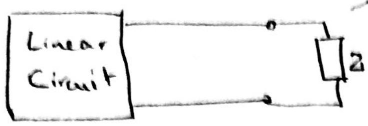
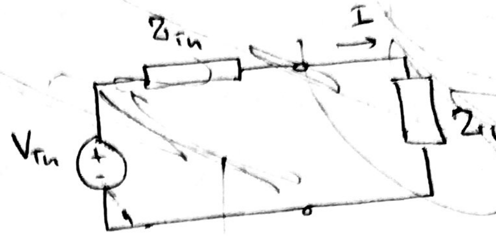
★ Max. ~~Average Power~~ Transfer:

* We studied that to deliver max. power transfer to a load we use Thevenin theorem and that $R_L = R_{Th}$.
* now we see this concept for ac circuits.
* Consider below:



Circuit with ac Load.



Thevenin equivalent circuit.

* load is represented by Z_L.
* Rectangular form of Thevenin Impedance 'Z_{Th}' and load impedance is 'Z_L'

$$Z_{Th} = R_{Th} + jX_{Th} \quad , \quad Z_L = R_L + jX_L$$

* The current through load is:

$$I = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{V_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)}$$

* The average power delivered to load is:

$$P = \frac{1}{2}|I|^2 R_L$$

$$Z = (R_{Th}+R_L) + j(X_{Th}+X_L)$$
$$|Z| = \sqrt{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$
$$|Z|^2 = (R_{Th}+R_L)^2 + (X_{Th}+X_L)^2$$

$$P = \frac{R_L}{2} \cdot \frac{|V_{Th}|^2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2} \quad \text{— (1)}$$

* Objective is to adjust load parameters R_L and X_L to deliver max. P. So.
* So we take $\partial P / \partial X_L$ and set it to zero.

$$\frac{\partial P}{\partial X_L} = \frac{|V_{Th}|^2}{2} \frac{\partial}{\partial R_L}\left(\frac{R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}\right)$$

$$= \frac{|V_{Th}|^2}{2}\left(\frac{\left((R_{Th}+R_L)^2 + (X_{Th}+X_L)^2\right)(0) - (R_L)\left(2(0) + 2(X_{Th}+X_L)(1+0)\right)}{\left((R_{Th}+R_L)^2 + (X_{Th}+X_L)^2\right)^2}\right)$$

$$= \frac{|V_{Th}|^2}{\cancel{2}}\left(-\frac{R_L(\cancel{2}X_{Th}+\cancel{2}X_L)}{((R_{Th}+R_L)^2+(X_{Th}+X_L)^2)^2}\right)$$

$$= \frac{-|V_{Th}|^2 R_L(X_{Th}+X_L)}{((R_{Th}+R_L)^2+(X_{Th}+X_L)^2)^2} \quad \text{when set} = 0$$

So $\boxed{X_{Th} = -X_L}$ —— (a)

* Solving $\partial P/\partial R_L$ is:

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2}{2}\frac{\partial}{\partial R_L}\left(\frac{R_L}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2}\right)$$

$$= \frac{|V_{Th}|^2}{2}\left(\frac{((R_{Th}+R_L)^2+(X_{Th}+X_L)^2 - R_L(2(R_{Th}+R_L)(0+1)+(0)))}{((R_{Th}+R_L)^2+(X_{Th}+X_L)^2)^2}\right)$$

$$= \frac{|V_{Th}|^2}{2}\left(\frac{R_{Th}^2+R_L^2+\cancel{2R_{Th}R_L}+X_{Th}^2+X_L^2+2X_{Th}X_L-\cancel{2R_LR_{Th}}-2R_L^2)}{((R_{Th}+R_L)^2+(X_{Th}+X_L)^2)^2}\right)$$

$$= \frac{|V_{Th}|^2}{2}\left(\frac{R_{Th}^2-R_L^2+(X_{Th}+X_L)^2)}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2)^2}\right) \quad \text{we set it} = 0$$

$$\boxed{R_L = \sqrt{R_{Th}^2+(X_{Th}+X_L)^2}} \text{ —— (b)}$$

* From (a) and (b) we conclude for max. Power transfer that Z_L must be selected so that $X_L = -X_{Th}$ and $R_L = R_{Th}$ i.e..

