☆ Apparent Power and Power Factor:-

→ we know that voltage and current at the terminals of circuit are : $v(t) = V_m \cos(\omega t + \theta_v)$, $i(t) = I_m \cos(\omega t + \theta_i)$

or in phasor form:

$V = V_m \angle \theta_v$ and $I = I_m \angle \theta_i$. the average power is:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

or the rms power is.

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

$$P = S \cos(\theta_v - \theta_i).$$

where, $\boxed{S = V_{rms} I_{rms}}$

This S is called apparent power.

The factor $\cos(\theta_v - \theta_i)$ is called power factor.

* So, "The apparent power (in VA) is the product of the rms values of voltage and current".
* "Power factor is dimensionless, since it is the ratio of average power to the apparent power,

$$\boxed{pf = \frac{P}{S} = \cos(\theta_v - \theta_i)}$$

* The angle $\theta_v - \theta_i$ is called the "power factor angle".

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

Since, $V_{rms} = \frac{V}{\sqrt{2}} = V_{rms} \angle \theta_v$

and $I_{rms} = \frac{I}{\sqrt{2}} = I_{rm} \angle \theta_i$

$V = V_m \angle \theta_v$
$V = \sqrt{2}\, V_{rms} \angle \theta_v$
$\frac{V}{\sqrt{2}} = V_{rms} \angle \theta_v$
$V_{rms} = V_{rms} \angle \theta_v$

So the impedance is:

$$Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$$

* Power factor ranges b/w 0 and 1.

<u>PF = 1</u>: for purely resistive load

Current and voltage are in phase, i.e $\theta_v - \theta_i = 0$

→ This means that $P = S$

ave. power = apparent power.

<u>PF = 0</u>: for purely reactive load.

$\theta_v - \theta_i = \pm 90°$ and $P = 0$

* when current leads voltage which means capacitive load. (leading PF)
* when voltage leads current which means inductive load (lagging PF)

<u>Exp 11.9</u>: $i(t) = 4\cos(100\pi t + 10°)A$, $v(t) = 120\cos(100\pi t - 20°)V$.

Find S and PF of load

<u>Sol</u>: The apparent power $S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240VA$

The power factor $PF = \cos(\theta_v - \theta_i)$

$= \cos(-20° - 10°) = 0.866$ (leading).

PF is leading as current is leading voltage.

Alternatively to find PF.

$$Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i = \frac{\cancel{120}^{30}/\cancel{\sqrt{2}}}{4/\cancel{\sqrt{2}}} \angle -20° - 10°$$

$$Z = 30 \angle -30° = 25.98 - j15\ \Omega.$$

$PF = \cancel{\cos(\theta_v - \theta_i)} = \cos(-30°) = 0.866$ — leading.

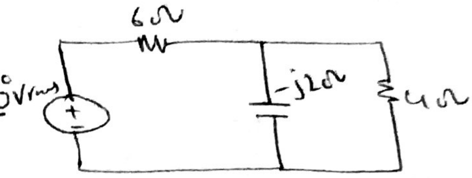
From Z we have $R = 25.98$ and $X_c = -15$

$$X_c = -15 = -\frac{1}{\omega C}$$

or $$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu F$$

Exp 11.10: PF = ? P = ?

Total impedance:



$$Z = 6 + 4 \| (-j2) = 6 + \frac{-j2 \times 4}{4 - j2}$$

$$= 6.8 - j1.6 = 7 \angle -13.24° \,\Omega.$$

$$Z = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$$

The PF is:

$\cancel{PF = \cos(0° +)}$ $PF = \cos(-13.24)$

$PF = 0.9734$ (leading)

Since impedance is capacitive. The rms value of current is:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{30 \angle 0°}{7 \angle -13.24°} = 4.286 \angle 13.24° \text{ A}$$

The average power supplied by sources is:

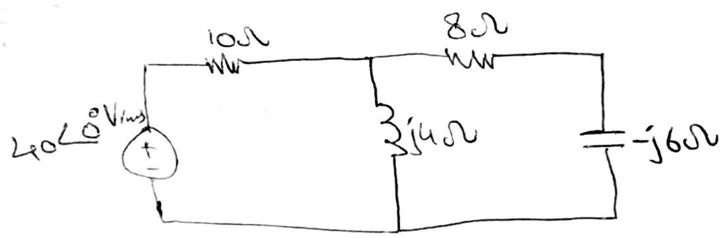
$P = V_{rms} I_{rms} Pf = (3)(4.286)(0.9734).$

$P = 125 \,W$

or $P = I_{rms}^2 R = (4.2862)^2 (6.8) = 125 W.$

PP 11.10: Calculate the power factor? and avg. power P = ?

