What happens if a predicate contains variable: can we tell if it is true or false?
- Not usually; we need to know an interpretation for the variables
- A predicate that contains no variables is a proposition
- Predicates of arity 1 are called properties.
- E.g. Man(x)
- Mortal(x)
- Malfunctioning(x)
- Predicate that have arity 0 called primitive propositions.

FOL

- Qunatifiers
  - All men have a mother
  - Every natural number has a prime factor

  - ∀----- the universal quantifier;
    - is read “for all”
  - ∃----- the existential quantifier:
    - Is read “there exists”
  - $\exists_1$---- the unique quantifier
    - Is read “there exists a unique”

FOL

- Quantifier signature - predicate

- Where
- Quantifier is one of
- Signature is variable
- And predicate

Examples

- ∀x: Man –Mortal(x)
  - For all x of type Man, x is mortal
  - All men are mortal
- ∃m: Monitor – MonitorState(m, ready)
  - There exists a monitor that is in a ready state
- ∃n: N-n=(n*n)
  - Some natural number is equal to its own square
- ∃c: EC- Borders (c, Albania)
  - Some EC country borders Albania

Prolog Introduction

- Predicates
  - **man(Ahmed)**
  - **father(Ahmed, Belal)**
  - **brother(Belal, Chand)**
  - **brother(Chand, Delawar)**
  - **owns(Belal, car)**
  - **tall(Belal)**
  - **hates(Ahmed, Chand)**
  - **family()**
- Formulae
  - **$\forall$X,Y,Z(man(X) $\wedge$ man(Y) $\wedge$ father(Z,Y) $\wedge$ father(Z,X) $\Rightarrow$ brother(X,Y))**
- Variables
  - **X, Y** and **Z**
- Constants
  - **Ahmed, Belal, Chand, Delawar** and **car**

Prolog code

```
man(ahmed).
father(ahmed, belal).
brother(ahmed, chand).
owns(belal, car).
tall(belal).
hates(ahmed, chand).
family().

brother(X,Y):-
      man(X),
      man(Y),
      father(Z,Y),
      father(Z,X).
```

Example goal (to be given on command line in PIE)

```
brother(ahmed,belal).
Or
brother(ahmed,X).
Or
father(X,Y).
Etc.
```

Prolog Program

- Clauses
  - Definition(s) of Predicate = Sentence OR Phrase
  - Types
    - Rules (function definitions)
      - 0 or more conditions (statements)
    - Facts
      - 0 conditions
      - All parameters constant
  - Examples
    - Fact

```
brother(ahmed, chand).
```

    - Rule

```
brother(X,Y):-
    man(X),
    man(Y),
    father(Z,Y),
    father(Z,X).
```