# Chapter # 11.

## ⇒ AC Power Analysis:-

★ Instantaneous & Average Power
★ Max. Average Power Transfer
★ Effective or RMS value
★ Apparent Power and Power Factor.
★ Complex Power.
★ Power Factor Correction.

### → Importance of Power:-

* Electrical power is most efficiently generated, transmitted and distributed in ac form.
* all electrical and electronic devices have certain power ratings
* most common form of electric power is 50 or 60 Hz ac power

### → Instantaneous Power:-

* "Instantaneous Power (in watts) is the power at any instant of time
* "Instantaneous power $p(t)$ absorbed by an element is the product of the instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it". i.e.

$$\boxed{p(t) = v(t)\,i(t)}$$

* Suppose the voltage and current at terminals of circuit be:

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

→ phase angles
→ amplitudes

* The instantaneous power absorbed is:

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

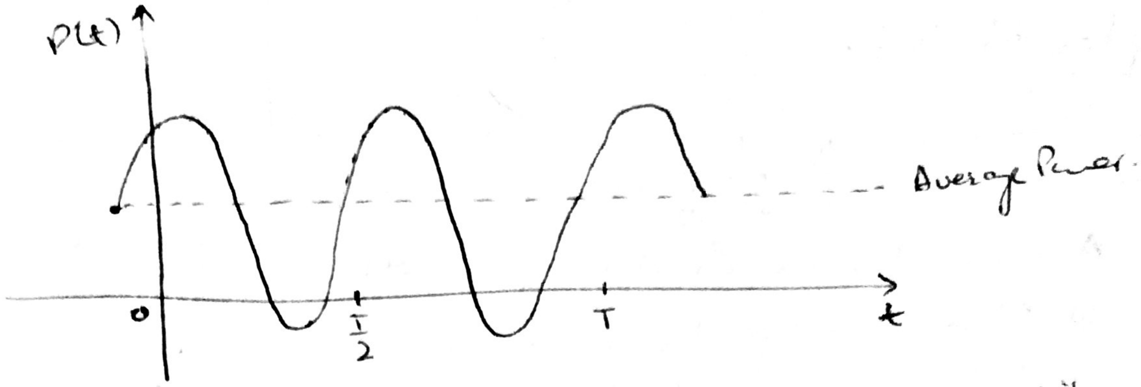
or

$$\boxed{p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$

$\because \cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$p(t) =$ constant power + sinusoidal power (freq $2\omega$)

* when $p(t) > 0$ power is absorbed by circuit
* $p(t) < 0$ " " " source.




" instantaneous power $p(t)$ entering a circuit".

* The instantaneous power changes with time, is therefore difficult to measure.

→ Average Power:-

* " The average power (in watts) is the average of instantaneous power over one period".

$$p(t) = v(t)\,i(t) \quad \text{instantaneous power}$$

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt} \quad \text{Average power.}$$

$v(t) = V_m \cos(\omega t + \theta_v)$ , $i(t) = I_m \cos(\omega t + \theta_i)$

So,
$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,\frac{1}{T}\int_0^T dt + \frac{1}{2} V_m I_m \underbrace{\frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt}_{=0}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + 0$$

So, $$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

* Now we take phasor forms of $v(t)$ and $i(t)$ as:

$v(t) = V_m \cos(\omega t + \theta_v) \Rightarrow V = V_m \angle \theta_v$

$i(t) = I_m \cos(\omega t + \theta_i) \Rightarrow I = I_m \angle \theta_i$

*

* Calculate P using phasor form of V and I as:

$$\frac{1}{2} V I^* = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$$

$$= \frac{1}{2} V_m I_m \left[\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i)\right]$$

* The real part above is same as we saw in P average power. So,

$$\boxed{P = \frac{1}{2} \mathrm{Re}[V I^*] = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}$$

* Consider two cases: $\underline{\theta_v = \theta_i}$ (volt. & current in phase)

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |I|^2 R$$

means that for pure resistive load (circuit) absorbs power all the time.

* $\theta_v - \theta_i = \pm 90°$. we have reactive load.

$$P = \frac{1}{2} V_m I_m \cos 90° = 0$$

means a pure reactive load (circuit) absorbs no average power.

__Exp 11.1:__ $v(t) = 120 \cos(377t + 45°)$ V and $i(t) = 10 \cos(377t - 10°)$ A find instantaneous and average power absorbed?.

