

AC Bridges:-

Bridge Circuit:-

A bridge circuit is a special electric circuit consisting of 4 arms and 4 nodes arranged in a predefined manner.

→ In those 4 arms, if parameters of 3 arms are known, then we can easily find the parameters of the 4th arm.

There are basically 2 types of bridges:-

i):- DC Bridge

ii):- AC Bridge

i):- DC Bridge:-

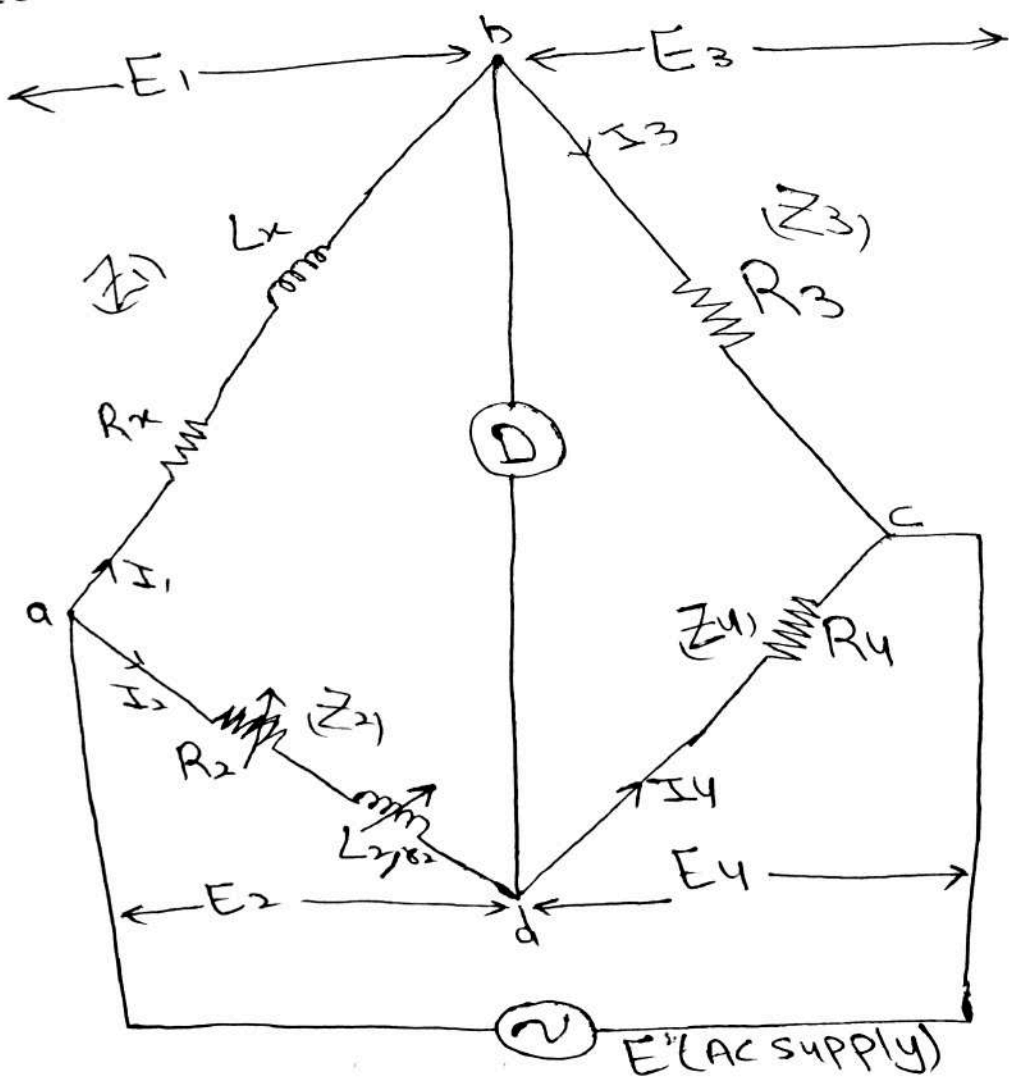
- a):- It contain DC supply only.
- b):- Parameter included is only (R)
- c):- phasor is not included.

ii):- AC Bridge:-

- a):- It contain AC supply
- b):- parameter included are R, L and C
- c):- phasor's are included.

