Digital Logic & design (Theory) Lecture No.6

CHAPTER OUTLINE

4–7 Boolean Expressions and Truth Tables

All standard Boolean expressions can be easily converted into truth table format using binary values for each term in the expression. The truth table is a common way of presenting, in a concise format, the logical operation of a circuit. Also, standard SOP or POS expressions can be determined from a truth table. You will find truth tables in data sheets and other literature related to the operation of digital circuits.

Converting SOP Expressions to Truth Table Format

Recall from Section 4–6 that an SOP expression is equal to 1 only if at least one of the product terms is equal to 1. A truth table is simply a list of the possible combinations of input variable values and the corresponding output values (1 or 0). For an expression with a domain of two variables, there are four different combinations of those variables ($2^2 = 4$). For an expression with a domain of three variables, there are eight different combinations of those variables ($2^3 = 8$). For an expression with a domain of four variables, there are sixteen different combinations of those variables ($2^4 = 16$), and so on.

The first step in constructing a truth table is to list all possible combinations of binary values of the variables in the expression. Next, convert the SOP expression to standard form if it is not already. Finally, place a 1 in the output column (X) for each binary value that makes the standard SOP expression a 1 and place a 0 for all the remaining binary values. This procedure is illustrated in Example 4–20.

Converting POS Expressions to Truth Table Format

Recall that a POS expression is equal to 0 only if at least one of the sum terms is equal to 0. To construct a truth table from a POS expression, list all the possible combinations of binary values of the variables just as was done for the SOP expression. Next, convert the POS expression to standard form if it is not already. Finally, place a 0 in the output column (X) for each binary value that makes the expression a 0 and place a 1 for all the remaining binary values. This procedure is illustrated in Example 4–21.

EXAMPLE 4–20

Develop a truth table for the standard SOP expression $\overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + ABC$.

Solution

There are three variables in the domain, so there are eight possible combinations of binary values of the variables as listed in the left three columns of Table 4–6. The binary values that make the product terms in the expressions equal to 1 are

TABLE 4–6

Inputs			Output	
A	B	C	X	Product Term
0	0	0	0	
0	0	1	1	$\overline{A}\,\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\,\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

$\overline{A}\,\overline{B}C$: 001; $A\overline{B}\,\overline{C}$: 100; and ABC: 111. For each of these binary values, place a 1 in the output column as shown in the table. For each of the remaining binary combinations, place a 0 in the output column.

EXAMPLE 4–21

Determine the truth table for the following standard POS expression:

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

Solution

There are three variables in the domain and the eight possible binary values are listed in the left three columns of Table 4–7. The binary values that make the sum terms in the expression equal to 0 are $A + B + C$: 000; $A + \overline{B} + C$: 010; $A + \overline{B} + \overline{C}$: 011; $\overline{A} + B + \overline{C}$: 101; and $\overline{A} + \overline{B} + C$: 110. For each of these binary values, place a 0 in the output column as shown in the table. For each of the remaining binary combinations, place a 1 in the output column.

TABLE 4–7

Inputs			Output	
A	B	C	X	Sum Term
0	0	0	0	$(A + B + C)$
0	0	1	1	
0	1	0	0	$(A + \overline{B} + C)$
0	1	1	0	$(A + \overline{B} + \overline{C})$
1	0	0	1	
1	0	1	0	$(\overline{A} + B + \overline{C})$
1	1	0	0	$(\overline{A} + \overline{B} + C)$
1	1	1	1	

Notice that the truth table in this example is the same as the one in Example 4–20. This means that the SOP expression in the previous example and the POS expression in this example are equivalent.

Determining Standard Expressions from a Truth Table

To determine the standard SOP expression represented by a truth table, list the binary values of the input variables for which the output is 1. Convert each binary value to the corresponding product term by replacing each 1 with the corresponding variable and each 0 with the corresponding variable complement. For example, the binary value 1010 is converted to a product term as follows:

$$1010 \longrightarrow A\overline{B}C\overline{D}$$

If you substitute, you can see that the product term is 1:

$$A\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

To determine the standard POS expression represented by a truth table, list the binary values for which the output is 0. Convert each binary value to the corresponding sum term by replacing each 1 with the corresponding variable complement and each 0 with the corresponding variable. For example, the binary value 1001 is converted to a sum term as follows:

$$1001 \longrightarrow \overline{A} + B + C + \overline{D}$$

If you substitute, you can see that the sum term is 0:

$$\overline{A} + B + C + \overline{D} = \overline{1} + 0 + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

EXAMPLE 4–22

From the truth table in Table 4–8, determine the standard SOP expression and the equivalent standard POS expression.

