★ Kirchhoff's Laws in Frequency Domain :-

- Both KCL and KVL can be expressed in freq. domain.
- For KVL, let $v_1, v_2, \ldots v_n$ be voltages across closed loop. Then.

$$v_1 + v_2 + v_3 + \ldots v_n = 0 \quad \text{—— (9.51)}$$

In the sinusoidal steady state, each voltage above can be written as:

$$V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \ldots + V_{mn}\cos(\omega t + \theta_n) = 0 \quad \text{— (9.52)}$$

This can be written as:

$$Re(V_{m1}e^{j\theta_1}e^{j\omega t}) + Re(V_{m2}e^{j\theta_2}e^{j\omega t}) + \ldots + Re(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

$$Re\left[(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \ldots + V_{mn}e^{j\theta_n})e^{j\omega t}\right] = 0 \quad \text{—— (9.53)}$$

If we let $V_k = V_{mk}e^{j\theta_k}$, then (phasor part)

$$Re[\underbrace{(V_1 + V_2 + \ldots + V_n)}_{\text{Phasor Part}}\underbrace{e^{j\omega t}}_{\text{time domain.}}] = 0 \quad \text{—— (9.54)}$$

Since $e^{j\omega t} \neq 0$,

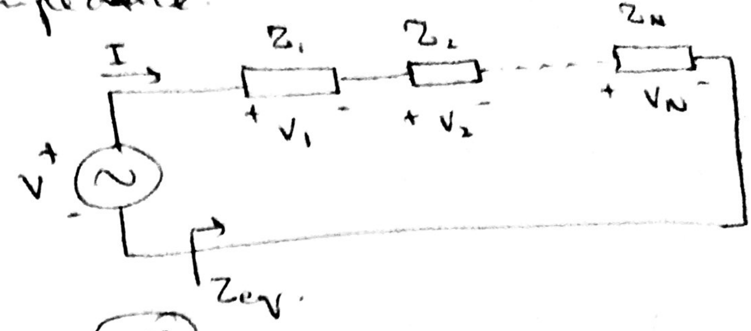
$$V_1 + V_2 + \ldots + V_n = 0 \quad \text{—— (9.55)}$$

- Similarly we can do same procedure for find KCL in phasor domain. For. $i_1 + i_2 \ldots + i_n = 0$ — (9.56) are currents entering and leaving in time t.,
- Their phasor forms will be $I_1 + I_2 + \ldots + I_n = 0$ (9.57)

A Impedance Combinations:-

→ Impedances in Series:-

- Consider N series-connected impedance.
- Current I is same through all impedances.
- Apply KVL around loop.

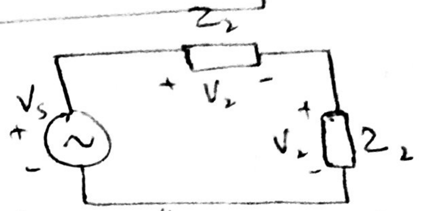


$$V = V_1 + V_2 + \dots + V_N$$
$$= I(Z_1 + Z_2 + \dots + Z_N) \quad \text{—— (9.58)}$$

or $V = I(Z_{eq}) \Rightarrow$ $\boxed{Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + \dots + Z_N}$ —— (9.59)

- If $N = 2$, then,

$$I = \frac{V}{Z_1 + Z_2} \quad \text{—— (9.60)}$$



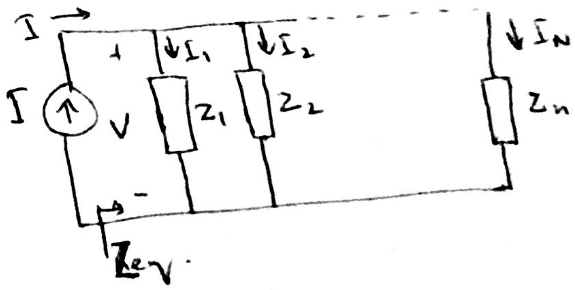
"Voltage Division"

Since, Voltage division in V_1 and V_2 as below:
$V_1 = I Z_1$ and $V_2 = I Z_2$.