$$\boxed{Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*}$$

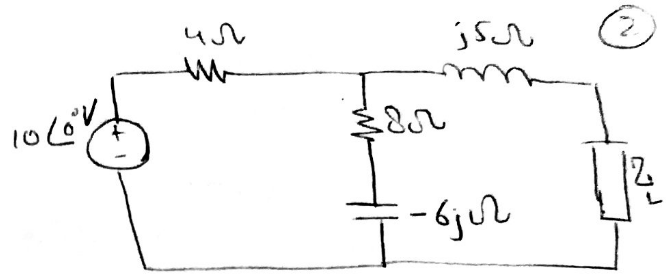
* So if we put $R_L = R_{Th}$ and $X_L = -X_{Th}$ in eq (1):

$$\boxed{P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}} ✓$$

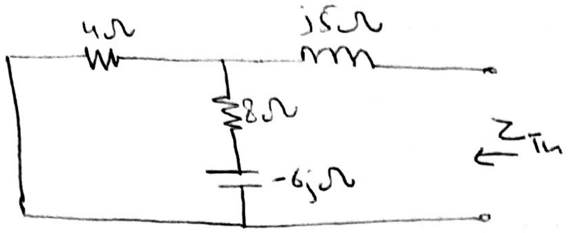
* Incase when load is purely real i.e $X_L = 0$ and eq (b) becomes:

$$\boxed{R_L = \sqrt{R_{Th}^2+X_{Th}^2} = |Z_{Th}|} ✓$$ for resistive load. $R_L = |Z_{Th}|$

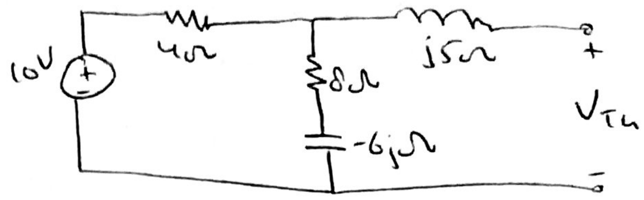
Exp - 11.5 : Find Z_L and P_{max}. ②



Solution: First of all we find Z_{Th} and V_{TH}.



"for Z_{TH}"



"for V_{Th}"

Z_{Th}:

$$Z_{Th} = j5 + 4 \| (8 - j6)$$

$$= j5 + \frac{4(8-j6)}{4+8-j6} = j5 + \frac{32-24j}{12-j6} = 2.933 + j4.467\,\Omega$$

V_{Th}:

$$V_{Th} = 10\left(\frac{8-j6}{4+8-j6}\right) \text{ by voltage division}$$

$\because$ $V_L = \frac{R_L}{R_{Th}+R_L} V_{Th}$

$V_{Th} = V_L\left(\frac{R_{Th}+R_L}{R_L}\right)$

$$= \left(\frac{8-j6}{12-j6}\right)10$$

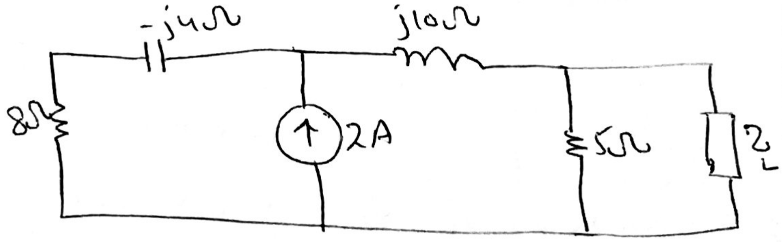
$$= \frac{80-j60}{12-j6} = 7.454\angle{-10.3^\circ}\ V$$

∴ As per conditions that for max. power transfer. $Z_L = Z_{Th}^*$ So $Z_L = 2.933 - j4.467\,\Omega$

To find max. power transfer we use formula:

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\ W.$$

PP. 11.5 slide : Find Z_L and P_{max}:



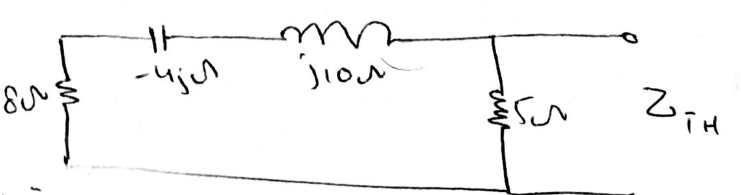
Soln: First Z_{TH} and V_{TH}

Z_{TH}:

