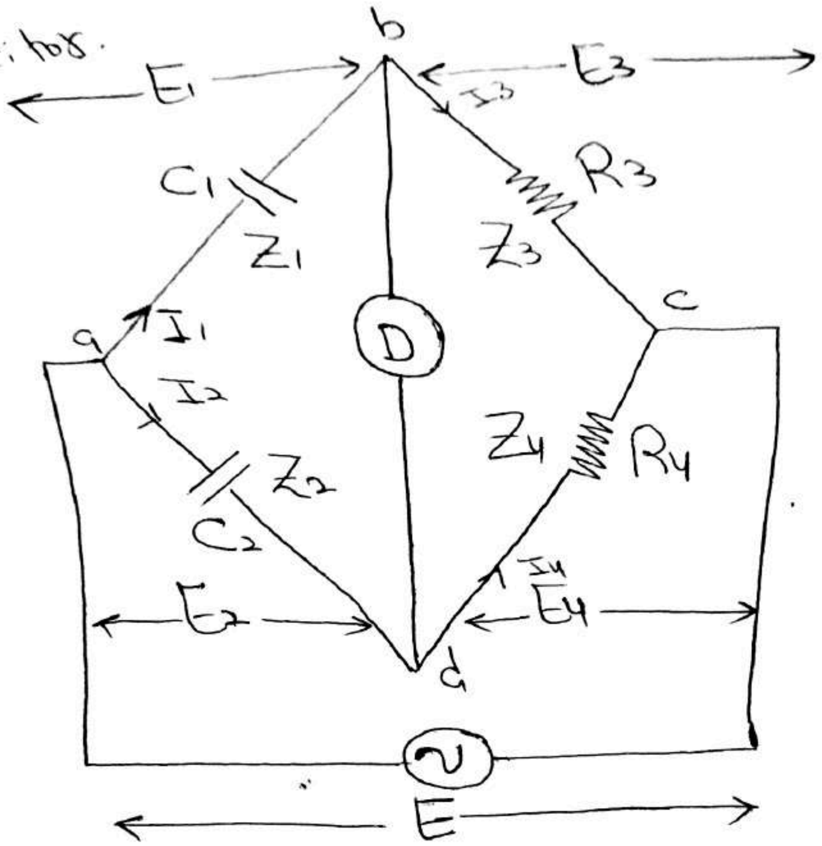
De Sauty's Bridge :-

→ This bridge is used for the measurement of capacitance of a capacitor by comparing it with a standard capacitor.



Arm a-b:- unknown capacitance (C_1)

$$Z_1 = \frac{1}{j\omega C_1}$$

Arm a-d: Standard capacitor (C_2)

$$Z_2 = \frac{1}{j\omega C_2}$$

Arm b-c:- Known non-inductive resistance (R_3)

$$Z_3 = R_3$$

Arm c-d:- Known non-inductive resistance (R_4)

$$Z_4 = R_4$$

At balance Condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

putting the values in above equation:-

$$\left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_2}\right) R_3$$

$$\left(\frac{1}{C_1}\right) R_4 = \left(\frac{1}{C_2}\right) R_3$$

or $$\frac{R_4}{R_3} = \frac{C_1}{C_2}$$

$$\boxed{C_1 = C_2 \cdot \frac{R_4}{R_3}}$$

where C_1 = Unknown capacitance
C_2 = Standard capacitance

→ Note that these bridge can only measure capacitance of those capacitors which are free from dielectric losses or which are ideal.