Slide



Total impedance as seen by source is:

$$Z = 10 + j4 \| (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2}$$

$Z = 12.69 \angle 20.62°$

The $PF = \cos(20.62°) = 0.936$ (lagging).

means current is lagging and load is inductive.

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40 \angle 0°}{12.69 \angle 20.62°} = 3.152 \angle -20.62$$

Average Power is: $P = I_{rms}^2 R = (3.152)^2 (11.87) = 118 W$

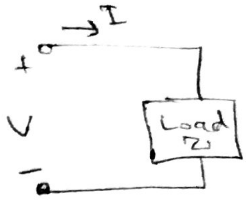
where R is resistive part of Z.

or $P = V_{rms} I_{rms} Pf = (40)(3.152)(0.936) = 118 W.$

⟹ Complex Power :-

* is important in power analysis because it contains all the information pertaining to the power absorbed by a given load.

* Consider an ac load ⟶



Phasor form of V and I are:

$$V = V_m \angle \theta_v, \quad I = I_m \angle \theta_i$$

* So the complex power absorbed by ac load is: the product of the voltage and complex conjugate of the current, or.

$$S = \frac{1}{2} V I^*$$

In terms of rms,

$$S = V_{rms} I_{rms}^*$$

where,

$$V_{rms} = \frac{V}{\sqrt{2}} = V_{rms} \angle \theta_v$$

$$I_{rms} = \frac{I}{\sqrt{2}} = I_{rms} \angle \theta_i$$

$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

Thus we may write 'S' as:

$$S = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

From above we can say that magnitude of complex power is apparent power. Also angle of complex power is the power factor angle.

* Complex power in terms of load impedance Z.

$$Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$$

Thus. $V_{rms} = Z\, I_{rms}$ and Put it into 'S'.

$$S = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*} = V_{rms} I_{rms}^*$$

$$\boxed{\begin{aligned} S &= V_{rms} I_{rms}^* \\ Z &= \frac{V_{rms}}{I_{rms}} \\ S &= V_{rms} \cdot \frac{V_{rms}}{Z^*} \\ S &= \frac{V_{rms}^2}{Z^*} \end{aligned}}$$

Since $Z = R + jX$, so, we have

$$S = I^2_{rms}(R + jX) = P + jQ$$

where P and Q are real and imaginary parts of the complex power i.e.

$P = Re(S) = I^2_{rms}R$ (is avg/real power, depends on load R)

$Q = Im(S) = I^2_{rms}X$. (is reactive power, depends on load reactance X).

We can also write:

$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$, $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

P is in watts. deliver to load. and is useful power.

Q " " VAR energy exchange b/w source & reactive part of load.

Remember! If: $Q = 0$ for resistive loads (Pf unity)

$Q < 0$ " capacitive " (leading pf)

$Q > 0$ " inductive " (lagging pf).

* "Complex power (in VA) is the product of the rms voltage phasor and rms current phasor complex conjugate. As a complex quantity, it's real part is real power P and it's imaginary part is reactive power Q."

$$\text{Complex Power} = S = P + jQ = V_{rms}(I_{rms})^*$$
$$= |V_{rms}||I_{rms}| \angle \theta_v - \theta_i$$

$$\text{Apparent Power} = S = |S| = |V_{rms}||I_{rms}| = \sqrt{P^2 + Q^2}$$

$$\text{Real } " = P = Re(S) = S\cos(\theta_v - \theta_i)$$

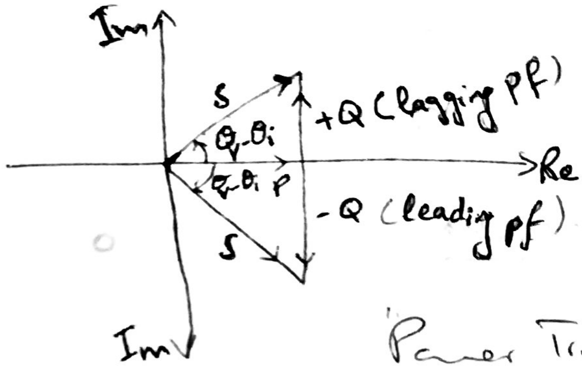
$$\text{Reactive } " = Q = Im(S) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

