Formal Methods in Software Engineering

Engr. Madeha Mushtaq
Department of Computer Science
Iqra National University

Logic Problem

- In order to better understand the logical rules and the logical connectors, let us look at a logic problem.
- On an island there are two types of residents: Knights and knaves.
- Knights always speak truth and knaves always tell a lie.
- Suppose a stranger arrives there and asks someone “Are you a knight”.
- The person replies, “If I am a knight then I will eat my hat.”

Logic Problem

- Now we have to proof that the person will eat his hat.
- Using the logical rules, we have to proof that our hypothesis is true.
- First step is to convert the problem into propositions and see how we can model/formulate this problem.
- Lets use the symbol A to denote the proposition ”A is a knight.”
- Let H be the proposition, “A eats his hat”.

Logic Problem

- A is a knight: A
- A eats his hat: H

- If I am a knight then I'll eat my hat:
  - $A \Rightarrow H$

Logic Problem

- The person X is equivalent to the statement S, “If I am a knight then I will eat my hat.”
- X ⇔ S
- Person X is equivalent to the statement because, if X is knight the statement is true, if X is knave the statement is false.
- So X and S both have the same truth value.
- So if person A is making the statement A => H, we will have
- A ⇔ A =>H

Logic Problem

- We have seen that $(X \Leftrightarrow S)$
- Therefore $(A \Leftrightarrow A \Rightarrow H)$

- Now our objective is to logically deduce H.

Logic Problem

- First we will try to proof it using truth table.

A	H	$A \Rightarrow H$	$A \Leftrightarrow (A \Rightarrow H)$

Truth Table Columns

Proof Using Truth Table

A	H	$A \Rightarrow H$
T	T	T
F	T	T
T	F	F
F	F	T

Proof Using Truth Table

A	$A \Rightarrow H$	$A \Leftrightarrow (A \Rightarrow H)$
T	T	T
F	T	F
T	F	F
F	T	F

Proof Using Truth Table

A	H	$A \Rightarrow H$	$A \Leftrightarrow (A \Rightarrow H)$
T	T	T	T
F	T	T	F
T	F	F	F
F	F	T	F

- Hence there is only one valid row and in that valid row, the value of H is true, means the proposition H, ”A eats his hat” is true.
- Hence we conclude that A has to eat his eat.

Problems with Truth Table

- The problem with truth tables is that you can make a truth table if number of propositions are small.
- But if number of variables are large, it is not easy to make a truth table.
- For example, if we have 2 variables, we will have 2^2 = 4 rows in the truth table.
- But what if we have 10 variables, then we will have 2^10= 1K rows.
- It won’t be possible to make a truth table.
- So for large variables, we can use deduction and equivalence.

Proof using equivalences

- $A \Leftrightarrow (A \Rightarrow H)$

  $\equiv$ $A \Leftrightarrow$ (not A or H)

  $\equiv$ (A and (not A or H)) or

  (not A and not (not A or H))

Proof using equivalences

- A and (not A or H)   //Apply Distributive law

≡ (A and not A) or (A and H)

≡ false or (A and H)

≡ A and H

Proof using equivalences

- not A and not (not A or H) //Apply De-Morgans Law

  ≡ not A and (A and not H) //Apply Associative Law

  ≡ (not A and A) and not H

  ≡ false and not H

  ≡ false

Proof using equivalences

- Hence

  A ⇔ (not A or H)

  ≡ (A and H) or false

  ≡ A and H
- Therefore A⇔ A => H is equivalent to A and H.
- Hence our outcome is A and H i.e. A is a knight and H (A has to eat his hat).

Rules of Inference and Logical Deduction

- There are two category of rules that we will use for logical deduction and proofs.
  - Introduction
  - Elimination

Rules of Inference and Logical Deduction

- This is the first rule of Introduction.
- Given two propositions p, q, that are true.
- We can conclude that p and q is true.

Introduction
p q ――――― p and q

Rules of Inference and Logical Deduction

- This is the second rule of Introduction.
- If we are given only one proposition, say only p or only q.
- If p is given, means p is true. We can say that p or q is true.
- Similarly if q is given i.e. q is true. We can say that p or q is true.

Introduction	
p __________ p or q	q __________ p or q

Rules of Inference and Logical Deduction

- Another important rule of introduction is, If we have q and q is true. We can say p => q.
- As we have seen in implication truth table, if q is true, p=> q will always be true.
- No matter what is the value of p.

Introduction
[p] q ——— $p \Rightarrow q$

Rules of Inference and Logical Deduction

- So first rule of Elimination is: given p and q, means p and q is true. We can conclude p is true.
- Similarly, given p and q, means p and q is true. We can conclude q is true.
- So we can eliminate either p or q.
- Because if P and q is true, means p as well as q is true independent of each other.

Elimination	
p and q	p and q
p	q

Rules of Inference and Logical Deduction

- This is the second rule of Elimination.
- If we are given that p is true and also p =>q is true.
- We can conclude that q must be true.

Elimination
$$\frac{p \qquad p \Rightarrow q}{q}$$

Rules of Inference and Logical Deduction

- Another important rule of elimination is, if we are given p and not p.
- What does it mean? It means we have contradiction.
- We have two contradictory statements. So the outcome will be false.
- Similarly if we start from False i.e. if we have a false argument or assumption.
- From a false assumption , we can conclude anything.

Elimination	
$\frac{p \quad \text{not } p}{\text{false}}$	$\frac{\text{false}}{p}$

End of Slides