Surveying-II
CE-207 (T)

CURVES

Lecture No 1

Department of civil engineering
UET Peshawar

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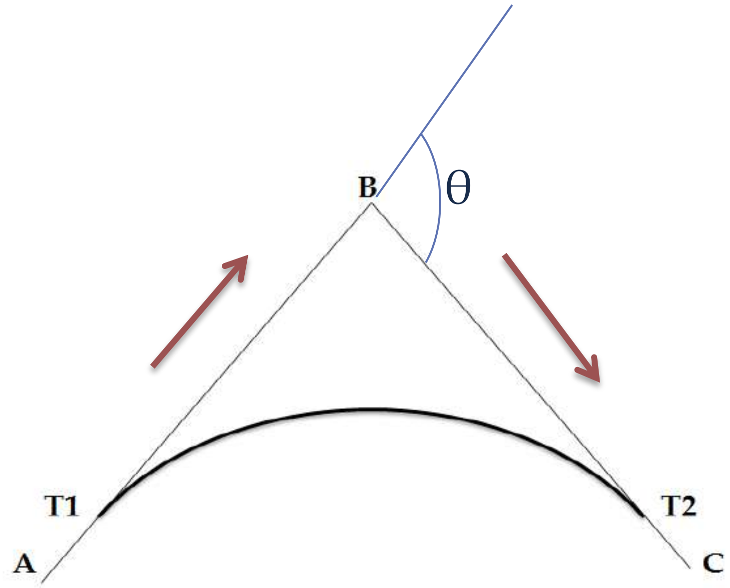
Curves

- Curves are usually employed in lines of communication in order that the change in direction at the intersection of the straight lines shall be gradual.
- The lines connected by the curves are tangent to it and are called Tangents or Straights.
- The curves are generally circular arcs but parabolic arcs are often used in some countries for this purpose.
- Most types of transportation routes, such as highways, railroads, and pipelines, are connected by curves in both horizontal and vertical planes.

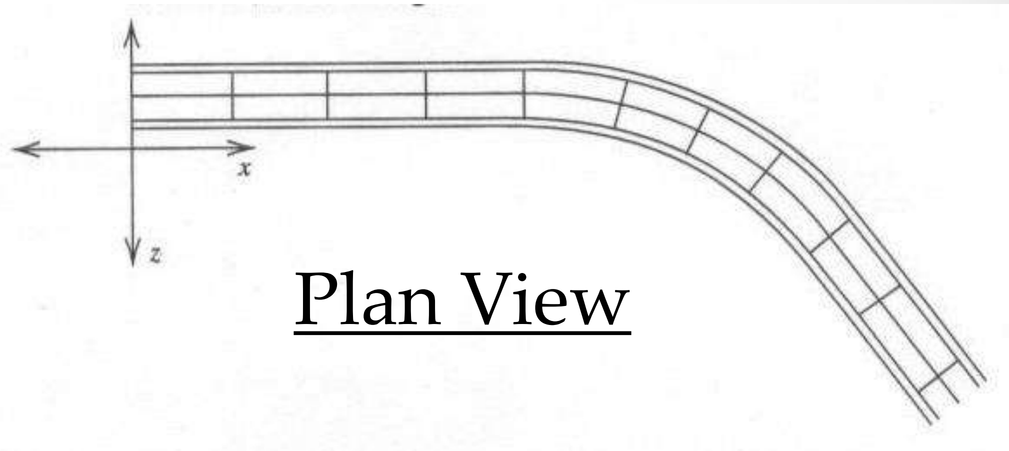
Curves

- The purpose of the curves is to deflect a vehicle travelling along one of the straights safely and comfortably through a deflection angle θ to enable it to continue its journey along the other straight.



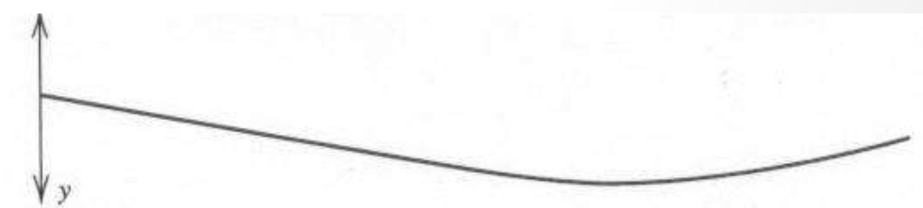
Curves

Horizontal Alignment



Plan View

Vertical Alignment



Profile View

Curves

Classification of Curves

- Curves
  - Horizontal Curves
    - Simple Curves
    - Compound Curves
    - Transition Curves
    - Reverse Curves
  - Vertical Curves

Simple Curves, Compound Curves, Transition Curves — Circular Curves

Reverse Curves — Spiral Curves (non-circular curves)

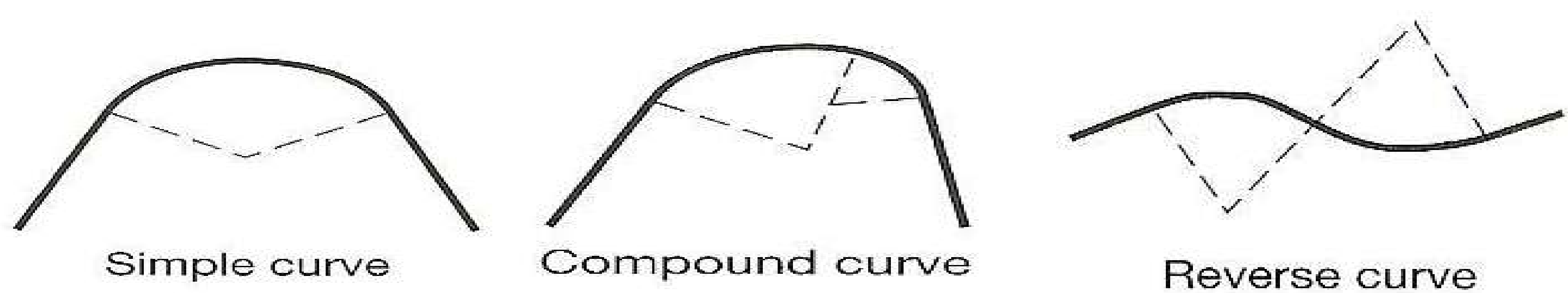
Curves

Classification of Curves

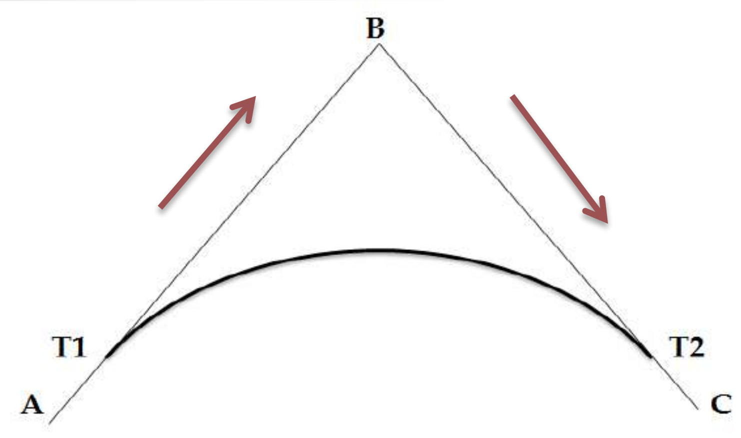
1) Simple Curves: Consist of single Arc Connecting two straights.