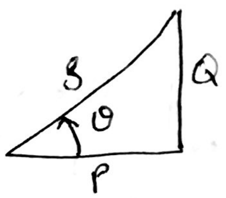
* Power Triangle :- The Complex power S contains all the information power of a load. It is

- Real ~~pow~~ power is P
- imaginary " " Q
- magnitude " apparent power S.
- phase angle is cosine of it's power factor.

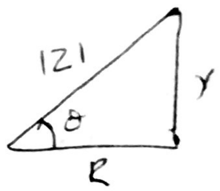
* Power Triangle has 4 items:
  - apparent power / complex power
  - real power
  - reactive "
  - Pf angle.



"Power Triangle"



"Power Triangle"



"Impedance Triangle"

* when 'S' lies in first quadrant → Inductive load and Pf is lagging.
* when 'S' lies in 4th " → capacitive " and Pf is leading.

Exp 11.11:- $v(t) = 60\cos(\omega t - 10^\circ)\,V$, $i(t) = 1.5\cos(\omega t + 50^\circ)\,A$ (1)

(a) Find complex and apparent powers

(b) Real and active "

(c) PF and load impedance

Soln: (a) First we solve for rms value of V and I. i.e.

$V_{rms} = \frac{60}{\sqrt{2}} \angle -10^\circ$, $I_{rms} = \frac{1.5}{\sqrt{2}} \angle +50^\circ$

The Complex power is:

$S = V_{rms} I_{rms}^* = \left(\frac{60}{\sqrt{2}} \angle -10^\circ\right)\left(\frac{1.5}{\sqrt{2}} \angle -50^\circ\right) = 45 \angle -60^\circ \; VA$

The Apparent power S

$S = |S| = 45 \; VA$

(b) We can express Complex power into rectangular form as:

$S = 45 \angle -60^\circ$

$= 45\left[\cos(-60^\circ) + j\sin(-60^\circ)\right] = 22.5 - j\,38.97$

Since, $S = P + Qj$, so the real power is: $P = 22.5\,W$

and reactive power is $Q = -38.97$ VAR.

(c) The PF is: $Pf = \cos(-60^\circ) = 0.5$ (leading)

It is leading because reactive power is negative.

The load impedance is:

$Z = \frac{V}{I} = \frac{60 \angle -10^\circ}{1.5 \angle 50^\circ} = 40 \angle -60^\circ \; \Omega.$

which is capacitive impedance.

Exp 11.2 $Pf = 0.856$, $S = 12{,}000$ VA Find ave. and reactive power, peak current and load impedance

Soln:

$Pf = \cos\theta \Rightarrow \cos\theta = 0.856 \Rightarrow \theta = \cos^{-1} 0.856 = 31.13°$.

(a) Apparent power $= S = 12{,}000$ VA,

Average / Real power $P = S\cos\theta = (12{,}000)\cos(31.13°)$

$P = 10.272$ kW.

Reactive Power $Q = S\sin\theta = (12{,}000)\sin(31.13°)$

$Q = 6.204$ kVA

(b) Since Pf is lagging, the Complex Power is

$S = P + Qj = 10.272 + j6.204$ kVA

From $S = V_{rms} I^*_{rms}$ we get,

$I^*_{rms} = \dfrac{S}{V_{rms}} = \dfrac{10.272 + j6204}{120\angle 0°} = 85.6 + j51.7$ A

$I^*_{rms} = 100\angle 31.13°$ A

Thus $I_{rms} = 100\angle -31.13°$ and the peak current is: $I_m = \sqrt{2}\, I_{rms} = \sqrt{2}\, 100 = 141.4$ A

(c) Load Impedance.

$Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{120\angle 0°}{100\angle -31.13°} = 1.2\angle 31.13°\ \Omega$.

which is inductive impedance.

Chapter 11 Problems: 11.7, 11.17, 11.20, 11.57, 11.1, 11.2, 11.5, 11.27, 11.24, 11.31, 11.44, 11.41, 11.48, 11.49, 11.50