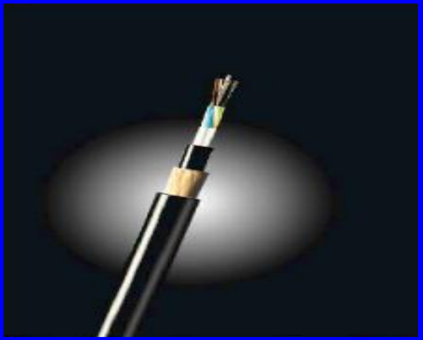
Chapter 3

Optical Fibers – (Multimode) Basics

- **How to conduct light ?**
- **Attenuation (衰减) ?**
- **Dispersion (色散) ?**
- **Bandwidth (带宽) ?**

History

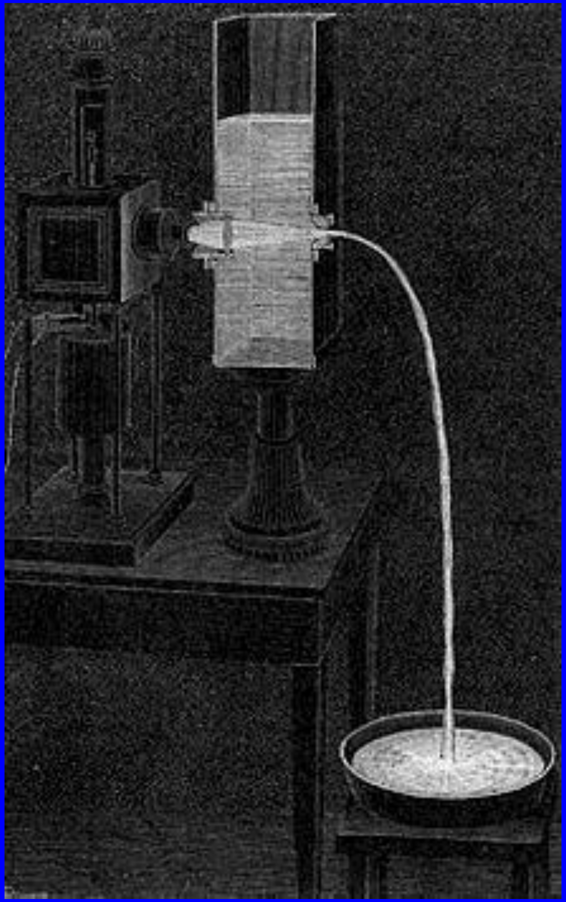
- Although glass fibers were made in the 1920s, their use became practical in the 1950s when the cladding layer was used.
- In 1966, 高錕 suggested to reduce the loss of optical fiber for fiber-optic communications.
- In 1970, Corning Ltd. fabricated the step-index optical fiber.
- In 1972, the attenuation of optical fiber was improved from 20dB/km to 4 dB/km with the improvement of material and fabication procedure.
- Further progress resulted by 1979, the loss of fiber was reduced to be only 0.2dB/km near the 1.55-um spectral region.



3.1 How optical fibers conduct light

Daniel Colladon first described this "light fountain" or "light pipe" in an 1842 article titled *On the reflections of a ray of light inside a parabolic liquid stream*. This particular illustration comes from a later article by Colladon, in 1884.

What's the principle?



◆ Step-Index Fiber (阶跃光纤）

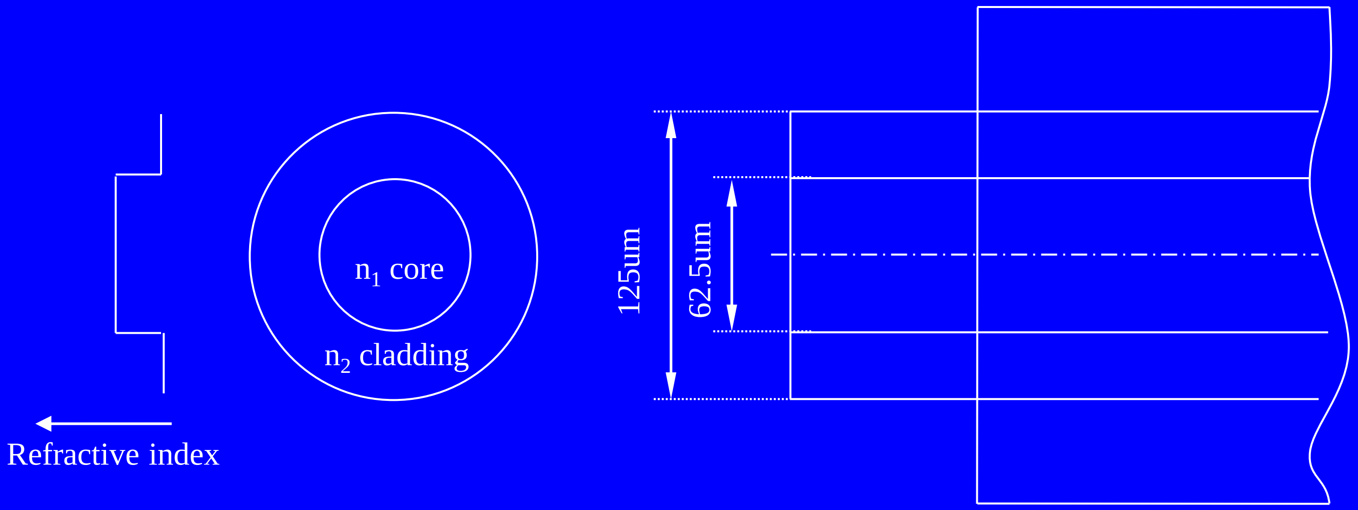
***Bare Fiber*：** Fiber Core + Cladding + Coating