So, $\boxed{V_1 = \frac{Z_1}{Z_1 + Z_2} V \quad \text{and} \quad V_2 = \frac{Z_2}{Z_1 + Z_2} V.}$ —— (9.61)

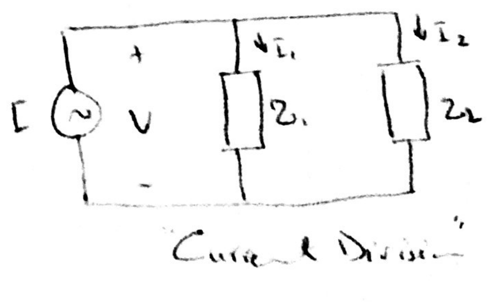
→ Impedances In Parallel:-

- Consider N parallel-connected impedance.
- Voltage across each impedance is same.
- Apply KCL.



$$I = I_1 + I_2 + \dots + I_N = \frac{V}{Z_1} + \frac{V}{Z_2} + \dots + \frac{V}{Z_N}$$
$$= V\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_N}\right)$$

$I = V \cdot \frac{1}{Z_{eq}}$



"Current Division"

- The equivalent admittance is:

$$Y_{eq} = Y_1 + Y_2 + \dots + Y_N$$

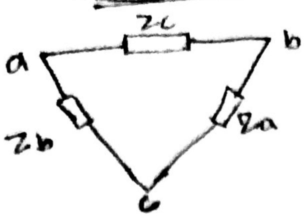
- when $N = 2$, the equivalent admittance is:

$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{Y_1 + Y_2} = \frac{1}{1/Z_1 + 1/Z_2} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

- Since Current Division in I_1 and $I_2 \Rightarrow I_1 = \frac{V}{Z_1}$, $I_2 = \frac{V}{Z_2}$

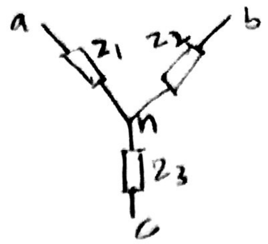
$$I_1 = \frac{I Z_{eq}}{Z_1} = \frac{I Z_1 Z_2}{(Z_1 + Z_2) Z_1} = \frac{I Z_2}{Z_1 + Z_2}, \quad I_2 = \frac{I Z_{eq}}{Z_2} = \frac{I Z_1 Z_2}{(Z_1 + Z_2) Z_2} = \frac{I Z_1}{Z_1 + Z_2}$$

'Δ' Delta Circuit



$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

Star or Y Circuit



Z_4 = $\dfrac{\text{Right} * \text{Left}}{\text{Right} + \text{Left} + \text{Bottom}}$.

$$Z_1 = \frac{Z_bZ_c}{Z_a+Z_b+Z_c}$$

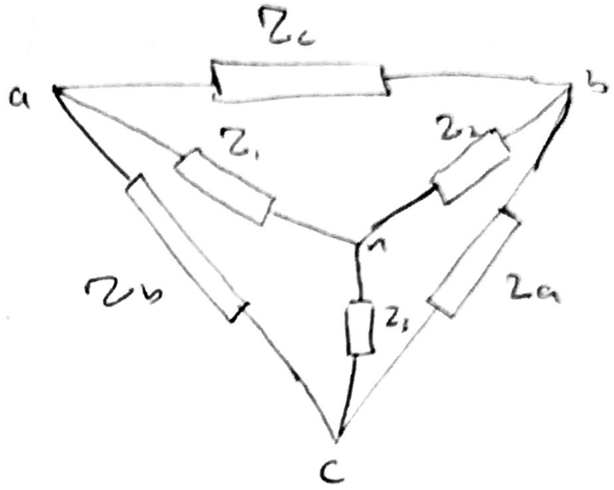
* Impedances in Δ to Y and Y to Δ transformation:- (2)

→ Y to Δ Conversion:

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$



" Superimposed Y and Δ networks".

