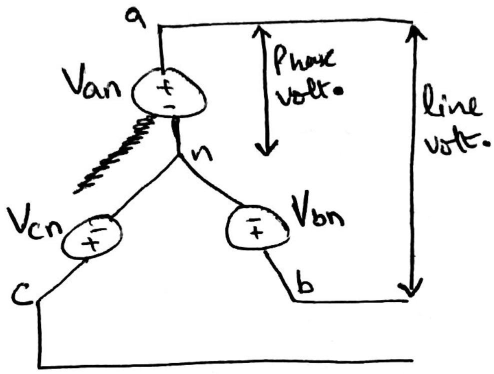
=> Three Phase Circuits:

=> Wye-Wye Balanced Connections:

⇒ key Concepts:

→ For voltage, look at the Source.



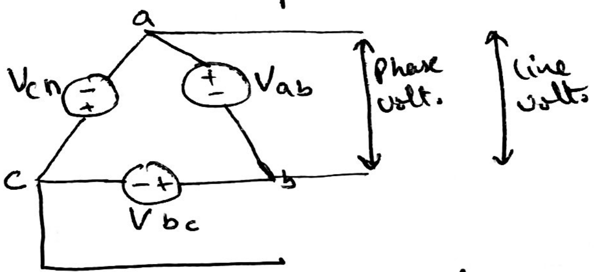
line volt.: is line to line volt. as, here b/w a and b, b and c is line volt.

Phase volt. is volt. across source, e.g a and n, c and n, b and n. are three phase voltages.

$\boxed{V_L = \sqrt{3} V_p \angle 30^\circ}$

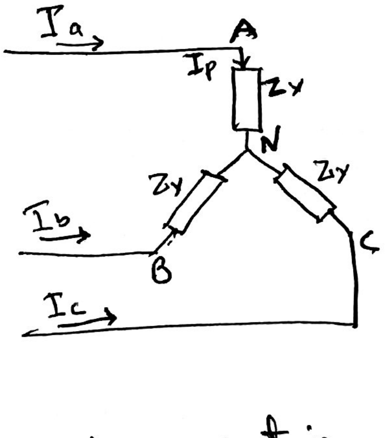
Remember V_L is $> V_p$

* So here line volt. and phase volt. are not same.



* So here line and phase volt. are same.

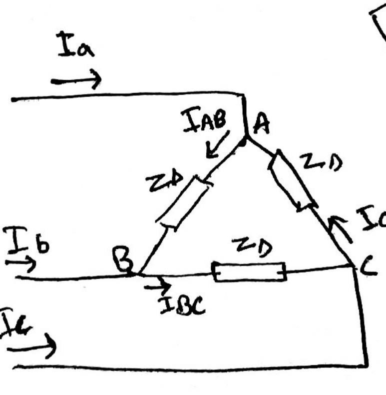
→ For currents, look at the load:



* The current through any of line is line current, e.g I_a, I_b, I_c
* The current through any of impedance is phase current.
* here line current and phase current are same. $I_L = I_p$.

Remember.

$\boxed{I_L = \sqrt{3}\, I_p \angle -30^\circ}$

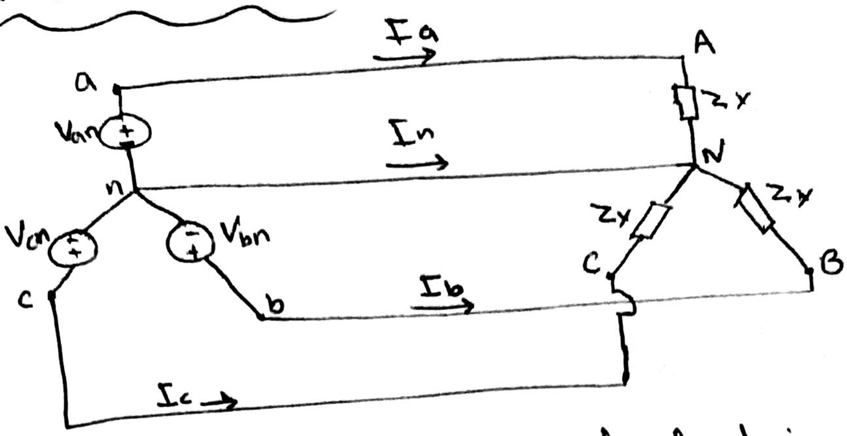


* here current is dividing into two paths so $I_L \neq I_p$

Line current:
I_a, I_b, I_c

phase current:
I_{AB}, I_{BC}, I_{CA}

⇒ Balanced Y - Y connection :-



* Here we see source and load impedance.

