Chapter 6

Algebraic Manipulation

Highest Common Factor: Highest common factor of two or more polynomials is the highest degree polynomial that can divide these polynomials exactly.

Least Common Factor: Least common factor of two or more polynomials is the lowest degree polynomial which is exactly divisible by these polynomials.

Example 1: Find HCF of x^2-y^2 & x^2-xy

Sol: Given x^2-y^2 & x^2-xy

Take $x^2-y^2=(x+y)(x-y)$

Now $x^2-xy=x(x-y)$

HCF = $x-y$

Example 2: Find HCF of $ax^2+5ax+6a$, ax^3+9ax^2+14ax & $15a(x^2-4)$

Sol: Given $ax^2+5ax+6a$, ax^3+9ax^2+14ax & $15a(x^2-4)$ Take

$$ax^2+5ax+6a=a\{x^2+5x+6\}$$
$$=a\{x^2+3x+2x+6\}$$
$$=a\{x(x+3)+2(x+3)\}$$
$$=a(x+2)(x+3)$$
$$ax^3+9ax^2+14ax=ax\{x^2+9x+14\}$$
$$=ax\{x^2+7x+2x+14\}$$
$$=ax\{x(x+7)+2(x+7)\}$$
$$=ax(x+2)(x+7)$$
$$15a(x^2-4)=15a(x^2-2^2)$$
$$=15a(x+2)(x-2)$$

HCF = $a(x+2)$

Example: Find HCF of $2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

Sol: Given $2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$ using division method

$$2x^3+7x^2+4x-4\overline{)\,2x^3+9x^2+11x+2\,}\quad 1$$
$$\underline{2x^3\pm7x^2\pm4x\mp4}$$
$$2x^2+7x+6$$

$$2x^2+7x+6\overline{)\,2x^3+7x^2+4x-4\,}\quad x$$
$$\underline{2x^3\pm7x^2\pm6x}$$
$$-2\,|\,-2x-4$$
$$x+2$$

$$x+2\overline{)\,2x^2+7x+6\,}\quad 2x$$
$$\underline{\pm2x^2\pm4x}$$
$$3x+6$$
$$\underline{\pm3x\pm6}$$

HCF = $x+2$

Example 4: Find HCF of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $3x^3+5x^2-6x-2$

Sol: Given P= $x^3-6x^2+11x-6$,

Q= $3x^3-5x^2+6x-4$ & R = $3x^3+5x^2-6x-2$

$$3x^3-5x^2+6x-4\overline{)\,3x^3+5x^2-6x-2\,}\quad 1$$
$$\underline{3x^3\mp5x^2\pm6x\mp4}$$
$$-2\,|\,10x^2-12x+2$$
$$5x^2-6x+1$$

$$5x^2-6x+1\overline{)\,3x^3-5x^2+6x-4\,}\quad 3x-7$$
$$\times5$$
$$15x^3-25x^2+30x-20$$
$$\underline{15x^3\mp18x^2\pm3x}$$
$$-7x^2+27x-20$$
$$\times5$$
$$-35x^2+135x-100$$
$$\underline{\mp35x^2\pm42x\mp7}$$
$$93\,|\,93x-93$$
$$x-1$$

$$x-1\overline{)\,5x^2-6x+1\,}\quad 5x-1$$
$$\underline{\pm5x^2\mp5x}$$
$$-x+1$$
$$\underline{\mp x\pm1}$$

HCF of Q And R is $x-1$

Now to check $x-1$ is also HCF of P

$$x-1\overline{)\,x^3-6x^2+11x-6\,}\quad x^2-5x+6$$
$$\underline{x^3\mp x^2}$$
$$-5x^2+11x$$
$$\underline{\mp5x^2\pm5x}$$
$$6x-6$$
$$\underline{\pm6x\mp6}$$

$\therefore$ HCF of P,Q &R is $x-1$

Exp 5: Find LCM of x^2+4x+4 & x^2+5x+6

Sol: Given x^2+4x+4 and x^2+5x+6

Take $x^2+4x+4=x^2+2.x.2+2^2$

$$=(x+2)^2=(x+2)(x+2)$$

and $x^2+5x+6=x^2+3x+2x+6$

$= x(x+3)+2x(x+3)$

$=(x+2)(x+3)$

LCM = common x non common

$=(x+2)\times(x+2)(x+3)$

$=(x+2)^2(x+3)$

Exp 6: Find LCM of x^2-4x+3, x^2-3x+2 and x^2-5x+6

Sol: Given P= x^2-4x+3, Q= x^2-3x+2 & R= x^2-5x+6

Take P= $x^2-4x+3 = x^2-3x-1x+3$

$= x(x-3)-1(x-3)$

$=(x-1)(x-3)$

Now Q = $x^2-3x+2 = x^2-2x-1x+2$

$= x(x-2)-1(x-2)$

$=(x-1)(x-2)$

And R = $x^2-5x+6 = x^2-2x-3x+6$

$= x(x-3)-2(x-3)$

$=(x-3)(x-2)$

Common factor of P & Q = $x-1$

Common factor of Q & R = $x-2$

Common factor of R & P = $x-3$

LCM = common x non common

$=(x-1)(x-2)(x-3)$

Theorem: if A and B are two given polynomials and their HCF and LCM are represented by H and L respectively then $A\times B = H\times L$

Proof: H is a common factors of A and B,

Let $\frac{A}{H}=a$ and $\frac{B}{H}=b$ clearly

$A = H\times a$ ....(1) and $B = H\times b$ ....(2)

Since LCM= common x non-common factors

i.e. $L = H\times a\times b$ Multiply both sides by H

$H\times L = H\times H\times a\times b$

$H\times L = H\times a\times H\times b$ multiplication commutes

$H\times L = A\times B$ From eq (1) and eq (2)

Exp7: Find LCM of $x^3-6x^2+11x-6$ & x^3-4x+3

Sol: Given $x^3-6x^2+11x-6$ and x^3-4x+3

$$\begin{array}{r|l} x^3-4x+3 & x^3-6x^2+11x-6 \quad (1 \\ & \underline{x^3 \quad \mp 4x \pm 3} \\ -3 & -6x^2+15x-9 \\ & 2x^2-5x+3 \end{array}$$

$$\begin{array}{r|l} 2x^2-5x+3 & x^3 \quad -4x+3 \quad (x+5 \\ & \times 2 \\ & 2x^3 \quad -8x+6 \\ & \underline{\pm 2x^3 \mp 5x^2 \pm 3x} \\ & 5x^2-11x+6 \\ & \times 2 \\ & 10x^2-22x+12 \\ & \underline{\pm 10x^2 \mp 25x \pm 15} \\ 3 & 3x-3 \\ & x-1 \end{array}$$

$$\begin{array}{r|l} x-1 & 2x^2-5x+3 \quad (2x+6 \\ & \underline{2x^2 \mp 2x} \\ & -3x+3 \\ & \underline{\mp 3x \pm 3} \end{array}$$

HCF = $x-1$

$L = \frac{A\times B}{H}$

$L = \frac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$

Divide any polynomial by HCF

$$\begin{array}{r|l} x-1 & x^3 \quad -4x+3 \quad (x^2+x-3 \\ & \underline{x^3 \mp x^2} \\ & x^2-4x+3 \\ & \underline{\mp x^2 \mp x} \\ & -3x+3 \\ & \underline{\mp 3x \pm 3} \end{array}$$

$\therefore L = (x^3-6x^2+11x-6)(x^2+x-3)$

Example 8: Find HCF and LCM of $3x^3-2x^2-3x+2$ and $6x^3-7x^2-x+2$

Sol: $3x^3-2x^2-3x+2$ & $6x^3-7x^2-x+2$

$$\begin{array}{r|l} 3x^3-2x^2-3x+2 & 6x^3-7x^2-x+2 \quad (2 \\ & \underline{\pm 6x^3 \mp 4x^2 \mp 6x \pm 4} \\ -1 & -3x^2+5x-2 \\ & 3x^2-5x+2 \end{array}$$

$$\begin{array}{r|l} 3x^2-5x+2 & 3x^3-2x^2-3x+2 \quad (x+1 \\ & \underline{\pm 3x^3 \mp 5x^2 \pm 2x} \\ & 3x^2-5x+2 \\ & \underline{\pm 3x^2 \mp 5x \pm 2} \end{array}$$

HCF = $3x^2-5x+2$

$L = \frac{A\times B}{H}$

$L = \frac{(3x^3-2x^2-3x+2)(6x^3-7x^2-x+2)}{3x^2-5x+2}$

Already HCF is divided by one of polynomial

$L = (x+1)(6x^3-7x^2-x+2)$

Example 9: if HCF and LCM of two polynomials are $x-3$ & $x^3-9x^2+26x-24$ respectively. Find polynomial when one polynomial is x^2-5x+6

Sol: Given HCF = $x-3$ & P = x^2-5x+6

And LCM $x^3-9x^2+26x-24$

Since H x L = P x Q

$Q=\frac{H\times L}{P}$ putting the values

$$Q=\frac{(x-3)(x^3-9x^2+26x-24)}{x^2-5x+6}$$

$$x^2-5x+6\,\big)\,x^3-9x^2+26x-24\quad (x-4)$$
$$x^3\mp5x^2\pm6x$$
$$-4x^2+20x-24$$
$$\mp4x^2\pm20x\mp24$$

$\therefore Q=(x-3)(x-4)=x^2-7x+12$

Hence second polynomial = $x^2-7x+12$

Example 10: If HCF and LCM of two polynomials are $x-1$ and x^3+4x^2+x-6 respectively. Find polynomials of degree 2.