→ Δ to Y Conversion:

$$Z_1 = \frac{Z_bZ_c}{Z_a+Z_b+Z_c}$$

$$Z_2 = \frac{Z_cZ_a}{Z_a+Z_b+Z_c}$$

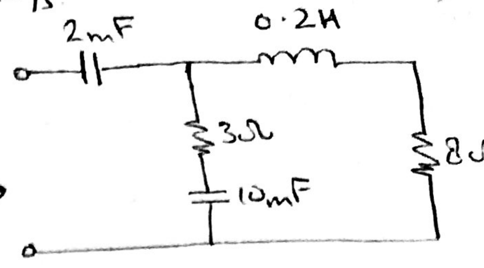
$$Z_3 = \frac{Z_aZ_b}{Z_a+Z_b+Z_c}$$

• "A Δ or Y circuit is said to be 'balanced' if it has equal impedances in all branches". e.g. consider each impedance is 1Ω then.

$Z_\Delta = 3Z_Y$ or $Z_Y = \frac{1}{3} Z_\Delta$

and circuit is balanced.

Exp 9:10:- Find input impedance. Assume $\omega = 50$ rad/s.



We set Z_1 = Impedance of 2mF

Z_2 = " " 3Ω and 10mF

Z_3 = " " 0.2H and 8Ω.

Then,

$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j\,50\times 2\times 10^{-3}} = -j\frac{1}{100\times 10^{-3}} = -j\frac{10\cancel{00}}{1\cancel{00}} = -j10\,\Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j(50)(10\times 10^{-3})} = 3 + \frac{10\cancel{0}}{j5\cancel{0}\cancel{0}} = 3 + \frac{20}{j}$$

$$= \cancel{\frac{3j+2}{j}} = 3 - 2j\ \Omega$$

$$Z_3 = 8 + j\omega L = 8 + j(50)(0.2) = 8 + j10\ \Omega.$$

The input impedance is:

$$Z_{in} = Z_1 + Z_2 \| Z_3 = -j10 + \frac{(3-j2)(8+j10)}{3-2j+8+j10}$$

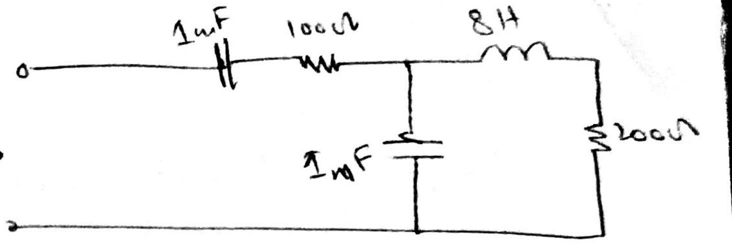
$$= -j10 + \frac{(24+j30-j16-20)}{11+8j} = -j10 + \frac{44+14j}{11+8j}$$

$$= -j10 + \frac{(44+14j)(11-8j)}{(11)^2+(8)^2} = -j10 + \frac{484-352j+154j+112}{121+64}$$

$$= -j10 + \frac{596-198j}{185} = -j10 + 3.221 - 1.07j$$

$$Z_{in} = 3.221 - j11.07.$$

PP. 9.10 : Find Z_{in} , $\omega = 10$ rad/s
Book



Soln:

$Z_1 = 1mF$ & 100Ω

$Z_2 = 1mF$

$Z_3 = 8H$ & 200Ω

Then $Z_2 = \frac{1}{j\omega C} = \frac{1}{j(10)1\times10^{-3}} = \frac{1000}{j10} = -j100\ \Omega$

$Z_1 = 100 + \frac{1}{j\omega C} = 100 + \frac{1}{j10\times10^{-3}} = 100 - j100\ \Omega$

$Z_3 = 200 + j\omega L = 200 + j(10)8 = 200 + j80.$

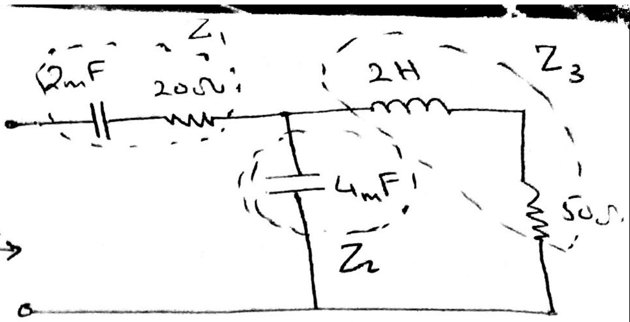
Then $Z_{in} = Z_1 + Z_2 \| Z_3$

$= 100 - j100 + \frac{(-j100)(200+j80)}{-j100+200+j80} = 100 - j100 + \frac{(-j20000+8000)}{200-20j}$

