Example 1(b): Design slab, beams, girder, columns and footings of a 90′ × 60′ Hall. Minimum obstruction to mobility inside the Hall requires that only two columns can be allowed inside the Hall. Height of the hall is 20′.

Concrete compressive strength (f_c') = 3 ksi.

Steel yield strength (f_y) = 40 ksi.

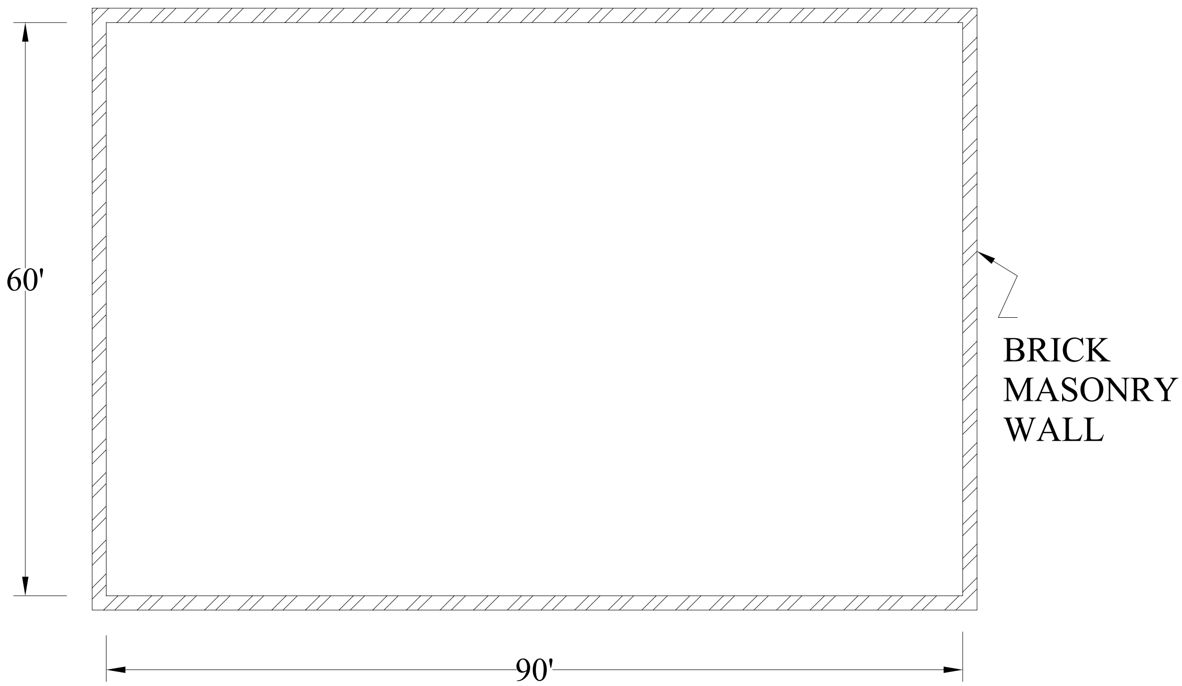


Figure 1: 90′ × 60′ Hall.

Solution: -

Second option for structural arrangement of the 90′ × 60′ Hall, figure 2:

- A girder is running along 90′ side of Hall.
- The girder is supported on columns at 30′ interval.
- Beams spaced at 10′ c/c run along 60′ side of Hall.
- Beams are supported on girder dividing the beams into two 30′ spans.
- As height of Hall is 20′, assume 18″ thick brick masonry walls.
- Assume 18″ × 18″ R.C.C. columns inside the Hall.

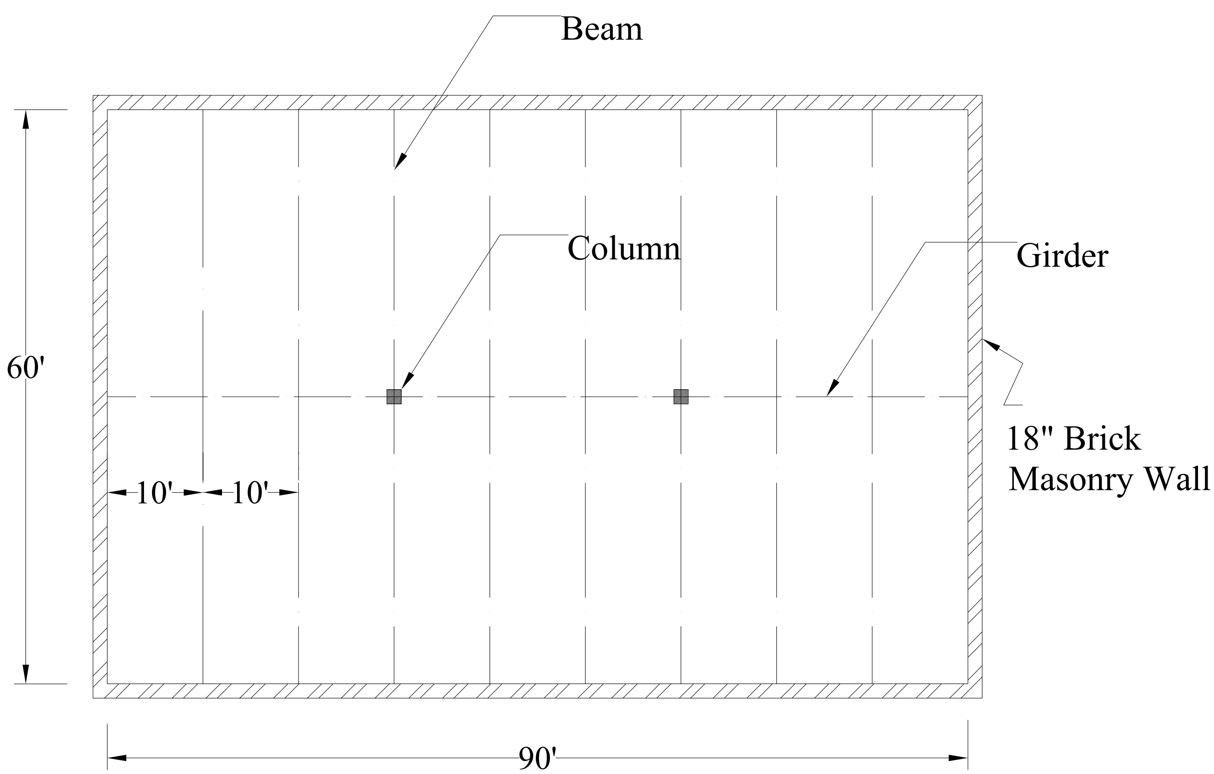


Figure 2: Structural Arrangement.

Class activity: *Is it suitable to place columns along girder at regions other than beam girder intersection……?*

(1) SLAB DESIGN:

Step No 1: Sizes.

Table 1.1: ACI formulae for thickness of continuous one way slab, ACI 9.5.2	
Case	Slab thickness (in)
End span (one end continuous)	$l/24$
Interior span (both ends continuous)	$l/28$
(i) l = Span length in inches. (ii)For f_y other than 60,000 psi, the values from above formulae shall be multiplied by $(0.4 + f_y/100000)$.	

Assume 6″ slab. Span length for end span of slab will be equal to clear span plus depth of member (slab), but need not exceed center to center distance between the supports.

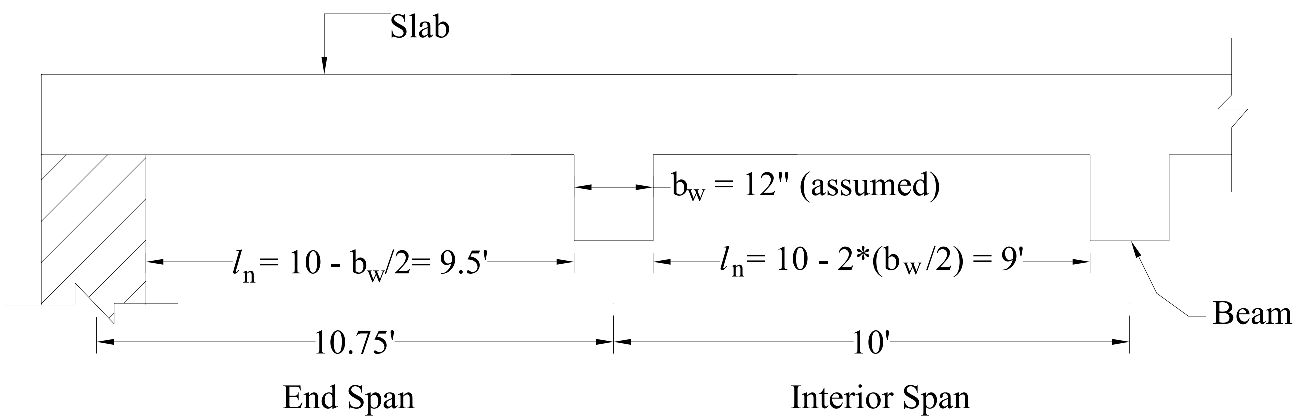


Figure 3: c/c & clear spans of slab.

Table 1.2: Span length of slab (figure 3)				
Case	c/c distance	Clear span (l_n)	l_n + depth of slab (ACI 8.7.1)	Span length(l)
End span (one end continuous)	10.75′	9.5'	9.5 + 0.5 = 10'	10′
Interior spans (both ends continuous)	10′	9'	n/a	9′

Table 1.3: Slab thickness calculation according to ACI 9.5.2.		
Span	Formula for thickness	Thickness of slab (in)
End span (one end continuous)	$l/24 \times (0.4+f_y/100000)$	(10/24) × (0.4 + 40000/100000) ×12= 4″
Interior span (both ends continuous)	$l/28 \times (0.4+f_y/100000)$	(9/28) × (0.4 + 40000/100000) ×12= 3″
l = Span length in inches.		

Therefore,

Slab thickness (h_f) = 4″ (Minimum requirement of ACI 9.5.2.1).

Take h_f = 6″

Effective depth (d) = h_f – 0.75 – (3/8)/2 = 5″ (for #3 main bar)

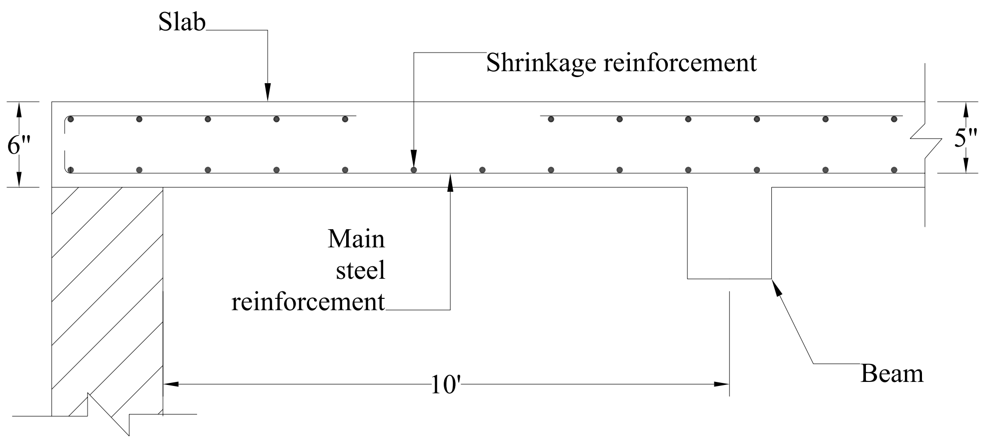


Figure 4: Effective depth of slab.

Step No 2: Loading.

Table 1.4: Dead Loads.			
Material	Thickness (in)	γ (kcf)	Load= γ × Thickness (ksf)
Slab	6	0.15	(6/12) × 0.15 = 0.075
Mud	3	0.12	(3/12) × 0.12 = 0.03
Tile	2	0.12	(2/12) × 0.12= 0.02

Service Dead Load (D.L) = 0.075 + 0.03 + 0.02

= 0.125 ksf

Service Live Load (L.L) = 40 psf or 0.04 ksf (for Hall)

Service Load (w_s) = D.L + L.L = 0.125 + 0.04 = 0.165 ksf

Factored Load (w_u) = 1.2D.L + 1.6L.L

= 1.2 × 0.125 + 1.6 × 0.04 = 0.214 ksf

Step No 3: Analysis.

Our slab system is:

- One-way,
- Clear spans less than 10′,
- Exterior ends of slab are discontinuous and unrestrained.

Refer to ACI 8.3.3 or page 396, Nilson 13th Ed. Following ACI moment coefficients apply:

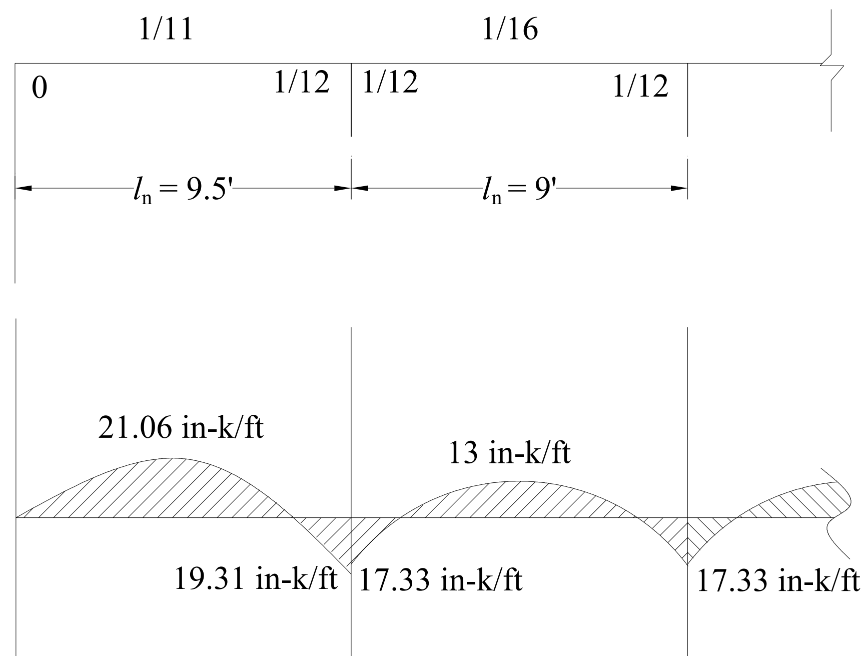


Figure 5: ACI moment coefficients for slab, ACI 8.3.3.

