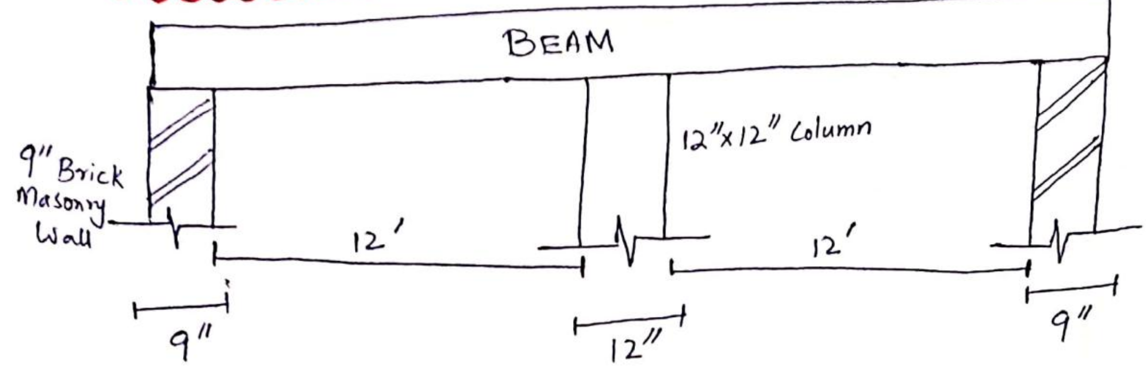
BEAM DESIGN



c/c distance b/w supports $= l = 12' + \frac{\frac{12''}{2}}{12} + \frac{\frac{9''}{2}}{12} = 12' + 0.5' + 0.375' = 12.875'$

Given data:-

Exterior Support = 9" Brick Masonry wall

$f_c' = 3\text{ Ksi} = 3000\text{ psi}$

$f_y = 40\text{ ksi} = 40000\text{ psi}$

Interior Support = Column Support = 12" x 12"

Step 1:- Sizes:-

$l = 12.875'$ (c/c distance b/w supports)

Minimum thickness of beam (ACI 9.5.2.1)

$h_{min} = 1.5' = 18''$

Also $h = \frac{l}{18.5}\left(0.4 + \frac{f_y}{100000}\right) = \frac{12.875}{18.5}\left(0.4 + \frac{40{,}000}{100000}\right) \times 12$

$h_{actual} = 6.68''$

Also Table 4.1 $h_{actual} = \frac{l}{28} = \frac{12.875}{28} = 0.46 \times 12 = 5.52''$

So Minimum thickness $h_{min} = 1.5'' = 18''$ will govern

effective depth $= d = h - 3 = 18 - 3 = 15''$

$d = 1.25'$

TABLE 4.1 Minimum Thickness of Nonprestressed Beams or One-Way Slabs Unless Deflections Are Computed[1,2]

	Minimum Thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Step# 2:- Loads

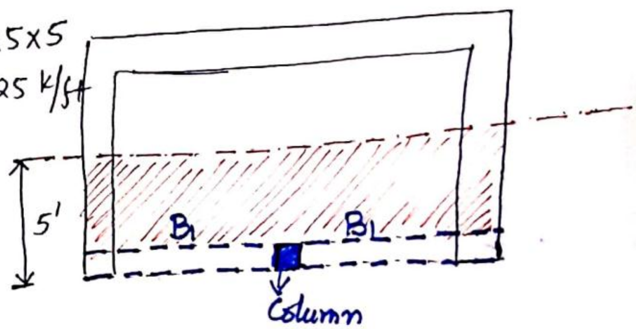
Material	thickness (inches)	γ (kcf)	load = γ x thickness
Slab	5	0.15	$0.15 \times \frac{5}{12} = 0.0625$
Mud	4	0.12	$0.12 \times \frac{4}{12} = 0.04$
Brick tile	2	0.12	$0.12 \times \frac{2}{12} = 0.02$