$$Z_{TH} = 5 \| (8 - 4j + 10j)$$

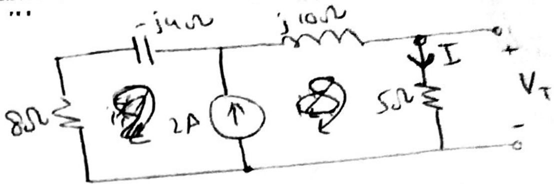
$$= 5 \| (8 + j6)$$

$$= \frac{(5)(8+j6)}{5+8+j6} = \frac{40+30j}{13+j6} = 3.415 + j0.7317$$



For max Power transfer: $Z_L = Z_{TH}^* = 3.415 - j0.7317$

V_{TH}: ~~Apply mesh analysis to loop 1.~~



~~$-2(8)+2(j4)=0$~~

~~$-2(8)+(-2+i_2)j4=0$~~

~~$-16-j8-i_2 j4=0$~~

~~$i_1 8\Omega - i_1 j4\Omega + i_2 10\Omega + i_2 5\Omega = 0 \Rightarrow$~~ ~~$13+6j=0$~~

~~$i_1(8-j4)+i_2(5+j10)=0$~~ $\qquad i_2 = i_1+2$

$i_1(8+j4)+(i_1+2)(5+j10)=0$

~~$8i_1+i_1 j4+5i_1+i_1 j10+10+20j=0$~~

~~$10+13i_1+i_1 j14+20j=0 \Rightarrow i_1(13+j14)=-10-20j$~~

$$i_1 = \frac{-10-20j}{13+j14}$$

Using current division $\qquad i_1 = \frac{R_2}{R_1+R_1} i$

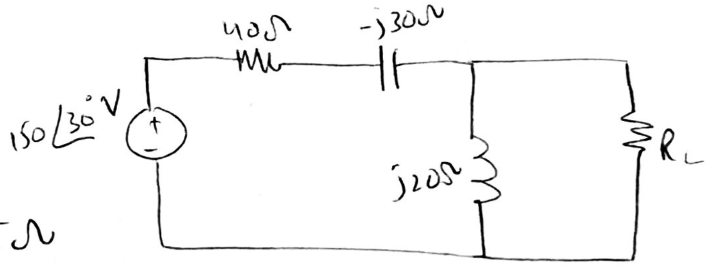
$$I = \frac{8-j4}{8-j4+j10+5}(2) = \frac{16-j8}{13+j6}$$

$$V_{TH} = 5I = 5\left(\frac{16-j8}{13+j6}\right) = 6.25\angle -51.34^\circ$$

P_{max}:

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}} = \frac{(6.25)^2}{8(3.415)} = 1.429\,W$$

Exp: 11.6 Find R_L and P_{max}.



Sol: Z_{TH}: $Z_{TH} = (40-j30)\parallel j20$

$$= \frac{j20(40-j30)}{j20+40-j30} = 9.412+j22.35\,\Omega$$

V_{TH}: By voltage division.

$$V_{TH} = \frac{j20}{j20+40-j30}(150\angle 30^\circ) = 72.76\angle 134^\circ\ V$$

purely resistive load.

Value of R_L that will absorb max ~~Pow~~ average power is:

$$R_L = |Z_{TH}| = \sqrt{(9.412)^2+(22.35)^2} = 24.25\,\Omega \qquad R_L = \sqrt{R_{TH}^2 + X_{TH}^2}$$

The current through load is:

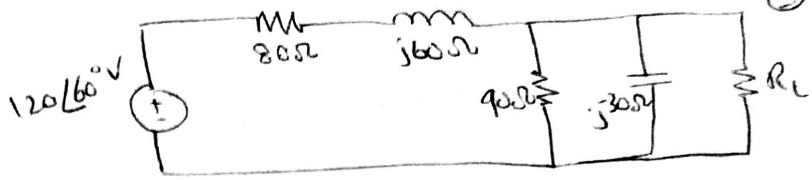
$$I = \frac{V_{TH}}{Z_{TH}+R_L} = \frac{72.76\angle 134^\circ}{33.66+j22.35} = 1.8\angle 100.42^\circ\ A$$

The max. average power absorbed by R_L is:

$$P_{max} = \frac{1}{2}|I^2|R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\,W$$

PP. 11.6: Find R_L and P_{max}

Slide Book



Sol: $Z_{Th} = ?$

$$Z_1 = 80 + j60 \quad \text{and} \quad Z_2 = 90 \,\|\, -j30 = \frac{(90)(-j30)}{90 - j30} = 9 - j27$$

$$Z_{Th} = Z_1 \| Z_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27} = 17.181 - j24.57\ \Omega$$