Core: Doped silica -> n_1

Cladding: Pure SiO_2 -> n_2

Coating： Plastic or carbon

To achieve TIR: $n_1 > n_2$



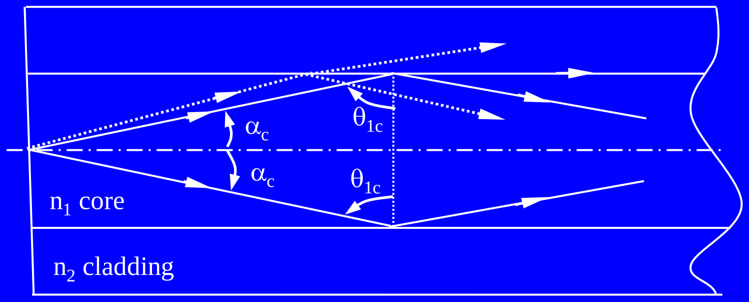
◆ Total Internal Reflection （全内反射）

Critical incident angle : θ_{1c}

(临界入射角)

Critical propagation angle : α_c

(临界传播角)



To save light inside an optical fiber, the propagation angle of rays should have

$$\alpha_z \le \alpha_c \,.$$

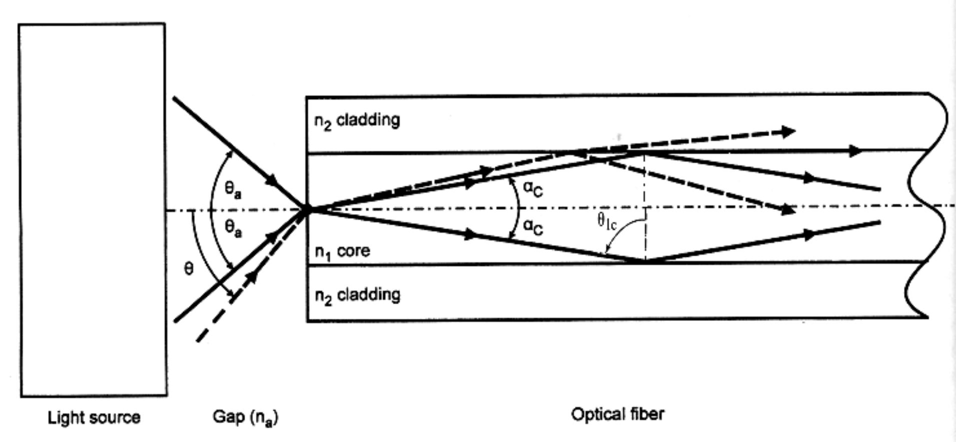
From Snell's Law, one has:

$$\sin\theta_{1c} = n_2/n_1 \text{, and } \cos\alpha_c = n_2/n_1.$$

Thus, one can derive:

$$a_c = \sin^{-1}\sqrt{[1-(n_2/n_1)^2]}$$

Launching the Light



Critical incident angle : $\sin\theta_{1c} = \dfrac{n_2}{n_1}$

Critical propagation angle : $\alpha_C = \sin^{-1}\sqrt{1-\left(\dfrac{n_2}{n_1}\right)^2}$

-> Acceptance angle (接收角): θ_a or $2\theta_a$

$$\boxed{\sin\theta_a = n_1 \sin\alpha_C}$$

◆ Numerical Aperture （数值孔径）

$NA = \sin\theta_a$ ：Describe the ability of an optical fiber to gather light from a source, and preserve this light insider the fiber.

$$= n_1 \sin(\alpha_c)$$

$$= n_1 \sqrt{(1-({}^{n_1}\!/_{n_2})^2)} = \sqrt{(n_1)^2 - (n_2)^2}$$

$n = (n_1 + n_2)/2$: Average refractive index.

$\Delta = (n_1 - n_2)/n$: Relative difference of the refractive indexes.

-> $NA = n\sqrt{2\Delta}$

: n_1 and n_2 are not important in themselves, but only in their average and relative difference.

Example:

Type-1:	Type-2:
$n_1 = 1.48$	$n_1 = 1.495$
$n_2 = 1.46$	$n_2 \approx 1.402$
$\dfrac{\Delta n}{n} = 1\%$	$\dfrac{\Delta n}{n} \approx 6\%$
$\Theta_{1C} = 80.57°$	$\Theta_{1C} = 69.68°$
$\alpha_C = 9.43°$	$\alpha_C = 20.32°$
$\Theta_a = 14.033°$	$\Theta_a = 31.27°$
NA = 0.2425	NA = 0.5192

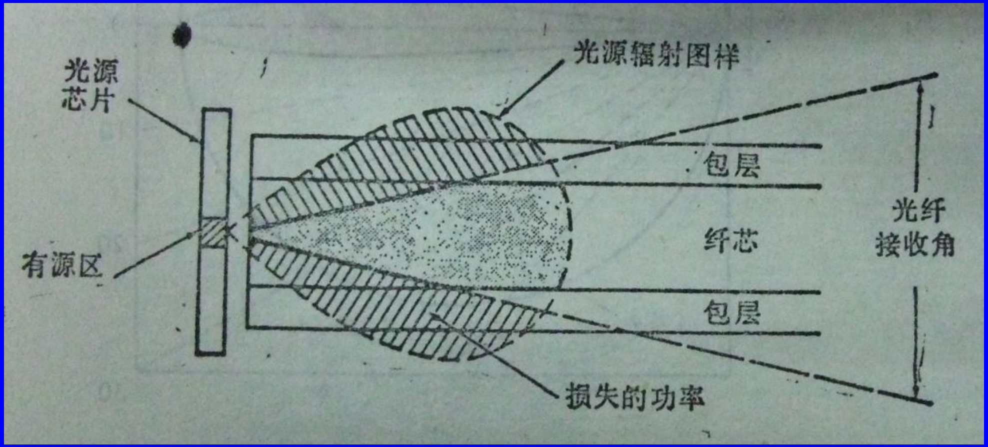
◆ Measurement of NA

-> for a fiber launched by a LED source
(Lambertian source: 朗伯光源)