2) Compound curves: Consist of 2 arcs of different radii, bending in the same direction and lying on the same sides of their common tangents, their centers being on the same side of the curve.

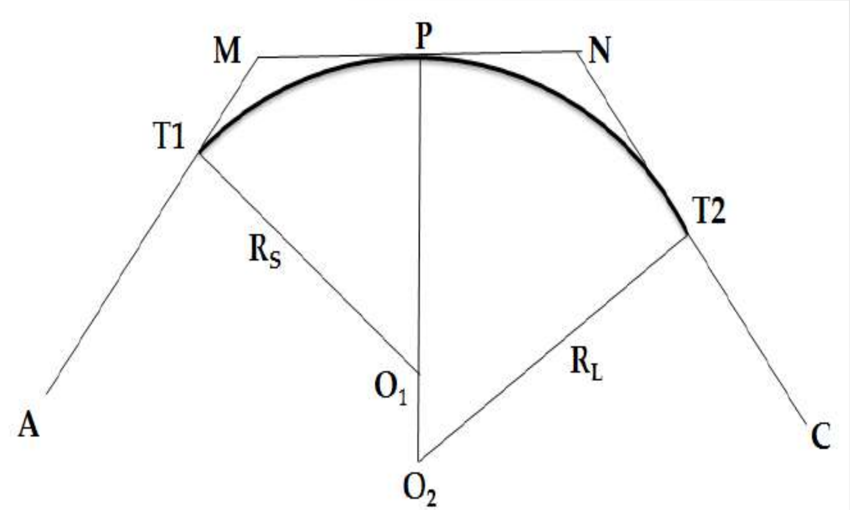
3) Reverse curves: Consist of 2 arcs of equal or unequal radii, bending in opposite direction with common tangent at their junction (meeting Point), their center lying on the opposite sides of the curve.