__Soln:__ The instantaneous power is given by:

$$p = vi = 1200 \cos(377t + 45°) \cos(377t - 10°)$$

$$p = 600 \left[\cos(754t + 35°) + \cos 55°\right]$$ $\because \cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$$p = 344.2 + 600 \cos(754t + 35°) \text{ W}$$

To find average power:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(10) \cos[45° - (-10°)]$$

$$\underbrace{P = 344.2 \text{ W}}$$

is the constant part of 'p' instantaneous power.

PP 11.1 slide: $v(t) = 80\cos(10t + 20^\circ)$ , $i(t) = 15\sin(10t + 60^\circ)$

find $p(t)$ and $P$ ?

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{1}{2}(80)(15)\cos(20^\circ - 60^\circ) + \frac{1}{2}(80)(15)\cos(20t + 20^\circ + 60^\circ)$$

$$= 600\cos(-40^\circ) + 600\sin(20t + 80^\circ)$$

$$= 600\cos(-40^\circ) +$$

$$i(t) = 15\sin(10t + 60^\circ)$$

$$= 15\cos(10t - 30^\circ)$$

$$p(t) = \frac{1}{2}(80)(15)\cos(50^\circ) + \frac{1}{2}(80)(15)\cos(20t - 10^\circ)$$

$$= 600\cos(50^\circ) + 600\cos(20t - 10^\circ)$$

$$= 385.7 + 600\cos(20t - 10^\circ)\ \omega$$

Now we find instantaneous power as:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2}(80)(15)\cos(20^\circ + 30^\circ)$$

$$= 600\cos(50^\circ)$$

$P = 385.7\ \omega$ average power.

PP 11.1 Book: $v(t) = 330\cos(10t + 20^\circ)V$ , $i(t) = 33\sin(10t + 60^\circ)A$

find $p(t) = ?$ , $P = ?$

$$i(t) = 33\sin(10t + 60^\circ) = 33\cos(10t - 30^\circ)$$

$$p(t) = \frac{1}{2}(330)(33)\cos(-50^\circ) + \frac{1}{2}(330)(33)\cos(20t - 10^\circ)$$

$$= 5445\cos(-50^\circ) + 5445\cos(20t - 10^\circ)$$

$$= 3499.97 + 5445\cos(20t - 10^\circ)\ \omega.$$

Average power $P = \frac{1}{2}(330)(33)\cos(+50^\circ)$

$$= 3499.97\ \omega.$$

**Exp 11.2**: Calculate average power absorbed by an impedance $Z = 30 - j70\ \Omega$ when voltage $V = 120\angle 0°$.

Solution: The current the impedance is:

$$I = \frac{V}{Z} = \frac{120\angle 0°}{30 - j70°} = \frac{120\angle 0°}{76.16\angle -66.8°}$$

$$I = 1.576\angle 66.8°\ A$$

The average power is $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$$P = \frac{1}{2}(120)(1.576)\cos(0 - 66.8°)$$

$$P = 37.24\ W$$

**PP 11.2**: $I = 33\angle 30°$, $Z = 40\angle -22°\ \Omega$. find P.
Book

$$I = \frac{V}{Z} \Rightarrow \quad V = IZ = 33\angle 30° \cdot 40\angle -22°$$

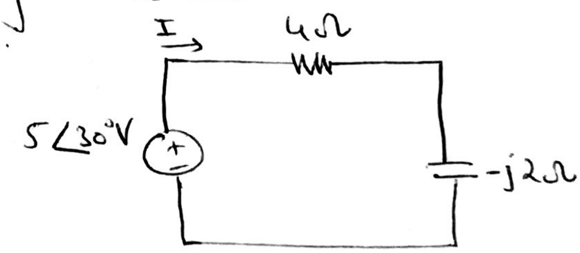
$$V = 1320\angle 8°$$

and avg. power $P = \frac{1}{2}(1320)(33)\cos(8° - 30°)$

$$= 21780\cos(-22°)$$

$$P = 20194.06\ W$$

**Exp 11.3**: Find avg. power supplied by source and average power absorbed by resistor.




Solution: Current I is given by:

$$I = \frac{5\angle 30°}{4 - j2} = \frac{5\angle 30°}{4.472\angle -26.57°}$$

$$I = 1.118\angle 56.57°\ A$$

The average power supplied by source is:

$$P = \frac{1}{2}(5)(1.118)\cos(30° - 56.57°) = 2.5\ W$$

The current through resistor is:

$$I_R = I = 1.118\angle 56.57°\ A$$

and voltage across it is

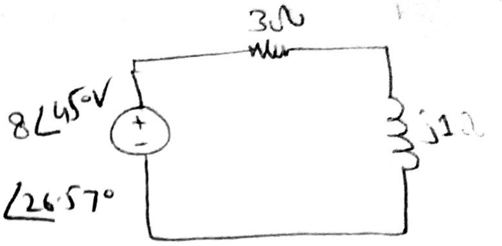
$$V_R = 4 I_R = 4.472\angle 56.57°\ V$$

The average power absorbed is

$$P = \frac{1}{2} V_m I_m = \frac{1}{2}(4.472)(1.118) = 2.5\ W$$

Same

PP11.3 slide : Find avg. power absorbed? by resistor and inductor? Find power supplied?