TABLE 4–8

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Solution

There are four 1s in the output column and the corresponding binary values are 011, 100, 110, and 111. Convert these binary values to product terms as follows:

$$011 \longrightarrow \overline{A}BC$$
$$100 \longrightarrow A\overline{B}\,\overline{C}$$
$$110 \longrightarrow AB\overline{C}$$
$$111 \longrightarrow ABC$$

The resulting standard SOP expression for the output X is

$$X = \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

For the POS expression, the output is 0 for binary values 000, 001, 010, and 101. Convert these binary values to sum terms as follows:

$$000 \longrightarrow A + B + C$$
$$001 \longrightarrow A + B + \overline{C}$$
$$010 \longrightarrow A + \overline{B} + C$$
$$101 \longrightarrow \overline{A} + B + \overline{C}$$

The resulting standard POS expression for the output X is

$$X = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})$$

4–8 The Karnaugh Map

A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression. As you have seen, the effectiveness of algebraic simplification depends on your familiarity with all the laws, rules, and theorems of Boolean algebra and on your ability to apply them. The Karnaugh map, on the other hand, provides a "cookbook" method for simplification. Other simplification techniques include the Quine-McCluskey method and the Espresso algorithm.

A **Karnaugh map** is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value. Instead of being organized into columns and rows like a truth table, the Karnaugh map is an array of **cells** in which each cell represents a binary value of the input variables. The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells. Karnaugh maps can be used for expressions with two, three, four, and five variables, but we will discuss only 3-variable and 4-variable situations to illustrate the principles. *A discus-sion of 5-variable Karnaugh maps is available on the website*.

The number of cells in a Karnaugh map, as well as the number of rows in a truth table, is equal to the total number of possible input variable combinations. For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.

The 3-Variable Karnaugh Map

The 3-variable Karnaugh map is an array of eight cells, as shown in Figure 4–25(a). In this case, A, B, and C are used for the variables although other letters could be used. Binary values of A and B are along the left side (notice the sequence) and the values of C are across the top. The value of a given cell is the binary values of A and B at the left in the same row combined with the value of C at the top in the same column. For example, the cell in the upper left corner has a binary value of 000 and the cell in the lower right corner has a binary value of 101. Figure 4–25(b) shows the standard product terms that are represented by each cell in the Karnaugh map.

AB \ C	0	1
00		
01		
11		
10		

(a)

AB \ C	0	1
00	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$
01	$\overline{A}B\overline{C}$	$\overline{A}BC$
11	$AB\overline{C}$	ABC
10	$A\overline{B}\,\overline{C}$	$A\overline{B}C$

(b)

FIGURE 4–25 A 3-variable Karnaugh map showing Boolean product terms for each cell.

The 4-Variable Karnaugh Map

The 4-variable Karnaugh map is an array of sixteen cells, as shown in Figure 4–26(a). Binary values of *A* and *B* are along the left side and the values of *C* and *D* are across the top. The value of a given cell is the binary values of *A* and *B* at the left in the same row combined with the binary values of *C* and *D* at the top in the same column. For example, the cell in the upper right corner has a binary value of 0010 and the cell in the lower right corner has a binary value of 1010. Figure 4–26(b) shows the standard product terms that are represented by each cell in the 4-variable Karnaugh map.

Cell Adjacency

The cells in a Karnaugh map are arranged so that there is only a single-variable change between adjacent cells. **Adjacency** is defined by a single-variable change. In the 3-variable map the 010 cell is adjacent to the 000 cell, the 011 cell, and the 110 cell. The 010 cell is not adjacent to the 001 cell, the 111 cell, the 100 cell, or the 101 cell.

Physically, each cell is adjacent to the cells that are immediately next to it on any of its four sides. A cell is not adjacent to the cells that diagonally touch any of its corners. Also, the cells in the top row are adjacent to the corresponding cells in the bottom row and

AB \ *CD*	00	01	11	10
00				
01				
11				
10				

(a)

AB \ *CD*	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}\overline{B}CD$	$\overline{A}\overline{B}C\overline{D}$
01	$\overline{A}B\overline{C}\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
11	$AB\overline{C}\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\overline{C}\overline{D}$	$A\overline{B}\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

(b)

FIGURE 4–26 A 4-variable Karnaugh map.

the cells in the outer left column are adjacent to the corresponding cells in the outer right column. This is called "wrap-around" adjacency because you can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder. Figure 4–27 illustrates the cell adjacencies with a 4-variable map, although the same rules for adjacency apply to Karnaugh maps with any number of cells.

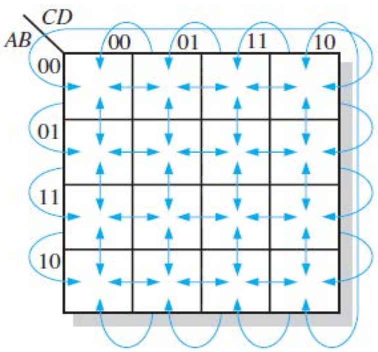


FIGURE 4–27 Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.

4–9 Karnaugh Map SOP Minimization

As stated in the last section, the Karnaugh map is used for simplifying Boolean expressions to their minimum form. A minimized SOP expression contains the fewest possible terms with the fewest possible variables per term. Generally, a minimum SOP expression can be implemented with fewer logic gates than a standard expression. In this section, Karnaugh maps with up to four variables are covered.

Mapping a Standard SOP Expression

For an SOP expression in standard form, a 1 is placed on the Karnaugh map for each product term in the expression. Each 1 is placed in a cell corresponding to the value of a product term. For example, for the product term $A\overline{B}C$, a 1 goes in the 101 cell on a 3-variable map.