(1) AT INTERIOR SUPPORT (left of support):

Negative moment ($-M_{Lint}$) = Coefficient × ($w_u l_n^2$)

= (1/12) × {0.214 × (9.5)^2}

= 1.609 ft-k/ft = 19.31 in-k/ft

(2) AT INTERIOR SUPPORT (right of support):

Negative moment ($-M_{Rint}$) = Coefficient × ($w_u l_n^2$)

= (1/12) × {0.214 × (9)^2}

= 1.44 ft-k/ft = 17.33 in-k/ft

(3) AT EXTERIOR MID SPAN:

Positive moment ($+M_{Mext}$) = Coefficient × ($w_u l_n^2$)

= (1/11) × {0.214 × (9.5)^2}

= 1.755 ft-k/ft = 21.06 in-k/ft

(4) AT INTERIOR MID SPAN:

Positive moment ($+M_{Mint}$) = Coefficient × ($w_u l_n^2$)

= (1/16) × {0.214 × (9)^2}

= 1.08 ft-k/ft = 13.00 in-k/ft

Step No 4: Design.

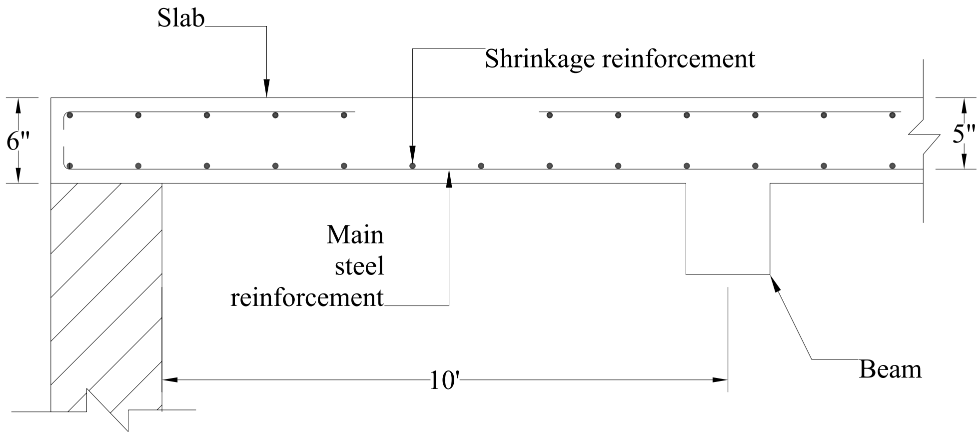


Figure 6: Reinforcement placement in slab.

Class activity: *Purpose of shrinkage reinforcement……?*

$A_{smin} = 0.002bh_f$ (for f_y 40 ksi, ACI 10.5.4)

$= 0.002 \times 12 \times 6 = 0.144$ in^2/ft

$a = A_{smin}f_y/(0.85f_c'b)$

$= 0.144 \times 40/(0.85 \times 3 \times 12) = 0.188''$

$\Phi M_n = \Phi A_{smin}f_y(d - a/2)$

$= 0.9 \times 0.144 \times 40 \times (5 - 0.188/2) = 25.4$ in-k/ft

ΦM_n as calculated from A_{smin} is greater than all moments as calculated in Step No 3.

Therefore, $A_s = A_{smin} = 0.144$ in^2/ft

Using ½″ Φ (#4) {# 13, 13 mm}, with bar area $A_b = 0.20$ in^2

Spacing = Area of one bar $(A_b)/A_s$

= (0.20 in^2/0.144 in^2/ft) x 12 = 16.67 in

Using 3/8″ Φ (#3) {#10, 10 mm}, with bar area $A_b = 0.11$ in^2

Spacing = Area of one bar $(A_b)/A_s$

= (0.11 in^2/0.144 in^2/ft) × 12 = 9.16″ ≈ 9″

Finally use #3 @ 9″ c/c (#10 @ 225 mm c/c). This will work for both Positive and Negative steel as A_{smin} governs.

Shrinkage reinforcement or temperature reinforcement (A_{st}):

$A_{st} = 0.002bh_f$

$A_{st} = 0.002 \times 12 \times 6 = 0.144$ in^2/ft

Shrinkage reinforcement is same as main reinforcement, because:

$A_{st} = A_{smin} = 0.144$ in^2

- Maximum spacing for main steel reinforcement in one way slab according to ACI 7.6.5 is minimum of:

  i) $3h_f = 3 \times 6 = 18''$

  ii) $18''$

  Therefore 9″ spacing is O.K.

- Maximum spacing for temperature steel reinforcement in one way slab according to ACI 7.12.2.2 is minimum of:

  i) $5h_f = 5 \times 6 = 30''$

  ii) $18''$

  Therefore 9″ spacing is O.K.

(2) <u>BEAM DESIGN (2 span, continuous):</u>

<u>Data Given:</u>

Exterior supports = 18″ brick masonry wall.

$f_c' = 3$ ksi

$f_y = 40$ ksi

Beams c/c spacing = 10′

Column dimensions = 18″ × 18″

Step No 1: Beam depth.

According to ACI 9.5.2.1, table 9.5 (a):

Minimum thickness of beam with one end continuous = $h_{min} = l/18.5$

l = clear span (l_n) + depth of member (beam) ≤ c/c distance between supports [ACI 8.7].

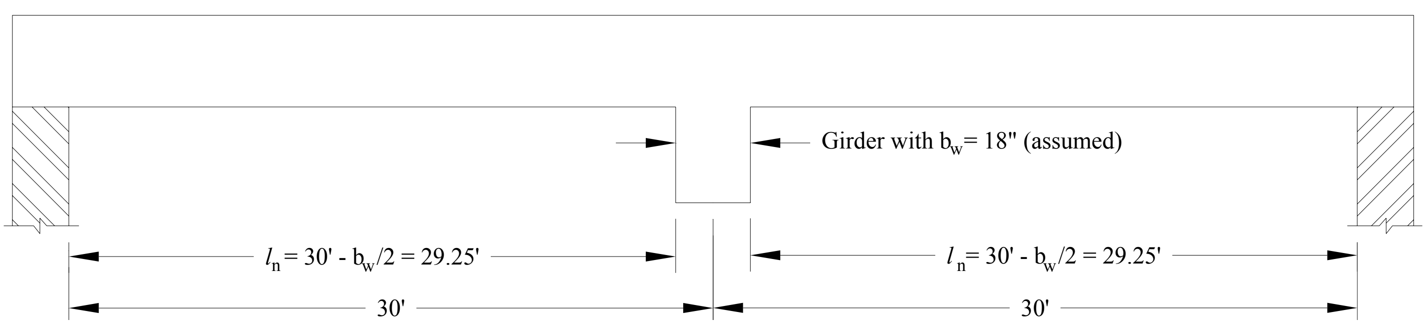


Figure 7: c/c distance & clear spans of beam.

Table 1.4: Clear span of beam ($b_w = 12''$ assumed)	
Case	Clear span (l_n)
End span (one end continuous)	30 – (18/12)/2 = 29.25′

Let depth of beam = 2′

l_n + depth of beam = 29.25′ + 2′ = 31.25′

c/c distance between beam supports = 30 + (18/12)/2 = 30.75′

Therefore l = 30.75′

Depth (h) = (30.75/18.5) × (0.4 + 40000/100000) x 12

= 15.95″ (Minimum requirement of ACI 9.5.2.1).

Take h = 2′ = 24″

d = h – 3 = 21″

Step No 2: Loads.

Service Dead Load (D.L) = 0.075 + 0.03 + 0.02 = 0.125 ksf (See table 1.3 above)

Service Live Load (L.L) = 40 psf or 0.04 ksf (for Hall)

Beam is supporting 10′ slab. Therefore load per running foot will be as follows:

Service Dead Load from slab = 0.125 × 10 = 1.25 k/ft

Service Dead Load from beam's self weight = $h_w b_w \gamma_c$

= (18 × 12/144) × 0.15 = 0.225 k/ft

Total Dead Load = 1.25 + 0.225 =1.475 k/ft

Service Live Load = 0.04 × 10 = 0.4 k/ft

w_s = D.L + L.L = 1.475 + 0.4 = 1.875 k/ft

w_u = 1.2D.L + 1.6L.L

= 1.2 × 1.475 + 1.6 × 0.4 = 2.41 k/ft

Step No 3: Analysis.

Refer to ACI 8.3.3 or page 396, Nilson 13th Ed, for ACI moment and shear coefficients.

(1) AT INTERIOR SUPPORT:

Negative moment ($-M_{Lint}$) = Coefficient × ($w_u l_n^2$)

= (1/9) × {2.41 × (29.25)^2}

= 229.08 ft-k = 2749 in-k

(2) AT MID SPAN:

Positive moment ($+M_{Mext}$) = Coefficient × ($w_u l_n^2$)

$= (1/11) \times \{2.41 \times (29.25)^2\}$

$= 187.42$ ft-k $= 2249$ in-k

$V_{max\ (int)} = 1.15 w_u l_n/2 = 1.15 \times 2.41 \times 29.25/2 = 40.5$ k

$V_{u(int)} = V_{max\ (int)} - w_u d = 40.5 - 2.41 \times 1.75 = 36.28$ k

$V_{max(ext)} = w_u l_n/2 = 2.41 \times 29.25/2 = 35.25$ k

$V_{u(ext)} = V_{max\ (ext)} - w_u d = 35.25 - 2.41 \times 1.75 = 31.03$ k

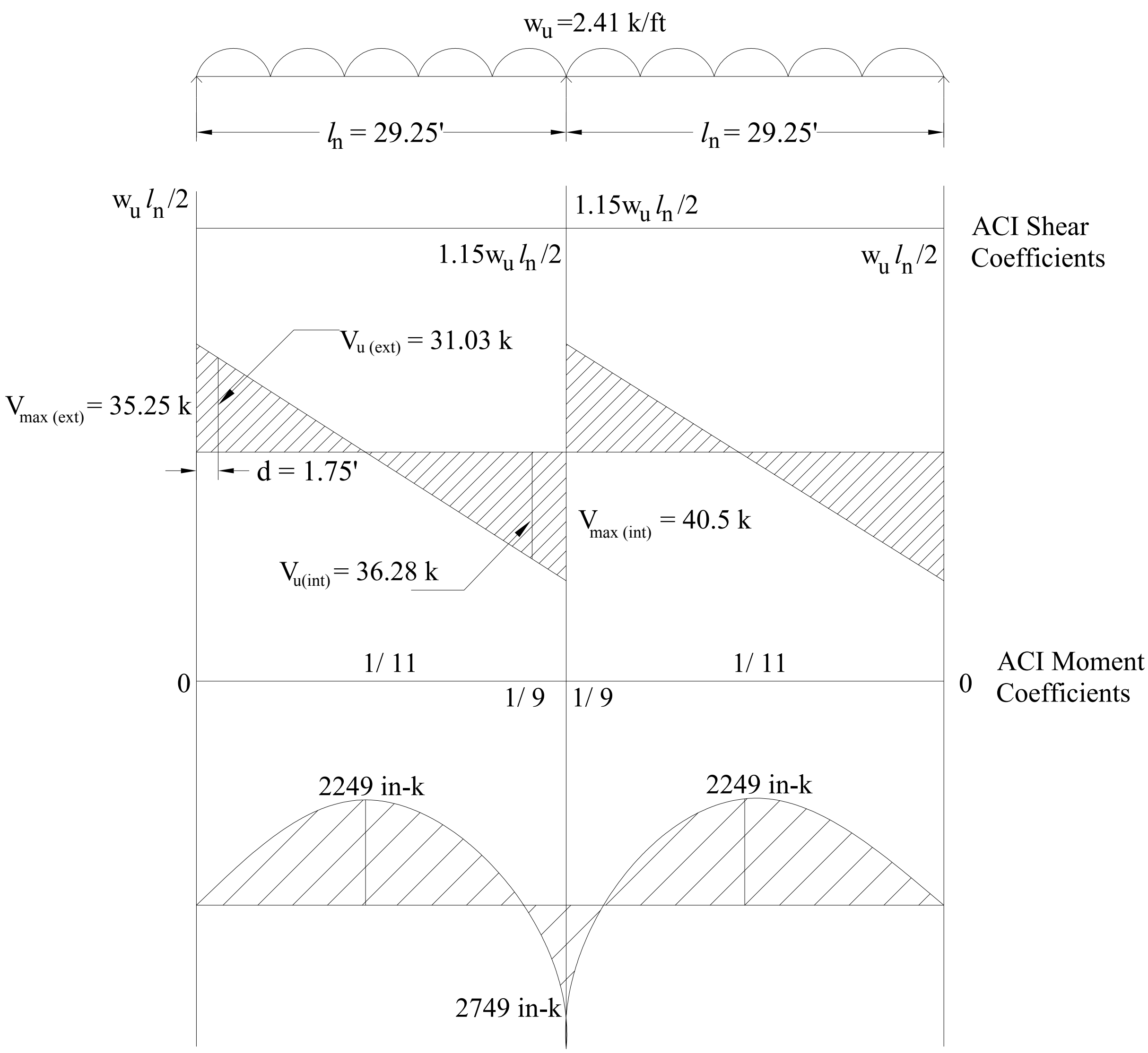


Figure 8: Approximate shear force and bending moment diagrams of beam.