$$P(\theta) = P_0 \cos\theta$$

$$\frac{P_{in}}{P_0} = \frac{\int_0^{2\pi}\int_0^{\theta_a} P_0 \cos\theta \sin\theta d\theta d\varphi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} P_0 \cos\theta \sin\theta d\theta d\varphi} = \frac{\sin^2\theta_a}{\sin^2\frac{\pi}{2}} = (NA)^2$$

-> $\boxed{NA = \sqrt{P_{in} / P_0}}$



Question :

Numerical aperture, NA, is the characteristic of an optical fiber to gather light from a source.

When the core diameter is larger, NA should be

a. larger

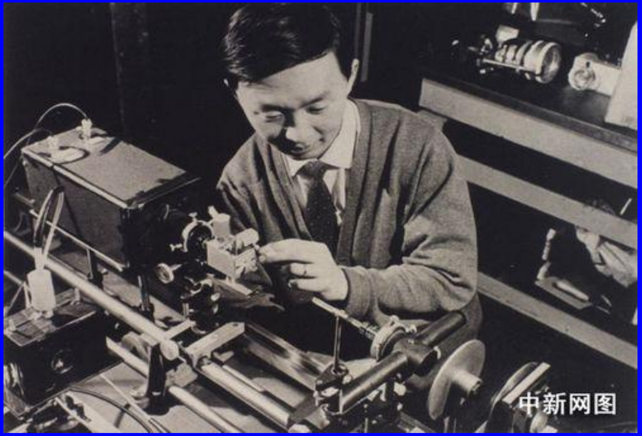
b. smaller

c. the same

Nobel Prize in Physics

In 1965, Charles K. Kao with Hockham concluded that the fundamental limitation for glass light attenuation is below **20 *dB/km*** (*decibels per kilometer*, is a measure of the attenuation of a signal over a distance), which is a key threshold value for optical communications.

Attenuation !!



中新网图

3.2 Attenuation

: power loss for reasons other than failure to achieve total internal reflection.

- **Attenuation** （衰减）
  - Bending Losses (弯曲损耗)
    - Macro-bending loss
    - Micro-bending loss
  - Scattering （散射）
  - Absorption （吸收）
    - Intrinsic absorption
    - Impurity absorption

Bending loss

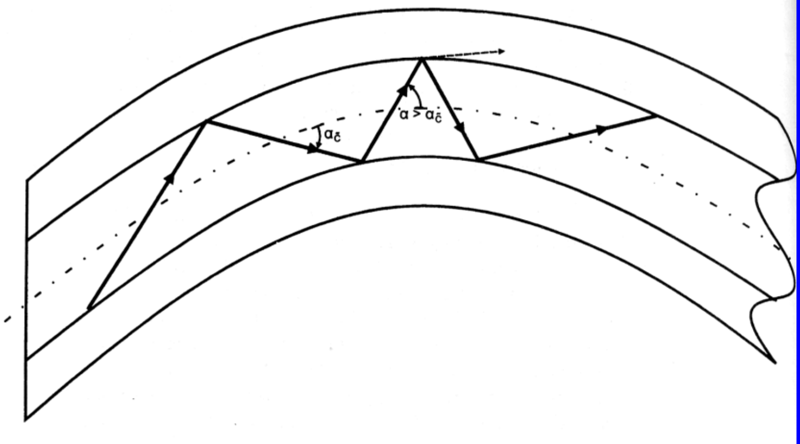
-> The flexibility of optical fiber is an advantage, but it brings some problems too!

-> It will induce the failure of TIR.

- **Macro-bending:**

  The curvature of the entire fiber changes the propagation angle to be more than critical angle.

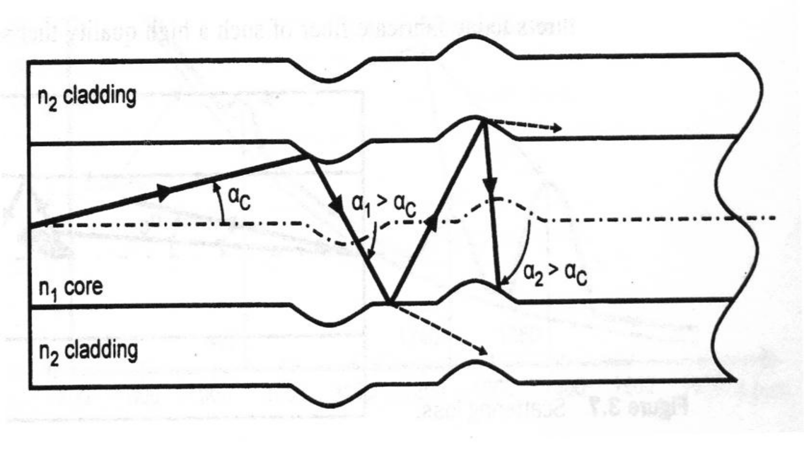
  -> minimum bending radius: 19mm or 13mm



- **Micro-bending:**

  The microconvexity or microdent will change the propagation directions.