When an SOP expression is completely mapped, there will be a number of 1s on the Karnaugh map equal to the number of product terms in the standard SOP expression. The cells that do not have a 1 are the cells for which the expression is 0. Usually, when working with SOP expressions, the 0s are left off the map. The following steps and the illustration in Figure 4–28 show the mapping process.

Step 1: Determine the binary value of each product term in the standard SOP expression. After some practice, you can usually do the evaluation of terms mentally.

Step 2: As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

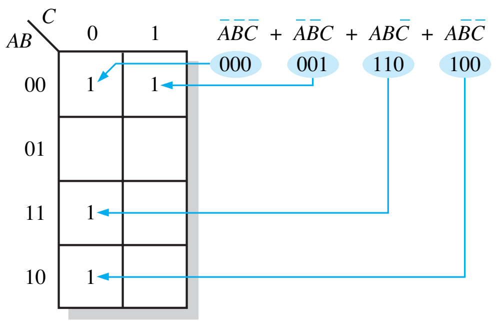


FIGURE 4–28 Example of mapping a standard SOP expression.

EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

Solution

Evaluate the expression as shown below. Place a 1 on the 3-variable Karnaugh map in Figure 4–29 for each standard product term in the expression.

$$\begin{array}{cccc} \overline{A}\,\overline{B}C & + \ \overline{A}B\overline{C} & + \ AB\overline{C} & + \ ABC \\ 001 & 010 & 110 & 111 \end{array}$$

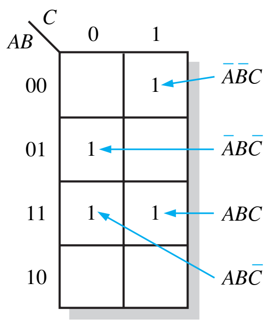


FIGURE 4–29

EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Solution

Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–30 for each standard product term in the expression.

$\overline{A}\,\overline{B}CD$	$\overline{A}B\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$AB\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$A\overline{B}C\overline{D}$
0011	0100	1101	1111	1100	0001	1010

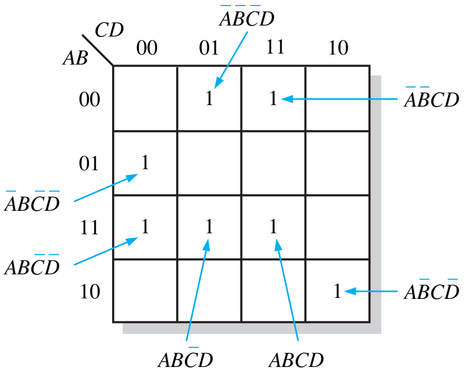


FIGURE 4–30

Mapping a Nonstandard SOP Expression

A Boolean expression must first be in standard form before you use a Karnaugh map. If an expression is not in standard form, then it must be converted to standard form by the procedure covered in Section 4–6 or by numerical expansion. Since an expression should be evaluated before mapping anyway, numerical expansion is probably the most efficient approach.

Numerical Expansion of a Nonstandard Product Term

Recall that a nonstandard product term has one or more missing variables. For example, assume that one of the product terms in a certain 3-variable SOP expression is $A\overline{B}$. This term can be expanded numerically to standard form as follows. First, write the binary value of the two variables and attach a 0 for the missing variable $\overline{C}$: 100. Next, write the binary value of the two variables and attach a 1 for the missing variable C: 101. The two resulting binary numbers are the values of the standard SOP terms $A\overline{B}\,\overline{C}$ and $A\overline{B}C$.

As another example, assume that one of the product terms in a 3-variable expression is B (remember that a single variable counts as a product term in an SOP expression). This term can be expanded numerically to standard form as follows. Write the binary value of the variable; then attach all possible values for the missing variables A and C as follows:

B
010
011
110
111

The four resulting binary numbers are the values of the standard SOP terms $\overline{A}B\overline{C}$, $\overline{A}BC$, $AB\overline{C}$, and ABC.

EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

Solution

The SOP expression is obviously not in standard form because each product term does not have three variables. The first term is missing two variables, the second term is missing one variable, and the third term is standard. First expand the terms numerically as follows:

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		

Map each of the resulting binary values by placing a 1 in the appropriate cell of the 3-variable Karnaugh map in Figure 4–31.

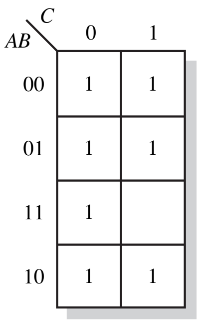


FIGURE 4–31

EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

Solution

The SOP expression is obviously not in standard form because each product term does not have four variables. The first and second terms are both missing two variables, the third term is missing one variable, and the rest of the terms are standard. First expand the terms by including all combinations of the missing variables numerically as follows:

$\overline{B}\,\overline{C}$	+ $A\overline{B}$	+ $AB\overline{C}$	+ $A\overline{B}C\overline{D}$	+ $\overline{A}\,\overline{B}\,\overline{C}D$	+ $A\overline{B}CD$
0000	1000	1100	1010	0001	1011
0001	1001	1101			
1000	1010				
1001	1011				

Map each of the resulting binary values by placing a 1 in the appropriate cell of the 4-variable Karnaugh map in Figure 4–32. Notice that some of the values in the expanded expression are redundant.