Sol: HCF = $x-1$ & LCM = x^3+4x^2+x-6

Since $L=H\times$ non-common factors so,

$$x-1\,\big)\,x^3+4x^2+x-6\quad (x^2+5x+6)$$
$$\pm x^3\mp x^2$$
$$5x^2+x-6$$
$$\pm5x^2\mp5x$$
$$6x-6$$
$$\pm6x\mp6$$

Then $L=H\times$ non-common factors, putting

$$x^3+4x^2+x-6=(x-1)(x^2+5x+6)$$
$$=(x-1)(x^2+3x+2x+6)$$
$$=(x-1)\{x(x+3)+2(x+3)\}$$
$$=(x-1)(x+2)(x+3)$$

$P=(x-1)(x+3)$ $\quad$ $Q=(x-1)(x+2)$

$=x^2+3x-x-3$ $\quad$ $=x^2+2x-1x-2$

$=x^2+2x-3$ $\quad$ $=x^2+x-2$

Which are the required polynomial of degree 2

Example 11: the sum of numbers is 120 and their HCF is 12 find the numbers.

Sol; Given that HCF = 12, So

Two number are P = $12x$ & Q = $12y$ with

x & y are non-common prime fraters

From the 1st set of fact P + Q =120

Putting the values of P and Q

$$12x+12y=120$$
$$x+y=12$$

Now we find two non-common factors whose sum is 10, all the possibilities are (1,9) , (2,8), (3,7), (4,6), (5,5)

There are (1,9) and (3,7) non-common factors

Either numbers P and Q take pair (1,9)

P = $12\times1=12$ $\quad$ Q = $12\times9=108$

or numbers P and Q take pair (3,7)

P = $12\times3=36$ $\quad$ Q = $12\times7=84$

Rule 1: Highest Common Factor:

$4=2\times2$

$6=2\times3$ $\quad$ Only common factors

$HCF=2$

Rule2: Least Common Factor:

$4=2\times2$

$6=2\times3$

$LCM=2\times2\times3=12$

LCM = common factors × non-common factors

$A\times B=H\times L$

$4\times6=2\times12$

$24=24$ $\quad$ Satisfied

Exercise 6.1

Q1. Find H.C.F of following by factorization method.

i). $(x+6)^2$ and x^2-36

Solution: given $(x+6)^2$ and x^2-36

First of all factorize polynomials,

$(x+6)^2=(x+6)(x+6)$

Now x^2-36

$x^2-6^2=(x-6)(x+6)$

So HCF of the given polynomials $x+6$

ii). x^4-y^4 and $x^4+2x^2y^2+y^4$

Solution: First we have to factorize

Take $\quad x^4-y^4=(x^2)^2-(y^2)^2$

$$=(x^2+y^2)(x^2-y^2)$$
$$=(x^2+y^2)(x+y)(x-y)$$

Now

$$x^4+2x^2y^2+y^4=(x^2)^2+2(x^2)(y^2)+(y^2)^2$$
$$=(x^2+y^2)^2$$
$$=(x^2+y^2)(x^2+y^2)$$

So the HCF of given polynomials (x^2+y^2)

iii). $x-3, x^2-9, (x-3)^2$

Solution: factorize the given polynomials

First $x-3=x-3$

Second $x^2-9=x^2-3^2$

$$=(x-3)(x+3)$$

Third $(x-3)^2=(x-3)(x-3)$

HCF = $x-3$

iv): $2^3 3^2(x-y)^3(x+2y)^2$,

$2^3 3^2(x-y)^2(x+2y)^3$ & $3^2(x-y)^2(x+2y)$

Sol: Given $2^3 3^2 (x-y)^3 (x+2y)^2$,

$2^3 3^2 (x-y)^2 (x+2y)^3$ & $3^2 (x-y)^2 (x+2y)$

When the given polynomial having factors with powers so HCF will be common factors with least powers so,

$HCF = 3^2 (x-y)^2 (x+2y)$

v). $2x^4 - 2y^4,\ 6x^2 + 12xy + 12xy + 6y^2$ and $9x^3 + 9y^3$

Sol: Given $2x^4 - 2y^4,\ 6x^2 + 12xy + 12xy + 6y^2$ and $9x^3 + 9y^3$

Take $2x^4 - 2y^4 = 2\left[x^4 - y^4\right]$

$= 2\left[(x^2)^2 - (y^2)^2\right]$

$= 2(x^2 + y^2)\left[(x)^2 - (y)^2\right]$

$= 2(x^2 + y^2)(x+y)(x-y)$

Now

$6x^2 + 12xy + 12xy + 6y^2 = 6x(x+2y) + 6y(2x+y)$

Now $9x^3 + 9y^3 = 9\left[x^3 + y^3\right]$

$= 9\left[(x)^3 + (y)^3\right]$

$= 9(x+y)(x^2 - xy + y^2)$

HCF = $x + y$

Q2. Find the H.C.F by division method

i). $x^2 - x - 6$ and $x^2 - 2x - 3$

Sol: Let P(x)= $x^2 - x - 6$ & Q(x)= $x^2 - 2x - 3$ then

$x^2 - 2x - 3 \overline{\smash{)}\, x^2 - x - 6}\ (1$

$\pm x^2 \mp 2x \mp 3$

$x - 3 \overline{\smash{)}\, x^2 - 2x - 3}\ (x + 1$

$\pm x^2 \mp 3x$

$x - 3$

$\pm x \mp 3$

$\times$

Hence HCF = x - 3

ii). $y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

Sol: Let P(x)= $y^3 - 3y + 2$ & Q(x)= $y^3 - 5y^2 + 7y - 3$

$y^3 - 5y^2 + 7y - 3 \overline{\smash{)}\, y^3 + 0.y^2 - 3y + 2}\ (1$

$\pm y^3 \mp 5y^2 \pm 7y \mp 3$

$5 \quad 5y^2 - 10y + 5$

$y^2 - 2y + 1 \overline{\smash{)}\, y^3 - 5y^2 + 7y - 3}\ (y + 3$

$\pm y^3 \mp 2y^2 \pm y$

$3y^2 + 6y - 3$

$\pm 3y^2 \pm 6y \mp 3$

$\times$

Hence HCF = $y^2 - 2y + 1$

iii). $2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$

Solution: $2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$

$x^5 + x^4 - 3x^3 - 3x^2 \overline{\smash{)}\, 2x^5 - 4x^4 + 0x^3 + 0x^2 - 6x}\ (2$

$\pm 2x^5 \pm 2x^4 \mp 6x^3 \mp 6x^2$

$-6\ (\ -6x^4 + 6x^3 + 6x^2 - 6x$

$x^4 - x^3 - x^2 + x$

$x^4 - x^3 - x^2 + x \overline{\smash{)}\, x^5 + x^4 - 3x^3 - 3x^2}\ \ (x + 2$

$\pm x^5 \mp x^4 \mp x^3 \pm x^2$

$2x^4 - 2x^3 - 4x^2$

$\pm 2x^4 \mp 2x^3 \mp 2x^2 \pm 2x$

$-2| -2x^2 - 2x$

$x^2 + x$

$x^2 + x \overline{\smash{)}\, x^4 - x^3 - x^2 + x}\,|\, x^2 - 2x$

$\pm x^4 \pm x^3$

$-2x^3 - x^2 + x$

$\mp 2x^3 \mp 2x^2$

$x^2 + x$

$\pm x^2 \pm x$

Therefore HCF = $x^2 + x = x(x+1)$

iv). $2x^3 + 10x^2 + 5x + 25$ & $x^3 + 5x^2 - x - 5$

Solution: Let P(x) = $2x^3 + 10x^2 + 5x + 25$ and Q(x) = $x^3 + 5x^2 - x - 5$

$x^3 + 5x^2 - x - 5 \overline{\smash{)}\, 2x^3 + 10x^2 + 5x + 25}\ (2$

$\pm 2x^3 \pm 10x^2 \mp 2x \mp 10$

$7 | 7x + 35$

$x + 5 \overline{\smash{)}\, x^3 + 5x^2 - x - 5}\ (x^2 - 1$

$\pm x^3 \pm 5x^2$

$-x - 5$

$\mp x \mp 5$

$\times$

Hence HCF = x + 5

Q3. Find LCM by factorization

i). $x + y, x^2 - y^2$

Sol: given $x + y, x^2 - y^2$

First $x + y = x + y$

Second $x^2 - y^2 = (x+y)(x-y)$

HCF = $x + y$

LCM = $(x+y)(x-y) = x^2 - y^2$

ii). $x^3 - y^3, x - y$

Sol: given $x^3 - y^3, x - y$

First $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Second $x - y = x - y$