Service Dead Load (D.L) = 0.0625 + 0.04 + 0.02 = 0.1225 ksf

Service live Load (L.L) = 40 psf = 0.04 ksf

Beam is supporting 5′ slab (approximately) per running foot therefore

Service Dead load from Slab = 0.1225 x 5
= 0.6125 k/ft



→ Per running foot

Self wt of beam = $h \times b_w \gamma_c$

$$= \left(\frac{18 \times 12}{144}\right) \times 0.15$$

= 0.225 k/ft

Total Dead load = 0.6125 + 0.225
= 0.8375 k/ft

Also Service live load for 5′ Slab per running foot

Service live Load = 0.04 x 5 = 0.2 k/ft

Factored load:-

$$W_u = 1.2\ D.L + 1.6\ L.L$$

$$W_u = (1.2 \times 0.8375) + (1.6 \times 0.2)$$

$$\boxed{W_u = 1.325\ k/ft}$$

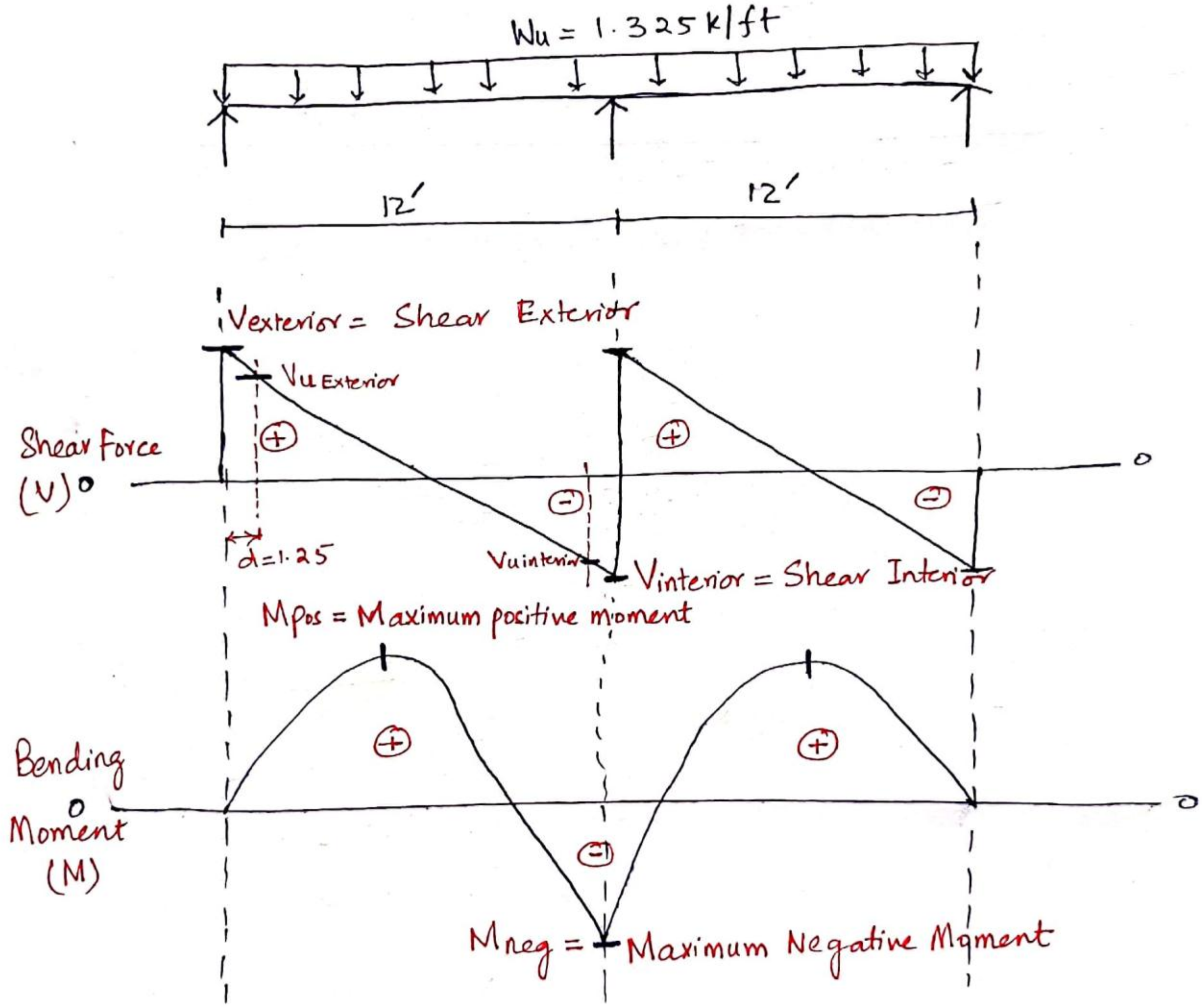
TABLE 12.1
Moment and shear values using ACI coefficients†