AB \ CD	00	01	11	10
00	1	1		
01				
11	1	1		
10	1	1	1	1

FIGURE 4–32

Karnaugh Map Simplification of SOP Expressions

The process that results in an expression containing the fewest possible terms with the few-est possible variables is called **minimization**. After an SOP expression has been mapped, a minimum SOP expression is obtained by grouping the 1s and determining the minimum SOP expression from the map.

Grouping the 1s

You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing 1s. The goal is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

EXAMPLE 4–27

Group the 1s in each of the Karnaugh maps in Figure 4–33.

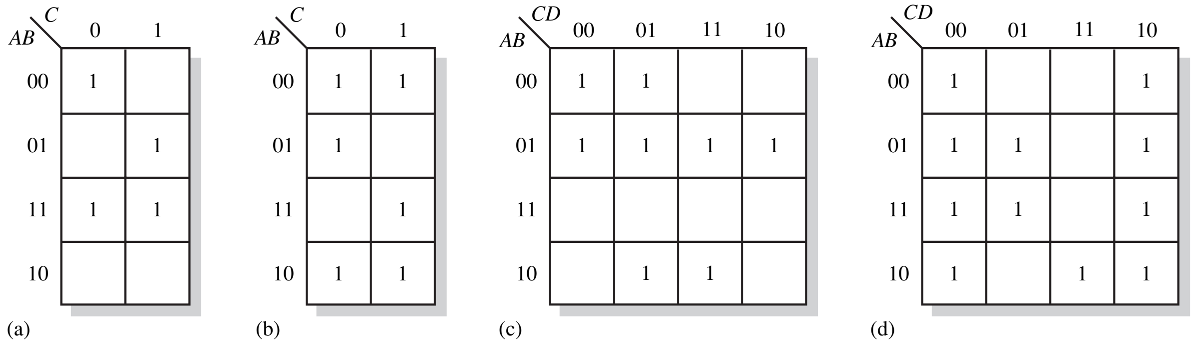


FIGURE 4–33

Solution

The groupings are shown in Figure 4–34. In some cases, there may be more than one way to group the 1s to form maximum groupings.

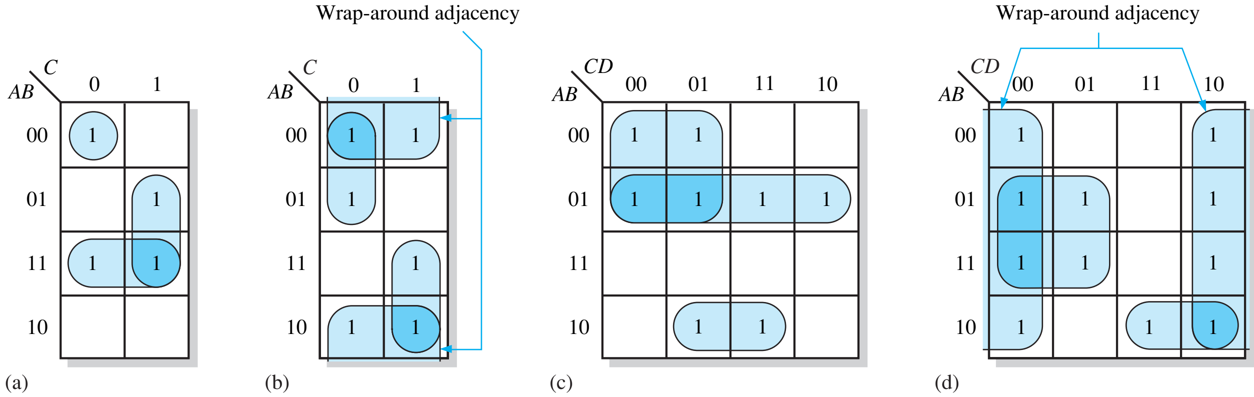


FIGURE 4–34

Determining the Minimum SOP Expression from the Map

When all the 1s representing the standard product terms in an expression are properly mapped and grouped, the process of determining the resulting minimum SOP expression begins. The following rules are applied to find the minimum product terms and the minimum SOP expression:

1. Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are called *contradictory variables.*
2. Determine the minimum product term for each group.
   - **(a)** For a 3-variable map:
     - **(1)** A 1-cell group yields a 3-variable product term
     - **(2)** A 2-cell group yields a 2-variable product term
     - **(3)** A 4-cell group yields a 1-variable term
     - **(4)** An 8-cell group yields a value of 1 for the expression
   - **(b)** For a 4-variable map:
     - **(1)** A 1-cell group yields a 4-variable product term
     - **(2)** A 2-cell group yields a 3-variable product term
     - **(3)** A 4-cell group yields a 2-variable product term
     - **(4)** An 8-cell group yields a 1-variable term
     - **(5)** A 16-cell group yields a value of 1 for the expression
3. When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression.