Proof of Balance Equation:-

when the bridge is balanced:- potential at point b = potential at point d

this means that:-

$E_1 = E_2$ & $E_3 = E_4$

$I_1 Z_1 = I_2 Z_2$

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1}$$

$I_3 Z_3 = I_4 Z_4$

$$\frac{I_3}{I_4} = \frac{Z_4}{Z_3}$$

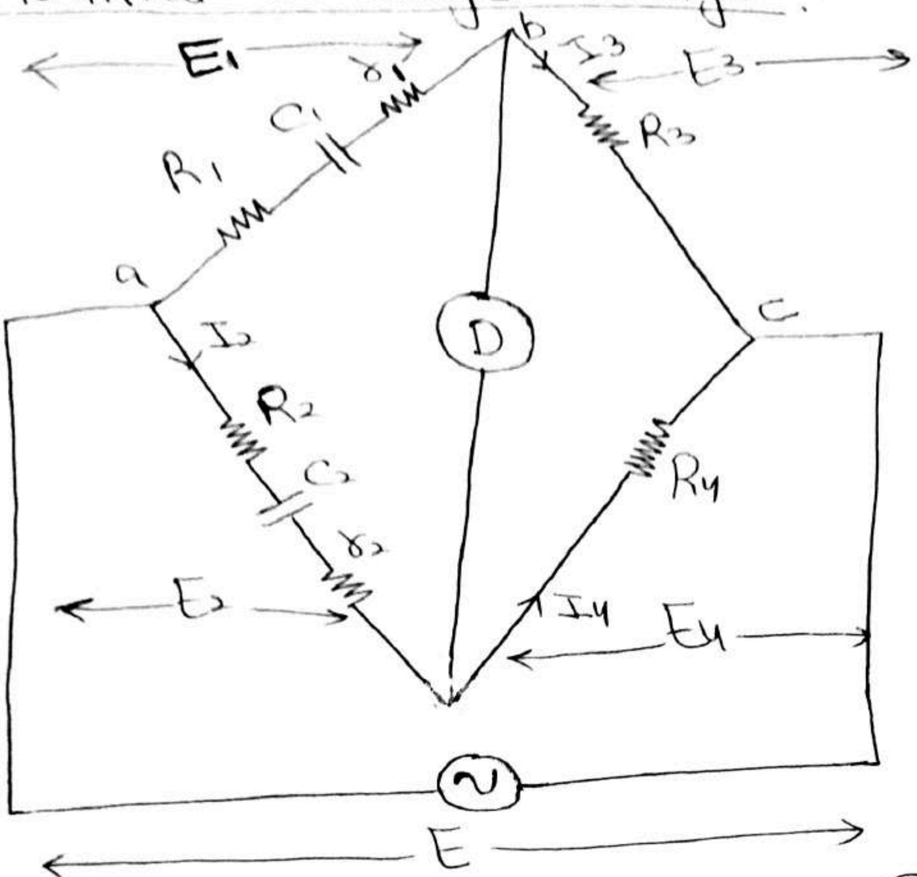
as $I_1 = I_3$ & $I_2 = I_4$

Hence $$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$\frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$\boxed{Z_2 Z_3 = Z_1 Z_4}$$

Modified De-Sauty's Bridge :-



Arm a-b :- unknown capacitance (C_1) connected in series with resistance (r_1) representing loss component and resistance (R_1).

Arm a-d :- Standard capacitor (C_2) connected in series with resistance (r_2) representing loss component and resistance (R_2).

R_3 & R_4 are known non-inductive resistances.

Now:-

$$Z_1 = R_1 + r_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2 + r_2 + \frac{1}{j\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

at balance condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$\underbrace{(R_1 + r_1) R_4}_{\text{Real part}} + \underbrace{\frac{R_4}{j\omega C_1}}_{\text{imaginary part}} = \underbrace{(R_2 + r_2) R_3}_{\text{Real part}} + \underbrace{\frac{R_3}{j\omega C_2}}_{\text{Imaginary part}}$$

equating Real part:- $(R_1 + r_1) R_4 = (R_2 + r_2) R_3$

$$\Rightarrow \quad \frac{R_4}{R_3} = \frac{R_2 + r_2}{R_1 + r_1} \quad \text{—— (1)}$$

equating imaginary part:-

$$\frac{R_4}{\cancel{j\omega} C_1} = \frac{R_3}{\cancel{j\omega} C_2}$$

$$\frac{C_1}{C_2} = \frac{R_4}{R_3} \quad \text{—— (2)}$$

comparing eqn (1) & (2)