Positive moment	
End spans	
If discontinuous end is unrestrained	$\frac{1}{11} w_u l_n^2$
If discontinuous end is integral with the support	$\frac{1}{14} w_u l_n^2$
Interior spans	$\frac{1}{16} w_u l_n^2$
Negative moment at exterior face of first interior support	
Two spans	$\frac{1}{9} w_u l_n^2$
More than two spans	$\frac{1}{10} w_u l_n^2$
Negative moment at other faces of interior supports	$\frac{1}{11} w_u l_n^2$
Negative moment at face of all supports for (1) slabs with spans not exceeding 10 ft and (2) beams and girders where ratio of sum of column stiffness to beam stiffness exceeds 8 at each end of the span	$\frac{1}{12} w_u l_n^2$
Negative moment at interior faces of exterior supports for members built integrally with their supports	
Where the support is a spandrel beam or girder	$\frac{1}{24} w_u l_n^2$
Where the support is a column	$\frac{1}{16} w_u l_n^2$
Shear in end members at first interior support	$1.15 \frac{w_u l_n}{2}$
Shear at all other supports	$\frac{w_u l_n}{2}$

† w_u = total factored load per unit length of beam or per unit area of slab.
l_n = clear span for positive moment and shear and the average of the two adjacent clear spans for negative moment.

Step # 3:- Shear and Moments

Using Table 12.1 Moments and shear values using ACI coefficients (Nelson 14th Edition) Design of Concrete Structures (Page # 407) - Attached



Shears:- $V_{Exterior} = \dfrac{W_u l_n}{2} = \dfrac{1.325 \times 12}{2} = 7.95\,K$

$$V_{u\,Exterior} = V_{Ext} - d^2 = 7.95 - (1.25)^2 = 6.39\,K$$

$$V_{interior} = 1.15\,\frac{W_u l_n}{2} = \frac{1.15 \times 1.325 \times 12}{2} = 9.14\,K$$

$$V_{u\,interior} = V_{interior} - d^2 = 9.14 - (1.25)^2 = 7.58\,K$$

Moments:-

Negative Moment = Coefficient × $W_u l_n^2$

$M_{neg} = \frac{1}{9} \times (1.325 \times 12^2)$

$M_{neg} = 21.2$ ft-K ×12

$\boxed{M_{neg} = 254.4 \text{ in-K}}$

Positive Moment = Coefficient × $W_u l_n^2$

$M_{pos} = \frac{1}{11} \times (1.325 \times 12^2)$

$M_{pos} = 17.34$ ft-K ×12

$\boxed{M_{pos} = 208.14 \text{ in-K}}$

Step#4 Design (Moment/Flexure)

Flexural Design

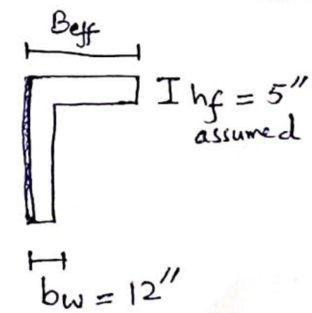
1 For Positive Moment:-

Step(a):- According to ACI 8.10, beff for L-beam is minimum of

(i) $6h_f + b_w = 6(5) + 12 = 42''$

(ii) $b_w + \frac{\text{Span length of beam}}{12} = 12 + \frac{(12.875 \times 12)}{12}$

$= 24.875''$



So $\boxed{b_{eff} = 24.875''}$

Step(b):- Check if beam is to be designed as rectangular beam or L beam

∴ Assumed $a = h_f = 5''$

Trial #1:- $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{208.14}{0.9 \times 40 (15 - \frac{5}{2})} \Rightarrow \boxed{A_s = 0.462 \text{ in}^2}$

Re-Calculate $a = \frac{A_s f_y}{0.85 f_c' b_{eff}} = \frac{0.462 \times 40}{0.85 \times 3 \times 24.875} \Rightarrow a = 0.29'' < h_f = 5''$ therefore Design Beam (Rectangular)