EXAMPLE 4–28

Determine the product terms for the Karnaugh map in Figure 4–35 and write the resulting minimum SOP expression.

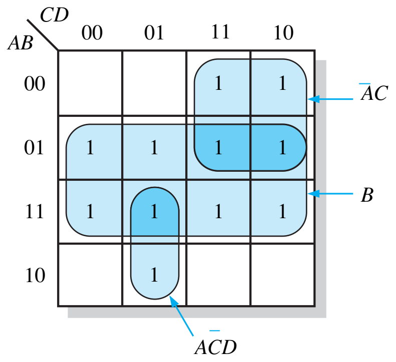


FIGURE 4–35

Solution

Eliminate variables that are in a grouping in both complemented and uncomplemented forms. In Figure 4–35, the product term for the 8-cell group is B because the cells within that group contain both A and $\overline{A}$, C and $\overline{C}$, and D and $\overline{D}$, which are eliminated. The 4-cell group contains B, $\overline{B}$, D, and $\overline{D}$, leaving the variables $\overline{A}$ and C, which form the product term $\overline{A}C$. The 2-cell group contains B and $\overline{B}$, leaving variables A, $\overline{C}$, and D which form the product term $A\overline{C}D$. Notice how overlapping is used to maximize the size of the groups. The resulting minimum SOP expression is the sum of these product terms:

$$B + \overline{A}C + A\overline{C}D$$

EXAMPLE 4–29

Determine the product terms for each of the Karnaugh maps in Figure 4–36 and write the resulting minimum SOP expression.

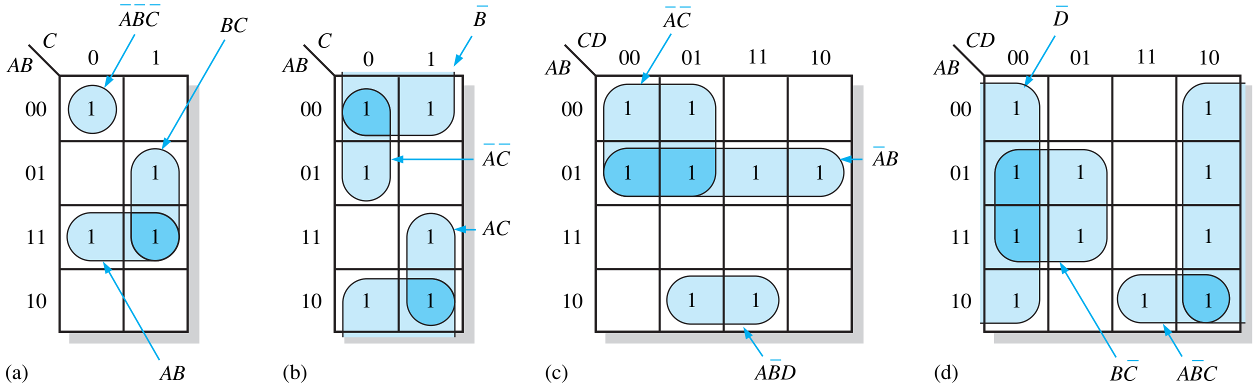


FIGURE 4–36

Solution

The resulting minimum product term for each group is shown in Figure 4–36. The minimum SOP expressions for each of the Karnaugh maps in the figure are

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$

(b) $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$

(d) $\overline{D} + A\overline{B}C + B\overline{C}$

EXAMPLE 4–30

Use a Karnaugh map to minimize the following standard SOP expression:

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

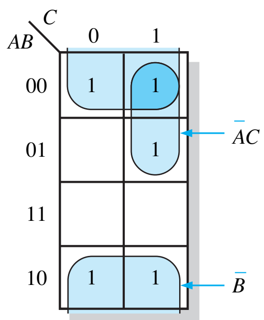


FIGURE 4–37

Solution

The binary values of the expression are 101 + 011 + 001 + 000 + 100

Map the standard SOP expression and group the cells as shown in Figure 4–37.

Notice the "wrap around" 4-cell group that includes the top row and the bottom row of 1s. The remaining 1 is absorbed in an overlapping group of two cells. The group of four 1s produces a single variable term, $\overline{B}$. This is determined by observing that within the group, $\overline{B}$ is the only variable that does not change from cell to cell. The group of two 1s produces a 2-variable term $\overline{A}C$. This is determined by observing that within the group, $\overline{A}$ and C do not change from one cell to the next. The product term for each group is shown. The resulting minimum SOP expression is $\overline{B} + \overline{A}C$

Keep in mind that this minimum expression is equivalent to the original standard expression.

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

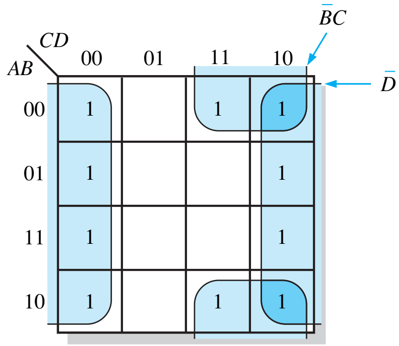


FIGURE 4–38

Solution

The first term $\overline{B}\,\overline{C}\,\overline{D}$ must be expanded into $A\overline{B}\,\overline{C}\,\overline{D}$ and $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ to get the standard SOP expression, which is then mapped; the cells are grouped as shown in Figure 4–38.