$= 100 - j100 + \frac{(-j20000+8000)(200+20j)}{(200)^2+(20)^2} = 100 - j100 + 49.505 - 95.049j$

$Z_{in} = 149.5 - 195.049j$

PP 11: Determine Z_{in} and $\omega = 10$ rad/s



Solution:- Let, Z_1 = Impedance of 2mF & 20Ω

Z_2 = " " 4mF

Z_3 = " " 2H & 50Ω

$$Z_1 = 20 + \frac{1}{j\omega C} = 20 + \frac{1}{j(10)(2\times10^{-3})} = 20 - j50\ \Omega$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(4\times10^{-3})} = -j25\ \Omega$$

$$Z_3 = 50 + j\omega L = 50 + j(10)(2) = 50 + j20$$

$$Z_{in} = Z_1 + Z_2 \| Z_3 = Z_1 + \frac{Z_2 Z_3}{Z_1 + Z_3} = 20 - j50 + \frac{-j25 \times (50 + j20)}{-j25 + 50 + j20}$$

$$= 20 - j50 + \frac{5 - 1250j}{50 - 5j} = 20 - j50 + \frac{(5 - 1250j)(50 - 5j)}{(50)^2 + (5)^2}$$

$$\cancel{= 20 - j50 + \frac{250 - 25j - 62500j - 6250}{2525}}$$

$$(50 - 5j)(50 + 5j)$$

$$2500 + 250j - 250j + 25$$

$$\cancel{= 20 - j50 + \frac{-6000 - 62525j}{2525}}$$

$$= 20 - j50 + \frac{500 - 1250j}{50 - 5j} = 20 - j50 + \frac{(500 - 1250j)(50 + 5j)}{2525}$$

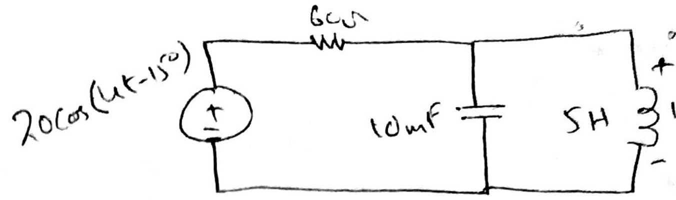
$$= 20 - j50 + \frac{25000 + 2500j - 62500j + 6250}{2525}$$

$$20 - j50 \cancel{+ \frac{18750 - 65000j}{2525}} + \frac{31250 - 6000j}{2525}$$

$$= 20 - j50 + 12.38 - 23.76j$$

$$Z_{in} = 32.38 - j73.76$$

Exp 9.11 Find $v_o(t)$



First we convert time domain terms to phasor domain.

$v_s = 20\cos(4t - 15°)$

$V_s = 20\angle -15°$, $\omega = 4$

$10mF \Rightarrow \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}}$

$5H \Rightarrow j\omega L = j4 \times 5 = j20\,\Omega$

Let Z_1 = impedance of 60Ω , Z_2 = Impedance 10mF & 5H

Then $Z_1 = 60\,\Omega$

and $Z_2 = -j25 \| j20 = \frac{-j25 \times j20}{-j25 + j20} = \frac{500}{-5j} = j100\,\Omega$.

Use voltage division method.