Discussion: *Can the ACI analysis be applied to each and every case…?*

Step No 4: Design.

(A) Flexural Design:

(1) For Positive Moment:

Step (a): According to ACI 8.10, b_{eff} is minimum of:

(i) $16h_f + b_w = 16 \times 6 + 12 = 108''$

(ii) (c/c span of beam)/4 = $(30.75/4) \times 12 = 92.25''$

(iii) c/c spacing between beams = $10 \times 12 = 120''$

Therefore, $b_{eff} = 92.25''$

Step (b): Check if beam is to be designed as rectangular beam or T-beam.

Trial #1:

(i) Assume $a = h_f = 6''$

$A_s = M_u / \{\Phi f_y(d - a/2)\}$

$A_s = 2249 / \{0.9 \times 40 \times (21 - 6/2)\} = 3.47 \text{ in}^2$

(ii) Re-calculate "a":

$a = A_s f_y / (0.85 f_c' b_{eff})$

$a = 3.47 \times 40 / (0.85 \times 3 \times 92.25) = 0.6'' < h_f$

Therefore design beam as rectangular beam.

Trial #2:

$A_s = 2249 / \{0.9 \times 40 \times (21 - 0.6/2)\} = 3.01 \text{ in}^2$

$a = 3.01 \times 40 / (0.85 \times 3 \times 92.25) = 0.51''$

This value is close enough to the previously calculated value of "a", therefore, $A_s = 3.01 \text{ in}^2$ O.K.

Step (c): Check for maximum and minimum reinforcement.

$A_{smax} = \rho_{max} b_w d$

$\rho_{max} = 0.85 \times 0.85 \times (3/40) \times \{0.003/(0.003 + 0.005)\} = 0.0203$

$A_{smax} = 0.0203 \times 12 \times 21 = 5.11 \text{ in}^2$

$A_{smin} = \rho_{min} b_w d$

$A_{smin} = 0.005 \times 12 \times 21 = 1.26 \text{ in}^2$

$A_{smin} < A_s < A_{smax}$, O.K.

Using $1''$ Φ (#8) {#25, 25 mm}, with bar area $A_b = 0.79 \text{ in}^2$

No. of bars = $A_s/A_b = 3.01/0.79 = 3.81 \approx 4$ bars

Use 4 #8 bars {4 # 25 bars, 25 mm}.

(2) For Interior Negative Moment:

Step (a): Now we take $b_w = 12''$ instead of b_{eff} for calculation of "a" because of flange in tension.

(i) $M_u = 2749$ in-k

$b_w = 12''$

$h = 24''$

$d = 21''$

Trial #1:

$A_s = M_u / \{\Phi f_y(d - a/2)\}$

Let a = 0.2d

$A_s = 2749 / [0.9 \times 40 \times \{21 - (0.2 \times 21)/2)\}] = 3.65 \text{ in}^2$

Trial #2:

$a = 3.65 \times 40 / (0.85 \times 3 \times 12) = 4.77''$

$A_s = 2749 / \{0.9 \times 40 \times (21 - 4.77/2)\} = 4.10 \text{ in}^2$

Trial #3:

$a = 4.1 \times 40 / (0.85 \times 3 \times 12) = 5.36''$

$A_s = 2749 / \{0.9 \times 40 \times (21 - 5.36/2)\} = 4.16 \text{ in}^2$

Trial #4:

$a = 4.16 \times 40 / (0.85 \times 3 \times 12) = 5.44''$

$A_s = 2749 / \{0.9 \times 40 \times (21 - 5.44/2)\} = 4.17 \text{ in}^2$

Step(b): Check for maximum and minimum reinforcement.

A_{smin} (1.26 in^2) < A_s (4.17 in^2) < A_{smax} (5.11 in^2), O.K.

Using 1″ Φ(#8) {#25, 25 mm}, with bar area $A_b = 0.79 \text{ in}^2$

No. Of bars = A_s/A_b

$= 4.17/0.79 = 5.27 \approx 6$ bars

Use 6 #8 bars {6 #25 bars, 25 mm}

(B) Shear Design for beam:

Step (a):

$d = 21'' = 1.75'$

$V_{u\,(ext)} = 31.03$ k

$V_{u\,(int)} = 36.28$ k

<u>Step (b):</u>

$\Phi V_c = \Phi 2\sqrt{(f_c')}b_w d$

$= \{0.75 \times 2 \times \sqrt{(3000)} \times 12 \times 21\}/1000 = 20.7$ k

$\Phi V_c < V_{u(ext)}$ and $V_{u(int)}$ {Shear reinforcement is required}

<u>Step (c):</u> Spacing.

(a) For $V_{u(ext)}$:

$s_{dext} = \Phi A_v f_y d/ (V_{u(ext)} - \Phi V_c)$

{Use #3, 2 legged stirrups with $A_v = 0.11 \times 2 = 0.22$ in^2}

$s_{dext} = 0.75 \times 0.22 \times 40 \times 21/ (31.03 - 20.7) \approx 13''$

(b) For $V_{u(int)}$:

$s_{dint} = \Phi A_v f_y d/ (V_{u(int)} - \Phi V_c)$

Use #3, 2 legged stirrups,

$s_{dint} = 0.75 \times 0.22 \times 40 \times 21/ (36.28 - 20.7) \approx 9''$

<u>Step (d):</u> Maximum spacing and minimum reinforcement requirement as permitted by ACI 11.5.4 and 11.5.5.3 shall be minimum of:

(i) $A_v f_y/(50b_w) = 0.22 \times 40000/(50 \times 12) = 14.67''$

(ii) $d/2 = 21/2 = 10.5''$

(iii) $24''$

(iv) $A_v f_y/ 0.75\sqrt{(f_c')}b_w = 0.22 \times 40000/ \{(0.75 \times \sqrt{(3000)} \times 12\} = 17.85''$

<u>Other checks:</u>

(a) Check for depth of beam {ACI 11.5.6.9}:

$\Phi V_s \leq \Phi 8\sqrt{(f_c')}b_w d$

$\Phi 8\sqrt{(f_c')}b_w d = 0.75 \times 8 \times \sqrt{(3000)} \times 12 \times 21/1000$

$= 82.8$ k

$\Phi V_s = (\Phi A_v f_y d)/ s_d$

$= (0.75 \times 0.22 \times 40 \times 21)/9 = 15.4 \text{ k} < 82.8$ k, O.K.

So depth is O.K. If not, increase depth of beam.

(b) Check if "$\Phi V_s \leq \Phi 4\sqrt{(f_c')}b_w d$" {ACI 11.5.4.3}:

If "$\Phi V_s \leq \Phi 4\sqrt{(f_c')}b_w d$", the maximum spacing ($s_{max}$) is O.K. Otherwise reduce spacing by one half.

$\Phi 4\sqrt{(f_c')}b_w d = 0.75 \times 4 \times \sqrt{(3000)} \times 12 \times 21/1000 = 41.4$ k

$\Phi V_s = (\Phi A_v f_y d)/ s_d$

$= (0.75 \times 0.22 \times 40 \times 21)/9 = 15.4 \text{ k} < 41.4 \text{ k}$, O.K.

Since maximum spacing allowed by ACI is 10.5″, therefore,

$\Phi V_n = \Phi V_c + \Phi V_s$

$\Phi V_s = (\Phi A_v f_y d)/ s_{max}$

$\Phi V_s = (0.75 \times 0.22 \text{ x } 40 \times 21/10.5) = 13.20 \text{ k}$

Therefore $\Phi V_n = 13.20 + 20.70 = 33.90 \text{ k} > \{V_{u\,(ext)}$ but $< V_{u\,(int)}\}$

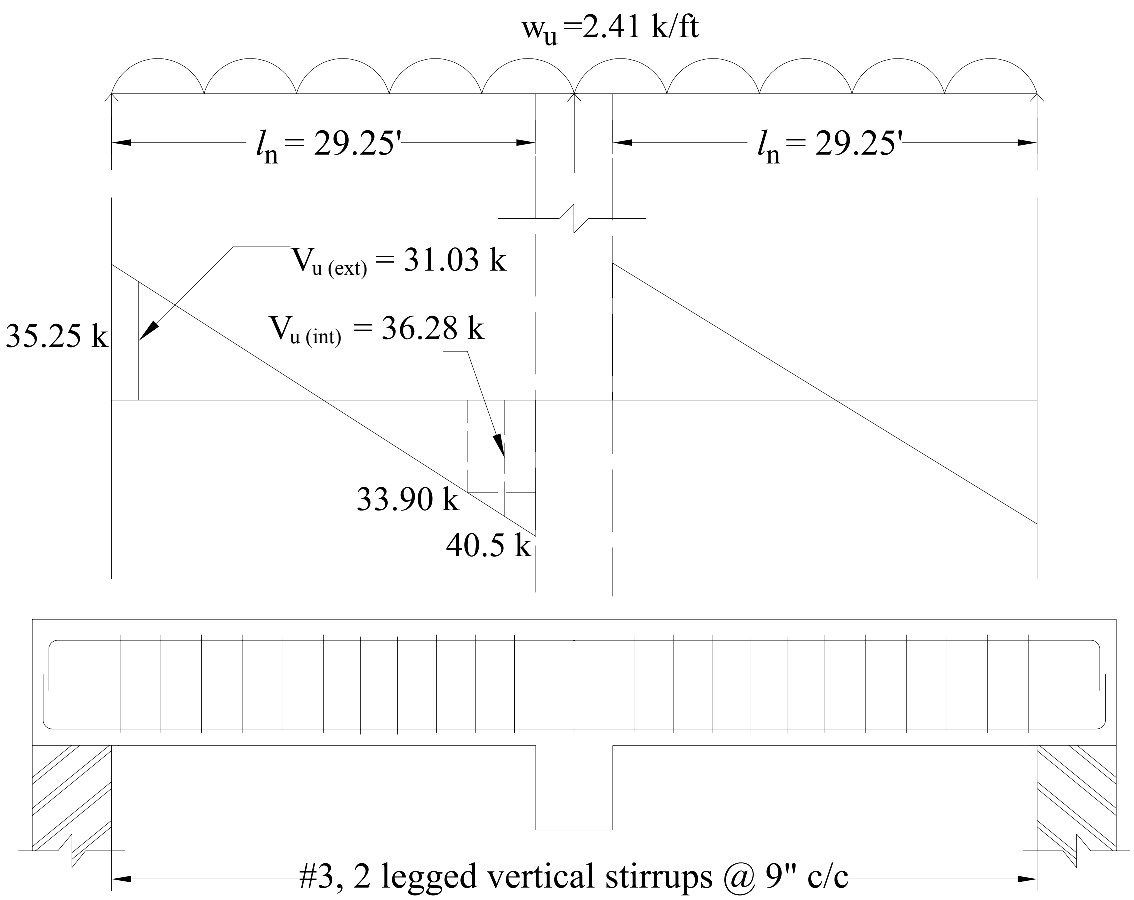


Figure 9: Stirrups arrangement in beam.

It means that for $V_{u(ext)}$ which is less than $\Phi V_n = 33.90$ k, 10.5″ of spacing can be provided. And theoretically for $V_{u(int)}$, 10.5″ of spacing can be provided from $\Phi V_c/2$ upto $V_u = 33.9$k. After $V_u = 33.9$k, $s_d = 9″$ must be provided.

But it will be practically feasible to provide # 3, 2 legged @ 9″ c/c {#10, 2 legged stirrups @ 225 mm c/c} throughout, starting at $s_d/2 = 9/2 = 4.5″$ from the face of the support at both ends.

(3) GIRDER DESIGN:

Beams load can be approximated as point loads on girder. The uniformly distributed load on girder is coming from self weight of girder rib plus weight of slab directly resting on girder.

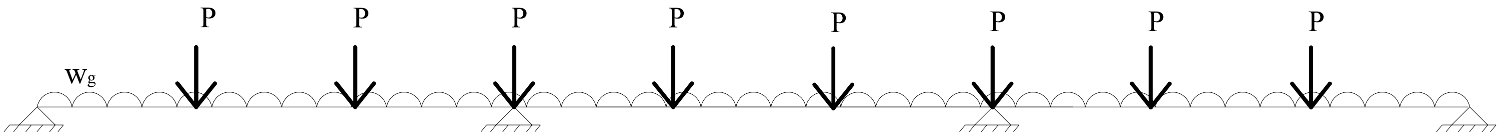


Figure 10: Load scheme of girder.

Step No 1: Sizes.

According to ACI 9.5.2.1, table 9.5 (a).

Table 1.5: ACI formulae for beam (girder) depth.	
Case	Depth, h_{min} (in)
End span (one end continuous)	$l/18.5$
Interior span (both ends continuous)	$l/21$
(i) l = Span length in inches. (ii) For f_y other than 60,000 psi, the values from above formulae shall be multiplied by $(0.4 + f_y/100000)$.	

Assume 3′ deep girder. Span length for end span of girder will be equal to clear span plus depth of member (girder), but need not exceed center to center distance between the supports.