Notice that both groups exhibit "wrap around" adjacency. The group of eight is formed because the cells in the outer columns are adjacent. The group of four is formed to pick up the remaining two 1s because the top and bottom cells are adjacent. The product term for each group is shown. The resulting minimum SOP expression is

$$\overline{D} + \overline{B}C$$

Keep in mind that this minimum expression is equivalent to the original standard expression.

Mapping Directly from a Truth Table

You have seen how to map a Boolean expression; now you will learn how to go directly from a truth table to a Karnaugh map. Recall that a truth table gives the output of a Boolean expression for all possible input variable combinations. An example of a Boolean expression and its truth table representation is shown in Figure 4–39. Notice in the truth table that the output X is 1 for four different input variable combinations. The 1s in the output column of the truth table are mapped directly onto a Karnaugh map into the cells corresponding to the values of the associated input variable combinations, as shown in Figure 4–39. In the figure you can see that the Boolean expression, the truth table, and the Karnaugh map are simply different ways to represent a logic function.

"Don't Care" Conditions

Sometimes a situation arises in which some input variable combinations are not allowed. For example, recall that in the BCD code covered in Chapter 2, there are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111. Since these unallowed states

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

Inputs A B C	Output X
0 0 0	1
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	1

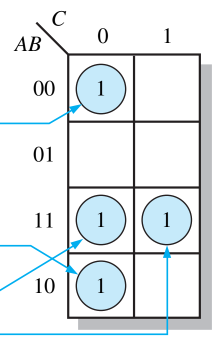


FIGURE 4–39 Example of mapping directly from a truth table to a Karnaugh map.

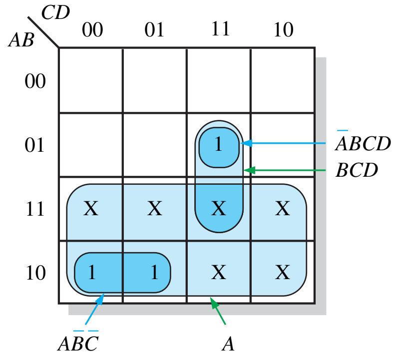
will never occur in an application involving the BCD code, they can be treated as **"don't care"** terms with respect to their effect on the output. That is, for these "don't care" terms either a 1 or a 0 may be assigned to the output; it really does not matter since they will never occur.

The "don't care" terms can be used to advantage on the Karnaugh map. Figure 4–40 shows that for each "don't care" term, an X is placed in the cell. When grouping the 1s, the Xs can be treated as 1s to make a larger grouping or as 0s if they cannot be used to advantage. The larger a group, the simpler the resulting term will be.

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\overline{B}\,\overline{C} + \overline{A}BCD$
With "don't cares" $Y = A + BCD$

FIGURE 4–40 Example of the use of "don't care" conditions to simplify an expression.

The truth table in Figure 4–40(a) describes a logic function that has a 1 output only when the BCD code for 7, 8, or 9 is present on the inputs. If the "don't cares" are used as 1s, the resulting expression for the function is $A + BCD$, as indicated in part (b). If the "don't cares" are not used as 1s, the resulting expression is $AB\overline{C} + \overline{A}BCD$; so you can see the advantage of using "don't care" terms to get the simplest expression.

EXAMPLE 4–32

In a 7-segment display, each of the seven segments is activated for various digits. For example, segment *a* is activated for the digits 0, 2, 3, 5, 6, 7, 8, and 9, as illustrated in Figure 4–41. Since each digit can be represented by a BCD code, derive an SOP expression for segment *a* using the variables *ABCD* and then minimize the expression using a Karnaugh map.

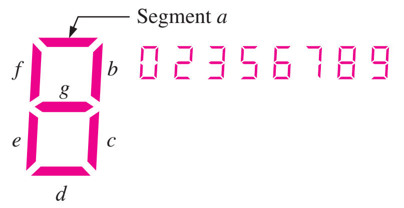


FIGURE 4–41 7-segment display.

Solution

The expression for segment *a* is

$$a = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D$$

Each term in the expression represents one of the digits in which segment *a* is used. The Karnaugh map minimization is shown in Figure 4–42. X's (don't cares) are entered for those states that do not occur in the BCD code.

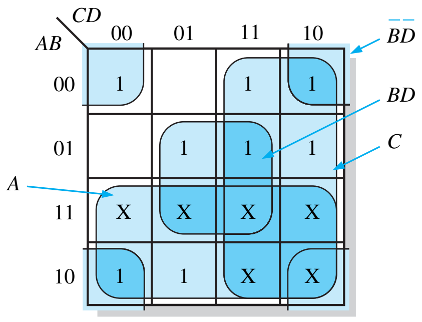


FIGURE 4–42

From the Karnaugh map, the minimized expression for segment *a* is

$$a = A + C + BD + \overline{B}\,\overline{D}$$

4–10 Karnaugh Map POS Minimization

In the last section, you studied the minimization of an SOP expression using a Karnaugh map. In this section, we focus on POS expressions. The approaches are much the same except that with POS expressions, 0s representing the standard sum terms are placed on the Karnaugh map instead of 1s.