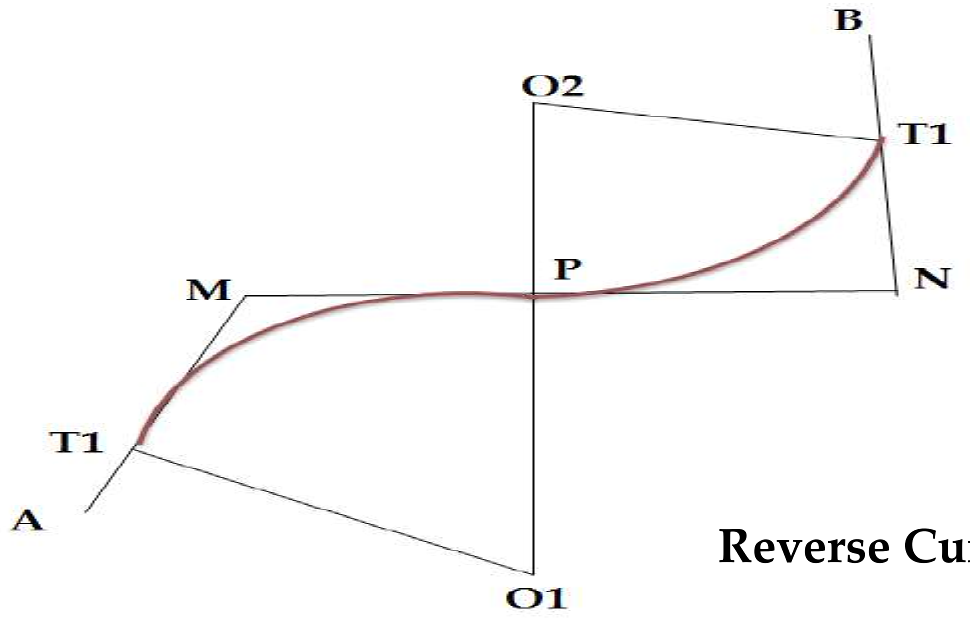
Curves



Simple Curve



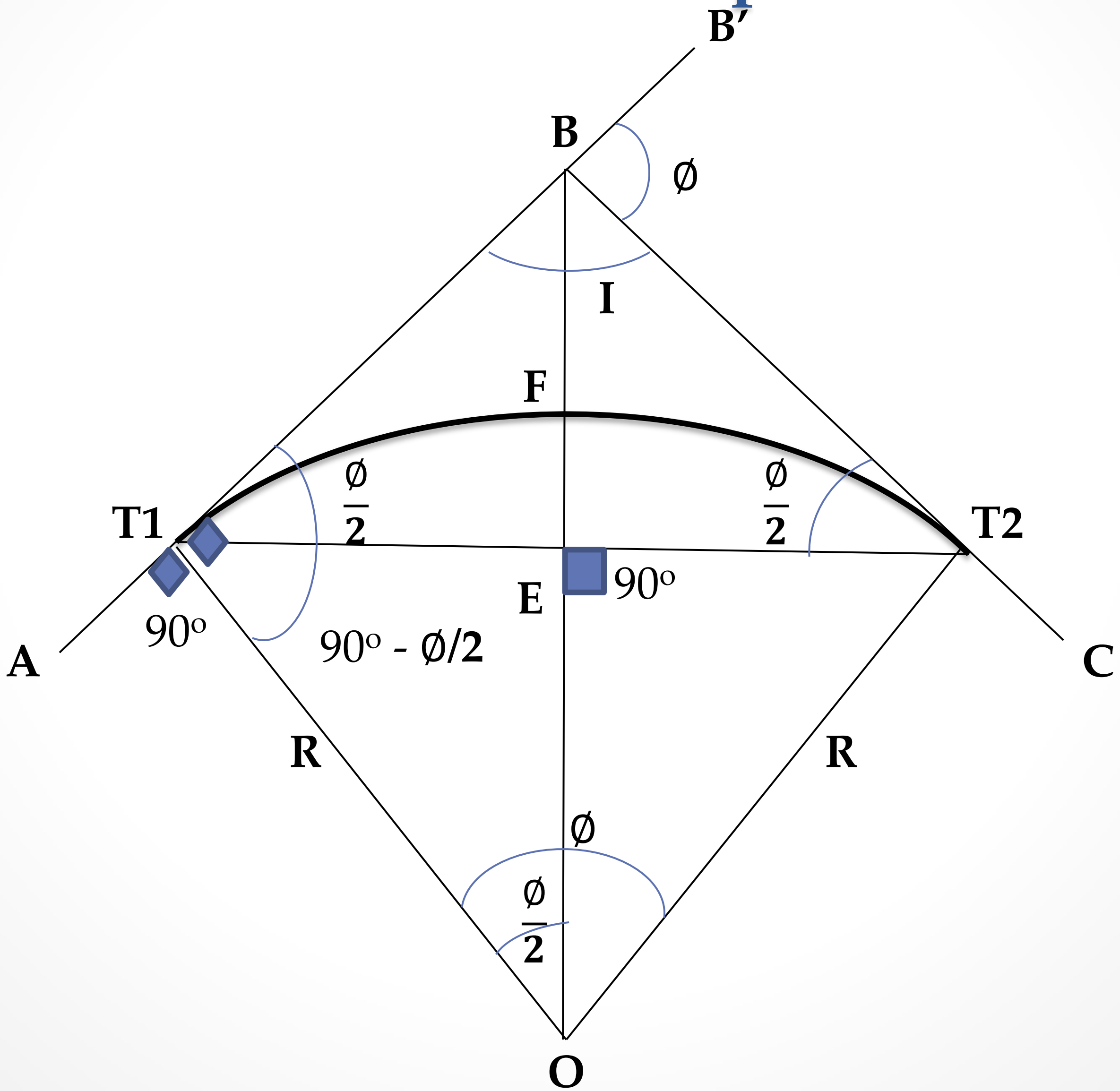
Compound Curve



Reverse Curve

Curves

Nomenclature of Simple Curves



Curves

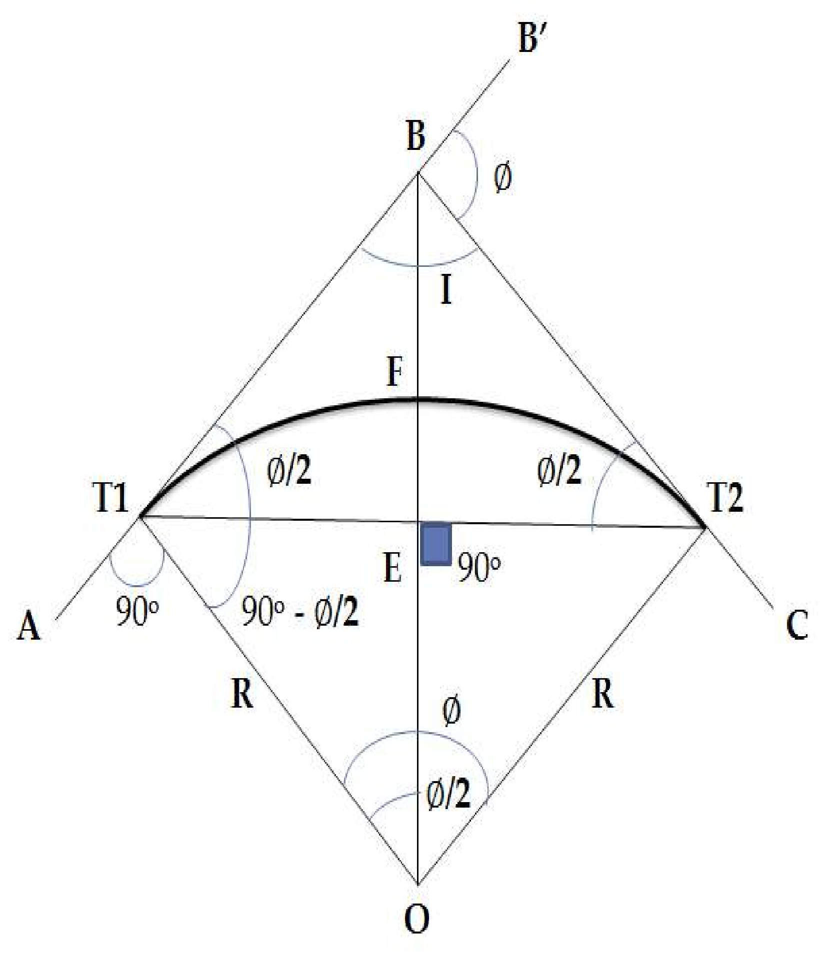
Nomenclature of Simple Curves

1) Tangents or Straights: The straight lines **AB** and **BC** which are connected by the curves are called the tangents or straights to curves.

2)Point of Intersection: (PI.) The Point **B** at which the 2 tangents **AB** and **BC** intersect or Vertex (V) .

3)Back Tangent: The tangent line **AB** is called 1st tangent or Rear tangents or Back tangent.

4) Forward Tangent: The tangents line **BC** is called 2nd tangent or Forward tangent.