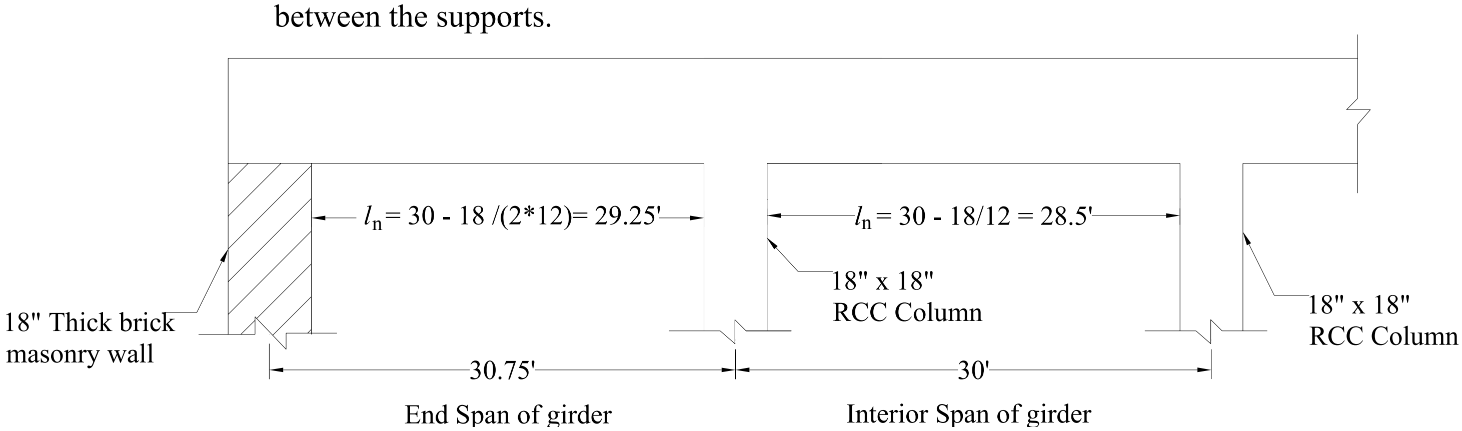


Figure 11: c/c & clear spans of girder.

Table 1.6: Span length of girder (figure 11)				
Case	c/c distance	Clear span (l_n)	l_n + depth of slab (ACI 8.7.1)	Span length(l)
End span (one end continuous)	30.75′	29.25'	29.25 + 3 = 32.25'	30.75′
Interior spans (both ends continuous)	30′	28.5'	n/a	28.5′

Table 1.7: Girder depth calculation.

Span	Formula for thickness	Thickness of slab (in)
End span (one end continuous)	$l/18.5 \times (0.4+f_y/100000)$	$(30.75/18.5)\times(0.4 + 40000/100000)\times12 \approx 16''$
Interior span (both ends continuous)	$l/21 \times (0.4+f_y/100000)$	$(28.5/21)\times(0.4 + 40000/100000)\times12 \approx 13''$
l = Span length in inches.		

Therefore, Girder depth (h) = 16″ (minimum requirement of ACI 9.5.2.1).

Take h = 36″

Effective depth of girder (d_g) = h – 3 = 33″

Discussion: *on relative stiffness of beam and girder.*

According to ACI 8.10, b_{eff} is minimum of:

(i) $16h_f + b_{wg} = 16 \times 6 + 18 = 114''$

(ii) (c/c span of girder)/4 = $(30.75/4) \times 12 = 92.25''$

(iii) c/c spacing between girder = not applicable

$$b_{eff} = 92.25''$$

Step No 2: Loads.

Refer figure 10 and 12.

(i) P is the point load on girder and is the reaction coming from the interior support of beam due to factored load.

P = 2 × {40.5} = 81 k (see shear force diagram for beam, fig. 8)

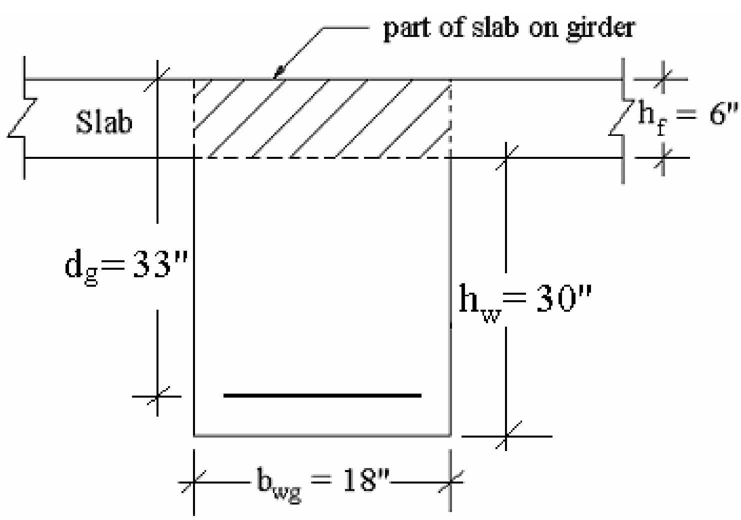


Figure 12: Girder cross-section.

<u>Note:</u> d_g = effective depth of girder = 33″

b_{wg} = width of girder web = 18″

h_{wg} = depth of girder web = 30″

(ii) $(U.D.L)_{self\ wt}$ = Factored self weight of girder rib

$=1.2h_{wg}b_{wg}\gamma_c$

$=1.2 \times (30 \times 18 \times 0.15)/144 = 0.675$ k/ft

(iii) Part of slab on girder $\{(U.D.L)_{sg}\}$:

$(U.D.L)_{sg} = w_{u\ (on\ slab)} \times b_{wg}$

$= 0.214 \times 18/12 = 0.321$ k/ft

Therefore $w_g = (U.D.L)_{self\ wt} + (U.D.L)_{sg}$

$= 0.675 + 0.321 = 1.0$ k/ft

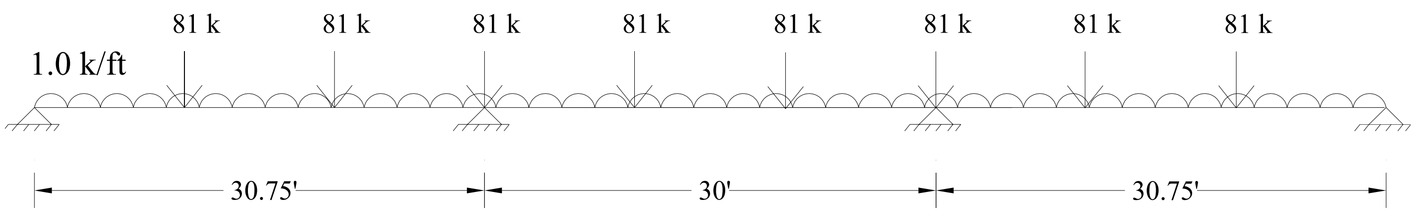


Figure 13: Factored loads on girder. Girder spans are according to ACI 8.7.

Step No 3: Analysis.

(i) Kinematics indeterminacy of girder:

K.I = 4 i.e. $(\theta_A, \theta_B, \theta_C, \theta_D)$

Table 1.7: Slope Deflection Method				
span	I (ft^4)	l (ft)	I/l	k
AB, BA	1	30.75	1/30.75	1
BC, CB	1	30	1/30	1.025
CD, DC	1	30.75	1/30.75	1

(ii) Fixed End moments.

$M_{F(AB)} = w_g l^2/12 + Pab^2/l^2 + Pba^2/l^2$

$M_{F(AB)} = 1.0 \times 30.75^2/12 + 81 \times 10.25 \times 20.5^2/30.75^2 + 81 \times 20.5 \times 10.25^2/30.75^2 = -632.297$ ft-k

$M_{F(BA)} = +\ 632.297$ ft-k

$M_{F(BC)} = w_g l^2/12 + Pab^2/l^2 + Pba^2/l^2 = -\ 615$ ft-k

$M_{F(CB)} = w_g l^2/12 + Pab^2/l^2 + Pba^2/l^2 = +\ 615$ ft-k

$M_{F(CD)} = -\ 632.297$ ft-k

$M_{F(DC)} = +\ 632.297$ ft-k

(iii) Slope deflection equation.

$M_{AB} = M_{FAB} + k_{AB}(2\ \theta_A + \theta_B)$

$M_{BA} = M_{FBA} + k_{BA}(2\ \theta_B + \theta_A)$

$M_{BC} = M_{FBC} + k_{BC}(2\ \theta_B + \theta_C)$

$M_{CB} = M_{FCB} + k_{CB}(2\ \theta_C + \theta_B)$

$M_{CD} = M_{FCD} + k_{CD}(2\ \theta_C + \theta_D)$

$M_{DC} = M_{FDC} + k_{DC}(2\ \theta_D + \theta_C)$

(iv) Joint conditions.

Joint A: $M_{AB} = 0$………………..(a)

Joint B: $M_{BA} + M_{BC} = 0$…………(b)

Joint C: $M_{CB} + M_{CD} = 0$…………(c)

Joint D: $M_{DC} = 0$………………..(d)

(v) Simplifying slope deflection equations using joint conditions.

Eqn (a) ….. $2\ \theta_A + \theta_B = 632.297$

Eqn (b) ….. $4.05\ \theta_B + \theta_A + 1.025\theta_C = -\ 17.297$

Eqn (c) ….. $4.05\ \theta_C + 1.025\theta_B + \theta_D = 17.297$

Eqn (d) ….. $2\ \theta_D + \theta_C = -\ 632.297$

(vi) Solving the above four equations:

$\theta_A = 382.177$

$\theta_B = -\ 132.058$

$\theta_C = 132.058$

$\theta_D = -\ 382.177$

(vii) Put these values back to slope deflections to get moments.

$M_{AB} = 0$

$M_{BA} = 750.35$ ft-k

$M_{BC} = -750.35$ ft-k

$M_{CB} = 750.35$ ft-k

$M_{CD} = -750.35$ ft-k

$M_{DC} = 0$

(viii) Shear force and bending moment diagrams.

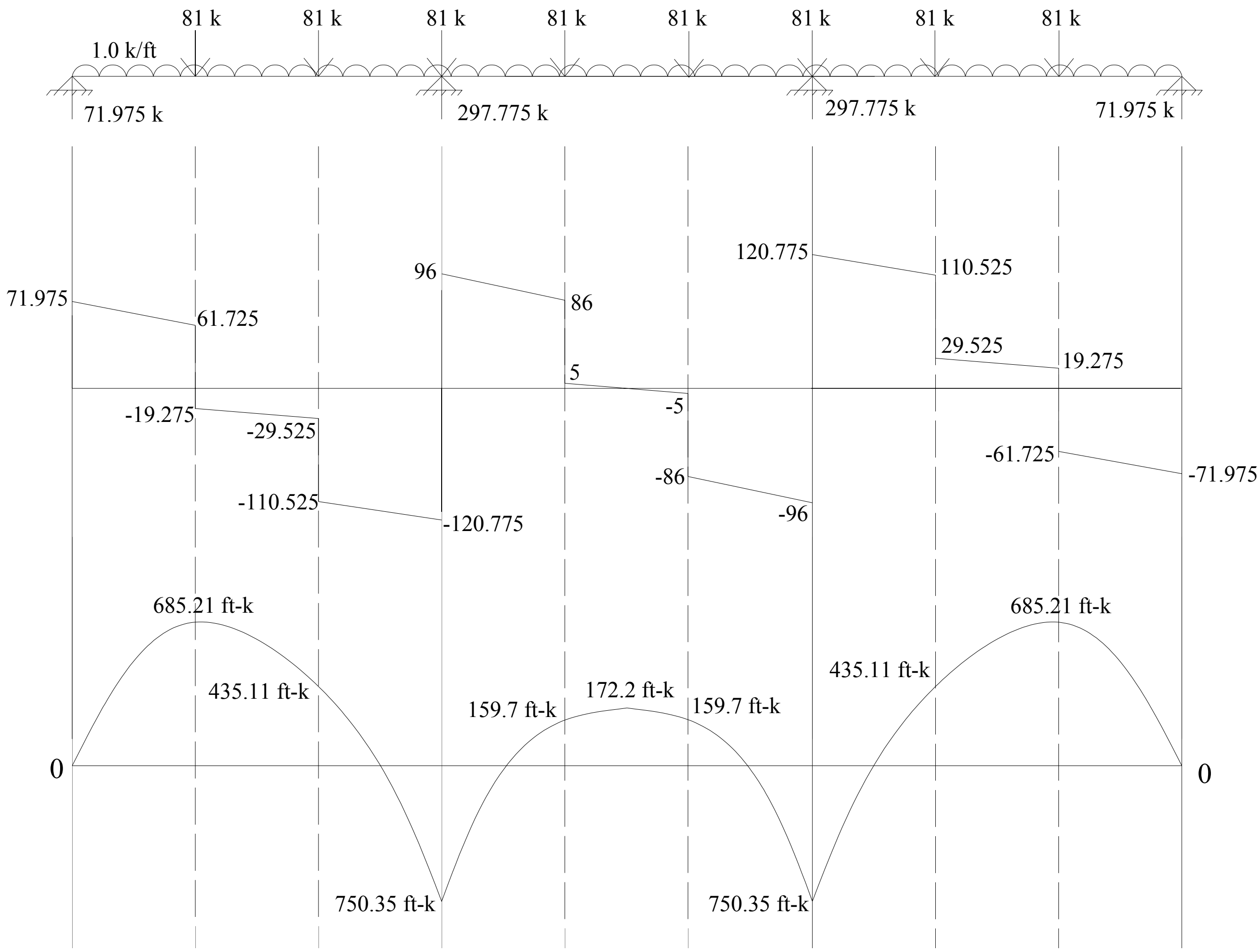


Figure 14: Shear Force and Bending Moment Diagrams for girder.

(ix) Moments for design purpose:

$+M_{ext} = 685.21$ ft-k = 8222 in-k

$+M_{int} = 172.2$ ft-k = 2066.4 in-k

$-M = -750.35$ ft-k = 9004 in-k

Step No 4: Design.

Step (a): Flexural Design.

(1) For Exterior Positive Moment ($+M_{ext}$):

1: Check if girder is to be designed as rectangular beam or T-beam.