- Phase voltages : Van, Vbn and Vcn
- Line conductors : a → A, b → B, c → C are called LINES.

- Line voltages : Vab, Vbc, Vca.

Source side → – The phase voltage is ($V_p = 230V$) Van.
and line voltage is ($V_L = 400V$) Vab

$V_L = \sqrt{3} \times 230 = 400$

So here $V_L \neq V_p$. i.e not same.

Load side → – The Line current and phase current are same.
see the phase current : (current through anyone load Z).

$V_{an} = V_p \angle 0^\circ$, $V_{bn} = V_p \angle -120^\circ$, $V_{cn} = V_p \angle +120^\circ$

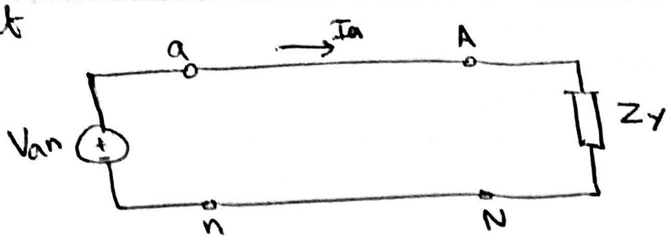
$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = V_p \angle 0^\circ - V_p \angle -120^\circ$$
$$= V_p (1 \angle 0^\circ - 1 \angle -120^\circ) = V_p (1 + 0.5 + j0.866)$$
$$= V_p (1.5 + j0.866) = V_p (1.732 \angle 30^\circ)$$

$$V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$$

Similarly, $V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_p \angle -90^\circ$

$V_{ca} = V_{cn} - V_{an} = V_{an} + V_{bn} = \sqrt{3}\, V_p \angle -210^\circ$

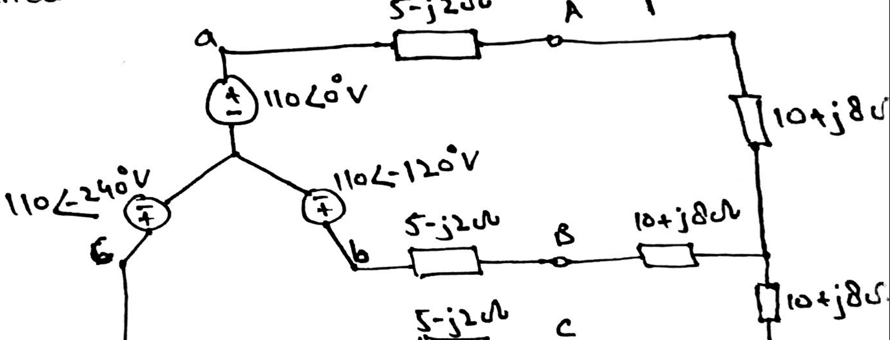
=> Single Phase Equivalent of Balanced Y-Y Connection



* Balanced three phase circuits can be analyzed on per phase basis.
* We look at one phase, say phase 'a' and analyze the single equivalent circuit.
* Because the circuit is balanced, we can easily obtain other phase values using their phase relationships.

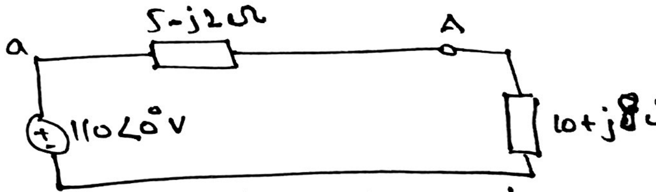
$$I_a = \frac{V_{an}}{Z_Y}$$

Q-1. Calculate the line current in the three wire Y-Y systems of figure below.