  -> coating or external force

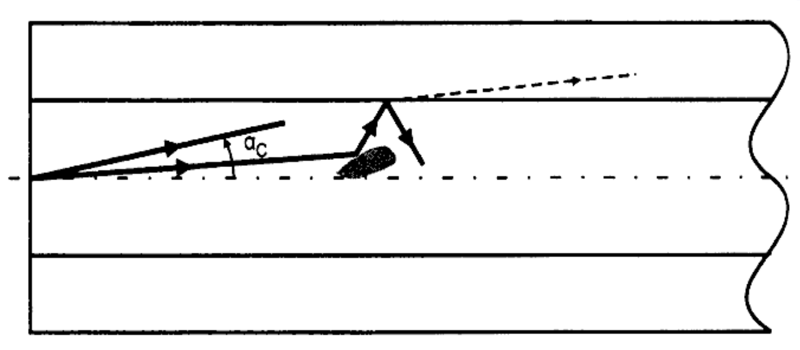


Scattering loss

Change of propagation direction induced by small changes in the core's refractive index

-> induce the failure of TIR again

Rayleigh scattering : $\propto \lambda^{-4}$



(Material) Absorption

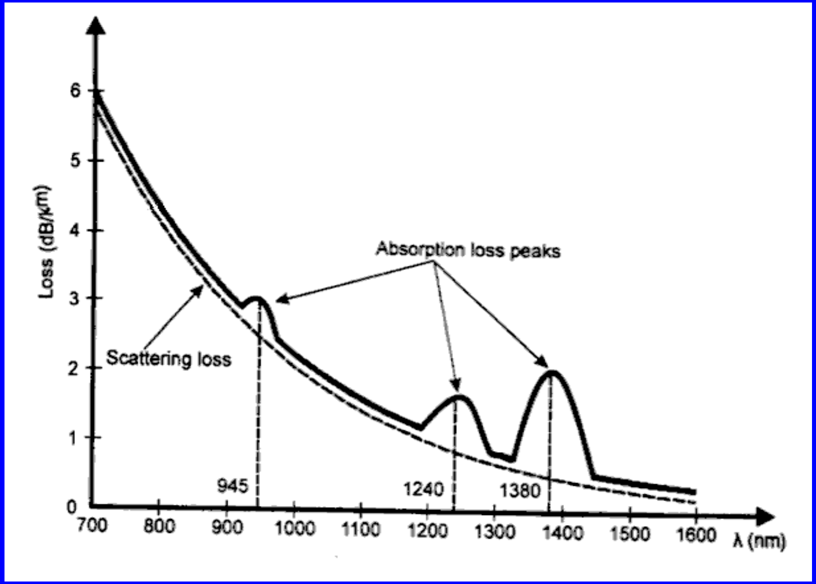
OH$^-$ molecules: 3 absorption peaks

Transparent windows:

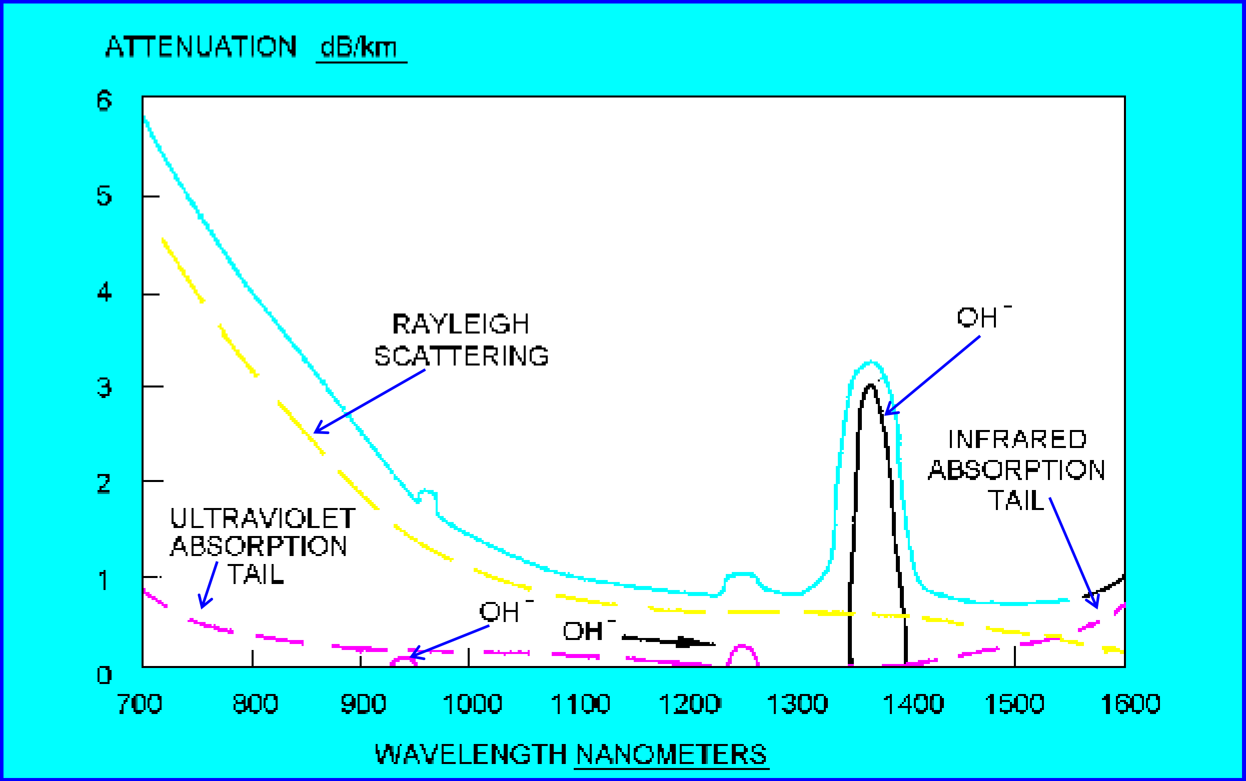
850 nm: 4 dB/km

1300nm: 0.5 dB/km

1550 nm: 0.3 dB/km



◆ Attenuation of Optical Fibers



-> : Dry fiber (无水光纤), or all-wave fiber (全波光纤), is optical fiber which hydroxide anion is eliminated.