- Trial #1:

  Assume $a = h_f = 6''$

  $A_s = M_u / \{\Phi f_y(d_g - a/2)\}$

  $A_s = 8222 / \{0.9 \times 40 \times (33-6/2)\} = 7.61 \text{ in}^2$

  Re-calculate "a":

  $a = A_s f_y / (0.85 f_c' b_{eff})$

  $a = 7.61 \times 40 / (0.85 \times 3 \times 92.25) = 1.29'' < h_f$

  Therefore design girder for positive moment as rectangular beam.

- Trial #2:

  $A_s = 8222 / \{0.9 \times 40 \times (33–1.29/2)\} = 7.06 \text{ in}^2$

  $a = 7.06 \times 40 / (0.85 \times 3 \times 92.25) = 1.20''$

- Trial #3:

  $A_s = 8222 / \{0.9 \times 40 \times (33–1.20/2)\} = 7.05 \text{ in}^2$, O.K.

2: Check for maximum and minimum reinforcement.

$A_{smax} = \rho_{max} b_w d_g$

$\rho_{max} = 0.85\beta_1(f_c'/f_y)\{\varepsilon_u/(\varepsilon_u + \varepsilon_y)\}$

$\rho_{max} = 0.85 \times 0.85 \times (3/40) \times \{0.003/(0.003 + 0.005)\} = 0.0203$

$A_{smax} = 0.0203 \times 18 \times 33 = 11.88 \text{ in}^2$

$A_{smin} = \rho_{min} b_w d_g = 0.005 \times 18 \times 33 = 2.97 \text{ in}^2$

A_{smin} (2.97 in^2) < A_s (6.81 in^2) < A_{smax} (11.88 in^2), O.K.

Using 1″ Φ (#8) {# 25, 25 mm}, with bar area $A_b = 0.79 \text{ in}^2$

No. of bars = A_s/A_b

= 7.05/0.79 = 8.92 ≈ 9 bars

Use 9 #8 bars {9 #25 bars, 25 mm}.

(2) For Interior Positive Moment ($+M_{int}$):

1: Check if girder is to be designed as rectangular beam or T-beam.

- Trial #1:

  Assume $a = h_f = 6''$

$A_s = M_u/\ \{\Phi f_y(d_g - a/2)\}$

$A_s = 2066.4/\ \{0.9 \times 40 \times (33 - 6/2)\} = 1.91\ in^2$

Re-calculate "a":

$a = A_s f_y/\ (0.85 f_c{}^{'}\ b_{eff})$

$a = 1.91 \times 40/\ (0.85 \times 3 \times 92.25) = 0.324'' < h_f$

Therefore design girder for positive moment as rectangular beam.

- Trial #2:

  $A_s = 2066.4/\ \{0.9 \times 40 \times (33 - 0.324/2)\} = 1.74\ in^2$

  $a = 1.74 \times 40/\ (0.85 \times 3 \times 92.25) = 0.295''$

- Trial #3:

  $A_s = 2066.4/\ \{0.9 \times 40 \times (33\text{-}0.295/2)\} = 1.74\ in^2$

$A_s = 1.74\ in^2 < (A_{smin} = 2.97\ in^2)$. So provide A_{smin}.

Using 1″ Φ (#8) {# 25, 25 mm}, with bar area $A_b = 0.79\ in^2$

No. of bars = $A_s/A_b = 2.97/0.79 = 3.75 \approx 4$ bars

Use 4 #8 bars {4 #25 bars, 25 mm}.

(3) For Negative Moment.

1: Now we take $b_{wg} = 18''$ instead of b_{eff} for calculation of "a" because of flange in tension.

. $M_u = 9004$ in-k

$h = 36''$; $b_{wg} = 18''$

$d_g = 33''$

- Trial #1:

  $A_s = M_u/\ \{\Phi f_y(d_g - a/2)\}$

  $A_s = 9004/\ (0.9 \times 40 \times \{33\text{-}\ (0.2 \times 33)/2)\} = 8.42\ in^2$

  $a = 8.42 \times 40/\ (0.85 \times 3 \times 18) = 7.34''$

- Trial #2:

  $A_s = 9004/\ \{0.9 \times 40 \times (33 - 7.34/2)\} = 8.53\ in^2$

  $a = 8.53 \times 40/\ \{0.85 \times 3 \times 18\} = 7.43''$

- Trial #3:

  $A_s = 9004/\ \{0.9 \times 40 \times (33 - 7.43/2)\} = 8.54\ in^2$

  $A_s = 8.54\ in^2$, O.K.

2: Check for maximum and minimum reinforcement.

A_{smin} (2.97 in^2) <A_s (8.54 in^2) <A_{smax} (11.88 in^2), O.K.

Using 1″ Φ (#8) {#25, 25 mm}, with bar area A_b = 1.0 in^2

No. of bars = A_s/A_b

= 8.54/0.79 = 10.81 ≈ 12 bars (for symmetry)

Use 12 #8 bars {12 #25 bars, 25 mm}.

Step (b): Shear Design for girder:

d_g = 33″ = 2.75′

$\Phi V_c = \Phi 2\sqrt{(f_c')} b_{wg} d_g$

$\Phi V_c = \{0.75 \times 2 \times \sqrt{(3000)} \times 18 \times 33\}/1000 = 48.808$ k

Maximum spacing and minimum reinforcement requirement as permitted by ACI 11.5.4 and 11.5.5.3 shall be minimum of:

(i) $A_v f_y/(50 b_w) = 0.22 \times 40000/(50 \times 18) = 9.77'' \approx 9.5''$

(ii) $d_g/2 = 33/2 = 16.5''$

(iii) 24″

(iv) $A_v f_y/ 0.75\sqrt{(f_c')} b_w = 0.22 \times 40000/ \{(0.75 \times \sqrt{(3000)} \times 18\} = 11.90''$

Since maximum spacing allowed by ACI is 9.5″,

$\Phi V_n = \Phi V_c + \Phi V_s$

$\Phi V_s = (\Phi A_v f_y d_g)/s_{max}$

$\Phi V_s = (0.75 \times 0.22 \times 40 \times 33/9.5) = 22.92$ k

$\Phi V_n = 48.808 + 22.92 = 71.72$ k > V_{max1}, V_{max2} and V_{max5} but < V_{max3} and V_{max4}

It means that maximum spacing as permitted by ACI governs for V_{max1}, V_{max2} and V_{max5}. Therefore for V_{max1}, V_{max2} and V_{max5}, s_d = 9.5″

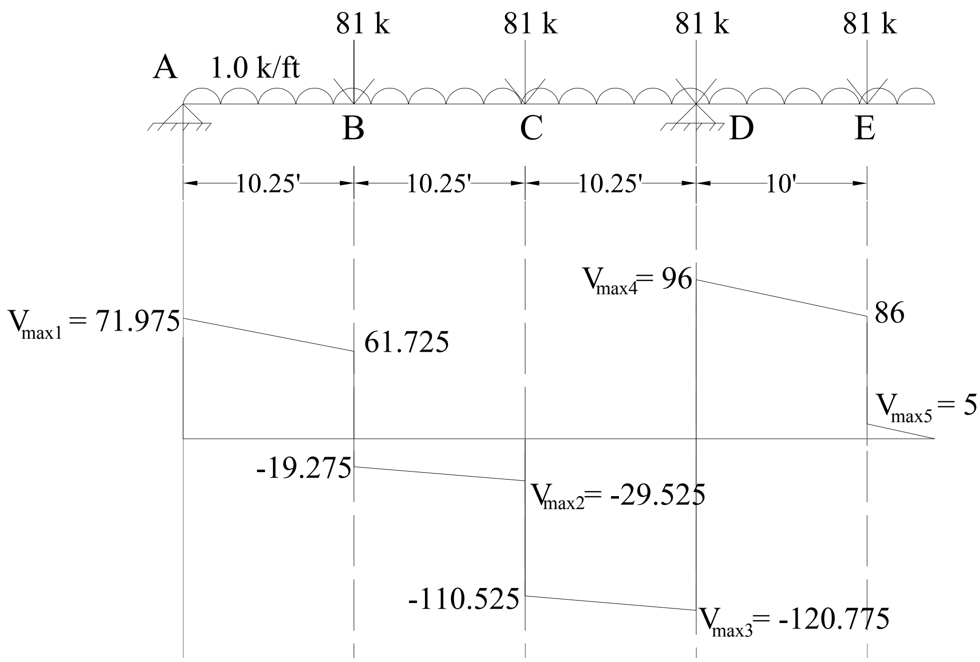


Figure 15: Shear Force Diagram for one half of girder.

<u>Spacing for V_{u3}:</u>

V_{max3} =120.775 k

V_{u3} = 117.5 k

$s_{d3} = \Phi A_v f_y d_g / (V_{u3} - \Phi V_c)$

Use #3, 2 legged stirrups with $A_v = 0.11 \text{ x } 2 = 0.22 \text{ in}^2$

$s_{d3} = 0.75 \times 0.22 \times 40 \times 33/ (117.50 - 48.808) = 3.17'' \approx 3''$ c/c

<u>Spacing for V_{u4}:</u>

V_{max4} = 96 k

V_{u4} = 94 k

$s_{d4} = \Phi A_v f_y d_g / (V_{u4} - \Phi V_c)$

Use #3, 2 legged stirrups,

$s_{d4} = 0.75 \times 0.22 \times 40 \times 33/ (94 - 48.808) = 4.8'' \approx 4.5''$ c/c

Take $s_{d4} = 3''$ c/c

<u>Other Checks:</u>

(1) Check for depth of girder (ACI 11.5.6.9).

$\Phi V_s \leq \Phi 8\sqrt{(f_c')} b_w d_g$

$\Phi 8\sqrt{(f_c')} b_w d_g = 0.75 \times 8 \times \sqrt{(3000)} \times 18 \times 33/1000 = 195.20$ k

$\Phi V_s = (\Phi A_v f_y d_g)/s_d$

$= (0.75 \times 0.22 \times 40 \times 33)/3 = 72.60$ k < 195.20 k, O.K.

So depth is O.K. If not, increase depth of girder.

(2) Check if "$\Phi V_s \leq \Phi 4\sqrt{(f_c')}b_w d$" {ACI 11.5.4.3}:

If "$\Phi V_s \leq \Phi 4\sqrt{(f_c')}b_w d$", the maximum spacing ($s_{max}$) is O.K. Otherwise reduce spacing by one half.

$\Phi 4\sqrt{(f_c')}b_w d_g = 0.75 \times 4 \times \sqrt{(3000)} \times 18 \times 33/1000 = 97.60$ k

$\Phi V_s = (\Phi A_v f_y d_g)/ s_d$

$= (0.75 \times 0.22 \times 40 \times 33)/3 = 72.60$ k < 97.60 k, O.K.

Refer to figure 15. Provide #3, 2 legged stirrups @ 9.5″ c/c {#10, 2 legged stirrups @ 240 mm c/c} from point A to C. And provide #3, 2 legged stirrups @ 3″ c/c {#10, 2 legged stirrups @ 75 mm c/c} from C to E. Start providing stirrups from section at distance. $s_d/2 = 9.5/2 = 4.75$″ from the face of the exterior support, and $s_d/2 = 3/2 = 1.5$″ from the face of the interior support.

Finally, however, for practically feasible placement of stirrups, provide #3, 2 legged stirrups @ 3″ c/c, 10′ from both sides of the support (E to D, and D to C), and 6″ c/c elsewhere.

Bar Cutoff: The actual analysis of the girder carried out by slope deflection method is as given below.

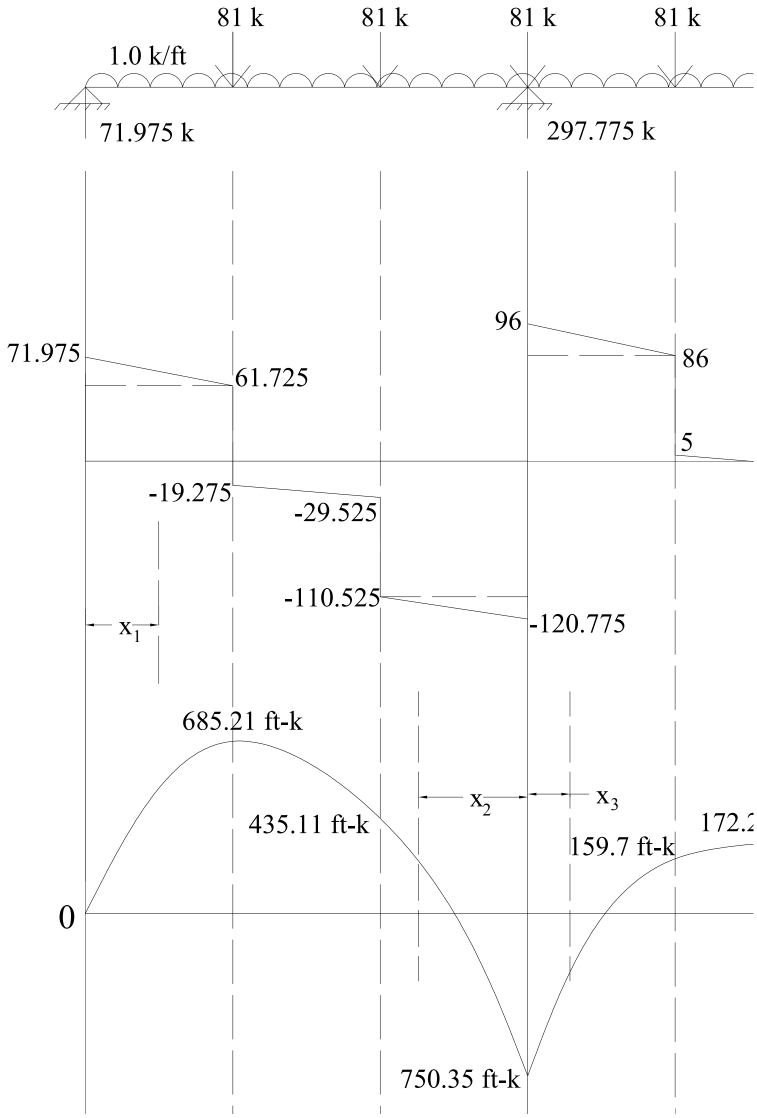


Figure 16: Shear force and bending moment diagram of half girder.