HCF = $x - y$

LCM = $(x-y)(x^2 + xy + y^2) = x^3 - y^3$

iii). x^5-x , x^5-x^2 & x^5-x^3

Sol: Given x^5-x , x^5-x^2 & x^5-x^3

$$x^5-x = x\left(x^4-1\right)$$
$$= x\left[\left(x^2\right)^2-1^2\right]$$
$$= x\left(x^2+1\right)\left[x^2-1\right]$$
$$= x\left(x^2+1\right)\left[x^2-1^2\right]$$
$$= x\left(x^2+1\right)(x+1)(x-1)$$

And $x^5-x^2 = x^2\left(x^3+1\right)$
$$= x^2\left(x^3+1^3\right)$$
$$= x^2(x+1)\left(x^2-x+1\right)$$

Now $x^5-x^3 = x^3\left(x^2-1\right)$
$$= x^3\left(x^2-1^2\right)$$
$$= x^3(x-1)(x+1)$$

HCF = $x(x+1)$

LCM $= x^3\left(x^2+1\right)(x+1)(x-1)\left(x^2-x+1\right)$

iv): $2^3 3^2 (x-y)^3 (x+2y)^2$, $2^3 3^2 (x-y)^2 (x+2y)^3$ & $3^2 (x-y)^2 (x+2y)$

Sol: Given $2^3 3^2 (x-y)^3 (x+2y)^2$, $2^3 3^2 (x-y)^2 (x+2y)^3$ & $3^2 (x-y)^2 (x+2y)$

When the given polynomial having factors with powers so LCM will be common factors with highest powers so,

$$HCF = 2^3.3^2.(x-y)^3.(x+2y)^3$$

Q4. Find L.C.M and H.C.F of following expressions

i). $x^3-2x^2-13x-10$ and $x^3-x^2-10x-8$

Sol: Given $x^3-2x^2-13x-10$ & $x^3-x^2-10x-8$

$$x^3-2x^2-13x-10\,\big)\,x^3-x^2-10x-8\,(1$$
$$\pm x^3 \mp 2x^2 \mp 13x \mp 10$$
$$x^2+3x+2\,\big)\,x^3-2x^2-13x-10\,(x-5$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-5x^2-15x-10$$
$$\mp 5x^2 \mp 15x \mp 10$$

HCF $= x^2+3x+2$

$$x^2+3x+2\,\big)\,x^3-x^2-10x-8\,(x-4$$
$$\pm x^3 \pm 3x^2 \pm 2x$$
$$-4x^2-12x-8$$
$$\mp 4x^2 \mp 12x \mp 8$$
$$\times$$

$$L=\frac{A\times B}{H}$$
$$L=\frac{\left(x^3-2x^2-13x-10\right)\left(x^3-x^2-10x-8\right)}{x^2+3x+2}$$
$$L=(x-4)\left(x^3-2x^2-13x-10\right)$$

ii). $2x^4-2x^3+x^2+3x-6$ & $4x^4-2x^3+3x-9$

Sol: Given $2x^4-2x^3+x^2+3x-6$ & $4x^4-2x^3+3x-9$ Now

$$2x^4-2x^3+x^2+3x-6\,\big)\,4x^4-2x^3+0.x^2+3x-9\,(2$$
$$\pm 4x^4 \mp 4x^3 \pm 2x^2 \pm 6x \mp 12$$
$$2x^3-2x^2-3x+3$$
$$2x^3-2x^2-3x+3\,\big)\,2x^4-2x^3+x^2+3x-6\,(x$$
$$\pm 2x^4 \mp 2x^3 \mp 3x^2 \pm 3x$$
$$2\,|\,4x^2-6$$
$$2x^2-3$$
$$2x^2-3\,\big)\,2x^3-2x^2-3x+3\,|\,x-1$$
$$\pm 2x^3 \quad \mp 3x$$
$$-2x^2 \quad +3$$
$$\mp 2x^2 \quad \pm 3$$

HCF = $2x^2-3$

$$2x^2-3\,\big)\,4x^4-2x^3+0.x^2+3x-9\,(2x^2-x+3$$
$$\pm 4x^4 \quad \mp 6x^2$$
$$-2x^3+6x^2+3x-9$$
$$\mp 2x^3 \quad \pm 3x$$
$$6x^2 \quad -9$$
$$\pm 6x^2 \quad \mp 9$$

$$L=\frac{A\times B}{H}$$
$$L=\frac{\left(2x^4-2x^3+x^2+3x-6\right)\left(4x^4-2x^3+3x-9\right)}{2x^2-3}$$
$$L=\left(2x^2-x+3\right)\left(2x^4-2x^3+x^2+3x-6\right)$$

iii). a^4-a^3-a+1 and a^4+a^2+1

Solution:

$$a^4-a^3-a+1\,\big)\,a^4+0.a^3+a^2+0.a+1\,|\,1$$
$$\pm a^4 \mp a^3 \quad \mp a \pm 1$$
$$a\,|\,a^3+a^2+a$$
$$a^2+a+1\,\big)\,a^4-a^3+0.a^2-a+1\,|\,a^2-2a+1$$
$$\pm a^4 \pm a^3 \pm a^2$$
$$-2a^3-a^2-a+1$$
$$\mp 2a^3 \mp 2a^2 \mp 2a$$
$$a^2+a+1$$
$$\pm a^2 \pm a \pm 1$$

HCF= a^2+a+1

$$L=\frac{A\times B}{H}$$
$$L=\frac{\left(a^4-a^3-a+1\right)\left(a^4+a^2+1\right)}{a^2+a+1}$$
$$L=\left(a^4+a^2+1\right)\left(a^2-2a+1\right)$$

iv). $1-x^2-x^4+x^5$ and $1+2x+x^2-x^4-x^5$

Sol: Given $1-x^2-x^4+x^5$ and $1+2x+x^2-x^4-x^5$

first arranging into descending order

$1-x^2-x^4+x^5=x^5-x^4-x^2+1$

$1+2x+x^2-x^4-x^5=-x^5-x^4+x^2+2x+1$

$x^5-x^4-x^2+1\overline{)-x^5-x^4+x^2+2x+1}\;-1$

$\mp x^5\pm x^4\pm x^2\quad\mp1$

$-2\overline{)\;-2x^4\quad+2x+2}$

$x^4-x-1\overline{)x^5-x^4-x^2\quad+1}\;x-1$

$\pm x^5\quad\mp x^2\mp x$

$-x^4+x+1$

$\mp x^4\pm x\pm1$

So H.C.F = x^4-x-1

$L=\dfrac{A\times B}{H}$

$L=\dfrac{\left(1+2x+x^2-x^4-x^5\right)\left(1-x^2-x^4+x^5\right)}{x^4-x-1}$

$L=\left(1+2x+x^2-x^4-x^5\right)\left(x-1\right)$

Q5. H.C.F and L.C.M of two polynomials are $x-2$ and x^3+3x^2-6x-8 respectively. If one of polynomial is x^2+2x-8, find second polynomial.

Sol: Given H.C.F = $x-2$

L.C.M = x^3+3x^2-6x-8

A = x^2+2x-8 We have to find B

$x+1$

$x^2+2x-8\overline{)x^3+3x^2-6x-8}$

$\pm x^3\pm2x^2\mp8x$

x^2+2x-8

$\pm x^2\pm2x\mp8$

$B=\dfrac{L\times H}{A}$

$L=\dfrac{\left(x^3+3x^2-6x-8\right)\left(x-2\right)}{x^2+2x-8}$

$L=\left(x+1\right)\left(x-2\right)$

Q6. If product of two polynomial is $x^4+5x^3-6x^2-2x-28$ and their H.C.F is $x-2$ Find L.C.M

Sol: Given PxQ= $x^4+5x^3-6x^2-2x-28$ &

H.C.F = $x-2$

$x^3+7x^2+8x+14$

$x-2\overline{)x^4+5x^3-6x^2-2x-28}$

$\pm x^4\mp2x^3$

$7x^3-\;\;6x^2$

$\pm7x^3\mp14x^2$

$8x^2-\;\;2x$

$\pm8x^2\mp16x$

$14x-28$

$\pm14x\mp28$

$L=\dfrac{A\times B}{H}$

$L=\dfrac{x^4+5x^3-6x^2-2x-28}{x-2}$

$L=x^3+7x^2+8x+14$

Q7. H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find polynomials of degree 2

Sol: Given H.C.F = $x+5$

L.C.M = $2x^3+11x^2+2x-15$

$2x^2+x-3$

$x+5\overline{)2x^3+11x^2+2x-15}$

$\pm2x^3\pm10x^2$

x^2+2x

$\pm x^2\pm5x$

$-3x-15$

$\mp3x\mp15$

$2x^3+11x^2+2x-15=\left(x+5\right)\left(2x^2+x-3\right)$

$L=\left(x+5\right)\left(2x^2+3x-2x-3\right)$

$L=\left(x+5\right)\left\{x\left(2x+3\right)-1\left(2x+3\right)\right\}$

$L=\left(x+5\right)\left(x-1\right)\left(2x+3\right)$

And H = $x+5$

$A=\left(x+5\right)\left(x-1\right)$

So $B=\left(x+5\right)\left(2x+3\right)$

Q8. If the product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and their L.C.M is $x^3+2x^2-11x-12$. Find their H.C.F