Trial #2:-

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{208.14}{0.9 \times 40 (15 - \frac{0.29}{2})} \Rightarrow \boxed{A_s = 0.389 \text{ in}^2}$$

$$a = \frac{A_s f_y}{0.85 \times f_c' b_{eff}} = \frac{0.389 \times 40}{0.85 \times 3 \times 24.875} \Rightarrow \boxed{a = 0.25''}$$

Value is close enough to the previously calculated value of "a" therefore

$$\boxed{A_{s_{actual}} = 0.389 \text{ in}^2}$$

Step(C):- Check for maximum and minimum Reinforcement

$$A_{s\,min} = \rho_{min} b_w d = 0.005 \times 12 \times 15$$

$$\boxed{A_{s\,min} = 0.9 \text{ in}^2}$$

$$A_{s\,actual} (0.389 \text{ in}^2) < A_{s\,min} (0.9 \text{ in}^2)$$

So $\boxed{A_{s\,Selected} = 0.9 \text{ in}^2}$

Using Appendix A Table A.12
use 3#5 bars ($A_s = 0.91 \text{ in}^2$)

Check for No of bars:-

$$\text{No of bar} = \frac{A_s}{A_b}$$

$$\text{No of bar} = \frac{0.91}{0.31} = 2.94$$

No of bars $\cong$ 3 $\boxed{\text{OK}}$

$$A_b = \frac{\pi}{4} d^2$$

$$d = \frac{5}{8} = 0.625''$$

$$A_b = \frac{\pi}{4} \times (0.625)^2$$

$$\boxed{A_b = 0.31 \text{ in}^2}$$

TABLE A.4 Areas of Groups of Standard Bars (in.^{2}) — U.S. Customary Units

Bar No.	Number of Bars 2	3	4	5	6	7	8	9	10
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

Bar No.	Number of Bars 11	12	13	14	15	16	17	18	19	20
4	2.16	2.36	2.55	2.75	2.95	3.14	3.34	3.53	3.73	3.93
5	3.37	3.68	3.99	4.30	4.60	4.91	5.22	5.52	5.83	6.14
6	4.86	5.30	5.74	6.19	6.63	7.07	7.51	7.95	8.39	8.84
7	6.61	7.22	7.82	8.42	9.02	9.62	10.22	10.82	11.43	12.03
8	8.64	9.43	10.21	11.00	11.78	12.57	13.35	14.14	14.92	15.71
9	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00
10	13.92	15.19	16.45	17.72	18.98	20.25	21.52	22.78	24.05	25.31
11	17.19	18.75	20.31	21.87	23.44	25.00	26.56	28.12	29.69	31.25
14	24.75	27.00	29.25	31.50	33.75	36.00	38.25	40.50	42.75	45.00
18	44.00	48.00	52.00	56.00	60.00	64.00	68.00	72.00	76.00	80.00

2 For Negative Moment :-

$$M_{neg} = M_u = 254.4 \text{ in-K}$$

$$b_w = 12'', \quad h = 18'', \quad d = 15''$$

trial#1:- $A_s = \dfrac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$ let $a = 0.2d = 0.2 \times 15$, $a = 3''$

$$A_s = \frac{254.4}{0.9 \times 40 \left(15 - \frac{3}{2}\right)} = 0.52 \text{ in}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{0.52 \times 40}{0.85 \times 3 \times 12} = 0.68''$$

trial #2:-

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{254.4}{0.9 \times 40 \left(15 - \frac{0.68}{2}\right)} = 0.48 \text{ in}^2$$

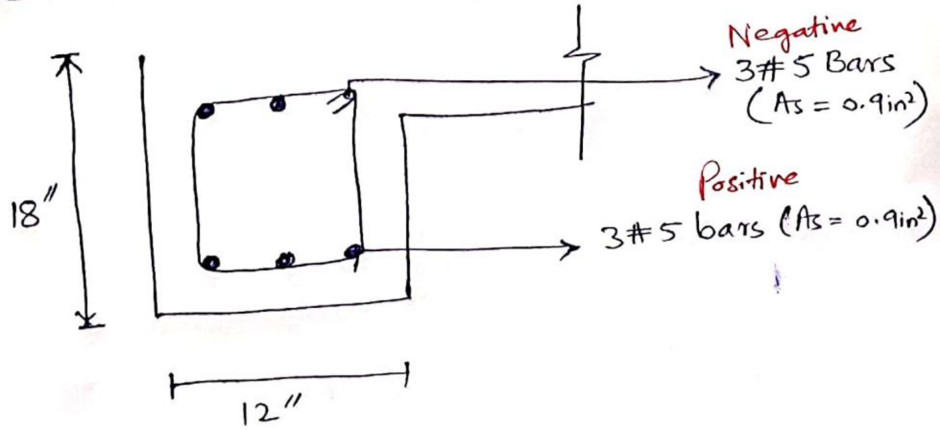
$$a = \frac{A_s f_y}{0.85 f_c' b_w} = \frac{0.48 \times 40}{0.85 \times 3 \times 12} = 0.63''$$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{254.4}{0.9 \times 40 \left(15 - \frac{0.63}{2}\right)} = 0.48 \text{ in}^2$$