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_s = \frac{j100}{60 + j100} \times (20\angle -15°)$$

$$= \frac{(j100)(60 - j100)}{3600 + 10000} (20\angle -15°) = \frac{j6000 + 10000}{13600} (20\angle -15°)$$

$$= (0.735 + 0.441j)(20\angle -15°)$$

0.5402 + 0.1946

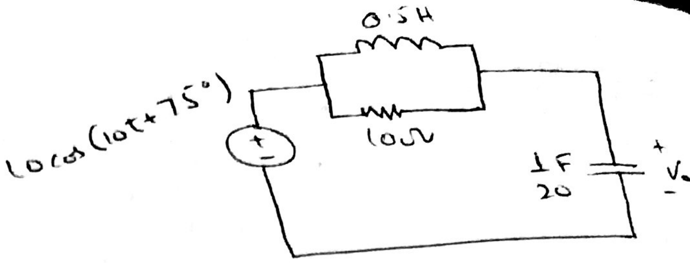
$$= (0.8575\angle 30.96°)(20\angle -15°)$$

$$= 17.15\angle 15.96°\ V$$

We convert to time domain. and get.

$v_o(t) = 17.15\cos(4t + 15.96°)$.

PP 9.11 Calculate V_o.



We convert time domain terms to freq. domain:

$v_t = 10\cos(10t+75°)$

$V_s = 10\angle 75°$, $\omega = 10$

$10\,\Omega \Rightarrow 10\,\Omega$

$0.5\,H \Rightarrow j\omega L = j \cdot 10 \cdot 0.5 = j5$

$\frac{1}{2}F \Rightarrow \frac{1}{j\omega C} = \frac{1}{j \cdot 10 \cdot \frac{1}{20}} = \frac{20}{j10} = -2j$

Assume. Z_1 = impedance of 0.5H and 10Ω

Z_2 = " " $\frac{1}{20}$F

$Z_1 = 10 \| j5 = \frac{10 \times j5}{10+j5} = \frac{50j}{10+j5} = 2+j4\,\Omega$

$Z_2 = -j2\,\Omega$

$V_o = \frac{Z_2}{Z_1+Z_2} V_s = \frac{-j2}{2+j4-j2}(10\angle 75°)$

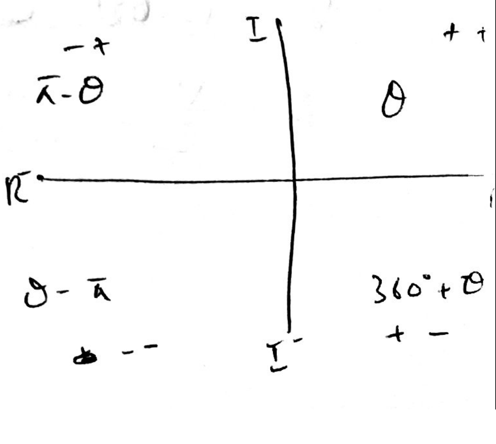
$= \frac{-j}{1+j}(10\angle 75°) = \frac{-j(1-j)}{1+1}(10\angle 75°)$

$= \frac{-j-1}{2}(10\angle 75°) = -0.5-0.5j\,(10\angle 75°)$

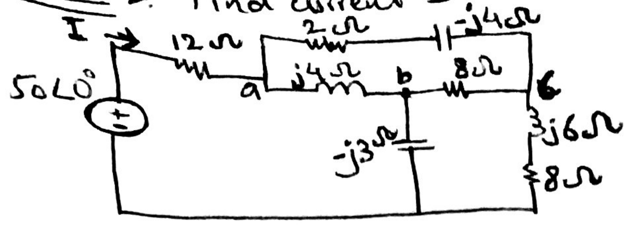
$= (0.707\angle -135°)(10\angle 75°)$

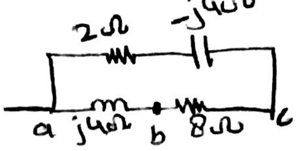
$= 7.07\angle -60°$

and $v(t) = 7.07\cos(10t-60°)$ V.