◆ Calculation of Total Attenuation

- **Loss in linear or decibels (分贝):** **(fiber, devices, …)**

$$Loss = P_{out} / P_{in}$$

$$Loss(dB) = -10\log_{10}(P_{out} / P_{in})$$

: notice the minus sign!!

- **Attenuation (loss) per fiber-length :** **(fiber!)**

(also called “Attenuation”, cable-loss factor, attenuation coefficient)

$$A(dB/km) = loss(dB) / fiberlength(km)$$

$$p_{out} = p_{in} \times 10^{-AL/10}$$

- **Maximum transmission distance**

$$L = (10/A)\log_{10}(P_{in}/P_{out})$$

-> the maximum transmission distance imposed by attenuation,

-> the minimum value of P_{out} is determined by the sensitivity of the receiver

- **Power unit *dBm*** : choose 1mW as the reference power

$$\underline{P_{out}}(dBm) = \underline{+10}\log(P_{out}/1mW)$$

（line 4, p. 56)

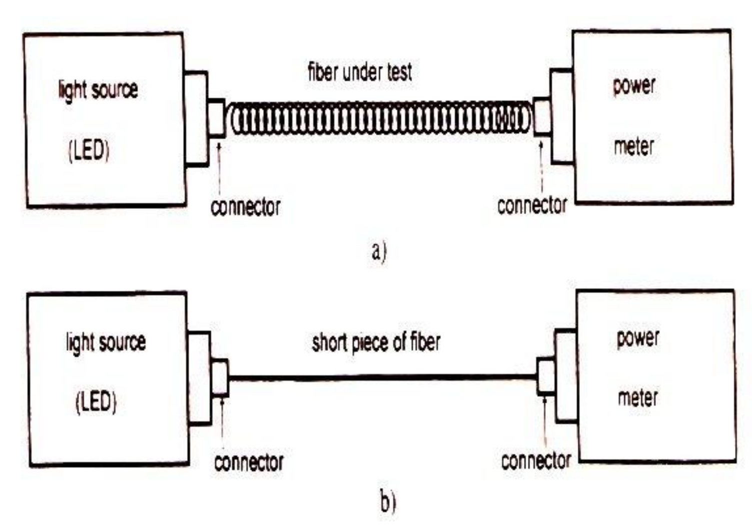
P55, Paragraph 2 from the second section: "First, it is a key….."

◆ Measuring Attenuation

Cut Method (截断法)

Measuring fiber attenuation with connection losses →

Measuring connection losses →



$$Loss(dB) = P_{in}(dBm) - P_{out}(dBm)$$

-> Measure the powers when the device under the test is "in and out".

More about ***Cut Method*** (截断法) *:*

-> The precision of this method is mainly determined by two factors:

(a) How accuracy you can reproduce connection losses

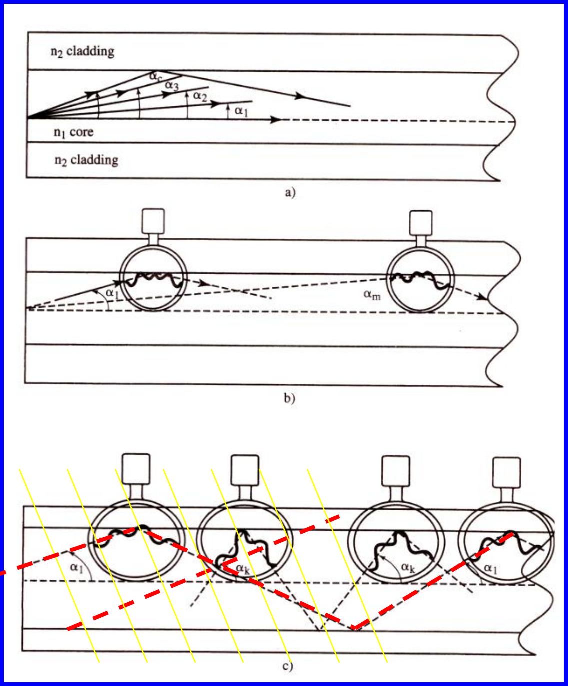
(b) How negligible is the attenuation introduced by a short piece of fiber.

-> When measuring attenuation in a multimode fiber, special care should be taken to use a light beam filling the entire cross-sectional area of the core (called overfilled launching) to make sure that (all possible modes are excited).

3.3 Intermodal and Chromatic Dispersion

Optical signal:-> Pulse width, bandwidth -> distortion?

- **Modes (模式)**



-> Modes as different beams with different propagation distances.

-> Different beams experience different phase shifts (1. Different phase fronts. 2. Reflection phase shift.)

-> Optical fiber supports only those modes that complete the full zigzag at the same repeated phase (stable!). The other one is not stable.

Wave α_1 reproduces itself after the whole cycle of propagation, but wave α_k does not.

More about Modes :

-> These different beams with different propagation angles are called modes.

-> We distinguish modes by their propagating angles and we use the word order to designate the specific mode.

-> The smaller the mode’s propagating angle, the lower the order of the mode.

-> The zero-order mode is also called fundamental mode.

-> The mode traveling at critical propagation angle is the highest order mode possible for this fiber.