Mapping a Standard POS Expression

For a POS expression in standard form, a 0 is placed on the Karnaugh map for each sum term in the expression. Each 0 is placed in a cell corresponding to the value of a sum term. For example, for the sum term $A + \overline{B} + C$, a 0 goes in the 010 cell on a 3-variable map.

When a POS expression is completely mapped, there will be a number of 0s on the Karnaugh map equal to the number of sum terms in the standard POS expression. The cells that do not have a 0 are the cells for which the expression is 1. Usually, when working with POS expressions, the 1s are left off. The following steps and the illustration in Figure 4–43 show the mapping process.

Step 1: Determine the binary value of each sum term in the standard POS expression. This is the binary value that makes the term equal to 0.

Step 2: As each sum term is evaluated, place a 0 on the Karnaugh map in the corresponding cell.

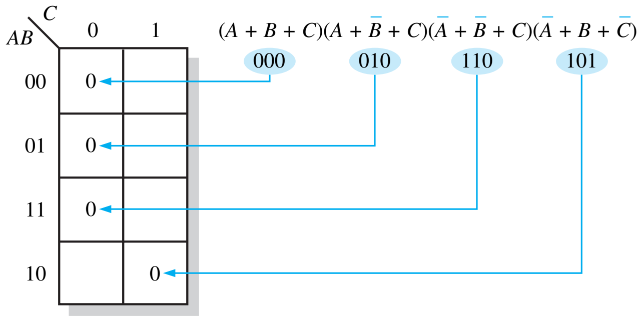


FIGURE 4–43 Example of mapping a standard POS expression.

EXAMPLE 4–33

Map the following standard POS expression on a Karnaugh map:

$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$

Solution

Evaluate the expression as shown below and place a 0 on the 4-variable Karnaugh map in Figure 4–44 for each standard sum term in the expression.

$$\underset{1100}{(\overline{A} + \overline{B} + C + D)}\underset{1011}{(\overline{A} + B + \overline{C} + \overline{D})}\underset{0010}{(A + B + \overline{C} + D)}\underset{1111}{(\overline{A} + \overline{B} + \overline{C} + \overline{D})}\underset{0011}{(A + B + \overline{C} + \overline{D})}$$

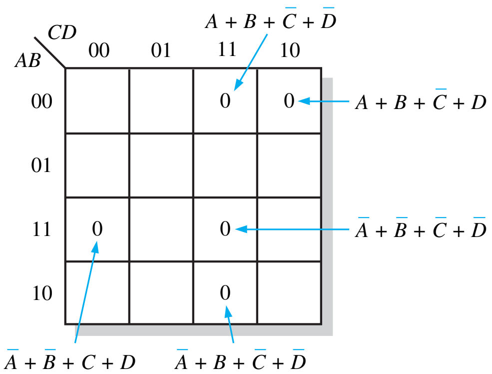


FIGURE 4–44

Karnaugh Map Simplification of POS Expressions

The process for minimizing a POS expression is basically the same as for an SOP expression except that you group 0s to produce minimum sum terms instead of grouping 1s to produce minimum product terms. The rules for grouping the 0s are the same as those for grouping the 1s that you learned in Section 4–9.

EXAMPLE 4–34

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.

Solution

The combinations of binary values of the expression are

$$(0 + 0 + 0)(0 + 0 + 1)(0 + 1 + 0)(0 + 1 + 1)(1 + 1 + 0)$$

Map the standard POS expression and group the cells as shown in Figure 4–45.

Notice how the 0 in the 110 cell is included into a 2-cell group by utilizing the 0 in the 4-cell group. The sum term for each blue group is shown in the figure and the resulting minimum POS expression is

$$A(\overline{B} + C)$$

Keep in mind that this minimum POS expression is equivalent to the original standard POS expression.

Grouping the 1s as shown by the gray areas yields an SOP expression that is equivalent to grouping the 0s.

$$AC + A\overline{B} = A(\overline{B} + C)$$

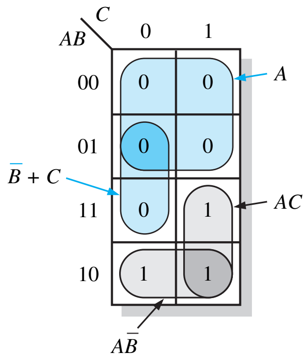


FIGURE 4–45

EXAMPLE 4–35

Use a Karnaugh map to minimize the following POS expression:

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

Solution

The first term must be expanded into $\overline{A} + B + C + D$ and $A + B + C + D$ to get a standard POS expression, which is then mapped; and the cells are grouped as shown in Figure 4–46. The sum term for each group is shown and the resulting minimum POS expression is

$$(C + D)(A + B + D)(\overline{A} + B + C)$$

Keep in mind that this minimum POS expression is equivalent to the original standard POS expression.