Curves

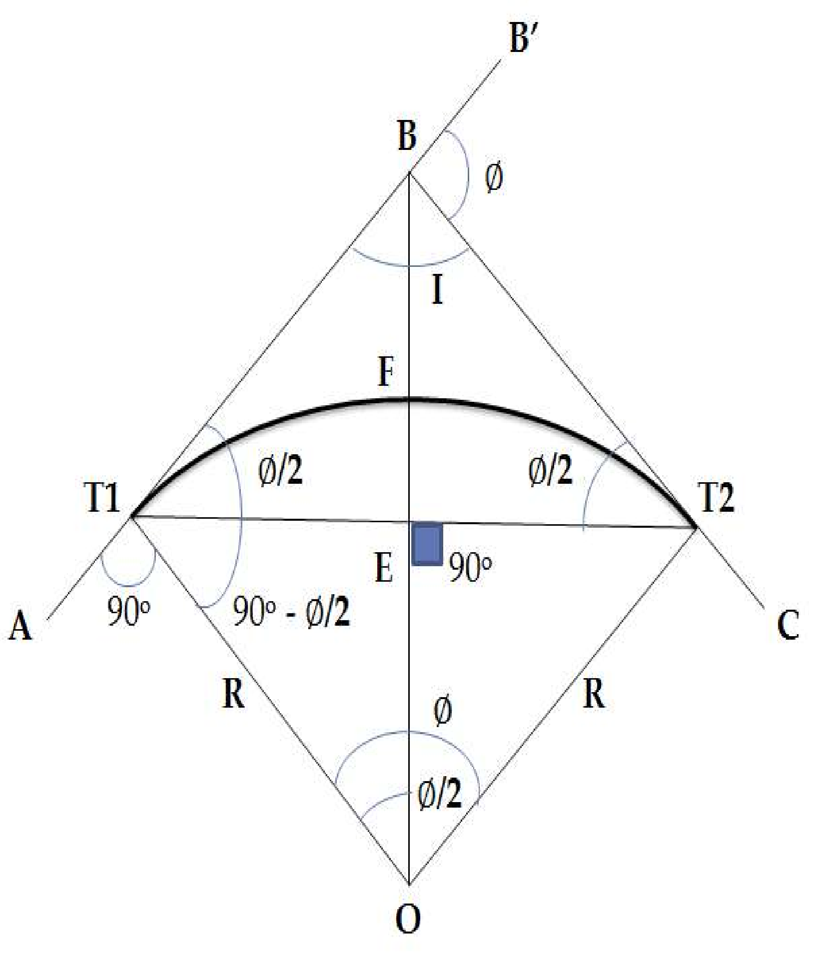
Nomenclature of Simple Curves

5) Tangents Points: The points $\mathbf{T_1}$ and $\mathbf{T_2}$ at which the curves touches the straights.

5.a) Point of Curve (P.C): The beginning of the curve $\mathbf{T_1}$ is called the point of curve or tangent curve (T.C).

5.b) Point of tangency (C.T): The end of curve $\mathbf{T_2}$ is called point of tangency or curve tangent (C.T).

6) Angle of Intersection: (I) The angle **ABC** between the tangent lines AB and BC. Denoted by **I**.



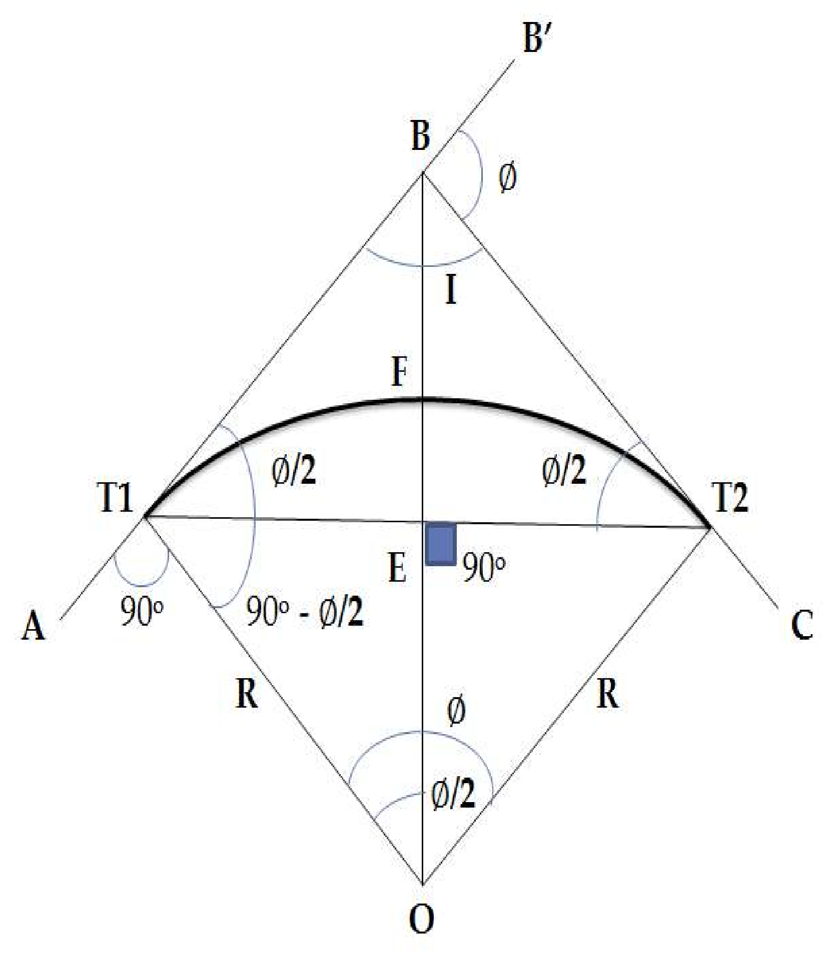
Curves

Nomenclature of Simple Curves

7) Angle of Deflection (∅): Then angle **B`BC** by which the forward (head tangent deflect from the Rear tangent.

8) Tangent Length: (BT_1 and BT_2) The distance from point of intersection B to the tangent points T_1 and T_2. These depend upon the radii of curves.

9) Long Cord: The line T_1T_2 joining the two tangents point T_1 and T_2 is called long chord. Denoted by ℓ.