$\boxed{A_{s\,actual} = 0.48 \text{ in}^2} < A_{s\,min}\ (0.9 \text{ in}^2)$

So $\boxed{A_{s\,min} = 0.9 \text{ in}^2}$

Use 3#5 bars ($A_s = 0.91 \text{ in}^2$) Appendix A Table A.12



Step# 5:- Shear Design for beam

$d = 15'' = 1.25'$

V_u (exterior) $= 6.39\,K$

V_u (interior) $= 7.58\,K$

(a) Check whether shear Reinforcement is needed or Not

$$\phi V_c = \frac{\phi 2\sqrt{f_c'}\,b_w d}{1000} = 0.75 \times 2\sqrt{3000} \times 12 \times 15/1000$$

$$\phi V_c = 14.78\,K > V_{u(Ext)} \text{ and } V_{u\,(Interior)}$$

theoretically, no shear Reinforcement is required but minimum will be provided.

(b) Minimum Spacing of Stirrups ACI 11.5.4 and 11.5.5.3

(i) $\dfrac{A_v f_y}{50 b_w} = \dfrac{0.22 \times 40000}{50 \times 12} = 14.67''$ c/c

(ii) $\dfrac{d}{2} = \dfrac{15}{2} = 7.5''$ c/c

(iii) $24''$ c/c

(iv) $\dfrac{A_v f_y}{0.75\sqrt{f_c'}\,b_w} = \dfrac{0.22 \times 40000}{0.75 \times \sqrt{3000} \times 12} = 17.85''$ c/c

$A_v = 2 \times$ Area of Assumed bar

Assumed bar #3

$\frac{\pi}{4} \times \left(\frac{3}{8}\right)^2 = 0.11''$

$A_v = 2 \times 0.11$

$A_v = 0.22\ in^2$

Minimum of Above

$$\boxed{S = 7.5''\ c/c}$$

(c) Check For Spacing under "Maximum Spacing requirement of ACI"

$$\phi V_s \leq \frac{\phi 4\sqrt{f_c'}\,b_w d}{1000} \quad \{ACI\ (11.5.4.3)\}$$

$$\phi 4\sqrt{f_c'}\,b_w d = \frac{0.75 \times 4 \times \sqrt{3000} \times 12 \times 15}{1000} = 29.54K$$

$$\phi V_s = \frac{\phi A_v f_y d}{S} = \frac{0.75 \times 0.22 \times 40 \times 15}{7.5} = 13.2K < 29.54K$$

$\boxed{OK}$

There fore Spacing given under "Maximum Spacing requirement of ACI" is OK-
Other wise reduce spacing by half.