Maxwell Inductance bridge :-

- This bridge was invented by Maxwell.
- This is used for the measurement of inductance.
- It measures an unknown inductance by comparing it with a known inductance.
- It is an AC bridge which consists of 4 arms containing 4 impedances in which 3 are known and one is unknown.
- It also contains a detector in order to detect the balance condition of the bridge and an AC supply.



Arm a-b :- unknown inductance (L_x) connected in series with a resistance (R_x).

Arm b-c :- Known non-inductive resistance (R_3)

Arm c-d :- Known non-inductive resistance (R_4).

Note :- non-inductive resistance means that (R_4) is a pure resistor which does not contain any inductance.

Arm d-a :- variable inductance (L_2) of fixed internal resistance (R_2) connected in series with variable resistance (R_2).

Balance equation :-

$$Z_1 Z_4 = Z_2 Z_3$$

where $Z_1 = R_x + j\omega L_x$

$$Z_2 = R_2 + \delta_2 + j\omega L_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

$$(R_x + j\omega L_x) R_4 = (R_2 + \delta_2 + j\omega L_2) R_3$$

$$R_x R_4 + j\omega L_x R_4 = R_2 R_3 + R_3 \delta_2 + j\omega L_2 R_3$$

$$R_x R_4 + j\omega L_x R_4 = (R_2 + \delta_2) R_3 + j\omega L_2 R_3$$

Proof of balance equation

when the bridge is balanced potential at point B = potential at D

this means that :-

$$E_1 = E_2 \quad E_3 = E_4$$

$$I_1 Z_1 = I_2 Z_2 \quad I_3 Z_3 = I_4 Z_4$$

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} \quad \frac{I_3}{I_4} = \frac{Z_4}{Z_3}$$

$$\text{as } I_1 = I_3 \quad \text{and } I_2 = I_4$$

Hence

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$\text{or } \frac{Z_2}{Z_1} = \frac{Z_4}{Z_3}$$

$$Z_1 Z_4 = Z_2 Z_3$$

Here $R_x R_y$ & $(R_2 + r_2) R_3$ are real terms
while $j\omega L_x R_y$ & $j\omega L_2 R_3$ are imaginary terms.

Hence:-

$$R_x R_y = (R_2 + r_2) R_3 \quad \& \quad j\omega L_x R_y = j\omega L_2 R_3$$

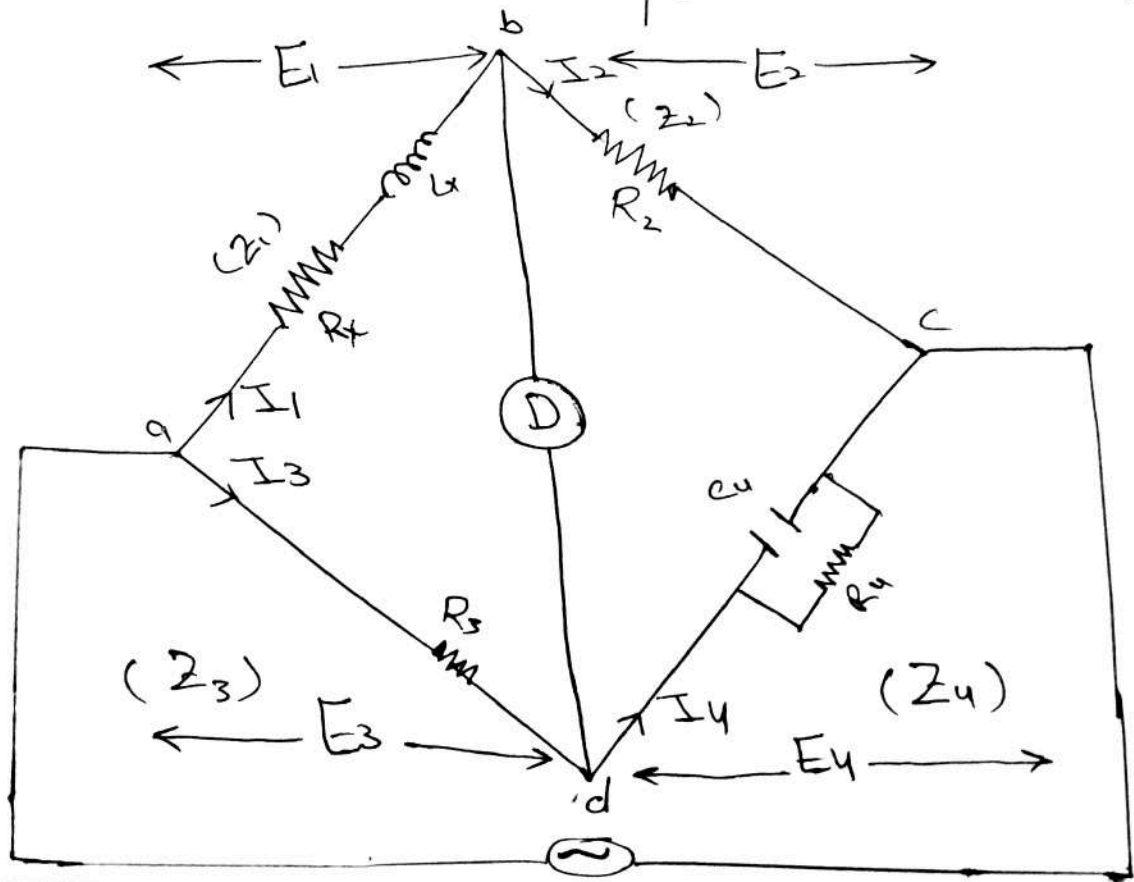
$$\Rightarrow \boxed{R_x = \frac{(R_2 + r_2) R_3}{R_y}}$$