Solution: A x B = $x^4+6x^3-3x^2-56x-48$

LCM = $x^3+2x^2-11x-12$

$x+4$

$x^3+2x^2-11x-12\overline{)x^4+6x^3-3x^2-56x-48}$

$\pm x^4\pm2x^3\mp11x^2\mp12x$

$4x^3+8x^2-44x-48$

$\pm4x^3\pm8x^2\mp44x\mp48$

$H=\dfrac{A\times B}{L}$

$H=\dfrac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$

$H=x+4$

Q9. Waqar wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

Sol: Given number of Bananas = 128

$128=2\times2\times2\times2\times2\times2\times2\times2$

$=2^8$

Number of apples = 176

$176=2\times2\times2\times2\times11$

$=2^4\times11$

HCF = $2^4=16$ children

Example 12: Simplify $\dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

Sol: given $\dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

$=\dfrac{x+y+x-y}{3x+2y}$

$=\dfrac{x+x}{3x+2y}$

$=\dfrac{2x}{3x+2y}$

Example 13: Simplify $\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

Sol: Given $\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

$=\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{(x-y)(x+y)}$

$=\dfrac{x-y}{x-y}\times\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{(x-y)(x+y)}$

$=\dfrac{(x-y)^2-x^2+2y^2}{(x-y)(x+y)}$

$=\dfrac{x^2+y^2-2xy-x^2+2y^2}{x^2-y^2}$

$=\dfrac{y^2+2y^2-2xy}{x^2-y^2}$

$=\dfrac{3y^2-2xy}{x^2-y^2}$

Exp 14: Simplify $\dfrac{x^2-xy+y^2}{x^3+y^3}+\dfrac{x^2+xy+y^2}{x^3-y^3}-\dfrac{1}{x^2-y^2}$

Sol: Given $\dfrac{x^2-xy+y^2}{x^3+y^3}+\dfrac{x^2+xy+y^2}{x^3-y^3}-\dfrac{1}{x^2-y^2}$

$=\dfrac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\dfrac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\dfrac{1}{x^2-y^2}$

$=\dfrac{1}{x+y}+\dfrac{1}{x-y}+\dfrac{1}{x^2-y^2}$

$=\dfrac{x-y}{x-y}\times\dfrac{1}{x+y}+\dfrac{1}{x-y}\times\dfrac{x+y}{x+y}-\dfrac{1}{x^2-y^2}$

$=\dfrac{x-y}{x^2-y^2}+\dfrac{x+y}{x^2-y^2}-\dfrac{1}{x^2-y^2}$

$=\dfrac{x-y+x+y-1}{x^2-y^2}$

$=\dfrac{2x-1}{x^2-y^2}$

Example 15: Simplify

$\dfrac{y}{y^2-y-2}-\dfrac{1}{y^2+5y-14}-\dfrac{2}{y^2+8y+7}$

Sol: Given $\dfrac{y}{y^2-y-2}-\dfrac{1}{y^2+5y-14}-\dfrac{2}{y^2+8y+7}$

$=\dfrac{y}{y^2-2y+y-2}-\dfrac{1}{y^2+7y-2y-14}-\dfrac{2}{y^2+7y+y+7}$

$=\dfrac{y}{y(y-2)+1(y-2)}-\dfrac{1}{y(y+7)-2(y+7)}-\dfrac{2}{y(y+7)+1(y+7)}$

$=\dfrac{y}{(y+1)(y-2)}-\dfrac{1}{(y-2)(y+7)}-\dfrac{2}{(y+1)(y+7)}$

$=\dfrac{y(y+7)-1(y+1)-2(y-2)}{(y+1)(y-2)(y+7)}$

$=\dfrac{y^2+7y-y-1-2y+4}{(y+1)(y-2)(y+7)}$

$=\dfrac{y^2+4y+3}{(y+1)(y-2)(y+7)}$

$=\dfrac{y^2+3y+y+3}{(y+1)(y-2)(y+7)}$

$=\dfrac{y(y+3)+1(y+3)}{(y+1)(y-2)(y+7)}$

$=\dfrac{(y+1)(y+3)}{(y+1)(y-2)(y+7)}$

$=\dfrac{y+3}{(y-2)(y+7)}$

Example 16: Simplify $\dfrac{x+4}{x-3}\times\dfrac{x^2-9}{x^2-x-2}$

Sol: Given $\dfrac{x+4}{x-3}\times\dfrac{x^2-9}{x^2-x-2}$

$=\dfrac{x+4}{x-3}\times\dfrac{x^2-3^2}{x^2-2x+1x-2}$

$=\dfrac{x+4}{x-3}\times\dfrac{x^2-3^2}{x(x-2)+1(x-2)}$

$=\dfrac{x+4}{x-3}\times\dfrac{(x-3)(x+3)}{(x+1)(x-2)}$

$=\dfrac{(x+4)(x+3)}{(x+1)(x-2)}$

Example 17: Multiply $\dfrac{x^2-2x}{2x^2+5x+3}$ by $\dfrac{2x^2-3x-9}{x^2-9}$ and write the answer in simplified

Sol: Multiply $\dfrac{x^2-2x}{2x^2+5x+3}$ by $\dfrac{2x^2-3x-9}{x^2-9}$

$\dfrac{x^2-2x}{2x^2+5x+3}\times\dfrac{2x^2-3x-9}{x^2-9}$

$=\dfrac{x(x-2)}{2x^2+3x+2x+3}\times\dfrac{2x^2-6x+3x-9}{x^2-3^2}$

$=\dfrac{x(x-2)}{x(2x+3)+1(2x+3)}\times\dfrac{2x(x-3)+3(x-3)}{x^2-3^2}$

$=\dfrac{x(x-2)}{(x+1)(2x+3)}\times\dfrac{(2x+3)(x-3)}{(x+3)(x-3)}$

$=\dfrac{x(x-2)}{(x+1)(x+3)}$

Example 18: $\left(\dfrac{x^3-y^3}{y^3}\times\dfrac{y}{x-y}\right)\div\dfrac{x^2+xy+y^2}{y^2}$

Sol: Given $\left(\dfrac{x^3-y^3}{y^3}\times\dfrac{y}{x-y}\right)\div\dfrac{x^2+xy+y^2}{y^2}$

$$=\frac{(x-y)(x^2+xy+y^2)}{y^3}\times\frac{y}{x-y}\times\frac{y^2}{x^2+xy+y^2}$$

$$=1$$

Example 19: $\left(\dfrac{3}{x-2}-\dfrac{1}{x+1}\right)\div\dfrac{x+4}{x-2}$

Sol: Given $\left(\dfrac{3}{x-2}-\dfrac{1}{x+1}\right)\div\dfrac{x+4}{x-2}$

$$=\left(\frac{3}{x-2}-\frac{1}{x+1}\right)\frac{x-2}{x+4}$$

$$=\left(\frac{x+1}{x+1}\times\frac{3}{x-2}-\frac{1}{x+1}\times\frac{x-2}{x-2}\right)\frac{x-2}{x+4}$$

$$=\left(\frac{3(x+1)}{(x+1)(x-2)}-\frac{x-2}{(x+1)(x-2)}\right)\frac{x-2}{x+4}$$

$$=\left(\frac{3(x+1)-(x-2)}{(x+1)(x-2)}\right)\frac{x-2}{x+4}$$

$$=\frac{3x+3-x+2}{(x+1)(x-2)}\times\frac{x-2}{x+4}$$

$$=\frac{2x+5}{(x+1)(x+4)}$$

Exercise 6.2

Q1. Simplify

i). $\dfrac{x}{x+y}+\dfrac{2y}{x+y}$

Sol: Given $\dfrac{x}{x+y}+\dfrac{2y}{x+y}=\dfrac{x+2y}{x+y}$

---

ii). $\dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

Sol: Given $\dfrac{x+y}{3x+2y}+\dfrac{x-y}{3x+2y}$

$$=\frac{x+y+x-y}{3x+2y}$$

$$=\frac{2x}{3x+2y}$$

---

iii). $\dfrac{3}{y-2}-\dfrac{2}{y+2}-\dfrac{y}{y^2-4}$

Solution: Given $\dfrac{3}{y-2}-\dfrac{2}{y+2}-\dfrac{y}{y^2-4}$

$$=\frac{y+2}{y+2}\frac{3}{y-2}-\frac{2}{y+2}\frac{y-2}{y-2}-\frac{y}{y^2-4}$$

$$=\frac{3(y+2)-2(y-2)}{(y+2)(y-2)}-\frac{y}{y^2-4}$$

$$=\frac{3y+6-2y+4}{y^2-2^2}-\frac{y}{y^2-4}$$

$$=\frac{y+10}{y^2-4}-\frac{y}{y^2-4}$$

$$=\frac{y+10-y}{y^2-4}=\frac{10}{y^2-4}$$

---

iv). $\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

Sol: Given $\dfrac{x-y}{x+y}-\dfrac{x^2-2y^2}{x^2-y^2}$

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x-y)(x+y)}$$

$$=\frac{x-y}{x-y}\times\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x-y)(x+y)}$$