Solution:

$$I = \frac{V}{Z} = \frac{8\angle 45^\circ}{3+j} = \frac{8\angle 45^\circ}{3.16\angle 18.43} = 2.53\angle 26.57^\circ$$

Current $I_R$ across the resistor is:

$$I_R = I = 2.53\angle 26.57^\circ \text{ A}$$

Voltage $V_R$ across the resistor is:

$$V_R = I_R R = (2.53\angle 26.57^\circ)3$$

$$V_R = 7.59\angle 26.57^\circ$$

So power absorbed by resistor is:

$$P_R = \frac{1}{2}V_m I_m = \frac{1}{2}(7.59)(2.53) = 9.6 W$$

Now we solve for current across inductor is:

$$I_L = I = 2.53\angle 26.57^\circ \text{ A}$$

$\theta_v - \theta_i$
$26.57 - 26.57$
$= 0$

Voltage $V_L$ across the inductor is:

$$V_L = I_L j = 2.53\angle 26.57^\circ \cdot j \qquad \because j = \angle 90^\circ$$

$$V_L = 2.53\angle 26.57^\circ + 90^\circ$$

$$V_L = 2.53\angle 116.57^\circ \text{ V.}$$

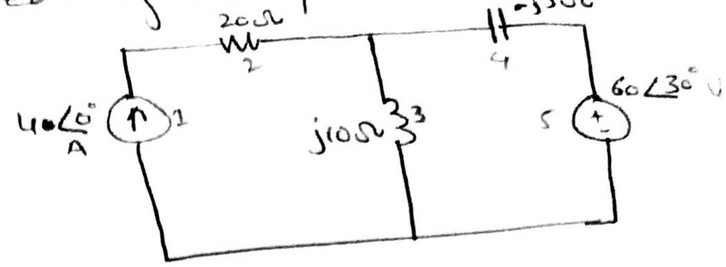
So the power absorbed by inductor is:

$$P_L = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$
$$= \frac{1}{2}(2.53)^2 \cos(90^\circ) = 0 W$$

The average power supplied is.

$$P = \frac{1}{2}(8)(2.53)\cos(45^\circ - 26.57^\circ) = 9.6 W$$

Exp 11.4 - Find average power generated by each source and the average power absorbed by each passive element.

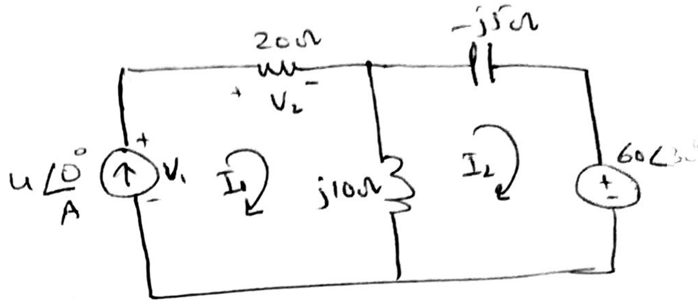



Solution:

We apply mesh analysis.

For mesh 1,

$I_1 = 4A$




For mesh 2,

$(j10 - j5)I_2 - j10 I_1 + 60\angle 30^\circ = 0$

$j5 I_2 = -60\angle 30^\circ + j10 I_1$

$j5 I_2 = -60\angle 30^\circ + j40$

$I_2 = \frac{-60\angle 30^\circ}{j5} + \frac{j40}{j5}$

$I_2 = -12\angle -60^\circ + 8 = -12(\cos(-60^\circ) + j\sin(-60^\circ)) + 8$

$= -12(0.5 + j(-0.866)) + 8 = \cancel{-6 + j0.866 + 8}$

$I_2 = -6 + 10.392j + 8 = 2 + 10.392j$

$I_2 = 10.58\angle 79.1^\circ$ A

For the voltage source, the current flowing from it is $I_2 = 10.58\angle 79.1^\circ$ A and the voltage across it is $60\angle 30^\circ$ V.