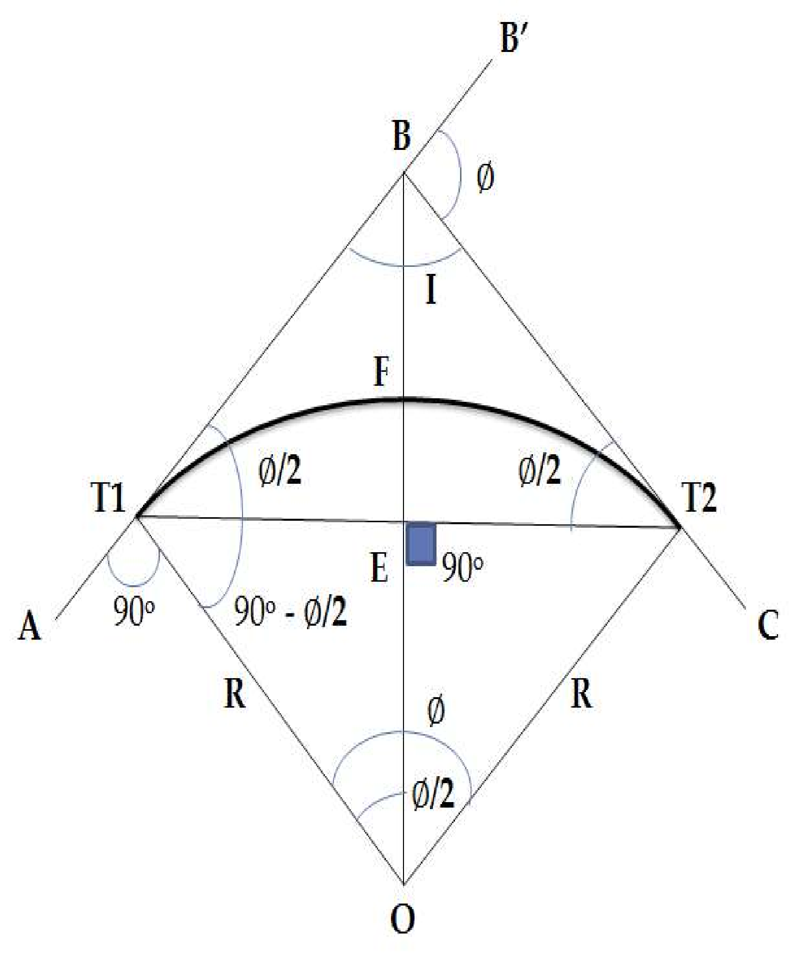
Curves

Nomenclature of Simple Curves

10) Length of Curve: the arc T_1FT_2 is called length of curve. Denoted by **L**.

11) Apex or Summit of Curve: The mid point **F** of the arc T_1FT_2 is called Apex of curve and lies on the bisection of angle of intersection. It is the junction of lines radii.

12) External Distance (BF): The distance **BF** from the point of intersection to the apex of the curve is called Apex distance or External distance.

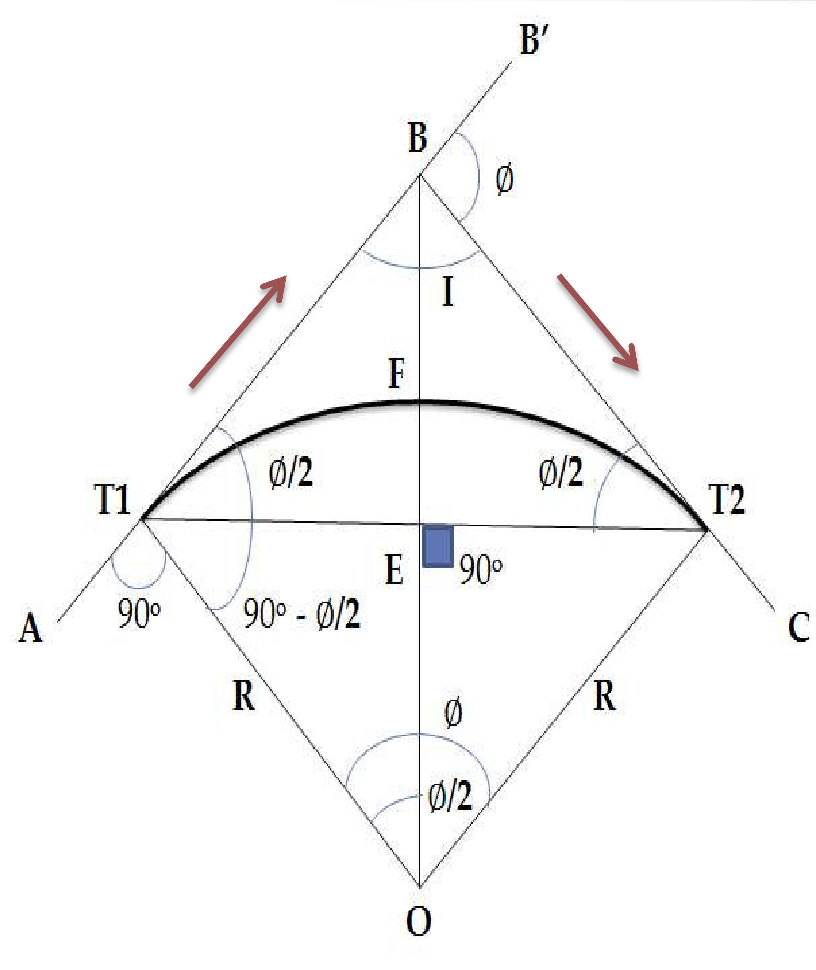


Curves

Nomenclature of Simple Curves

13) Central Angle: The angle T_1OT_2 subtended at the center of the curve by the arc T_1FT_2 is called central angle and is equal to the deflection angle.

14) Mid ordinate (EF): It is a ordinate from the mid point of the long chord to the mid point of the curve i.e distance **EF**. Also called Versed sine of the curve.



- If the curve deflect to the right of the direction of the progress of survey it is called Right-hand curve and id to the left , it is called Left-hand curve.
- The $\Delta\, BT_1T_2$ is an isosceles triangle and therefore the angle
- $\llcorner\, BT_1T_2 = \llcorner\, BT_2T_1 = \frac{\emptyset}{2}$

Curves

Elements of Simple Curves

a) $\llcorner T_1BT_2 + \llcorner B`BT_2 = 180^o$

$I + \emptyset = 180^o$

$\llcorner T_1OT_2 = \emptyset = 180^o - I$

b) Tangent lengths: (BT_1, BT_2)