Exp 9.12 Find current 'I' in the circuit below:-





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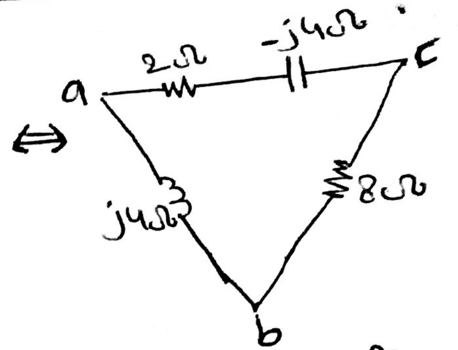
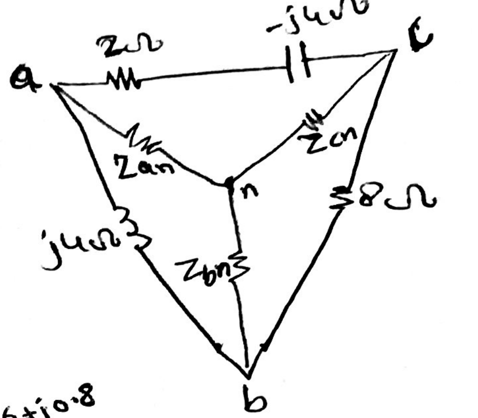


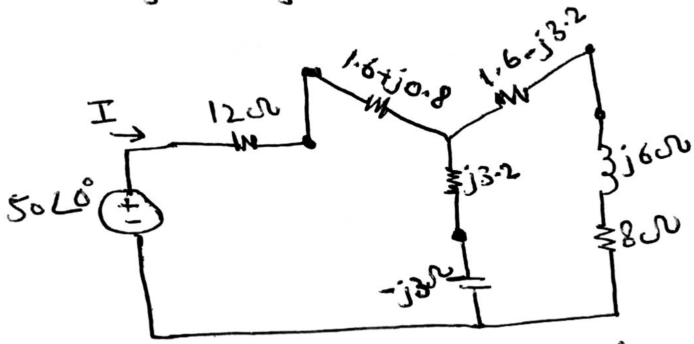
Now we find Z_{an}, Z_{bn}, Z_{cn} as:

$$Z_{an} = \frac{(j4)(2-j4)}{j4+(2-j4)+8} = \frac{16+8j}{10} = 1.6+j0.8\ \Omega$$

$$Z_{bn} = \frac{(j4)(8)}{j4+(2-j4)+8} = \frac{j32}{10} = j3.2\ \Omega$$

$$Z_{cn} = \frac{8(2-j4)}{j4+(2-j4)+8} = \frac{16-j32}{10} = 1.6-j3.2\ \Omega$$





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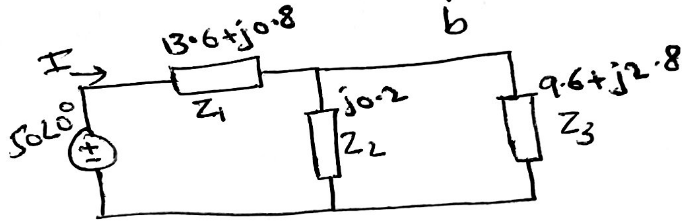

Now we calculate value of Z_1, Z_2, Z_3.

$Z_1 = 12+1.6+j0.8 = 13.6+j0.8$

$Z_2 = j3.2-j3 = j0.2$

$Z_3 = 1.6-j3.2+j6+8 = 9.6+j2.8$

we can find Z as: $Z = Z_1 + Z_2 \| Z_3 = Z_1 + \frac{Z_2 \cdot Z_3}{Z_2+Z_3}$

$$Z = 13.6+j0.8+\frac{(j0.2)(9.6+j2.8)}{(j0.2)+9.6+j2.8} = 13.6+j0.8+\frac{(j0.2)(9.6+j2.8)}{9.6+j3}$$

$$Z = 13.6+j0.8+\frac{(0.2\angle 90^\circ)(10\angle 16.26^\circ)}{10.058\angle 17.35^\circ} = 13.6+j0.8+0.1988\angle 88.91^\circ$$

$Z = 13.6+j0.8+j0.1984+0.0037 = 13.6037+0.998j$

$Z = 13.64\angle 4.19^\circ$

$$I = \frac{V}{Z} = \frac{50\angle 0^\circ}{13.64\angle 4.19^\circ} = 3.66\angle -4.19^\circ \text{ Amp}$$