So the average power is:

$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) = 207.8 W.$

According to PSC this average power is absorbed by the source i.e circuit is delivering average power to source.

For the current source the current through it is $I_1 = 4\angle 0^\circ$ and the voltage across it is:

$V_1 = 20I_1 + j10(I_1 - I_2) = 80 + j10(4 - 2 - j10.39)$

$V_1 = 183.9 + j20 = 184.984\angle 6.21^\circ$ V.

The average power supplied by current source is:

$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0^\circ) = -367.8 W$

It is -ve because the current source is supplying power to circuit.

For the resistor, the current through it is $I_1 = 4\angle 0°$ and the voltage across resistor is:

$$V_R = I_1 R = (20)(4\angle 0°) = 80\angle 0° V.$$

average Power absorbed by resistor is:

$$P_2 = \frac{1}{2}(80)(4) = 160W$$

For the capacitor:

current $= I_2 = 10.58\angle 79.1° A$

voltage $= V_c = I_2(-j5) = (5\angle -90°)(10.58\angle 79.1°)$

$$= 52.9\angle 79.1° - 90°$$

average Power absorbed by Capacitor is:

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90°) = 0$$

For the inductor:

Current $= I_1 - I_2 = 4 - 2 - j10.39 = 2 - j10.39 A = 10.58\angle -79.1°$

Voltage $V_L = (I_1 - I_2)j10 = (10.58\angle -79.1°)(j10)$

$$V_L = 105.8\angle -79.1° + 90° \text{ volt.}$$

Average power absorbed by inductor is:

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos(90°) = 0$$

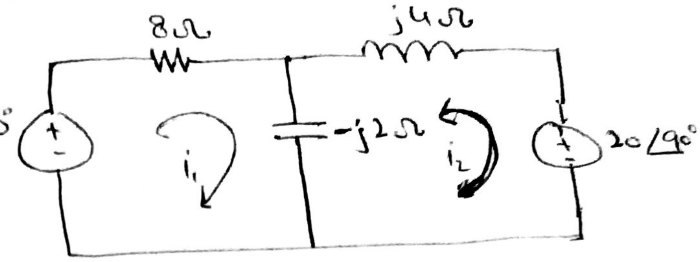
So, finally we have noticed that capacitor and inductor have absorbed 0 W average power.

Total Power supplied + Total power absorbed = 0

$$0 + 0 + -367.8 + 160 + 207.8 = 0$$

PP 11.4: Find power absorbed by each element in circuit. (5)
Book

Solution




Mesh 1:

$$-40+(8-j2)I_1+(-j2)I_2=0$$
$$-40+8I_1-j2I_1-j2I_2=0$$
$$-20+4I_1-jI_1-jI_2=0$$
$$(4-j)I_1-jI_2=20$$

Mesh 2:

$$-j20+j4I_2-j2(I_2+I_1)=0$$
$$-j20+4jI_2-j2I_2-j2I_1=0$$
$$-j20+(4j-j2)I_2-j2I_1=0$$
$$-j20+2jI_2-j2I_1=0$$
$$-j10+jI_2-jI_1=0$$
$$-jI_1+jI_2=j10$$

Convert the above eq. to matrix form.

$$\begin{bmatrix} 4-j & -j \\ -j & +j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

$$\Delta=2+j4 \quad , \quad \Delta_1=-10+j20 \quad , \quad \Delta_2=10+j60$$

$$I_1=\frac{\Delta_1}{\Delta}=5\angle 53.14^\circ \quad \text{and} \quad I_2=\frac{\Delta_2}{\Delta}=13.6\angle 17.11^\circ$$

For $40\angle 0^\circ$ source: $V_s=40\angle 0^\circ$ and $I_1=5\angle 53.14^\circ$

$$P_s=-\frac{1}{2}(40)(5)\cos(-53.14)^\circ=-60\text{w}$$

For $20\angle 90^\circ$ source: $V_s=20\angle 90^\circ$ , $I_2=13.6\angle 17.11^\circ$

$$P_s=-\frac{1}{2}(20)(13.6)\cos(90^\circ-17.11^\circ)=-40\text{w}$$

For resistor: $I = I_1 = 5 \angle 53.14^\circ$

$V = I_1 R = 5 \angle 53.14^\circ \times 8 = 40 \angle 53.14^\circ$

The average power absorbed is:

$$P = \frac{1}{2} Re|VI| = \frac{1}{2} 5 \times 40 = 100 W.$$

→ The average power absorbed by inductor and capacitor is zero.