Soln: • Here we have source in Y connection and have load in Y connection.

• We solve it for one phase and obtain I_a from the single phase analysis.



Single Phase equivalent circuit.

$$I_a = \frac{V_{an}}{Z_Y}$$

where $Z_Y = (5-j2)+(10+j8)$
$= 15+j6 = 16.155\angle 21.8^\circ$

So, $I_a = \dfrac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ A$

Since the source voltages in Fig. are in positive sequence, the line current is in also positive sequence:

$$I_b = I_a \angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$I_c = I_a \angle -240^\circ = 6.81\angle -261.8^\circ A = 6.81\angle 98.2^\circ A$$

Q.2 A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3\,\Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19\,\Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7\,\Omega$ per phase. Assuming a positive sequence for the source voltages and that $V_{an} = 120\angle 30^\circ$ V, find
(a) The line voltages, (b) the line currents.

Soln. (a)

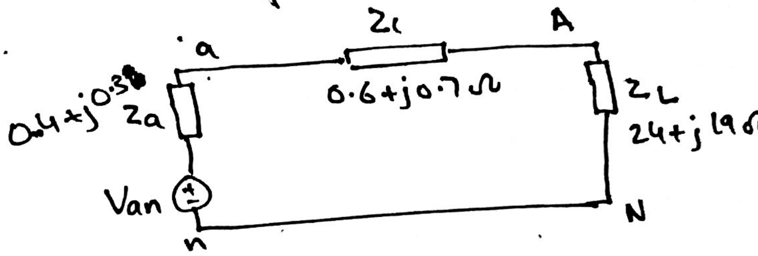
$$V_L = \sqrt{3}\, V_p \angle 30^\circ.$$

$$V_{ab} = \sqrt{3} \times 120 \angle 30^\circ . \angle 30^\circ = 207 \angle 60^\circ$$

$$V_{bc} = 207.8 \angle -60^\circ \text{ V}$$

$$V_{ca} = 207.8 \angle -180^\circ \text{ V}$$

(b) For line current just take one of the phase.



$$Z_y = 0.4 + j0.3 + 0.6 + j0.7 + 24 + j19.$$
$$= 25 + j20\,\Omega$$
$$= 32.02 \angle 38.65^\circ$$

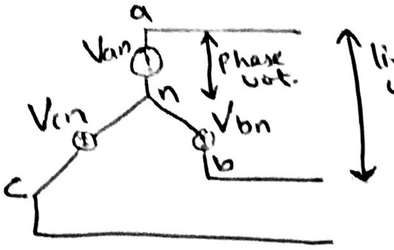
So, $$I_a = \frac{120\angle 30^\circ}{32.02\angle 38.65^\circ} = 3.75 \angle -8.65^\circ \text{ A}$$

$$I_b = I_a \angle -120^\circ = 3.75 \angle -128.66^\circ \text{A}$$

$$I_c = I_a \angle -240^\circ = 3.75 \angle 111.34^\circ \text{ A}$$

⇒ Balanced Wye – Delta (Y–Δ) Connection:-

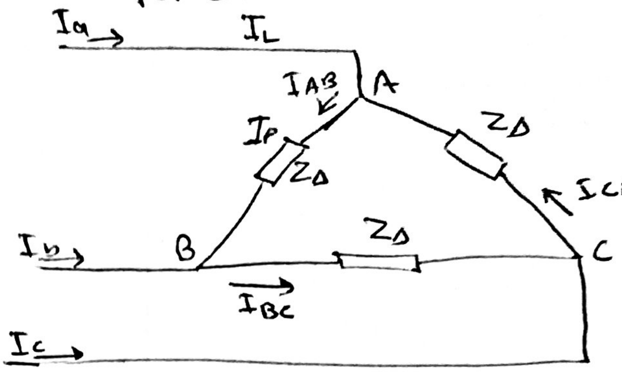
*key concepts:



"for voltages look at source"