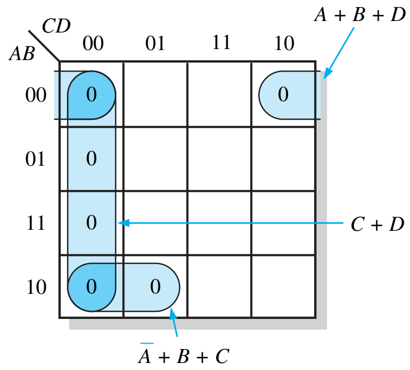


FIGURE 4–46

Converting Between POS and SOP Using the Karnaugh Map

When a POS expression is mapped, it can easily be converted to the equivalent SOP form directly from the Karnaugh map. Also, given a mapped SOP expression, an equivalent POS expression can be derived directly from the map. This provides a good way to compare both minimum forms of an expression to determine if one of them can be implemented with fewer gates than the other.

For a POS expression, all the cells that do not contain 0s contain 1s, from which the SOP expression is derived. Likewise, for an SOP expression, all the cells that do not contain 1s contain 0s, from which the POS expression is derived. Example 4–36 illustrates this conversion.

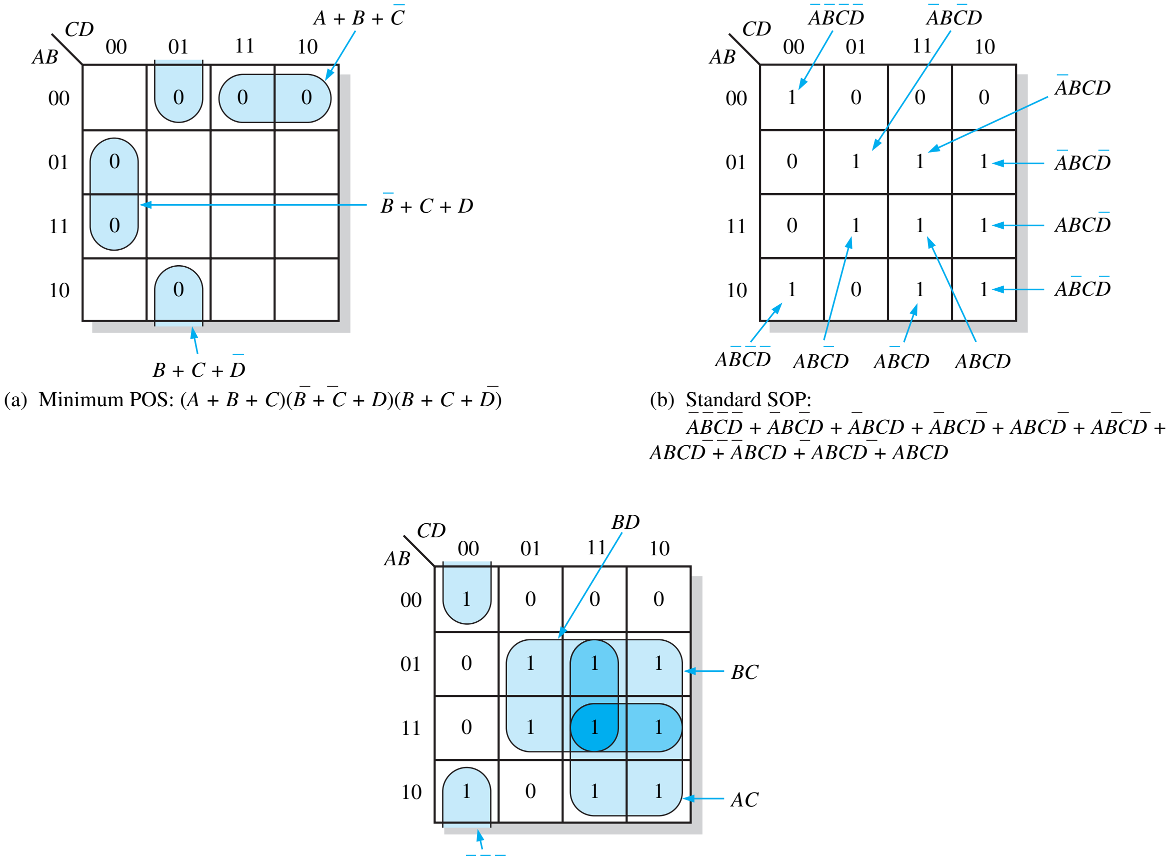
EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$

Solution

The 0s for the standard POS expression are mapped and grouped to obtain the minimum POS expression in Figure 4–47(a). In Figure 4–47(b), 1s are added to the cells that do not contain 0s. From each cell containing a 1, a standard product term is obtained as indicated. These product terms form the standard SOP expression. In Figure 4–47(c), the 1s are grouped and a minimum SOP expression is obtained.



(a) Minimum POS: $(A + B + C)(\overline{B} + \overline{C} + D)(B + C + \overline{D})$

(b) Standard SOP:
$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} +$
$AB\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD + ABCD$

(c) Minimum SOP: $AC + BC + BD + \overline{B}\,\overline{C}\,\overline{D}$

FIGURE 4–47