◆ V-number （V参数，归一化截止频率）

(*Normalized cut-off frequency*, *characteristic waveguide parameter*)

$$V = \frac{\pi d}{\lambda} NA$$

-> More light can be accommodated for larger core diameter d, larger NA and shorter wavelength

$$V = \frac{\pi d}{\lambda} \sqrt{(n_1)^2 - (n_2)^2}$$

-> n1, n2: refractive indexes of the core and cladding

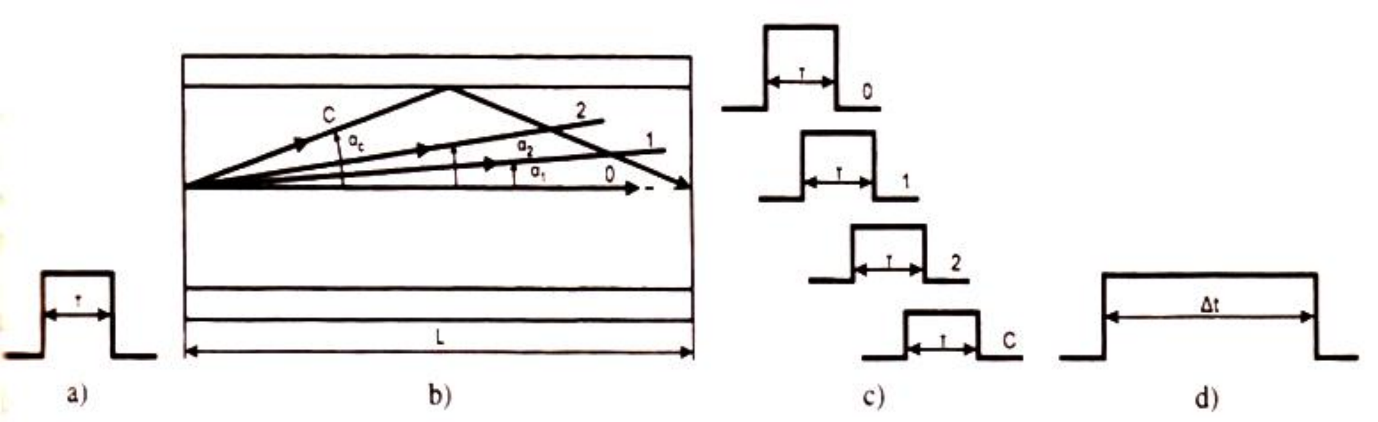
For a large V number (>20), **the number of modes** can be estimated by

⌧ Step-index fiber: $N = V^2/2$

⌧ Graded-index fiber: $N = V^2/4$

◆ Intermodal dispersion （模间色散）

(*different modes travels at different speeds*)



a) Original pulse; b) Modes in an optical fiber; c) Pluses delivered by an individual mode; d) Resulting pulse

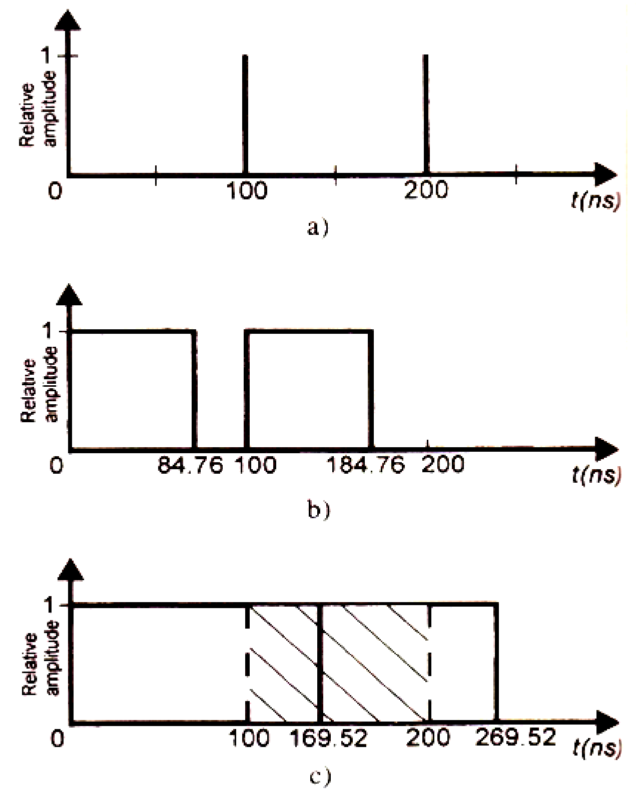
-> The zero-order (fundamental) mode needs time : $t_0 = L/v$ **,** $v = c/n_1$ **;**

-> The highest-order (critical) mode needs time： $t_c = L/(v\cos\alpha_c)$ （ $\cos\alpha_c = n_2/n_1$ ）

⌧ **Pulse spreading (脉冲展宽) stemming from intermodal dispersion:**

$$\Delta t_{SI} = t_c - t_0 = \frac{L}{2cn_2}(NA)^2$$ ： NA=0.275, $n_1 = 1.487$ => $\Delta t/L$ = 84.76 ns/km

◆ Restriction on Bit Rate



a) Input pulses;