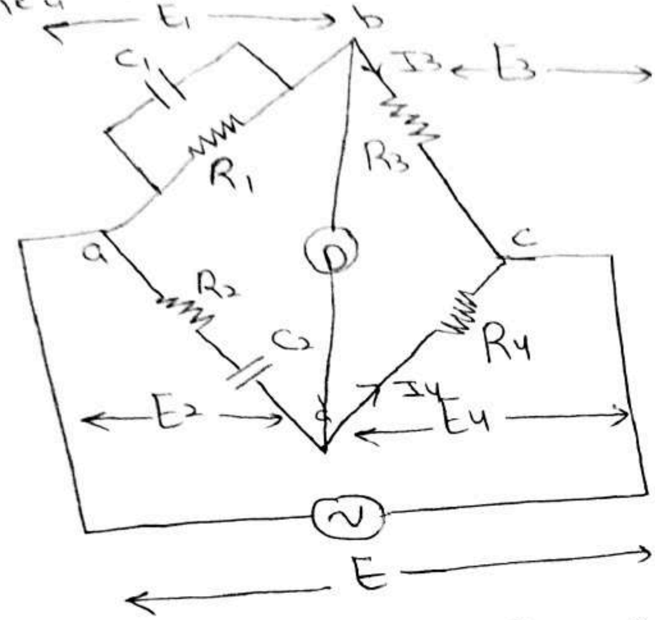
$$\boxed{\frac{C_1}{C_2} = \frac{R_4}{R_3} = \frac{R_2 + r_2}{R_1 + r_1}}$$

Wein's Bridge

→ It is used for the measurement of frequency in the range from 100Hz to 100KHz.

→ Its accuracy is between 0.1% to 0.5%.

→ named after the scientist wein who invented it.



Arm a-b:- Capacitance (C_1) connected in parallel with (R_1)

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{X_{C_1}} = \frac{1}{R_1} + JWC_1 \qquad \left(X_C = \frac{1}{JWC_1} \right)$$

$$\frac{1}{Z_1} = \frac{1 + JWC_1R_1}{R_1}$$

$$Z_1 = \frac{R_1}{1 + JWC_1R_1}$$

Arm b-c:- Known inductive Resistance (R_3)
Known

$$Z_3 = R_3$$

Arm c-d:- $Z_4 = R_4$

Arm a-d:- $Z_2 = R_2 + \frac{1}{JWC_2}$

at balance condition:-

$Z_1 Z_4 = Z_2 Z_3$

putting the values in above equation:-

$$\left(\frac{R_1}{1+JwC_1R_1}\right)R_4 = \left(R_2+\frac{1}{JwC_2}\right)R_3$$

$$\frac{R_4}{R_3} = \left(\frac{JwR_2C_2+1}{JwC_2}\right)\left(\frac{1+JwC_1R_1}{R_1}\right)$$

$$\frac{R_4}{R_3} = \frac{JwR_2C_2+J^2w^2C_2R_2C_1R_1+1+JwC_1R_1}{JwR_1C_2}$$

$$\frac{R_4}{R_3} = \frac{\cancel{Jw}R_2\cancel{C_2}}{\cancel{Jw}R_1\cancel{C_2}} + \frac{J^{\cancel{2}}w^{\cancel{2}}\cancel{C_2}R_2C_1\cancel{R_1}}{\cancel{Jw}\cancel{R_1}\cancel{C_2}} + \frac{1}{JwR_1C_2} + \frac{\cancel{Jw}C_1\cancel{R_1}}{\cancel{Jw}\cancel{R_1}C_2}$$

now equating real & imaginary parts:-

<u>Real parts:-</u>

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

<u>imaginary parts:-</u>

$$JwR_2C_1 + \frac{1}{JwR_1C_2} = 0 \rightarrow J^2w^2R_1C_1R_2C_2 + 1 = 0$$

$$+w^2R_1C_1R_2C_2 = +1$$

$$w = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

As we know that

$$w = 2\pi f$$

$$\text{or } \boxed{f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}}$$