In ΔT_1OB, $\tan\left(\frac{\emptyset}{2}\right) = BT_1 / OT_1$

$BT_1 = OT_1 \tan\left(\frac{\emptyset}{2}\right)$

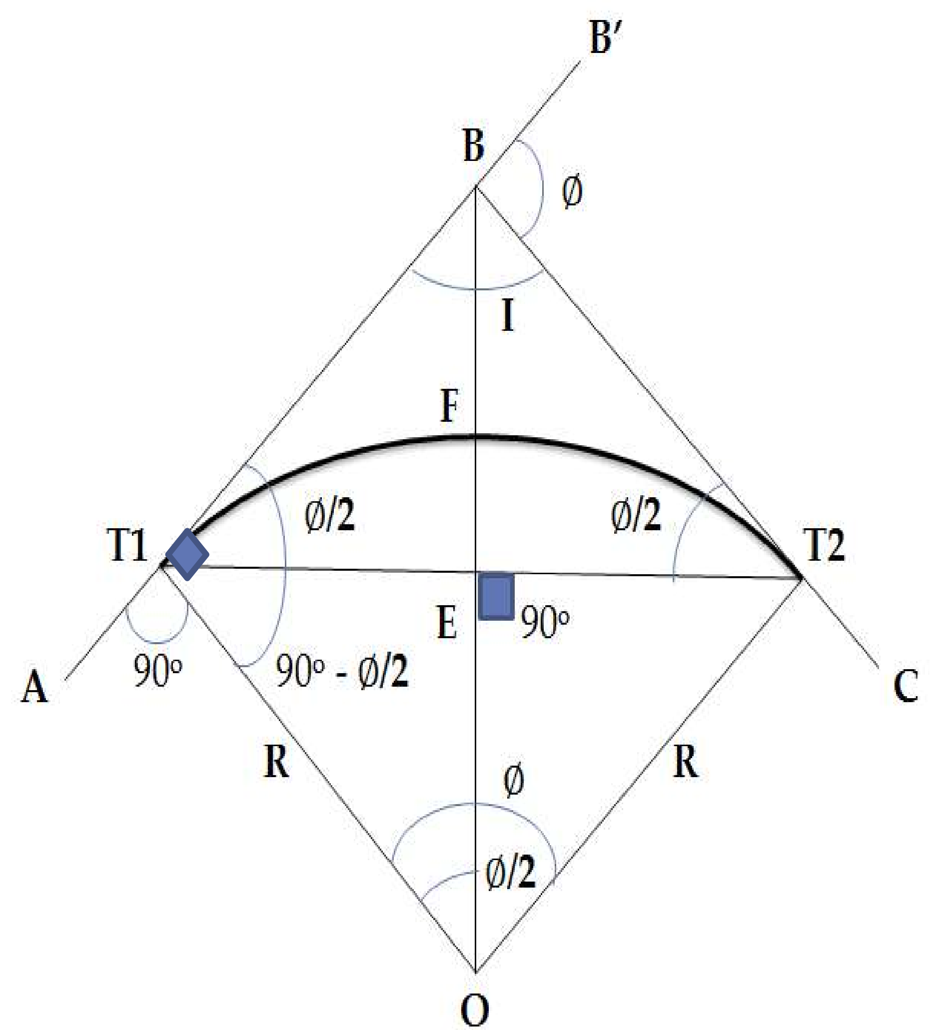
$BT_1 = BT_2 = R \tan\left(\frac{\emptyset}{2}\right)$

c) Length of Chord(ℓ):

In ΔT_1OE, $\sin\left(\frac{\emptyset}{2}\right) = T_1E / OT_1$

$T_1E = OT_1 \sin\left(\frac{\emptyset}{2}\right)$

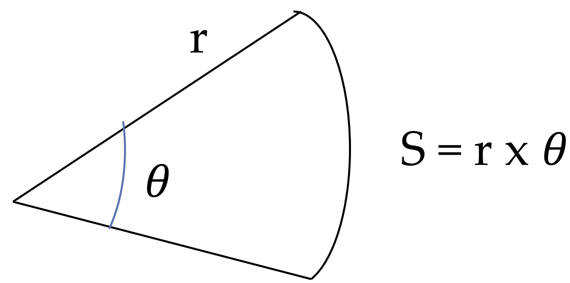
$T_1E = R \sin\left(\frac{\emptyset}{2}\right)$



Curves

Elements of Simple Curves

$\ell = 2\,T_1E = 2\,R\,\sin(\frac{\emptyset}{2})$



d) Length of Curve (L):

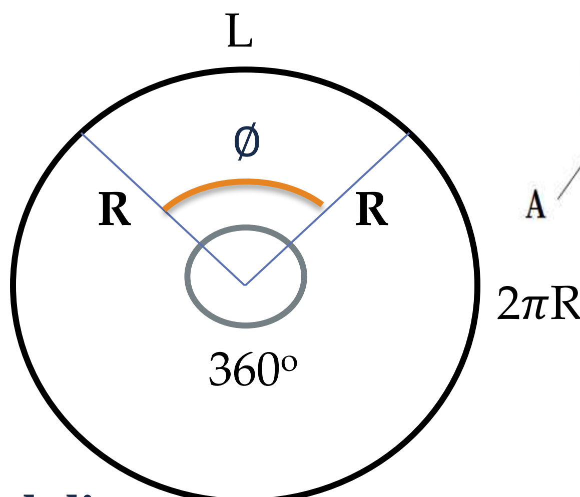
L = length of arc T_1FT_2

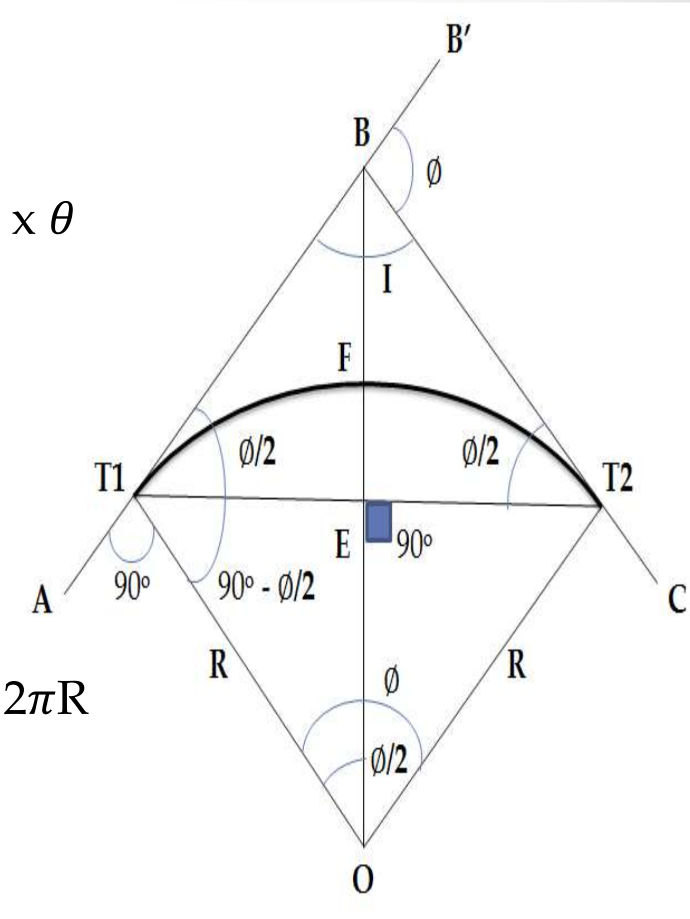
$L = R\,\emptyset\,(rad) = \frac{\pi\,R\,\emptyset}{180}$

Or

$L / 2\,\pi\,R = \emptyset / 360$

$L = 2\,\pi\,R\,\emptyset / 360 = \frac{\pi\,R\,\emptyset}{180}$





e) Apex distance or External distance:

BF = BO – OF

In ΔOT_1B, $\cos(\frac{\emptyset}{2}) = OT_1 / BO$

Curves

Elements of Simple Curves

$BO = OT_1 / \cos(\frac{\emptyset}{2}) = R / \cos(\frac{\emptyset}{2})$

$BO \quad = R \sec(\frac{\emptyset}{2})$

$BF = R \sec(\emptyset/2) - R$

$BF = R\ (\sec(\frac{\emptyset}{2}) - 1)$

$BF = R\ (\frac{1}{\cos(\frac{\emptyset}{2})} - 1)$

f) Mid ordinate or Versed sine of curve:

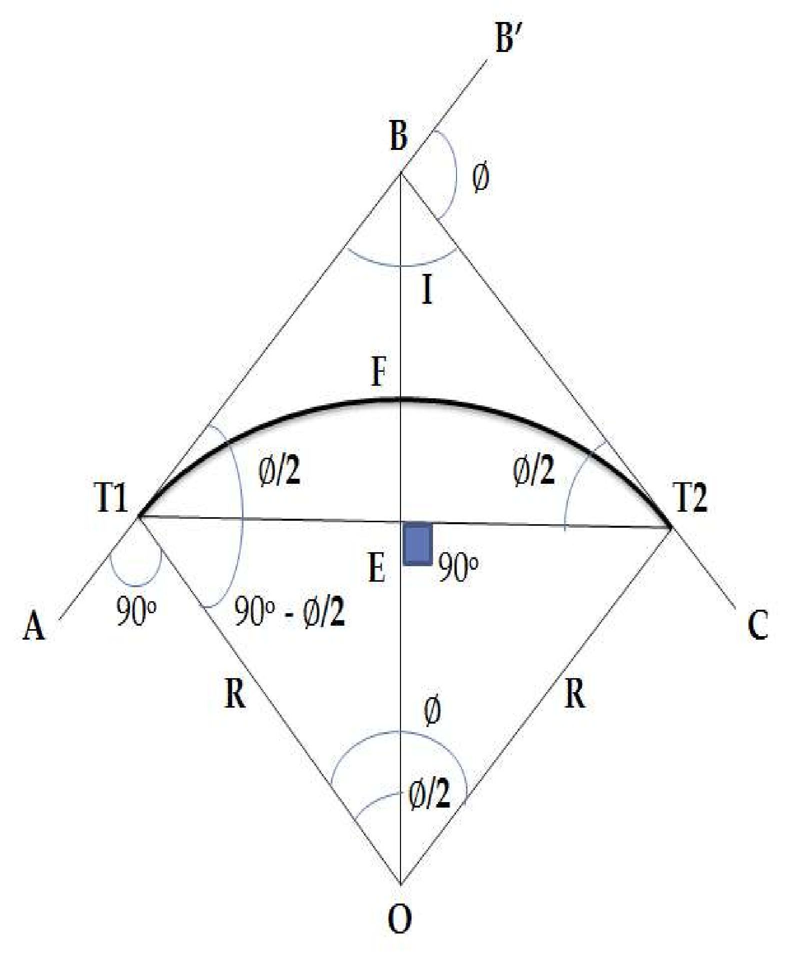
$EF = OF - OE$

In ΔT_1OE, $\cos(\emptyset/2) = OE / OT_1$

$OE = OT_1 \cos(\emptyset/2) = R \cos(\emptyset/2)$

$EF = R - R \cos(\emptyset/2)$

$EF = R\ (1 - \cos(\frac{\emptyset}{2}))$



Curves

Designation Of Curves

- In U.K a curve is defined by Radius which it expressed in terms of feet or chains(Gunter chain) e.g 12 chain curve, 24 chain curve.
- When expressed in feet the radius is taken as multiple of 100 e.g 200, 300, 400.. .
- In USA, Canada, India and Pakistan a curve is designated by a degree e.g 2 degree curve , 6 degree curve.

Degree Of Curves

Degree of curve is defined in 2 ways

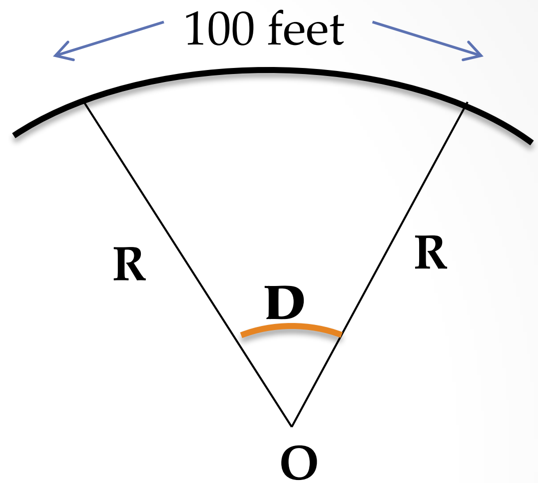
1) Arc Definition
2) Chord Definition

Curves

Degree Of Curves

1) Arc Definition:

" The degree of a curve is the central angle subtended by 100 feet of arc".



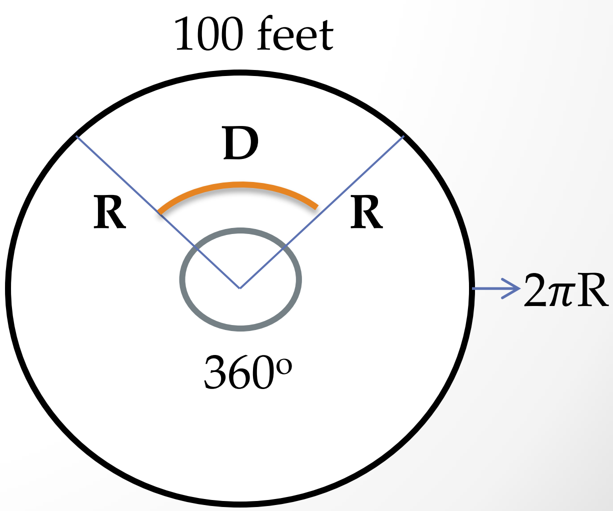
Let R = Radius of Curve

D = Degree of Curve

Then $\frac{D}{360} = \frac{100}{2\pi R}$

$$R = \frac{5729.58}{D} \text{ (feet)}$$

It is used in highways.