Bit rate : 10Mbit/s,
=>time cycle duration : 100 ns

b) Pulses after 1 km transmission

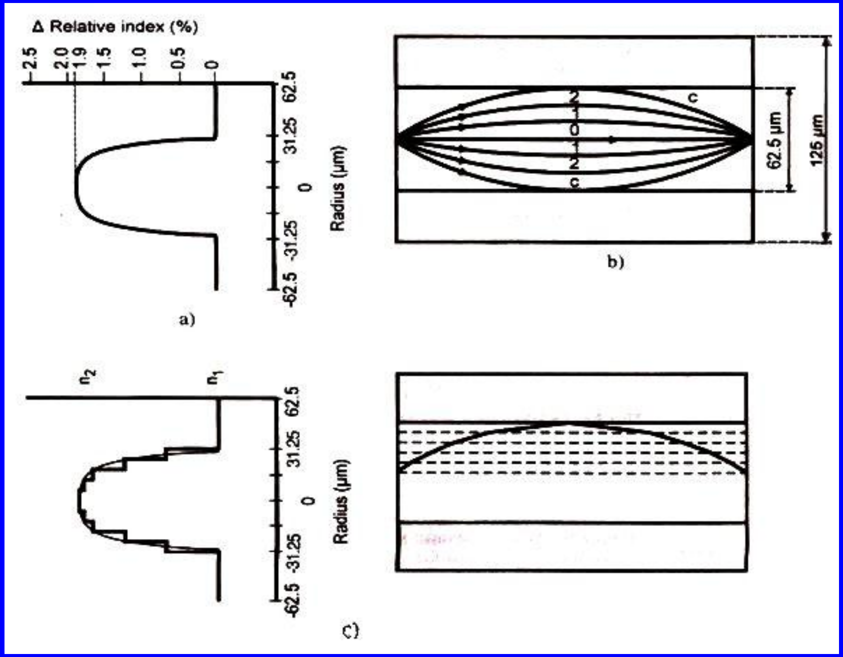
Pulse spreading: Δ t = 84.76 ns

c) Pulses after 2 km transmission

-> The maximum bit rate (for a transmission of 1 km)
= 1/Δt = 11.8Mbit/s.

-> Larger L => larger Δt => smaller bit rate.

◆ First Solution: Graded-index Fiber （渐变光纤）



a) Refractive index profile

b) Mode propagation

c) Principle of action (fabrication)
of graded index multimode fiber.

-> The refractive index of fiber core varies with the radius;

-> The beam traveling the farthest distance has the highest velocity and the beam traveling the shortest distance propagates at the slowest velocity.

- **Calculating pulse spreading for graded-index fiber**

$$\underline{\Delta t_{GI} = (LN_1\Delta^2)/(8c)\ \ ^*}$$

where N_1 is core group index of refraction

-> n_1 is a variable, i.e. $n_1(r)$

$$\Delta t_{GI} = (LNA^4)/(32cN_1^3)$$

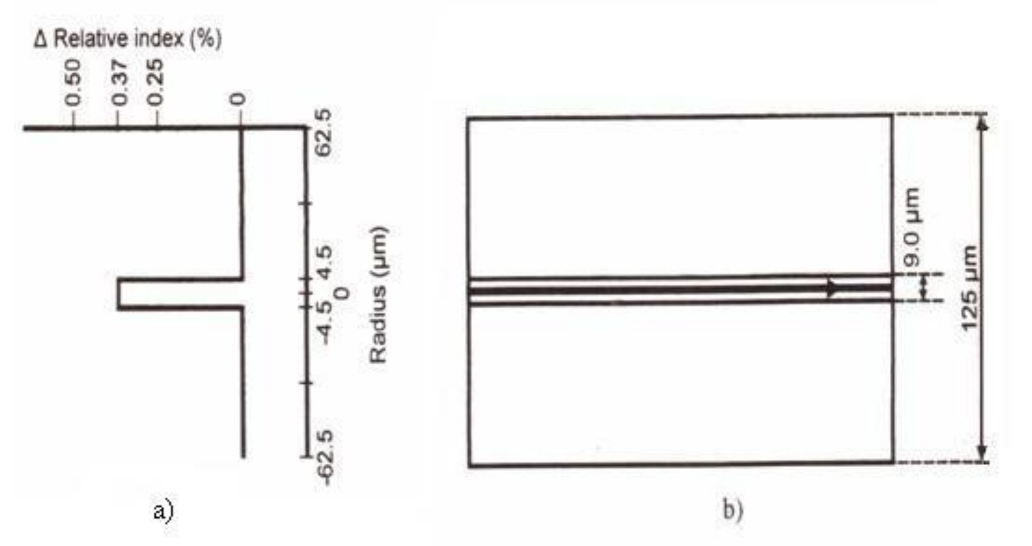
where approximation $n_1 = N_1$ was used.

$$\Delta t_{GI} = \Delta t_{SI}\underbrace{(\Delta/8)}$$

: A graded-index fiber has a modal dispersion $\Delta/8$ times less than that of a step-index fiber.

* Joseph C. Palais, Fiber Optic Communications, 4th ed., Englewood Cliffs, N.J.: Prentice Hall, 1998.

◆ A Better Solution : Single-mode Fiber (单模光纤)



Step-index Single Mode Optical fiber

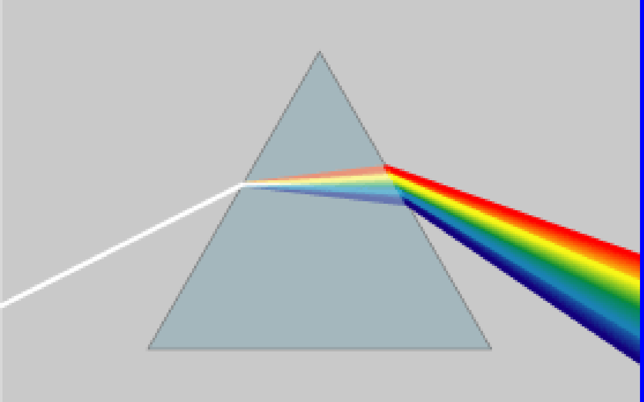
a) Refractive index profile

b) Mode propagation

-> Typically, the core diameter d is 8.3 um and the relative index Δ is 0.37%.
(for multimode fiber, d is around 62.5 um, and Δ is 2%)

-> A real single-mode condition is : $V \leq 2.405$

◆ Chromatic Dispersion （色度色散）



Why ?