$$\boxed{L_x = L_2 \frac{R_3}{R_y}}$$

● From this equation, we can see that the unknown inductance (L_x) can be calculated by comparing it with known inductance (L_2).

Maxwell Inductance Capacitance Bridge

- It is a type of AC Bridge.
- It measures inductance by comparing it with a standard capacitance.
- It is named after the scientist Maxwell.
- It consists of 4 arms having 4 impedances in which 3 impedances are known and one is unknown.
- It also consists of a detector which is used to obtain the balance condition of the bridge.
- It also consists of an AC power supply which is used to operate the bridge.



Now:-

$$Z_1 = R_x + j\omega L_x$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4}$$

$$\frac{1}{Z_4} = \frac{1}{R_4} + \frac{1}{X_c} \Rightarrow \frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4 \quad (\because X_c = \frac{1}{j\omega C})$$

$$\frac{1}{Z_4} = \frac{1 + j\omega R_4 C_4}{R_4} \Rightarrow Z_4 = \frac{R_4}{1 + j\omega R_4 C_4}$$

Balance equation:-

$$Z_1 Z_4 = Z_2 Z_3$$

putting values in above equation

$$(R_x + j\omega L_x) \left(\frac{R_4}{1 + j\omega C_4 R_4} \right) = R_2 R_3$$

$$R_x R_4 + j\omega L_x R_4 = R_2 R_3 + j\omega R_2 R_3 C_4 R_4$$

Here $R_x R_4$ & $R_2 R_3$ are Real terms while $j\omega L_x R_4$ & $j\omega R_2 R_3 C_4 R_4$ are imaginary terms.

$$R_x R_4 = R_2 R_3$$

$$j\omega L_x R_4 = j\omega R_2 R_3 C_4 R_4$$

$$L_x = R_2 R_3 C_4$$

$$\Rightarrow \boxed{R_x = \frac{R_2 R_3}{R_4}}$$

$$\boxed{L_x = R_2 R_3 C_4}$$

Hence $Z_1 = \frac{R_2 R_3}{R_4} + j\omega R_2 R_3 C_4$

Quality Factor of the coil is:-

- Every inductor possesses a small resistance in addition to its inductance
- The lower the value of this resistance (R), the better the quality of the coil
- The quality factor or Q factor of an inductor is defined as the "ratio of the coil to its resistance".
- Thus for an inductor, quality factor is expressed as:-

$$Q = \frac{WL}{R} \text{ or } \frac{X_L}{R} \text{ --- (1)}$$

For the given bridge, L_x is unknown and we want to find its Q .
So equation (1) can be written as:-

$$Q = \frac{WL_x}{R_x} \text{ --- (2)}$$

Putting the value of (L_x) and (R_x) in equation (2).