$$=\frac{(x-y)^2-x^2+2y^2}{(x-y)(x+y)}$$

$$=\frac{x^2+y^2-2xy-x^2+2y^2}{x^2-y^2}$$

$$=\frac{y^2+2y^2-2xy}{x^2-y^2}$$

$$=\frac{3y^2-2xy}{x^2-y^2}$$

---

v). $\dfrac{x}{2x^2+3xy+y^2}-\dfrac{x-y}{y^2-4x^2}+\dfrac{y}{2x^2+xy-y^2}$

Sol: Given $\dfrac{x}{2x^2+3xy+y^2}-\dfrac{x-y}{y^2-4x^2}+\dfrac{y}{2x^2+xy-y^2}$

$$=\frac{x}{2x^2+2xy+xy+y^2}-\frac{x-y}{-4x^2+y^2}+\frac{y}{2x^2+2xy-xy-y^2}$$

$$=\frac{x}{2x(x+y)+y(x+y)}-\frac{x-y}{-\left[(2x)^2-(y)^2\right]}+\frac{y}{2x(x+y)-y(x+y)}$$

$$=\frac{x}{(2x+y)(x+y)}+\frac{x-y}{(2x+y)(2x-y)}+\frac{y}{(2x-y)(x+y)}$$

$$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(2x+y)(x+y)(2x-y)}$$

$$=\frac{2x^2-xy+(x^2-y^2)+2xy+y^2}{(2x+y)(2x-y)(x+y)}$$

$$=\frac{2x^2-xy+x^2-y^2+2xy+y^2}{(4x^2-y^2)(x+y)}$$

$$=\frac{3x^2+xy}{(4x^2-y^2)(x+y)}$$

---

vi). $\dfrac{a}{3x-y}+\dfrac{a}{3x+y}-\dfrac{6ax}{9x^2-y^2}$

Sol: Given $\dfrac{a}{3x-y}+\dfrac{a}{3x+y}-\dfrac{6ax}{9x^2-y^2}$

$$=\frac{a(3x+y)+a(3x-y)}{(3x+y)(3x-y)}-\frac{6ax}{9x^2-y^2}$$

$$=\frac{3ax+ay+3ax-ay}{(3x)^2-(y)^2}-\frac{6ax}{9x^2-y^2}$$

$$=\frac{6ax}{9x^2-y^2}-\frac{6ax}{9x^2-y^2}=0$$

vii), $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

Sol: Given $\frac{y}{x-y}+\frac{y}{x+y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{x+y}{x+y}\frac{y}{x-y}+\frac{y}{x+y}\frac{x-y}{x-y}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{xy+y^2+xy-y^2}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{x^2+y^2}{x^2+y^2}\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}\frac{x^2-y^2}{x^2-y^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{2x^3y+2xy^3+2x^3y-2xy^3}{(x^2+y^2)(x^2-y^2)}+\frac{4x^3y}{x^4+y^4}$

$=\frac{4x^3y}{(x^2)^2-(y^2)^2}+\frac{4x^3y}{x^4+y^4}$

$=\frac{x^4+y^4}{x^4+y^4}\frac{4x^3y}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}\frac{x^4-y^4}{x^4-y^4}$

$=\frac{4x^7y+4x^3y^5+4x^7y-4x^3y^5}{(x^4)^2-(y^4)^2}$

$=\frac{8x^7y}{x^8-y^8}$

---

viii). $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

Sol: Given $\frac{1}{a^2+7a+10}+\frac{1}{a^2+10a+16}$

$=\frac{1}{a^2+5a+2a+10}+\frac{1}{a^2+8a+2a+16}$

$=\frac{1}{a(a+5)+2(a+5)}+\frac{1}{a(a+8)+2(a+8)}$

$=\frac{1}{(a+2)(a+5)}+\frac{1}{(a+2)(a+8)}$

$=\frac{(a+8)}{(a+8)}\frac{1}{(a+2)(a+5)}+\frac{1}{(a+2)(a+8)}\frac{(a+5)}{(a+5)}$

$=\frac{a+8+a+5}{(a+2)(a+5)(a+8)}$

$=\frac{2a+13}{(a+2)(a+5)(a+8)}$

---

ix). $\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

Sol: Given $\frac{1}{a-b}+\frac{1}{a+b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{a+b}{a+b}\frac{1}{a-b}+\frac{1}{a+b}\frac{a-b}{a-b}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{a+b+a-b}{a^2-b^2}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{2a}{a^2-b^2}+\frac{2a}{a^2+b^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{a^2+b^2}{a^2+b^2}\frac{2a}{a^2-b^2}+\frac{2a}{a^2+b^2}\frac{a^2-b^2}{a^2-b^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{2a^3+2ab^2+2a^3-2ab^2}{(a^2)^2-(b^2)^2}+\frac{4a^3}{a^4+b^4}$

$=\frac{4a^3}{a^4-b^4}+\frac{4a^3}{a^4+b^4}$

$=\frac{a^4+b^4}{a^4+b^4}\frac{4a^3}{a^4-b^4}+\frac{4a^3}{a^4+b^4}\frac{a^4-b^4}{a^4-b^4}$

$=\frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{(a^4)^2-(b^4)^2}$

$=\frac{8a^7}{a^8-b^8}$

---

x). $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

Sol: Given $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

$=\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{x^2-y^2}$

$=\frac{1}{x+y}+\frac{1}{x-y}+\frac{1}{x^2-y^2}$

$=\frac{x-y}{x-y}\times\frac{1}{x+y}+\frac{1}{x-y}\times\frac{x+y}{x+y}-\frac{1}{x^2-y^2}$

$=\frac{x-y}{x^2-y^2}+\frac{x+y}{x^2-y^2}-\frac{1}{x^2-y^2}$

$=\frac{x-y+x+y-1}{x^2-y^2}$

$=\frac{2x-1}{x^2-y^2}$

Q2. Simplify

i). $\frac{x^2-25}{5-x}$

Solution: We have $\frac{x^2-25}{5-x}$

$=\frac{x^2-5^2}{-x+5}$

$=\frac{(x-5)(x+5)}{-1(x-5)}$

$=-(x+5)$

---

ii). $\frac{x^2+5x+4}{4y^3}\times\frac{2y^2}{x^2+3x+2}$

Sol: Given $\frac{x^2+5x+4}{4y^3}\times\frac{2y^2}{x^2+3x+2}$

$=\frac{x^2+4x+1x+4}{4y^2.y}\times\frac{2y^2}{x^2+2x+1x+2}$

$=\frac{x(x+4)+1(x+4)}{2y\left[x(x+2)+1(x+2)\right]}$

$=\frac{(x+1)(x+4)}{2y(x+1)(x+2)}$

$=\frac{x+4}{2y(x+2)}$

---

iii). $\frac{x^2-5x+4}{x^2-3x-4}\div\frac{x^3-4x^2+x-4}{2x-1}$

Sol: Given $\frac{x^2-5x+4}{x^2-3x-4}\div\frac{x^3-4x^2+x-4}{2x-1}$

$=\frac{x^2-4x-1x+4}{x^2-4x+1x-4}\div\frac{x^2(x-4)+1(x-4)}{2x-1}$

$=\frac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)}\div\frac{(x^2+1)(x-4)}{2x-1}$