For positive bars at exterior support of exterior span: According to ACI, at least 1/4th (25%) of the positive reinforcement should continue into the support. However we continue 5 #8 bars out of 9 #8 bars, or in other words, we continue (55.5 %) of the total area of steel. Therefore:

Area of 5 #8 bars (A_s) = 5 × 0.79 = 3.95 in^2

$a = A_s f_y / (0.85 f_c' b) = 3.44$ in

$M_d = \Phi A_s f_y (d - a/2) = 4448$ in-k = 370.6 ft-k

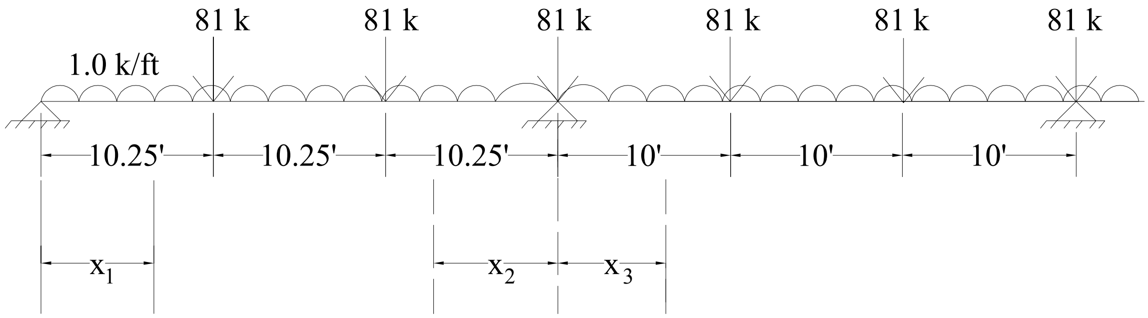


Figure 17: Exterior & interior spans of girder.

Write moment equation for exterior support of exterior span of girder.

$M_{x1} = 71.975x_1 - 1.0x_1^2/2 = 71.975x_1 - 0.5x_1^2$

At $M_{x1} = M_d = 370.6$ ft-k, $x_1 = 5'$

Actual cutoff point = x_1 – (greatest of d or $12d_b$)

= 5 – (33/12) = 2.25′ (from face of the exterior support)

For positive bars at Interior support of exterior span:

Refer to fig. 16, write moment equation for left side of interior support.

$M_{x2} = 435.11 - 110.525(10.25 - x_2) - 0.5(10.25 - x_2)^2$

At $M_{x2} = M_d = 370.6$ ft-k; $x_2 = 9.67'$

Actual cutoff point = x_2 – (greatest of d or $12d_b$)

= 9.67 – (33/12) ≈ 7′ (from face of the interior support)

No cutoff of positive bars will be done at the interior span and 5 #8 bars will be continued.

For negative bars at Interior face of interior support:

Refer to fig. 16, write moment equation for right side of interior support.

$M_{x3} = 172.2 - 86(10 - x_3) - 1.0(10 - x_3)^2/2$

At $M_{x3} = 0$; $x_3 = 8$ ft

Actual cutoff point = x_3 + (greatest of d, $12d_b$ or $l_n/16$) = 8 + (33/12) ≈ 10.75′

For negative bars at exterior face of interior support:

Moment equation for left side of interior support is:

$M_{x2} = 435.11 - 110.525(10.25 - x_2) - 0.5(10.25 - x_2)^2$

At $M_{x2} = 0$; $x_2 = 6.38$ ft

Actual cutoff point = x_2 + (greatest of d, $12d_b$ or $l_n/16$) = 6.38 + (33/12) ≈ 9.25′

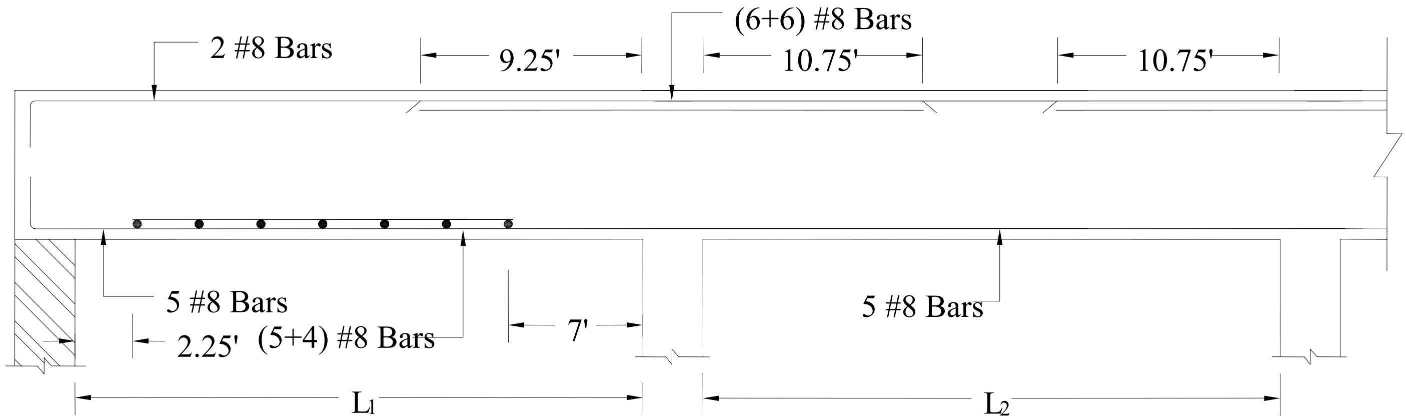


Figure 18: Bar cutoff scheme in girder.

(3) DESIGN OF COLUMN:

Gross area of column cross-section (A_g) = 18 × 18 =324 in^2

f_c' =3 ksi; f_y =40 ksi

i) Load on column:

P_u = 297.775 k (Reaction at interior support of girder due to factored load)

ii) Design:

Nominal strength (ΦP_n) of axially loaded column is:

$\Phi P_n = 0.80\Phi\{0.85f_c'(A_g–A_{st}) + A_{st}f_y\}$ {for tied column, ACI 10.3.6}

Let A_{st} = 1% of A_g (A_{st} is the main steel reinforcement area)

ΦP_n = 0.80 × 0.65 × {0.85 × 3 × (324 – 0.01 × 324) + 0.01 × 324 × 40}

= 492k > (P_u = 297.775 k), O.K.

A_{st} = 0.01 × 324 =3.24 in^2

Using 3/4″ Φ(#6) {# 19, 19 mm}, with bar area A_b =0.44 in^2

No. of bars = A_s/A_b = 3.24/0.44 = 7.36 ≈ 8 bars

Use 8 #6 bars {8 #19 bars, 19 mm}.

Tie bars: Using 3/8″ Φ (#3) {#10, 10 mm} tie bars for 3/4″ Φ (#6) {#19, 19 mm} main bars (ACI 7.10.5),

Spacing for Tie bars according to ACI 7.10.5.1 is minimum of:

(a) 16 × dia of main bar =16 × 3/4 =12″ c/c

(b) 48 × dia of tie bar = 48 × (3/8) =18″ c/c

(c) Least column dimension =18″ c/c

Finally use #3, tie bars @ 9″ c/c (#10, tie bars @ 225 mm c/c).

(5) DESIGN OF FOOTING:

Data Given:

Column size = 18″ × 18″

$f_c' = 3$ ksi

$f_y = 40$ ksi

$q_a = 2.204$ k/ft^2

Factored load on column = 297.775 k (Reaction at interior support of girder)

Step No 1: Sizes.

Assume h = 1.5′ = 18″

d_{avg} = h – 3 – (one bar dia)

= 18 – 3 – 1 =14″ (for #8 bar)

$b_o = 2\{(c + d_{avg}) + (c + d_{avg})\} = 2 \times \{(18 + 14) + (18 + 14)\} = 128$ in

Assume depth of base of footing from ground level (z) = 5′

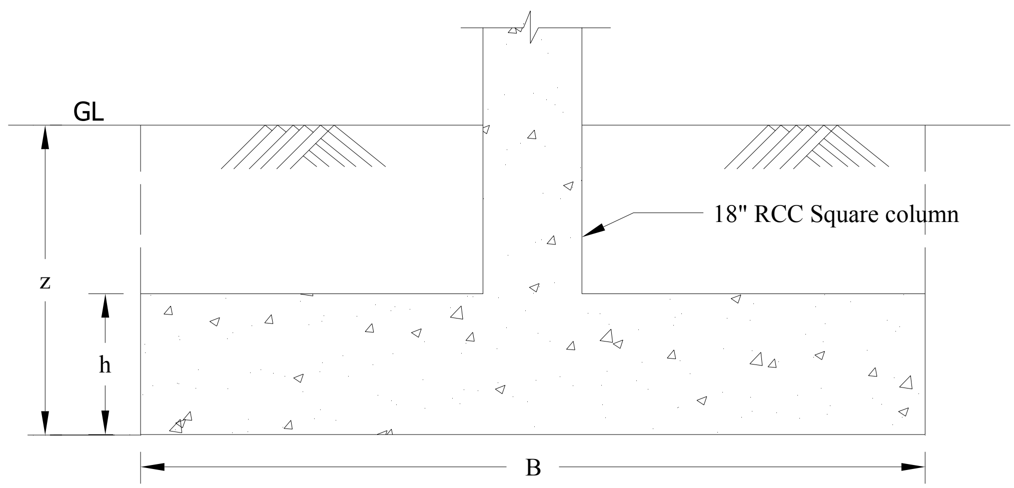


Figure 19: Footing sizes.

Since the space between the bottom of footing and the ground level is occupied partly by concrete and partly by soil, so the pressure (W) of this material at 5 ft depth is:

$W = \gamma_{fill}(z - h) + \gamma_c h$

$= 100 \times (5 – 1.5) + 150 \times (1.5) = 575$ psf

q_e (bearing pressure effective to carry column's service load):

$q_e = q_a – W$

= 2204 – 575 =1629 psf

Area of footing:

A_{req} = service load on column /q_e

Service load on column is the reaction at interior support of girder due to service load. Service Load on column can be calculated from analysis of girder under service load.

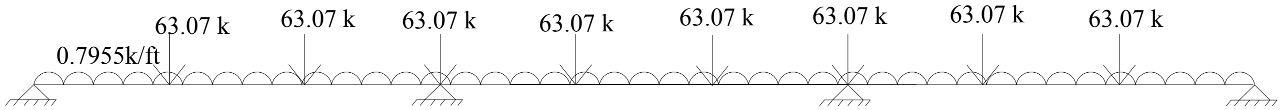


Figure 20: Service load on girder.

- P_s is the service point load and is the reaction coming from the mid support of beam.

  $P = 2\{1.15(w_s l_{nb}/2)\}$

  $= 2 \times \{1.15 \times (1.875 \times 29.25/2)\} = 63.07$ k

  Note: -

  w_s =service U.D.L on beam=ServiceD.L + ServiceL.L=1.475 + 0.4=1.875 k/ft

  l_{nb} = Clear span of beam = 29.25′

- Self weight of girder rib $(U.D.L)_{self\ wt} = h_{wg} b_{wg} \gamma_c$

  $= (30 \times 18 \times 0.15)/144 = 0.548$ k/ft

- Part of slab on girder $(U.D.L)_{sg} = w_{s(on\ slab)} \times b_{wg}$

  $= (0.125+0.04) \times 18/12 = 0.2475$ k/ft

  $w_g = (U.D.L)_{self\ wt} + (U.D.L)_{sg}$

  $= 0.548 + 0.2475 = 0.7955$ k/ft

Using the slope deflection method, the service load on column, which is the reaction at interior support of girder due to service load, comes out to be 233 k.

The service load on columns can also be calculated from ratio of service and factored loads as follows:

Service UDL on girder = 0.7955 k/ft

Factored UDL on girder = 1.0 k/ft

Ratio = Factored UDL on girder /Service UDL on girder = 1.26

Service point load on girder = 63.07 k

Factored point loads on girder = 81 k

Ratio = Factored point load on girder /Service point load on girder = 1.28

Therefore, if we divide reaction at interior support of girder by the ratio calculated above, it will give us a value of 234 k, which is very close to the actual analysis. Hence instead of detailed slope deflection method, this method can also be used.

Area of footing:

A_{req} = Service Load on column /q_e

$A_{req} = 233 / 1.629 = 142.58\ \text{ft}^2$

$B \times B = A_{req} = 142.58$

$B = 11.94' \approx 12'$

$A_{req}' = 12' \times 12'$ (Area Taken)

Step No 2: Loads.

q_u (bearing pressure for strength design of footing):

q_u = factored load on column / A_{req}'

$q_u = 297.775 / (12 \times 12) = 2.066$ ksf

Step No 3: Analysis.