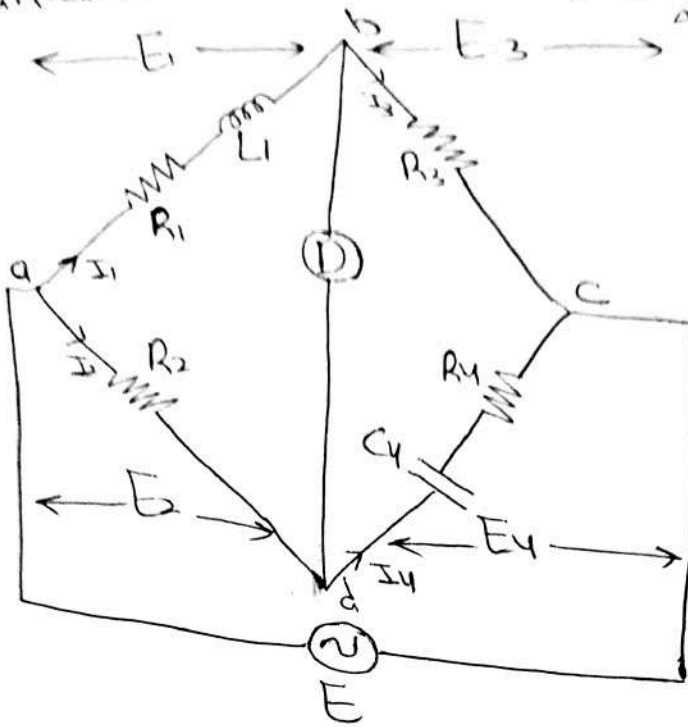
$$Q = \frac{W R_2 R_3 C_4}{\frac{R_2 R_3}{R_4}}$$

$$Q = (W R_2 R_3 C_4) \left(\frac{R_4}{R_2 R_3} \right)$$

$$\boxed{Q = W R_4 C_4}$$

Hay's Bridge:

- It is used for the measurement of inductance.
- modification of Maxwell's Bridge
- It consists of series combination of R_4, R_1, C_4 in the arm c-d.



- Here Arm a-b contains the unknown inductance (L_1) having resistance (R_1).

$$Z_1 = R_1 + j\omega L_1$$

- Arm b-c :- Known non-inductive resistance R_3

$$Z_3 = R_3$$

- Arm c-d :- Standard Capacitor (C_4) is connected in series with R_4 .

- Arm d-a :- Known non-inductive resistance (R_2):-
- $$Z_4 = R_4 + X_C = R_4 + \frac{1}{j\omega C_4} = R_4 - \frac{j}{\omega C_4} \quad (\because \frac{1}{j} = -j)$$
- $$Z_2 = R_2$$

$$Z_1 = R_1 + j\omega L_1, \quad Z_2 = R_2$$

$$Z_3 = R_3, \quad Z_4 = R_4 - \frac{j}{\omega C_4}$$

Balance equation

$$Z_1 Z_4 = Z_2 Z_3$$

putting values in the above equation.

$$(R_1 + j\omega L_1) \left(R_4 - \frac{j}{\omega C_4} \right) = R_2 R_3$$

$$R_1 R_4 - \frac{j R_1}{\omega C_4} + j\omega L_1 R_4 + \frac{L_1}{C_4} = R_2 R_3$$

equating the real and imaginary parts:

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad \text{--- (a)}$$

$$E_1 \quad j\omega L_1 R_4 - \frac{j R_1}{\omega C_4} = 0 \Rightarrow j\omega L_1 R_4 = \frac{j R_1}{\omega C_4}$$

putting R_1 in this equation (a)

$$R_1 = \omega^2 L_1 R_4 C_4 \quad \text{--- (b)}$$

$$\Rightarrow \omega^2 L_1 R_4^2 C_4 + \frac{L_1}{C_4} = R_2 R_3$$

$$L_1 \left(\frac{\omega^2 R_4^2 C_4^2 + 1}{C_4} \right) = R_2 R_3$$

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}$$

put this value in eq (b)

$$(b) \Rightarrow R_1 = \omega^2 \left(\frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \right) R_4 C_4 \Rightarrow R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}$$

For (1), quality factor can be expressed as:-

$$Q = \frac{WL_1}{R_1} \quad \text{--- (1)}$$

Putting the values of (1) $E_1 R_1$ in above equation:-

$$\begin{aligned} Q &= \frac{W R_2 R_3 C_4}{1 + W^2 R_4^2 C_4^2} \\ &= \frac{W^2 R_2 R_3 R_4 C_4^2}{1 + W^2 R_4^2 C_4^2} \\ &= \frac{W R_2 R_3 C_4}{W^2 R_2 R_3 R_4 C_4^2} \end{aligned}$$

$$Q = \frac{1}{WR_4 C_4}$$