-> Refractive index depends on wavelength, n = n(λ)

-> The velocity of light within a material is: v = c/n

-> The light with different wavelength travels along the fiber at different velocities.

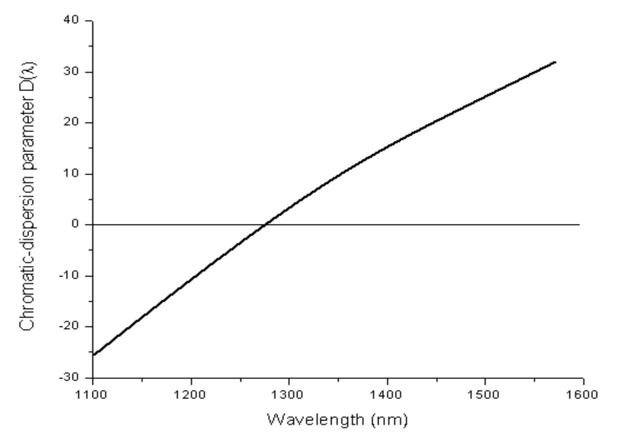
-> They will arrive at the receiver end at different times, <u>even if all of these beams propagates along the same path.</u>

- **Calculating pulse spreading**

$$\Delta t_{chrom} = D(\lambda) \cdot L \cdot \Delta\lambda$$

where $D(\lambda)$ is the chromatic-dispersion parameter （色散参数）in ps/nm.km.

$$D(\lambda) = \frac{S_0}{4}[\lambda - \frac{\lambda_0^4}{\lambda^3}]$$



-> λ_0 is the zero-dispersion wavelength : the wavelength at which D(λ) is zero.

-> S_0 is the zero-dispersion slope in ps/(nm^2.km).

- **Total pulse spreading caused by modal and chromatic dispersion**

$$\Delta t_{total} = \sqrt{(\Delta t_{\mathrm{modal}}^2 + \Delta t_{\mathrm{chrom}}^2)}$$

3.4 Bit Rate (比特率) and Bandwidth (带宽)

- **Bit rate**: The number of bits that can be transmitted per second over a channel.

$$BR = \frac{1}{T}(Hz)$$

: T — time interval between adjacent signals.

- **Bandwidth**: The frequency range within which a signal can be transmitted without significant deterioration.

$$\underline{BW = BR} \quad \text{or,} \quad BW = BR/2$$

which depends on the line codes, such as the non-return to zero (NZR) *etc*..

Dispersion and Bit Rate

for a practical standpoint, a coefficient 1/4 is general accepted in the industry.

$BR < 1/(4\Delta t)$ i.e. $T > 4\Delta t$

-> for a step-index multimode fiber, one has

$$BR_{SI} = 1/(4\Delta t_{SI}) = c/(4Ln_1\Delta) = cn_2/(2LNA^2)$$

-> for a graded-index multimode fiber, one has

$$BR_{GI} = 2c/(N_1 L\Delta^2)$$

-> If considering chromatic dispersion, one has

$$BR_{chrom} = 1/(4D(\lambda)L\Delta\lambda)$$

-> If considering the total bit rate, one has

$$BR_{total} = 1/4\sqrt{(\Delta t_{\mathrm{modal}}^2 + \Delta t_{\mathrm{chrom}}^2)}$$

◆ Reading a data sheet

Data sheet maybe different from each other, but four parts must be included.

- “Optical Characteristics” section
- “Geometric Characteristics” section
- “Environmental Specifications” section
- “Mechanical Specifications” section

◆ Homeworks

◆**3.7, 3.12, 3.21, 3.22, 3.25, 3.29, 3.30, 3.33, 3.47**

- **3.7** The core refractive index is 1.4513 and the cladding index is 1.4468. What is (1) the critical propagation angle? (2) the acceptance angle? (3) the numerical aperture?

- **3.12** For a specific fiber, NA = 0.2375 and n1 = 1.4860. Find n_2 (n cladding).

- **3.21** What does the term "transparent windows" mean? Specify three peak wavelengths for the transparent windows in modern optical fibers.

- **3.22** An optical fiber with attenuation of 0.25 dB/km is used for 20-km transmission. The light power launched into the fiber is 2mW. What is the output power?

- **3.25** Find the maximum transmission distance for a fiber link with an attenuation of 0.3 dB/km if the power launched in is 3mW and the receiver sensitivity is 100 μW.

◆ Homeworks

- **3.29** What is the number of modes for a graded-index fiber if d is 50 μm, NA is 0.200, and the operating wavelength is 1300 nm?
- **3.30** How many modes can support a step-index optical fiber whose d=8.3 μ m, $n_{1\text{core}}$=1.4513, $n_{2\text{clad}}$=1.4468, and λ=1550 nm?
- **3.33** Consider modal dispersion. For a step-index multimode fiber with NA=0.200 and n_1=1.486: a) Evaluate pulse spreading per 1 km length; b) Calculate the maximum number of bits per second that can be transmitted over 1 km.
- **3.47** A graded-index fiber has n1 = 1.486 and NA = 0.200. What is the bit rate for a 1-km link?

Reference:

1. Keiser, Gerd., Optical fiber communications,
Publisher Boston, Mass. : McGraw-Hill, c2000.
2. G. P. Agrawal, Fiber-optic communication systems,
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3. Haus, Hermann A., Waves and fields in optoelectronics,
Publisher Englewood Cliffs, NJ : Prentice-Hall, c1984.

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