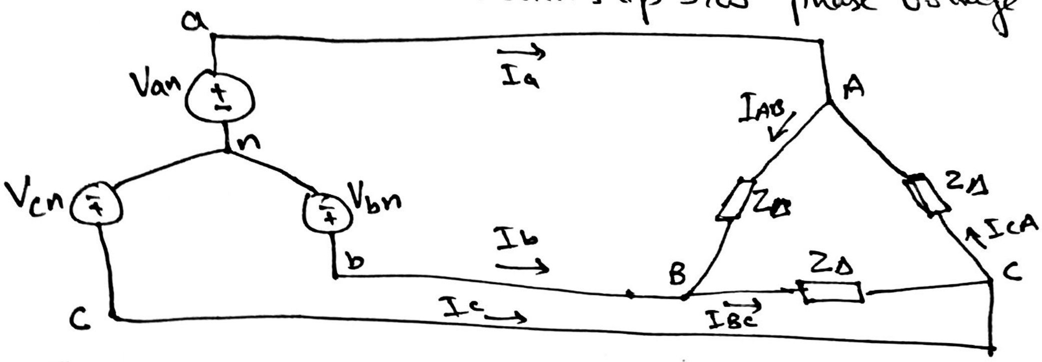
$V_L = \sqrt{3}\, V_P \angle 30^\circ$

For currents, look at the Load.



$I_L = \sqrt{3}\, I_P \angle -30^\circ$

STEP 1: Write down the relationship b/w phase voltage & Line voltage.



For positive sequence.

Phase voltages: $\quad$ $V_P = 230\,V$

$V_{an} = V_P \angle 0^\circ$

$V_{bn} = V_P \angle -120^\circ$

$V_{cn} = V_P \angle +120^\circ$

Line voltages.

$V_{ab} = \sqrt{3}\, V_P \angle 30^\circ = V_{AB}$

$V_{bc} = \sqrt{3}\, V_P \angle -90^\circ = V_{BC}$

$V_{ca} = \sqrt{3}\, V_P \angle 210^\circ = V_{CA}$

STEP 2: Solve for phase currents I_{AB}, I_{BC} and I_{CA}.

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}, \quad I_{BC} = \frac{V_{BC}}{Z_\Delta}, \quad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Apply KCL at node A:

$$I_a + I_{CA} = I_{AB}$$

Line current:

$I_a = I_{AB} - I_{CA}$, $\quad I_b = I_{BC} - I_{AB}$, $\quad I_c = I_{CA} - I_{BC}$

we can write: $I_{CA} = I_{AB} \angle 240^\circ$.

$\therefore\ I_a = I_{AB} - I_{CA} = I_{AB} - I_{AB} \angle 240^\circ$

$I_a = I_{AB}(1 + 0.5 - j0.866) = I_{AB}\, 1.732 \angle -30^\circ$

$$\boxed{I_a = I_{AB} \sqrt{3} \angle -30^\circ}$$

Similarly, $I_b = I_{BC} \sqrt{3} \angle -30^\circ$

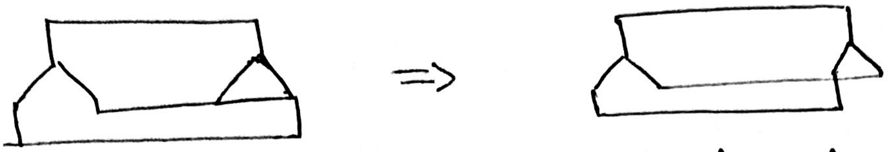
$I_c = I_{CA} \sqrt{3} \angle -30^\circ$

Alternat Way: To find current in Y-Δ connection is to transform the Δ connected load to an equivalent Y-connected load.

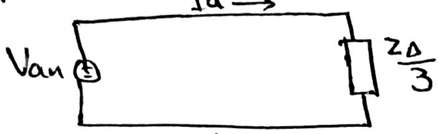
Using Δ-Y transform formula:

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$

Y-Δ



So, the three phase Y-Δ system can be replaced with equivalent single phase circuit.



$$\boxed{I_a = \frac{V_{an}}{Z_Y} = \frac{V_{an}}{Z_\Delta/3}}$$

→ much easier to solve.