$=\frac{(x-1)(x-4)}{(x+1)(x-4)}\times\frac{2x-1}{(x^2+1)(x-4)}$

$=\frac{(x-1)(2x-1)}{(x+1)(x^2+1)(x-4)}$

iv). $\dfrac{a(a+b)}{a^3-b^3}\times\dfrac{a^2+ab+b^2}{a^2+b^2}$

Sol: Given $\dfrac{a(a+b)}{a^3-b^3}\times\dfrac{a^2+ab+b^2}{a^2+b^2}$

$=\dfrac{a(a+b)}{(a-b)(a^2+ab+b^2)}\times\dfrac{a^2+ab+b^2}{a^2+b^2}$

$=\dfrac{a(a+b)}{(a-b)(a^2+b^2)}$

v). $\dfrac{7}{x^2-4}\div\dfrac{xy}{x+2}$

Sol: Given $\dfrac{7}{x^2-4}\div\dfrac{xy}{x+2}$

$=\dfrac{7}{x^2-2^2}\times\dfrac{x+2}{xy}$

$=\dfrac{7}{(x+2)(x-2)}\times\dfrac{x+2}{xy}$

$=\dfrac{7}{xy(x-2)}$

vi). $\dfrac{a^3-b^3}{a^4-b^4}\div\dfrac{a^2+ab+b^2}{a^2+b^2}$

Sol: Given $\dfrac{a^3-b^3}{a^4-b^4}\div\dfrac{a^2+ab+b^2}{a^2+b^2}$

$=\dfrac{(a-b)(a^2+ab+b^2)}{(a^2)^2-(b^2)^2}\times\dfrac{a^2+b^2}{a^2+ab+b^2}$

$=\dfrac{(a-b)(a^2+b^2)}{(a^2-b^2)(a^2+b^2)}$

$=\dfrac{(a-b)}{(a^2-b^2)}$

$=\dfrac{a-b}{(a-b)(a+b)}$

$=\dfrac{1}{a+b}$

vii). $\dfrac{2x}{3x-12}\div\dfrac{x^2-2x}{x^2-6x+8}$

Sol: Given $\dfrac{2x}{3x-12}\div\dfrac{x^2-2x}{x^2-6x+8}$

$=\dfrac{2x}{3x-12}\times\dfrac{x^2-6x+8}{x^2-2x}$

$=\dfrac{2x}{3(x-4)}\times\dfrac{x^2-4x-2x+8}{x(x-2)}$

$=\dfrac{2x}{3(x-4)}\times\dfrac{x(x-4)-2(x-4)}{x(x-2)}$

$=\dfrac{2x}{3(x-4)}\times\dfrac{(x-2)(x-4)}{x(x-2)}$

$=\dfrac{2}{3}$

viii). $\dfrac{a^4-8a}{2a^2+5a-3}\times\dfrac{2a-1}{a^2+2a+4}\div\dfrac{a^2-2a}{a+3}$

Sol: Given $\dfrac{a^4-8a}{2a^2+5a-3}\times\dfrac{2a-1}{a^2+2a+4}\div\dfrac{a^2-2a}{a+3}$

$=\dfrac{a(a^3-8)}{2a^2+6a-a-3}\times\dfrac{2a-1}{a^2+2a+4}\div\dfrac{a(a-2)}{a+3}$

$=\dfrac{a(a^3-2^3)}{2a(a+3)-1(a+3)}\times\dfrac{2a-1}{a^2+2a+4}\times\dfrac{a+3}{a(a-2)}$

$=\dfrac{a(a-2)(a^2+2a+4)}{(2a-1)(a+3)}\times\dfrac{2a-1}{a^2+2a+4}\times\dfrac{a+3}{a(a-2)}$

$=1$

ix). $\dfrac{9-x^2}{x^4+6x^3}\div\dfrac{x^3-2x^2-3x}{x^2+7x+6}$

Sol: Given $\dfrac{9-x^2}{x^4+6x^3}\div\dfrac{x^3-2x^2-3x}{x^2+7x+6}$

$=\dfrac{-x^2+9}{x^4+6x^3}\cdot\dfrac{x^2+7x+6}{x^3-2x^2-3x}$

$=\dfrac{-(x^2-9)}{x^3(x+6)}\cdot\dfrac{x^2+6x+1x+6}{x(x^2-2x-3)}$

$=\dfrac{-(x^2-3^2)}{x^3(x+6)}\cdot\dfrac{x(x+6)+1(x+6)}{x(x^2-3x+1x-3)}$

$=\dfrac{-(x-3)(x+3)}{x^3(x+6)}\cdot\dfrac{(x+1)(x+6)}{x(x(x-3)+1(x-3))}$

$=\dfrac{-(x-3)(x+3)}{x^3(x+6)}\cdot\dfrac{(x+1)(x+6)}{x(x+1)(x-3)}$

$=\dfrac{-(x+3)}{x^4}$

x). $\dfrac{ax+ab+cx+bc}{a^2-x^2}\times\dfrac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

Sol: Given $\dfrac{ax+ab+cx+bc}{a^2-x^2}\times\dfrac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

$=\dfrac{a(x+b)+c(x+b)}{(a+x)(a-x)}\times\dfrac{(x-a)^2}{x^2+bx+ax+ab}$

$=\dfrac{(a+c)(x+b)}{(a+x)(-x+a)}\times\dfrac{(x-a)^2}{x(x+b)+a(x+b)}$

$=\dfrac{(a+c)(x+b)}{-(a+x)(x-a)}\times\dfrac{(x-a)(x-a)}{(x+a)(x+b)}$

$=\dfrac{-(a+c)(x-a)}{(a+x)^2}$

Example 20: Find the square root by factorization.

$x^2+ax+\dfrac{a^2}{4}$

Sol: Given $x^2+ax+\dfrac{a^2}{4}$

$=(x)^2+2(x)\left(\dfrac{a}{2}\right)+\left(\dfrac{a}{2}\right)^2$

$=\left(x+\dfrac{a}{2}\right)^2$

Taking square root on both sides

$\sqrt{x^2+ax+\dfrac{a^2}{4}}=\sqrt{\left(x+\dfrac{a}{2}\right)^2}$

$\sqrt{x^2+ax+\dfrac{a^2}{4}}=\pm\left(x+\dfrac{a}{2}\right)$

Example 21: $x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$

Sol: Given $x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$

$=x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+27-2$

$=x^2+\frac{1}{x^2}+2.x.\frac{1}{x}-10\left(x+\frac{1}{x}\right)+25$

$=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(5)+5^2$

$=\left(x+\frac{1}{x}-5\right)^2$

Taking square root on both sides, we have

$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\sqrt{\left(x+\frac{1}{x}-5\right)^2}$

$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\left(x+\frac{1}{x}-5\right)$

Example 22: Find the square root by division method

$16x^4-24x^3+25x^2-12x+4$

Sol:

$$\begin{array}{r|l}
 & 4x^2 -3x +2 \\
\hline
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm16x^4 \\
\hline
8x^2-3x & -24x^3+25x^2 \\
 & \mp24x^3\pm9x^2 \\
\hline
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm16x^2\mp12x\pm4 \\
\hline
\end{array}$$

$\sqrt{16x^4-24x^3+25x^2-12x+4}=\pm\left(4x^2-3x+2\right)$

Example 23: Find the square root by division method

$\frac{x^4}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$

Sol: Rearranging

$\frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}$

$$\begin{array}{r|l}
 & \frac{x^2}{2} -2x +\frac{a}{3} \\
\hline
\frac{x^2}{2} & \frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\
 & \pm\frac{x^4}{4} \\
\hline
x^2-2x & -2x^3+4x^2 \\
 & \mp2x^3\pm4x^2 \\
\hline
x^2-4x+\frac{a}{3} & \frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\
 & \pm\frac{ax^2}{3}\mp\frac{4ax}{3}\pm\frac{a^2}{9} \\
\hline
\end{array}$$

$\therefore\sqrt{\frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}}=\pm\left(\frac{x^2}{2}-2x+\frac{a}{3}\right)$

Example 24: i). What should be added to

ii). What should be subtracted from

iii). For what value of x

the expression $9x^4-12x^3+10x^2-3x-3$ to make the perfect square

$$\begin{array}{r|l}
 & 3x^2 -2x +1 \\
\hline
3x^2 & 9x^4-12x^3+10x^2-3x-3 \\
 & \pm9x^4 \\
\hline
6x^2-2x & -12x^3+10x^2 \\
 & \mp12x^3\pm4x^2 \\
\hline
6x^2-4x+1 & 6x^2-3x-3 \\
 & \pm6x^2\mp4x\pm1 \\
\hline
 & x-4
\end{array}$$

To make given expression a perfect square, remainder should be zero. Hence

i). $-x+4$ should be added

ii). $x-4$ should be subtracted

iii). For value $x-4=0 \Rightarrow x=4$

Rule 4: $\sqrt{(a)^2\pm2(a)(b)+(b)^2}=\pm\sqrt{(a\pm b)^2}$

$\sqrt{(a)^2\pm2(a)(b)+(b)^2}=\pm(a\pm b)^2$

Rule 5: Square root by division method

$$\begin{array}{r|l}
 & a\pm b \\
\hline
a & a^2\pm2ab+b^2 \\
 & \pm a^2 \\
\hline
2a\pm b & \pm2ab+b^2 \\
 & \mp2ab\pm b^2 \\
\hline
 & 0
\end{array}$$

R.Work

$a\times a=a^2$

$2\times a=2a$

$\frac{\pm2ab}{2a}=\pm b$

$\pm b(2a+b)=\pm2ab\pm b^2$

Exercise 6.3

Q1. Find square root by factorization method

i). x^2+4x+4

Sol: Given $x^2+4x+4=x^2+2.x.2+2^2$

$=(x+2)^2$

Taking square root on both sides

$\sqrt{x^2+4x+4}=\pm(x+2)$

ii). $(x-y)^2+6(x-y)+9$

Sol; Given

$(x-y)^2+6(x-y)+9=(x-y)^2+2(x-y)(3)+3^2$

$(x-y)^2+6(x-y)+9=(x-y+3)^2$

Taking square root on both sides, we get

$\sqrt{(x-y)^2+6(x-y)+9}=\pm\sqrt{(x-y+3)^2}$

$\sqrt{(x-y)^2+6(x-y)+9}=\pm(x-y+3)$

iii). $x^2y^2-8xy+16$

Sol: $x^2y^2-8xy+16=(xy)^2-2(xy)(4)+4^2$

$=(xy-4)^2$

Taking square root on both sides

$\sqrt{x^2y^2-8xy+16}=\pm(xy-4)$

iv). $x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$

Sol: Given $x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$

Which is not a perfect square

$= x^2+\frac{1}{x^2}+2-8\left(x+\frac{1}{x}\right)+18-2$

$=\left(x+\frac{1}{x}\right)^2-8\left(x^2+\frac{1}{x^2}\right)+16$

$=\left(x+\frac{1}{x}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(4)+(4)^2$