(i) Punching shear:

$V_{up} = q_u B^2 - q_u(c + d_{avg})^2$

$V_{up} = 2.066 \times 12^2 - 2.066 \times \{(18+14)/12)\}^2$

$= 282.53$ k

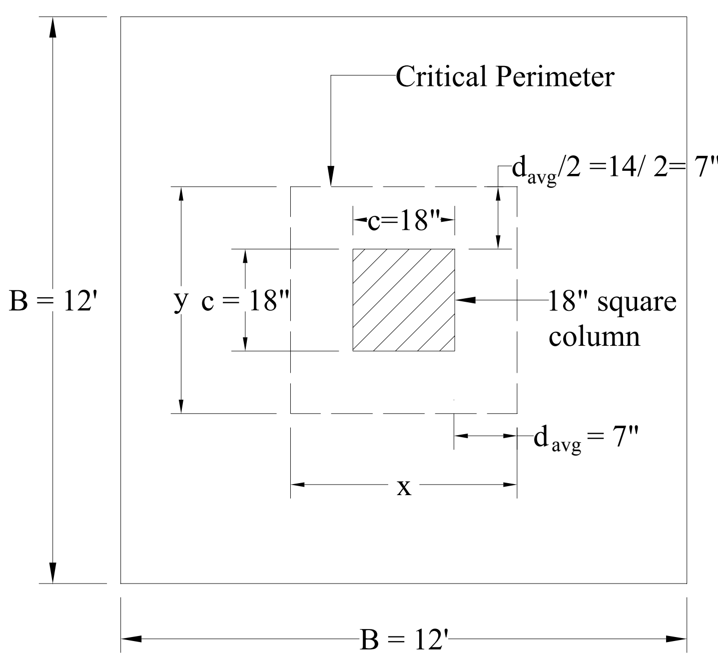


Figure 21: Critical Perimeter (for punching shear check)

(ii) Beam shear:

$V_{ud} = q_u\{((B - c)/2)\text{-}d_{avg}\}B$

$V_{ud} = 2.066 \times [\{(12\text{-}(18/12))/2\} \text{ - } (14/12)] \times 12 = 101.136$ k

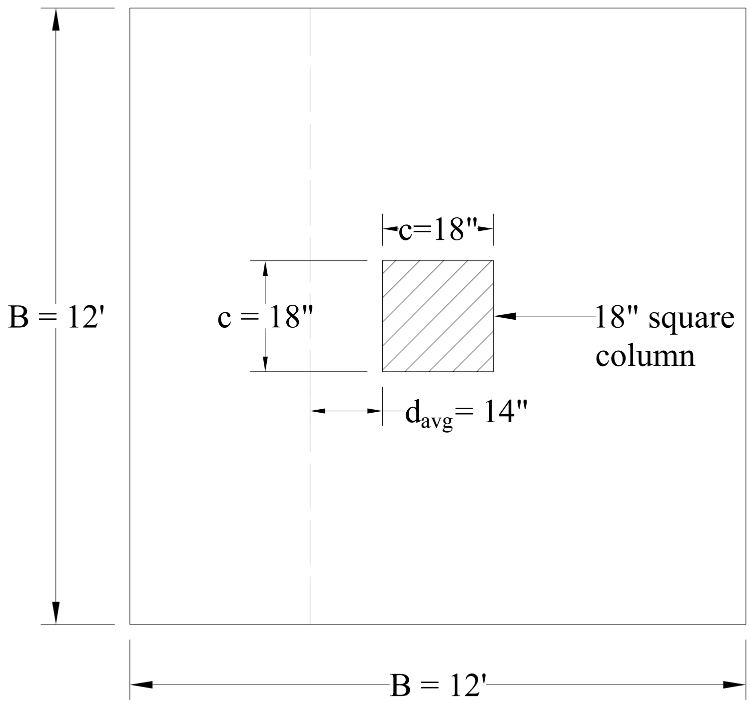


Figure 22: Beam shear calculation.

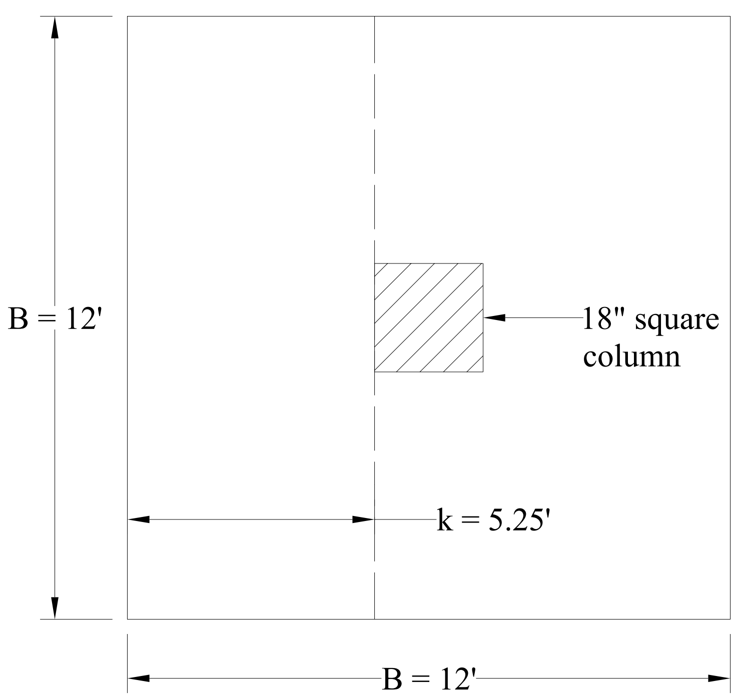


Figure 23: Bending moment calculation.

$M_u = q_u B k^2/2$

$k = (B – c)/2 = (12 \text{ x } 12 –18)/2 = 63 \text{ in} = 5.25'$

$M_u = 2.066 \times 12 \times 5.25 \times 5.25/2 = 341.33$ ft-k = 4095.95 in-k

Step No 4: Design.

(i) Design for punching shear:

$V_{up} = 282.53$ k

Punching shear capacity $(\Phi V_{cp}) = \Phi 4\sqrt{(f_c')} b_o d_{avg}$

$\Phi V_{cp} = 0.75 \times 4 \times \sqrt{(3000)} \times 128 \times 14/1000 = 294.45 \text{ k} > V_{up}$, O.K.

(ii) Design for beam shear:

$V_{ud} = 101.136$ k

Beam shear capacity $(\Phi V_{cd}) = \Phi 2\sqrt{(f_c')} B d_{avg}$

$\Phi V_{cd} = 0.75 \times 2 \times \sqrt{(3000)} \times (12 \times 12) \times 14/1000 = 165.63 \text{ k} > V_{ud}$, O.K.

(iii) Design for moment:

(a) Let $a = 0.2d_{avg} = 0.2 \times 14 = 2.8''$

$A_s = M_u/ \{\Phi f_y(d_{avg} – a/2)\}$

$A_s = 4095.95/ \{0.9 \times 40 \times (14 – 2.8/2)\} = 9.03 \text{ in}^2$

(b) $a = A_s f_y/ (0.85 f_c' B)$

$a = 9.03 \times 40/ (0.85 \times 3 \times 12 \times 12) = 0.98''$

$A_s = 4095.95/ \{0.9 \times 40 \times (14 – 0.98/2)\} = 8.42 \text{ in}^2$

(c) $a = 8.42 \times 40/ (0.85 \times 3 \times 12 \text{ x } 12) = 0.917''$

$A_s = 4095.95/ \{0.9 \times 40 \times (14 – 0.917/2)\} = 8.40 \text{ in}^2$

(d) $a = 8.40 \times 40/ (0.85 \times 3 \times 12 \times 12) = 0.915''$

$A_s = 4095.95/ \{0.9 \times 40 \times (14 – 0.915/2)\} = 8.40 \text{ in}^2$, O.K.

- Check the minimum reinforcement ratio:

$A_{smin} = \{3\sqrt{(f_c')}/f_y\} B d_{avg} \geq (200/f_y) B d_{avg}$

$= (3 \times \sqrt{(3000)} \times 12 \times 12 \times 14/40000) \geq (200/40000) \times 12 \times 12 \times 14$

$= 8.28 \text{ in}^2 < 10.08 \text{ in}^2$, not O.K.

So $A_{smin} = 10.08 \text{ in}^2$. As $A_{smin} > A_s$, thus A_{smin} governs. $A_s=10.08 \text{ in}^2$

Using 1″ Φ (#8) {#25, 25 mm}, with bar area $A_b = 0.79 \text{ in}^2$

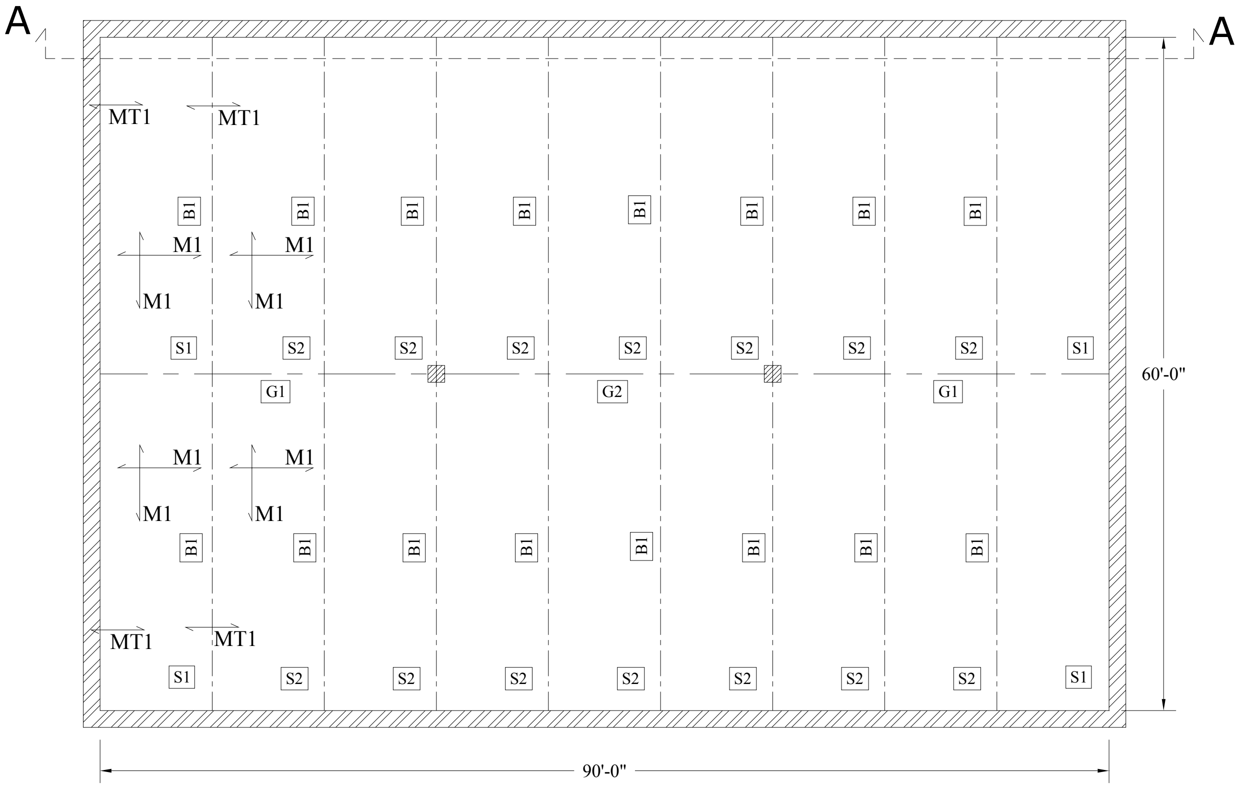
Spacing $= B \times A_b/A_{smin}$

$= 12 \times 12 \times 0.79/10.08 = 11.28$ in c/c ≈ 11 in c/c

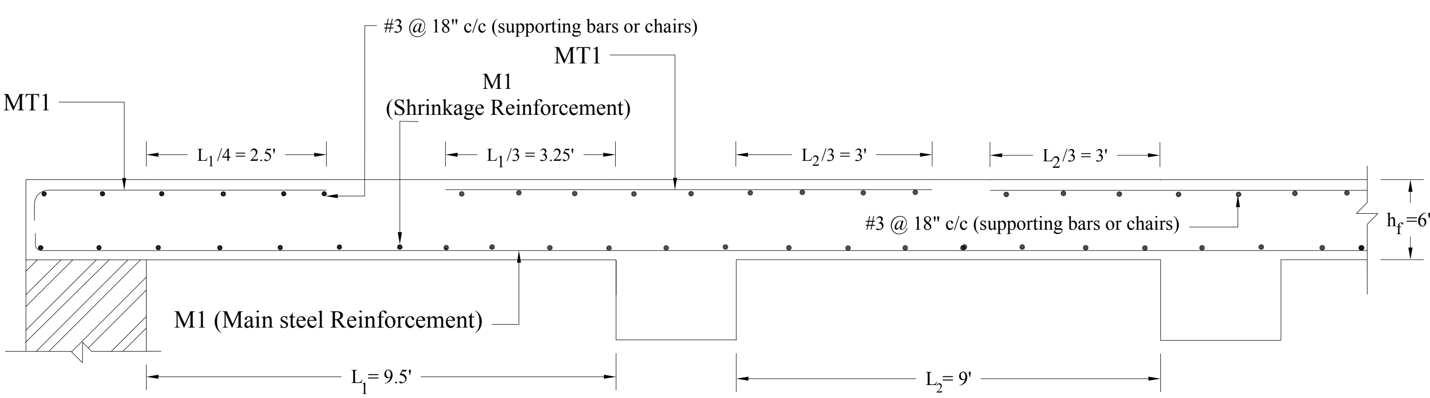
Use #8 @ 11″ c/c {#25 @ 275 mm c/c} both ways

(6) DRAFTING:

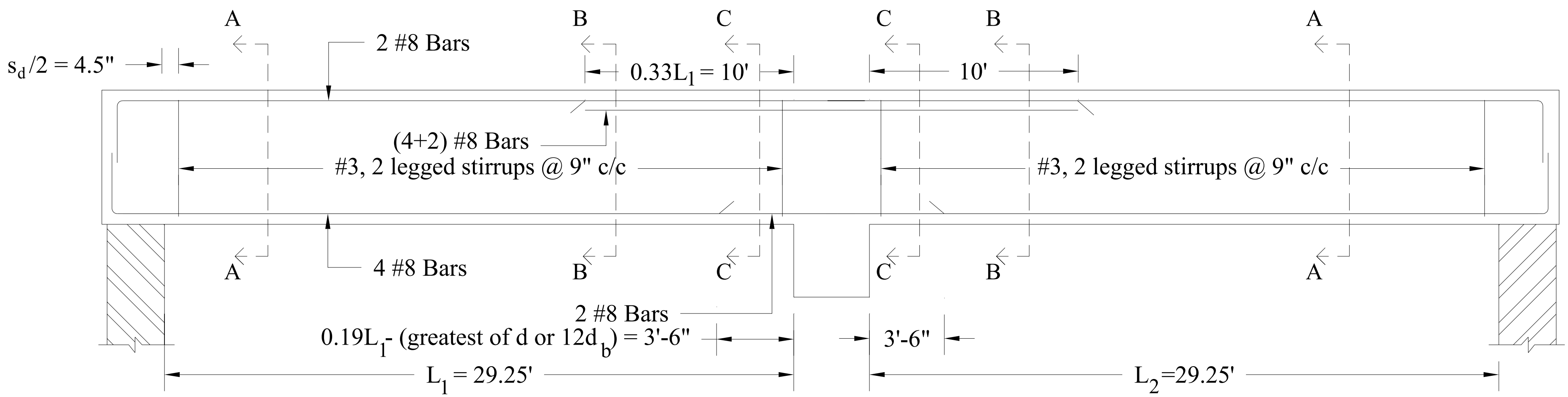
Slab S1 and S2:



Panel	Depth (in)	Mark	Bottom Reinforcement	Mark	Top reinforcement	
S1	6"	M1	3/8" Φ@ 9" c/c	MT1	3/8" Φ @ 9" c/c	Non continuous End
S2	6"	M1	3/8" Φ @ 9" c/c	MT1	3/8" Φ @ 9" c/c	Continuous End



Section A-A. Refer to figure 5.15, chapter 5, Nelson 13th Ed for bar cutoff.