(d) Check for depth of beam {ACI 11.5.6.9}

$$\phi V_s \leq \frac{\phi 8\sqrt{f_c'}\, b_w d}{1000}$$

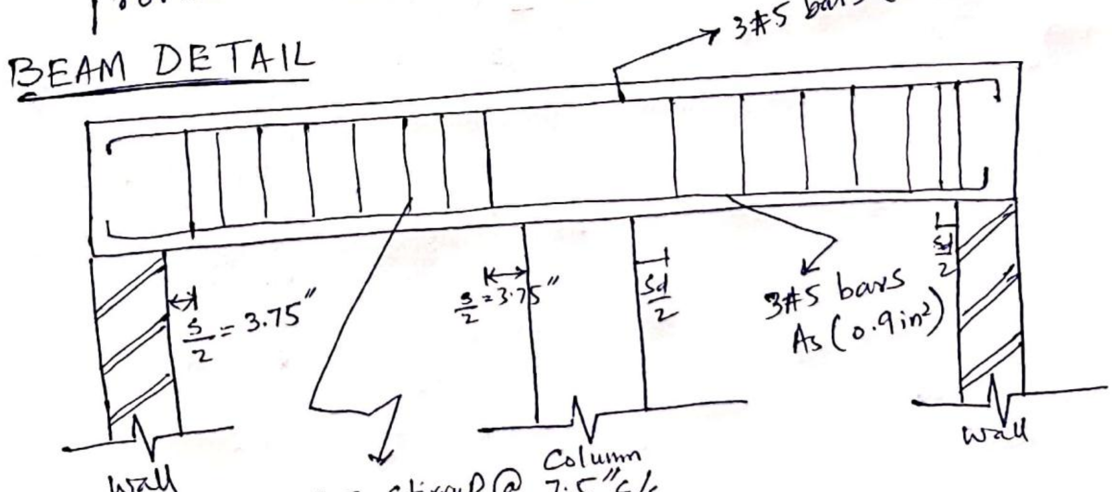
$$\frac{\phi 8\sqrt{f_c'}\, b_w d}{1000} = \frac{0.75 \times 8 \times \sqrt{3000} \times 12 \times 15}{1000} = 59.15\,K$$

$$\phi V_s = \frac{\phi A_v f_y d}{S} = \frac{0.75 \times 0.22 \times 40 \times 15}{7.5} = 13.2\,K < 59.15\,K \quad \boxed{OK}$$

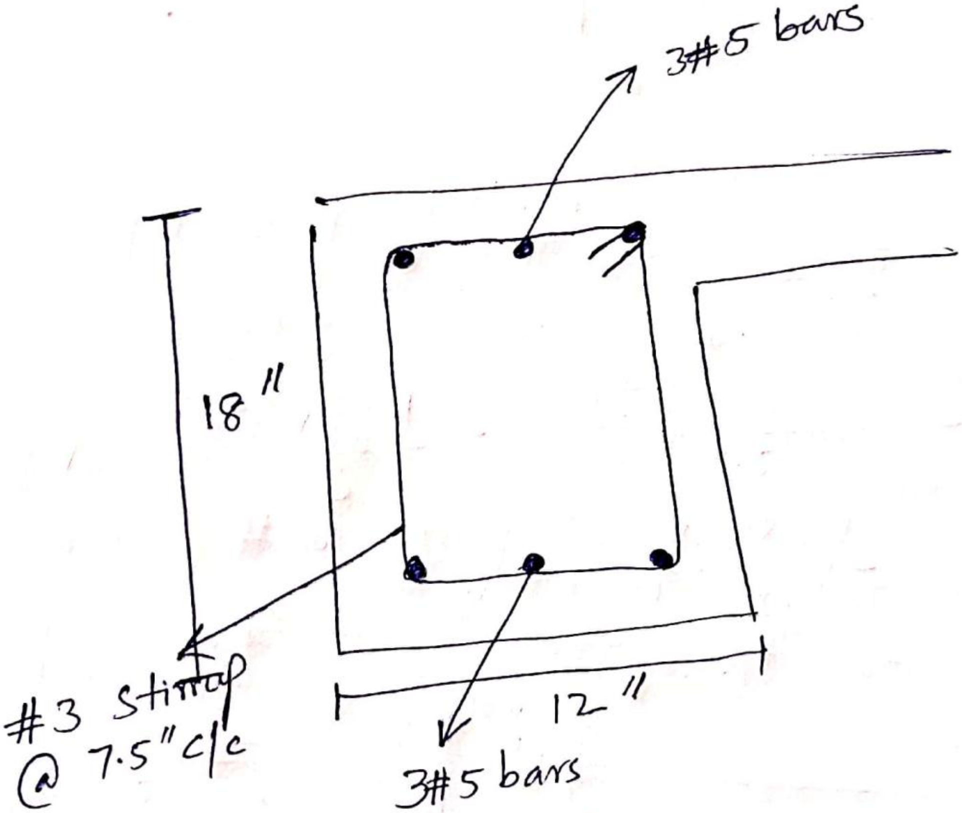
So depth is OK, if Not increase depth of beam

Provide #3 stirrup @ 7.5" c/c

BEAM DETAIL



$\frac{Sd}{2} = \frac{7.5}{2} = 3.75"$ from the face of the support



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