Q-1: A balanced abc sequence Y-connected source with $V_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $8+j4\ \Omega$ per phase. Calculate the phase and line currents.

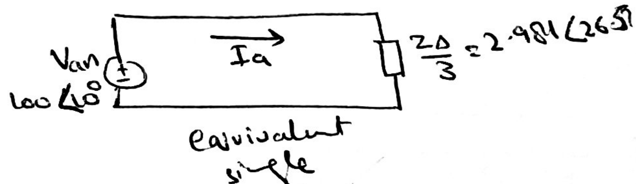
Soln: Convert Δ to Y at load side:

$V_{an} = 100\angle 10^\circ$ V , $Z_\Delta = 8+j4\ \Omega$.

$$Z_Y = \frac{Z_\Delta}{3} = \frac{8+j4}{3} = \frac{8.944\angle 26.57^\circ}{3} = 2.981\angle 26.57^\circ\ \Omega.$$

line current:

$$I_a = \frac{V_{an}}{Z_\Delta/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ}$$



$I_a = 33.54\angle -16.57^\circ$ A ✓

$I_b = I_a\angle -120^\circ = (33.54\angle -16.57^\circ)\angle -120^\circ$

$I_b = 33.54\angle -136.57^\circ$ A ✓

$I_c = I_a\angle +120^\circ = (33.54\angle -16.57^\circ)\angle 120^\circ$

$I_c = 33.54\angle 103.43^\circ$ A ✓

Phase Current: Line current $= \sqrt{3}$ Phase current $\angle -30^\circ$

Phase " $= \frac{\text{Line current}}{\sqrt{3}}\angle 30^\circ$

$$I_{AB} = \frac{33.54\angle -16.57^\circ}{\sqrt{3}}\cdot\angle 30^\circ = 19.36\angle 13.43^\circ \text{ A} ✓$$

$I_{BC} = I_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ$ A ✓

$I_{CA} = I_{AB}\angle 120^\circ = 19.36\angle 133.43^\circ$ A ✓

Q.2. One line voltage of the balanced Y-connected source is $V_{AB} = 180\angle -20^\circ V$. If source is connected to a Δ-connected load of $20\angle 40^\circ \Omega$, find the phase and line currents. Assume ABC sequence.

Sol:

line voltage $V_{AB} = 180\angle -20^\circ V$

$Z_\Delta = 20\angle 40^\circ \Omega$

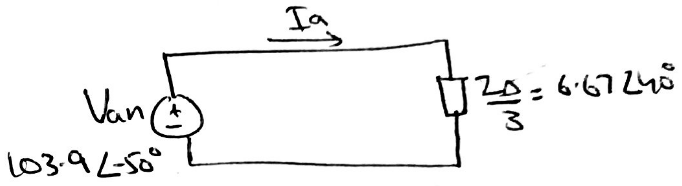
Using formula: $V_L = \sqrt{3}\, V_P \angle 30^\circ \Rightarrow V_P = \dfrac{V_L}{\sqrt{3}\angle 30^\circ}$

Phase voltage:

$$V_{an} = \frac{180\angle -20^\circ}{\sqrt{3}} \cdot \angle -30^\circ = 103.9\angle -50^\circ V$$

$$Z_Y = \frac{Z_\Delta}{3} = \frac{20\angle 40^\circ}{3} = 6.67\angle 40^\circ \Omega.$$

Line current:



$$I_a = \frac{V_{an}}{Z_a/3} = \frac{103.9\angle -50^\circ}{6.67\angle 40^\circ}$$

$$I_a = 15.57\angle -90^\circ A$$

$$I_b = I_a \angle -120^\circ = 15.59\angle +150^\circ A$$

$$I_c = I_a \angle +120^\circ = 15.59\angle 30^\circ A$$

Phase current:

$$I_{AB} = \frac{15.57\angle -90^\circ}{\sqrt{3}} \cdot \angle 30^\circ = 9\angle -60^\circ A$$

$$I_{BC} = I_{AB}\angle -120^\circ = 9\angle -180^\circ A$$

$$I_{CA} = I_{AB}\angle +120^\circ = 9\angle 60^\circ A.$$