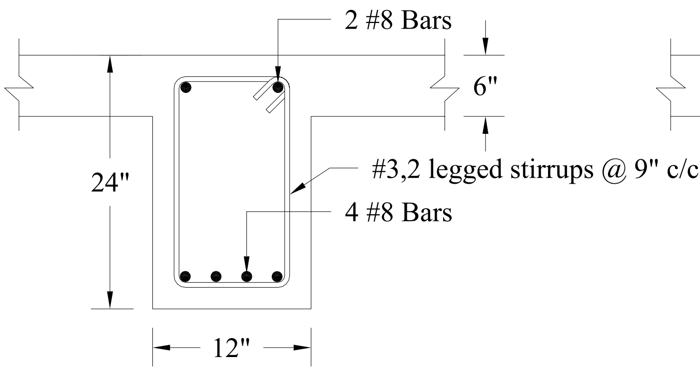


BEAM (B1)

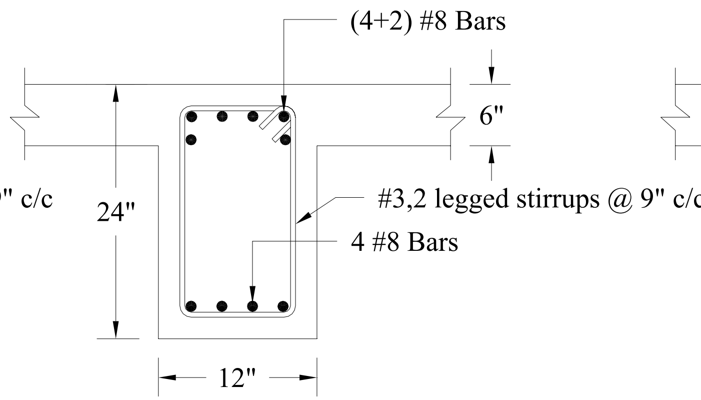
Notes: -

(1) Use graph A.3, Nelson 13th Ed for location of cut off for continuous beams.

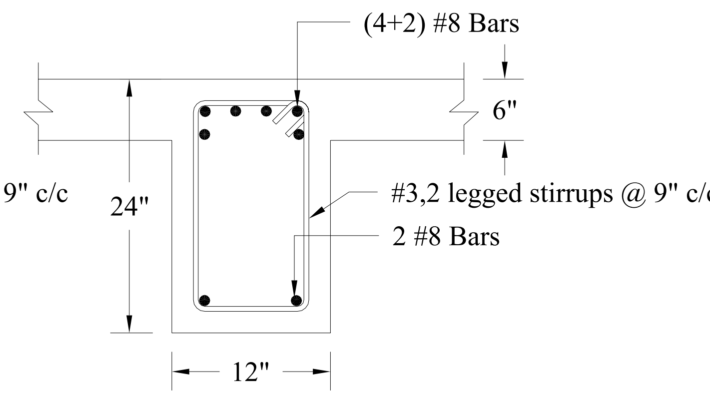
(2) Use table A.7, Nelson 13th Ed for maximum number of bars as a single layer in beam stem.



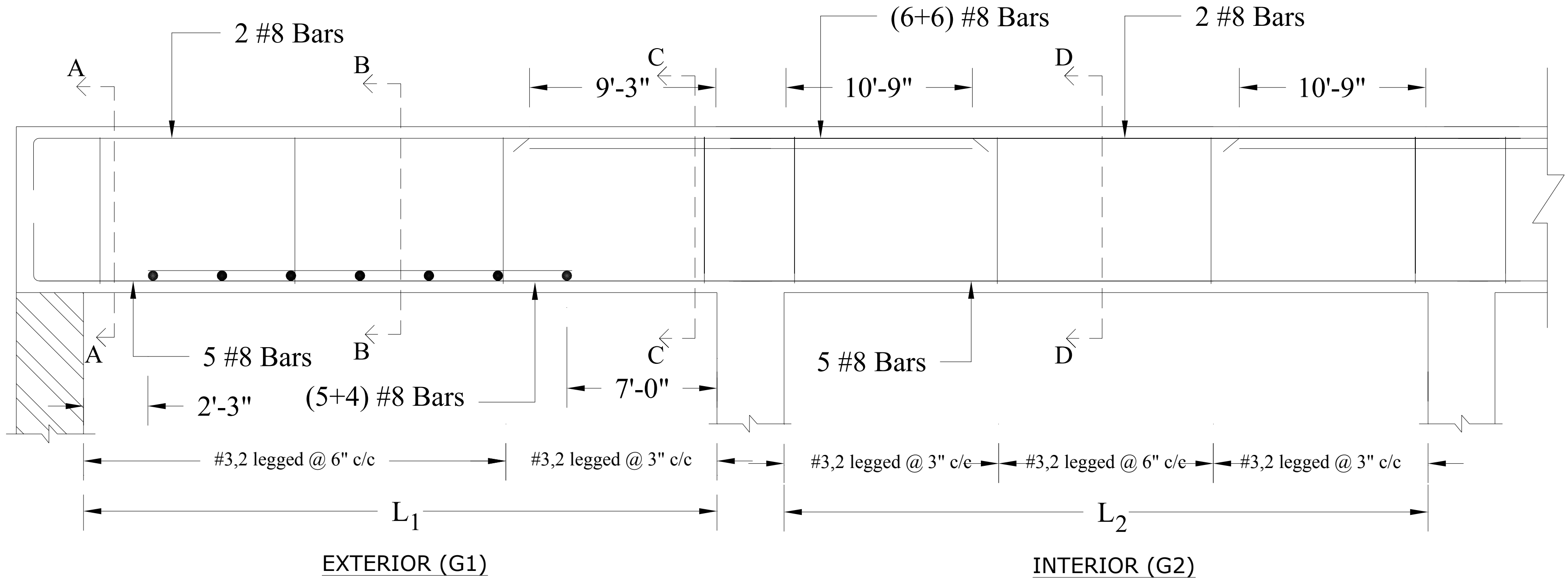
SECTION A-A



SECTION B-B

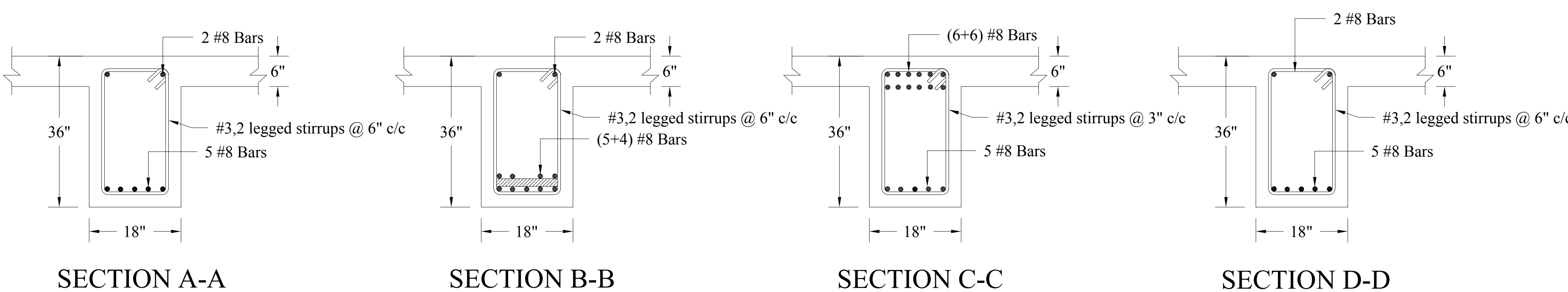


SECTION C-C

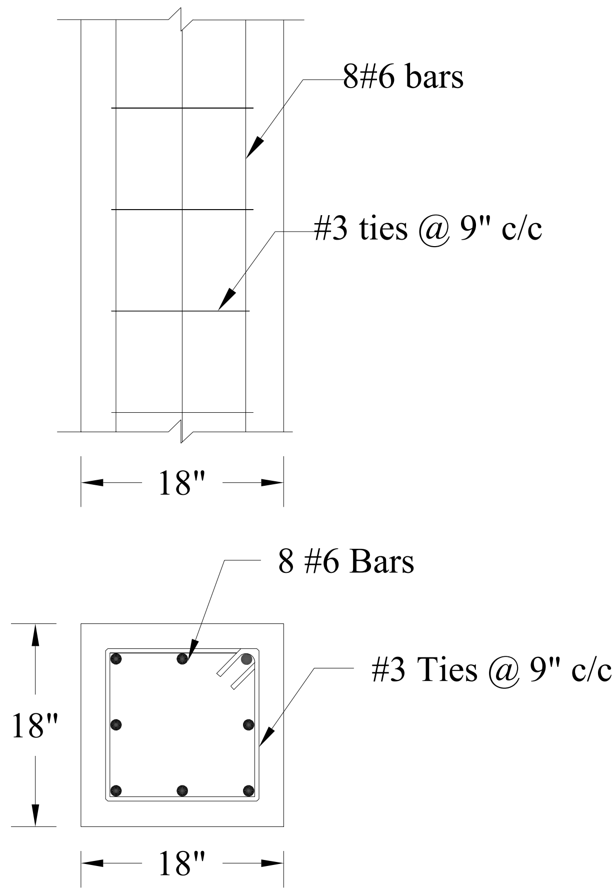


Notes: -

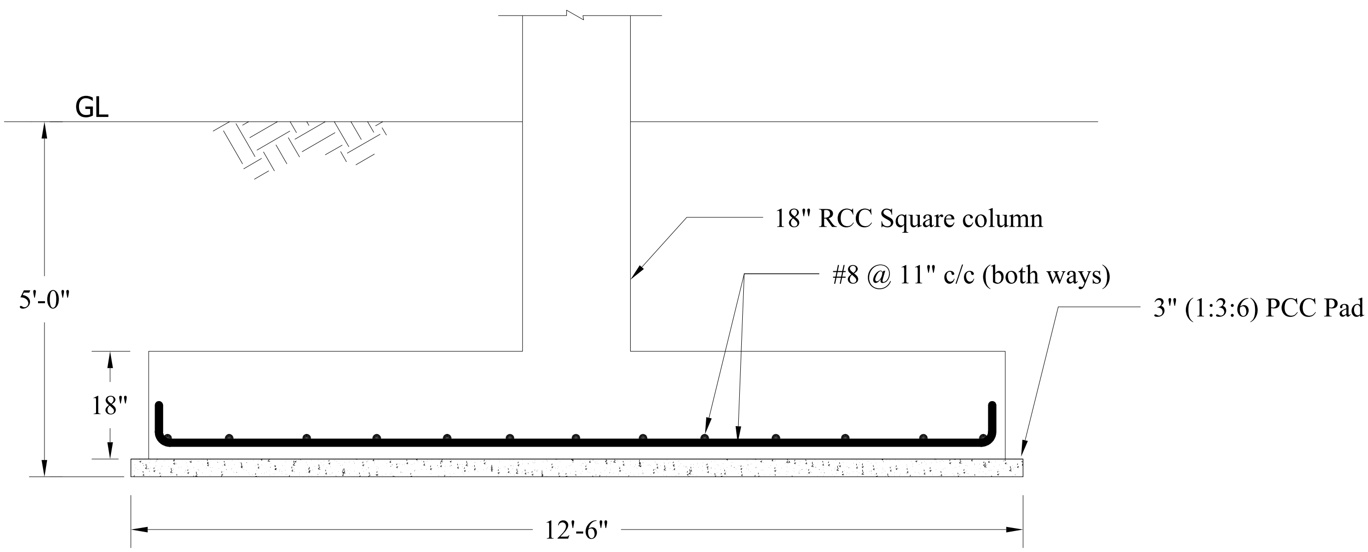
(1) Use Table A.&, Nelson 13th Ed for maximum number of bars as single layer in beam stem



Column:



Footing:



FOOTING DETAIL

References

- *Design of Concrete Structures by Nilson, Darwin and Dolan (13th ed.)*
- *ACI 318-02/05*