$V_{Th} = ?$

$$V_{Th} = \frac{Z_2}{Z_1 + Z_2}(120\angle 60^\circ) = \frac{9 - j27}{89 + j33}(120\angle 60^\circ)$$

$$V_{Th} = 35.98\angle -31.91^\circ$$

$$R_L = |Z_{Th}| = \sqrt{(17.181)^2 + (24.57)^2} = 30\ \Omega$$

The current through load is:

$$I = \frac{V_{Th}}{Z_{Th} + R_L} = \frac{35.98\angle -31.91^\circ}{30 + 17.181 - j24.57} = \frac{35.98\angle -31.91^\circ}{47.181 - j24.57}$$

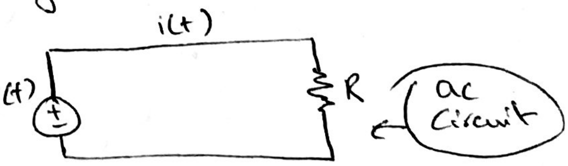
$$I = 0.6764\angle -4.4^\circ\ A$$

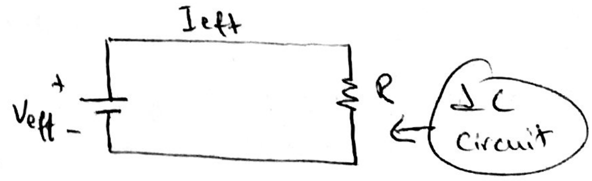
The max. average power absorbed by R_L is:

$$P_{max} = \frac{1}{2}|I|^2 R_L = \frac{1}{2}(0.6764)^2(30) = 6.863\ W.$$

Effective Value or RMS Value:-

* We find effective value to measure the effectiveness of voltage or current source in delivering power to the a resistive load.
* "Effective value of a periodic signal is it's root mean square (rms) value".
* Find I_{eff} ?





* The average power absorbed by resistor in ac circuit is:

$$P = \frac{1}{T}\int_0^T i^2 R\,dt \qquad \because \text{average Power} \Rightarrow P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{R}{T}\int_0^T i^2\,dt \quad \text{— (a)}$$

* Power absorbed by resistor in dc circuit.

$$P = I_{eff}^2 R \quad \text{— (b)}$$

* Equating both ⓐ and ⓑ

$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}}$$

* Similarly V_{eff} can be found as:

$$\boxed{V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt}}$$

* above means that "eff" value is the square root of the average of the periodic signal. So we can write:

$I_{eff} = I_{rms}$, $V_{eff} = V_{rms}$

* Suppose sinusoidal signal $i(t) = I_m \cos\omega t$, so it's I_{rms}:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2\omega t\, dt}$$

"Double angle identity"

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)\, dt} \; * \qquad \because \cos^2(x) = \frac{1}{2}\left[1+\cos(2x)\right]$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \quad (\text{for sinusoidal signals})$$

Similarly for $v(t) = V_m \cos\omega t$,

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad (\text{for sinusoidal signal}).$$

* So the average power can now be written as:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$\boxed{P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)}$$

* Similarly average power absorbed by resistor R is

$$\boxed{P = I_{rms}^2 R = \frac{V_{rms}^2}{R}}$$

$$* \quad = \sqrt{\frac{I_{rms}^2}{2\pi}\left[\int_0^T 1\, dt + \int_0^T \cos 2\omega t\, dt\right]}$$

$T = 2\pi$

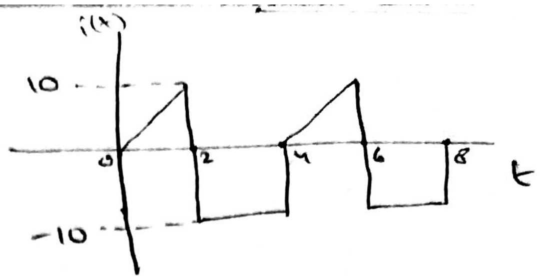
$$= \sqrt{\frac{I_{rms}^2}{2(2\pi)} t\Big|_0^{2\pi} + 0}$$

$$= \sqrt{\frac{I_{rms}^2 (2\pi)}{4\pi}}$$

$$= \frac{I_{rms}}{\sqrt{2}}$$

(9)

Exp 11.7 Find I_{rms}, $R = 2\Omega$, $P_r = ?$



Soln: $T = 4$.