$=\left(x+\frac{1}{x}-4\right)^2$

Taking square root on both sides

$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}=\pm\left(x+\frac{1}{x}-4\right)$

v). $x(x+1)(x+2)(x+3)+1$

Sol: Given $x(x+1)(x+2)(x+3)+1$

Rearranging accordingly a + b = c + d

$=x(x+3)(x+1)(x+2)+1$

$=\{x(x+3)\}\{x(x+2)+1(x+2)\}+1$

$=\{x^2+3x\}\{x^2+2x+1x+2\}+1$

$=(x^2+3x)(x^2+3x+2)+1$

Assume that $y=x^2+3x$ Then

$=y(y+2)+1$

$=y^2+2y+1$

$=y^2+2.y.1+1^2$

$=(y+1)^2$ $\quad\therefore y=x^2+3x$

$=(x^2+3x+1)^2$

Taking square root on both sides

$\sqrt{x(x+1)(x+2)(x+3)+1}=\pm(x^2+3x+1)$

vi). $\left(x^2+\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$

Sol: Given $\left(x^2+\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$

$=x^4+\frac{1}{x^4}+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$

$=x^4+\frac{1}{x^4}-2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+2+2$

$=\left(x^2-\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+4$

$=\left(x^2-\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9+16}{4}$

$=\left(x^2-\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{25}{4}$

$=\left(x^2-\frac{1}{x^2}\right)^2-2\left(x-\frac{1}{x}\right)\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2$

$=\left(x^2-\frac{1}{x^2}-\frac{5}{2}\right)^2$

Taking square root on both sides

$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\left(x^2-\frac{1}{x^2}-\frac{5}{2}\right)$

vii). $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Sol: given $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$

$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$

$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$

$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(2)^2$

$=\left(x^2+\frac{1}{x^2}-2\right)^2$

Taking square root on both sides

$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\left(x^2+\frac{1}{x^2}-2\right)$

viii). $\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Sol: Given $\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$

$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$

Taking square root on both sides

$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}=\pm\frac{2x^3-3y^3}{3x^2+4y^2}$

Q2. Find square root of following by division method

i). $4x^4-4x^3+13x^2-6x+9$

Sol: Given $4x^4-4x^3+13x^2-6x+9$, now

	$2x^2-x+3$
$2x^2$	$4x^4-4x^3+13x^2-6x+9$ $\pm4x^4$
$4x^2-x$	$-4x^3+13x^2$ $\mp4x^3\pm x^2$
$4x^2-2x+3$	$12x^2-6x+9$ $\pm12x^2\mp6x\pm9$

$\therefore\sqrt{4x^4-4x^3+13x^2-6x+9}=\pm(2x^2-x+3)$

ii). $x^4+x^3-\frac{31}{4}x^2-4x+16$

Sol: given $x^4+x^3-\frac{31}{4}x^2-4x+16$

$$x^2+\frac{x}{2}-4$$

x^2	$x^4+x^3-\frac{31}{4}x^2-4x+16$ $\pm x^4$
$2x^2+\frac{x}{2}$	$x^3-\frac{31}{4}x^2-4x+16$ $\mp x^3\pm\frac{1}{4}x^2$
$2x^2+x-4$	$-8x^2-4x+16$ $\mp 8x^2\mp 4x\pm 16$ $\times$

Required square root

$$\sqrt{x^4+x^3-\frac{31}{4}x^2-4x+16}=\pm\left(x^2+\frac{x}{2}-4\right)$$

iii). $x^2-2x+1+2xy-2y+y^2$

Sol: Given $x^2-2x+1+2xy-2y+y^2$

$$x-1+y$$

x	$x^2-2x+1+2xy-2y+y^2$ $\pm x^2$
$2x-1$	$-2x+1$ $\mp 2x\pm 1$
$2x-2+y$	$2xy-2y+y^2$ $\pm 2xy\mp 2y\pm y^2$

$$\therefore\sqrt{x^2-2x+1+2xy-2y+y^2}=\pm(x-1+y)$$

iv). $\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$

Sol: Given $\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$

$$x^2-\frac{1}{x^2}-6$$

$x^2-\frac{1}{x^2}$	$\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36$ $\pm\left(x^2-\frac{1}{x^2}\right)^2$
$2\left(x^2-\frac{1}{x^2}\right)-6$	$-12\left(x^2-\frac{1}{x^2}\right)+36$ $\mp 12\left(x^2-\frac{1}{x^2}\right)\pm 36$ $\times$

Required square root

$$\sqrt{\left(x^2-\frac{1}{x^2}\right)^2-12\left(x^2-\frac{1}{x^2}\right)+36}=\pm\left(x^2-\frac{1}{x^2}-6\right)$$

Q3. For what value of k the expression

$4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ will become perfect square.

Sol: Given $4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$

$$2x^2+8+\frac{8}{x^2}$$

$2x^2$	$4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 4x^4$
$4x^2+8$	$+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 32x^2\pm 64$
$4x^2+16+\frac{8}{x^2}$	$32+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 32\pm\frac{128}{x^2}\pm\frac{64}{x^4}$

$$\frac{k}{x^4}-\frac{64}{x^4}=0$$

$$\Rightarrow\frac{k}{x^4}=\frac{64}{x^4}$$

$$\Rightarrow k=64$$

Q3. $4x^4-12x^3+17x^2-13x+6$ will become perfect square

i). What should be added to

ii). What should be subtracted to

iii). For what value of x the expression

Solution: $2x^2-3x+2$

$2x^2$	$4x^4-12x^3+17x^2-13x+6$ $\pm 4x^4$
$4x^2-3x$	$-12x^3+17x^2-13x+6$ $\mp 12x^3\pm 9x^2$
$4x^2-6x+2$	$8x^2-13x+6$ $\pm 8x^2\mp 12x\pm 4$

$$-x+2=0$$

To make the given expression a perfect square, remainder should be zero.

Hence

i). $x-2$ should be added

ii). $-x+2$ should be subtracted

iii). For value $-x+2=0$

$$-x=-2$$

$$x=2$$

Q4. What should be subtracted and added to the expression $-4x^3+10x+7$ so that the expression is made perfect square.

Q5: i). Find the values of l and m for which expression will become perfect square

$x^4+4x^3+16x^2+lx+m$

Sol: Given $x^4+4x^3+16x^2+lx+m$

$$x^2+2x+6$$

x^2	$x^4+4x^3+16x^2+lx+m$ $\pm x^4$
$2x^2+2x$	$4x^3+16x^2$ $\pm 4x^3\pm 4x^2$
$2x^2+4x+6$	$12x^2+lx+m$ $\pm 12x^2\pm 24x\pm 36$

$$(l-24)x+m-36=0$$

For perfect square remainder should be zero

Therefore coefficient of x must be zero

$l-24=0 \quad \Rightarrow l=24$

And the constant term also be zero

$m-36=0 \quad \Rightarrow \quad m=36$

Q5: ii). Find the values of l and m for which expression will become perfect square

$49x^4-70x^3+109x^2+lx-m$

Sol: Given $49x^4-70x^3+109x^2+lx-m$

$$
\begin{array}{r|l}
 & 7x^2-5x+6 \\
\hline
7x^2 & 49x^4-70x^3+109x^2+lx-m \\
 & \pm 49x^4 \\
\hline
14x^2-5x & -70x^3+109x^2 \\
 & \mp 70x^3 \pm 25x^2 \\
\hline
14x^2-10x+6 & 84x^2+lx-m \\
 & \pm 84x^2 \mp 60x \pm 36 \\
\hline
 & (l+60)x-m-36=0
\end{array}
$$

For perfect square remainder should be zero

Therefore coefficient of x must be zero

$l+60=0 \quad \Rightarrow l=-60$

And the constant term also be zero

$-m-36=0 \quad \Rightarrow \quad m=-36$

Review Exercise 6

Q1. Select the correct answer.