Current waveform $i(t)$ is:

$$i(t) = \begin{cases} 5t & 0 < t < 2 \\ -10 & 2 < t < 4 \end{cases}$$

The rms value is:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)}$$

$$I_{rms} = 8.165\ A$$

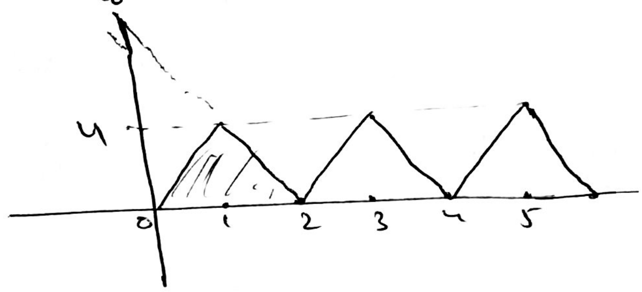
The power absorbed by resistor (2Ω) is:

$$P = I_{rms}^2 R = (8.165)^2 (2) = 133.3W.$$

Prop 11.8 Slide: Find $I_{rms} = ?$, $P_R = ?$

$R = 9\Omega$, $\boxed{T = 2}$

$$i(t) = \begin{cases} 4t & 0 < t < 1 \\ 8-4t & 1 < t < 2 \end{cases}$$

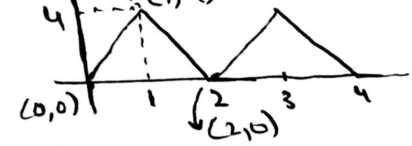


$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{2}\left[\int_0^1 (4t)^2 dt + \int_1^2 (8-4t)^2\,dt\right]}$$

$$= \sqrt{\frac{16}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (4-4t+t^2)\,dt\right]} = \sqrt{8\left(\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right)}$$

$$= \sqrt{\frac{16}{3}} = 2.309\ A$$

$$P = I_{rms}^2 R = 48\ \omega$$



→ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{4-0}{1-0} = 4$

$y = mx + c$ ⟹ (line eq. formula).

c → intersection.

$y = 4t$

→ $m = \dfrac{0-4}{2-1} = -\dfrac{4}{1} = -4$

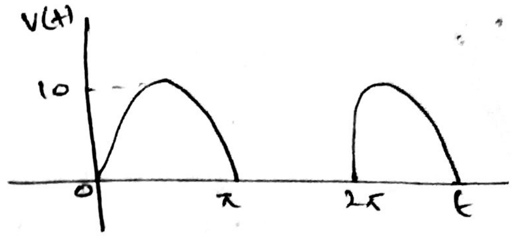
$y = mx + c = -4t + c$

and $c = 8$

So $y = -4t + 8$

$$\begin{cases} 4t & 0<t<1 \\ -4t+8 & 2<t<3 \end{cases}$$

Exp 11.8: $V_{rms} = ?$, $R = 10\,\Omega$, $P_R = ?$



$T = 2\pi$

$$v(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi}(10\sin t)^2\,dt + \int_{\pi}^{2\pi} 0^2\,dt\right]$$

As, $\sin^2 t = \frac{1}{2}(1-\cos 2t)$, Hence.

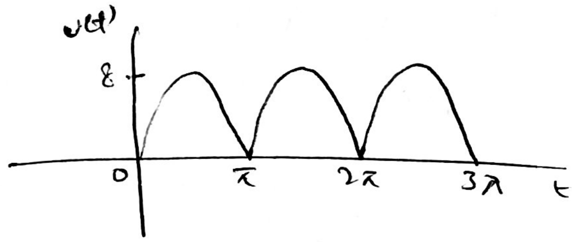
$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi}\frac{100}{2}(1-\cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Big|_0^{\pi}$$

$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25\,V$$

$V_{rms} = 5V$

and $P = \frac{V_{rms}^2}{R} = \frac{(5)^2}{10} = 2.5\,W.$

PP 11.8. $V_{rms} = ?$ $R = 6\,\Omega$. $P_R = ?$



Sol: $T = \pi$

$v(t) = 8\sin(t)$, $0 < t < \pi$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt = \frac{1}{\pi}\int_0^{\pi}(8\sin(t))^2\,dt$$

$$= \frac{64}{\pi}\int_0^{\pi}\frac{1}{2}\left[1 - \cos 2t\right]dt = 32.$$

$V_{rms} = 5.657\,V$

$$P = \frac{V_{rms}^2}{R} = \frac{32}{6} = 5.333\,W.$$