i). HCF of a^3-8b^3 and $a^2-4ab+b^2$ is

a). $a-2b$ b). $a^2+2ab+b^2$

c). $a+2b$ d). $(a+2b)^2$

ii). LCM of $(2x+3y)^5$ and $(2x+3y)^3$ is

a). $2x+3y$ b). $(2x+3y)^3$

c). $(2x+3y)^2$ d). $(2x+3y)^5$

iii). HCF of a^3-b^3 and a^2+ab+b^2 is

a). $a+b$ b). a^2+ab+b^2

c). $a-b$ d). $(a-b)^2$

iv). LCM of $(a-b)^4$ and $(a-b)^3$ is

a). $(a-b)$ b). $(a-b)^4$

c). $(a-b)^3$ d). $(a-b)^7$

v). Reduce to lowest term $\dfrac{10(x+3)(x-2)}{15(x-2)}$

a). $\dfrac{2(x+3)}{3}$ b). $\dfrac{10(x+3)}{15}$

c). $2x$ d). $2(x+3)$

vi). Simplified form of $\frac{b}{25a^2-b^2}-\frac{1}{5a-b}$ is

a). $\frac{5a}{25a^2-b^2}$ b). $\frac{+5a}{5a-b}$

c). $\frac{-5a}{5a+b}$ d). $\frac{-5a}{25a^2-b^2}$

vii). $\dfrac{5}{x^2-x-2}+\dfrac{3}{x^2+4x+3}=$

a). $\dfrac{8x+21}{(x-1)(x+2)(x+3)}$

b). $\dfrac{8x-3}{(x+1)(x-2)(x+3)}$

c). $\dfrac{8x+6}{(x-1)(x+2)(x+3)}$

d). $\dfrac{8x+9}{(x-1)(x+2)(x+3)}$

viii). $\dfrac{x^2-2x-3}{3x^2+x-2}$

a). $\dfrac{x-3}{3x-2}$ b). $\dfrac{-2x-3}{3x-2}$

c). $\dfrac{-2x-2}{x+1}$ d). $\dfrac{x-3}{3x+2}$

ix). $LCM=$

a). $\frac{HCF}{A\times B}$ b). $\frac{A\times B}{HCF}$

c). $\frac{A}{HCF}$ d). $\frac{B}{HCF}$

x). LCM of a^2-a+1 and a^3+1

a). $a+1$ b). a^2-a+1

c). a^3+1 d). a^2+a+1

Q2: i). Simplify $\dfrac{5}{2s+4}-\dfrac{3}{s^2+3s+2}+\dfrac{s}{s^2-s-2}$

Sol: Given $\dfrac{5}{2s+4}-\dfrac{3}{s^2+3s+2}+\dfrac{s}{s^2-s-2}$

$=\dfrac{5}{2s+4}-\dfrac{3}{s^2+2s+1s+2}+\dfrac{s}{s^2-2s+1s-2}$

$=\dfrac{5}{2s+4}-\dfrac{3}{s(s+2)+1(s+2)}+\dfrac{s}{s(s-2)+1(s-2)}$

$=\dfrac{5}{2(s+2)}-\dfrac{3}{(s+1)(s+2)}+\dfrac{s}{(s+1)(s-2)}$

$=\dfrac{5(s+1)(s-2)-6(s-2)+2s(s+2)}{2(s+2)(s+1)(s-2)}$

$=\dfrac{5(s^2-2s+1s-2)-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$

$=\dfrac{5s^2-10s+5s-10-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$

$=\dfrac{7s^2-7s+2}{2(s+2)(s+1)(s-2)}$

Q2: ii). $\dfrac{a}{(c-a)(a-b)}+\dfrac{b}{(a-b)(b-c)}+\dfrac{c}{(b-c)(c-a)}$

Sol: $\dfrac{a}{(c-a)(a-b)}+\dfrac{b}{(a-b)(b-c)}+\dfrac{c}{(b-c)(c-a)}$

$=\dfrac{a(b-c)+b(c-a)+c(a-b)}{(a-b)(b-c)(c-a)}$

$=\dfrac{ab-ac+bc-ab+ca-bc}{(a-b)(b-c)(c-a)}$

$=\dfrac{0}{(a-b)(b-c)(c-a)}$

$=0$

Q2: iii). Simplify $\dfrac{x^2-4}{xy^2}\cdot\dfrac{2xy}{x^2-4x+4}$

Sol: Given $\dfrac{x^2-4}{xy^2}\cdot\dfrac{2xy}{x^2-4x+4}$

$$=\frac{x^2-2^2}{xy^2}\cdot\frac{2xy}{x^2-2.x.2+2^2}$$

$$=\frac{(x-2)(x+2)}{xy.y}\cdot\frac{2xy}{(x-2)^2}$$

$$=\frac{2(x+2)}{y(x-2)}$$

Q2: iv). $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$

Solution: We have $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2)^2-(b^2)^2}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+b^2)}{(a^2-b^2)(a^2+b^2)}$$

$$=\frac{(a-b)}{(a^2-b^2)}$$

$$=\frac{a-b}{(a-b)(a+b)}$$

$$=\frac{1}{a+b}$$

Q3: Find LCM of $x^3-6x^2+11x-6$ & x^3-4x+3

Sol: Given $x^3-6x^2+11x-6$ and x^3-4x+3

$$\begin{array}{r|l} & \quad 1 \\ x^3-4x+3 & x^3-6x^2+11x-6 \\ & x^3 \quad\quad \mp 4x \pm 3 \\ \hline -3 & -6x^2+15x-9 \\ \hline & 2x^2-5x+3 \end{array}$$

$$\begin{array}{r|l} & \quad\quad x \\ 2x^2-5x+3 & x^3 \quad\quad -4x+3 \\ & \times 2 \\ \hline & 2x^3 \quad\quad -8x+6 \\ & \pm 2x^3 \mp 5x^2 \pm 3x \\ \hline & 5x^2-11x+6 \\ & \times 2 \\ \hline & 10x^2-22x+12 \\ & \pm 10x^2 \mp 25x \pm 15 \\ \hline 3 & 3x-3 \\ \hline & x-1 \end{array}$$

$$\begin{array}{r|l} & 2x+6 \\ x-1 & 2x^2-5x+3 \\ & 2x^2 \mp 2x \\ \hline & -3x+3 \\ & \mp 3x \pm 3 \\ \hline \end{array}$$

HCF = $x-1$

$$L=\frac{A\times B}{H}$$

$$L=\frac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$$

Divide any polynomial by HCF

$$\begin{array}{r|l} & x^2+x-3 \\ x-1 & x^3 \quad\quad -4x+3 \\ & x^3 \mp x^2 \\ \hline & x^2-4x+3 \\ & \mp x^2 \mp x \\ \hline & -3x+3 \\ & \mp 3x \pm 3 \\ \hline \end{array}$$

$$\therefore L=(x^3-6x^2+11x-6)(x^2+x-3)$$

Q4i): Find square root of $4x^2-12x+9$

Sol: Given $4x^2-12x+9$

$$=(2x)^2-2(2x)(3)+(3)^2$$

$$=(2x-3)^2$$

Taking Square root on both sides

$$\sqrt{4x^2-12x+9}=\pm(2x-3)$$

Q4ii): Find square root of

$x^4+4x^3+6x^2+4x+1$

Sol: Given $x^4+4x^3+6x^2+4x+1$ using division method

$$\begin{array}{r|l} & x^2+2x+1 \\ \hline x^2 & x^4+4x^3+6x^2+4x+1 \\ & \pm x^4 \\ \hline & 4x^3+6x^2 \\ & \pm 4x^3 \pm 4x^2 \\ \hline 2x^2+4x+1 & 2x^2+4x+1 \\ & \pm 2x^2 \pm 4x \pm 1 \\ \hline \end{array}$$

$$\therefore\sqrt{x^4+4x^3+6x^2+4x+1}=\pm(x^2+2x+1)$$

Q5: $\frac{x^3-y^3}{x^3+z^3}\times\frac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4}\times\frac{x^3+y^3}{x^2-y^2}$

Sol: $\frac{x^3-y^3}{x^3+z^3}\times\frac{x^2+xy+xz+yz}{x^4+x^2y^2+y^4}\times\frac{x^3+y^3}{x^2-y^2}$

$$=\frac{(x-y)(x^2+xy+y^2)}{(x+z)(x^2-xz+z^2)}\times\frac{x(x+y)+z(x+y)}{x^4+y^4+x^2y^2}\times\frac{(x+y)(x^2-xy+y^2)}{(x+y)(x-y)}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{(x+z)(x^2-xz+z^2)}\times\frac{(x+y)(x+z)}{(x^2)^2+(y^2)^2+x^2y^2}\times\frac{(x+y)(x^2-xy+y^2)}{(x+y)(x-y)}$$

$$=\frac{(x+y)(x^2+xy+y^2)(x^2-xy+y^2)}{(x^2-xz+z^2)\{(x^2)^2+(y^2)^2+2x^2y^2-2x^2y^2+x^2y^2\}}$$

$$=\frac{(x+y)(x^2+xy+y^2)(x^2-xy+y^2)}{(x^2-xz+z^2)\{(x^2+y^2)^2-x^2y^2\}}$$

$$=\frac{(x+y)(x^2+xy+y^2)(x^2-xy+y^2)}{(x^2-xz+z^2)\{(x^2+y^2)^2-(xy)^2\}}$$

$$=\frac{(x+y)(x^2+xy+y^2)(x^2-xy+y^2)}{(x^2-xz+z^2)(x^2+y^2+xy)(x^2+y^2-xy)}$$

$$=\frac{(x+y)}{(x^2-xz+z^2)}$$