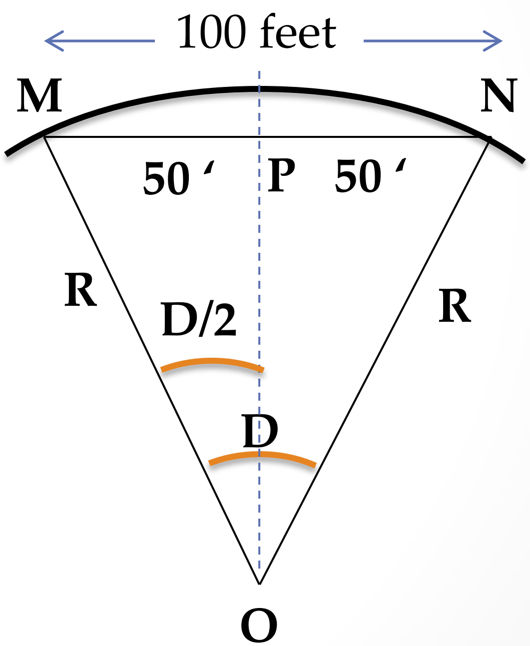


Curves

Degree Of Curves

2) Chord Definition:

" The degree of curve is the central angle subtended by 100 feet of chord".



From Δ**OPM**

$$\sin\left(\frac{D}{2}\right) = \frac{MP}{OM} = \frac{50}{R}$$

$$R = \frac{50}{\sin\left(\frac{D}{2}\right)} \text{ (feet)}$$

It is used in Railway.

Example: D = 1°

1) Arc Def: R = 5729.58 / D

= 5729.58 feet

2) Chord Def: R = 50 / sin (D/2)

= 5729.65 feet

Simple Curves

Method of Curve Ranging

- There are a number of different methods by which a centerline can be set out, all of which can be summarized in two categories:

- **Traditional methods:** which involve working along the centerline itself using the straights, intersection points and tangent points for reference.
- The equipment used for these methods include, tapes and theodolites or total stations.
- **Coordinate methods:** which use control networks as reference. These networks take the form of control points located on site some distance away from the centerline.
- For this method, theodolites, totals stations or GPS receivers can be used.

Simple Curves

Method of Curve Ranging

The methods for setting out curves may be divided into 2 classes according to the instrument employed .

1) Linear or Chain & Tape Method

2) Angular or Instrumental Method

Peg Interval:

Usual Practice--- Fix pegs at equal interval on the curve

20 m to 30 m (100 feet or one chain)

66 feet (Gunter's Chain)

Strictly speaking this interval must be measured as the Arc intercept b/w them, however it is necessarily measure along the chord. The curve consist of a series of chords rather than arcs.

Along the arc it is practically not possible that is why measured along the chord.

Simple Curves

Method of Curve Ranging

Peg Interval:

For difference in arc and chord to be negligible

$$\text{Length of chord} \ngtr \frac{R}{20} \text{ of curve}$$

R = Radius of curve

Length of unit chord = 30 m for flate curve (100 ft)

(peg interval) 20 m for sharp curve (50 ft)

10 m for very sharp curves (25 ft or less)

Simple Curves

Method of Curve Ranging

Location of Tangent points:

To locate T_1 and T_2

1) Fixed direction of tangents, produce them so as to meet at point B.

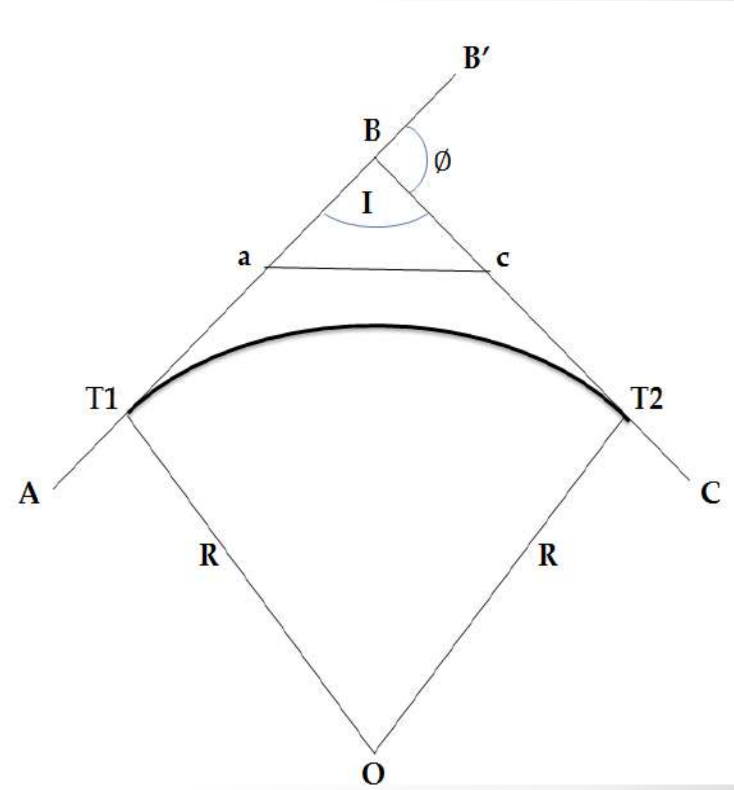
2) Set up theodolite at point B and measure T_1BT_2 (I).

Then deflection angle $\emptyset = 180^\circ - I$

3) Calculate tangents lengths by

$$BT_1 = BT_2 = R\tan\left(\frac{\emptyset}{2}\right)$$

4) Locate T_1 and T_2 points by measuring the tangent lengths backward and forward along tangent lines AB and BC.



Simple Curves

Method of Curve Ranging

Procedure:

- After locating the positions of the tangent points T_1 and T_2 ,their chainages may be determined.
- The chainage of T_1 is obtained by subtracting the tangent length from the known chainage of the intersection point B. And the chainage of T_2 is found by adding the length of curve to the chainage of T_1.
- Then the pegs are fixed at equal intervals on the curve.
- The interval between pegs is usually $30m$ or one chain length.
- The distances along the centre line of the curve are continuously measured from the point of beginning of the line up to the end .i.e the pegs along the centre line of the work should be at equal interval from the beginning of the line up to the end.

Simple Curves

Method of Curve Ranging

Procedure:

- There should be no break in the regularity of their spacing in passing from a tangent to a curve or from a curve to the tangent.
- For this reason ,the first peg on the curve is fixed at such a distance from the first tangent point (T_1) that its chainage becomes the whole number of chains i.e the whole number of peg interval.
- The length of the first sub chord is thus less than the peg interval and it is called a sub-chord.
- Similarly there will be a sub-chord at the end of the curve.
- Thus a curve usually consists of two sub-chords and a no. of full chords.

Simple Curves

Method of Curve Ranging

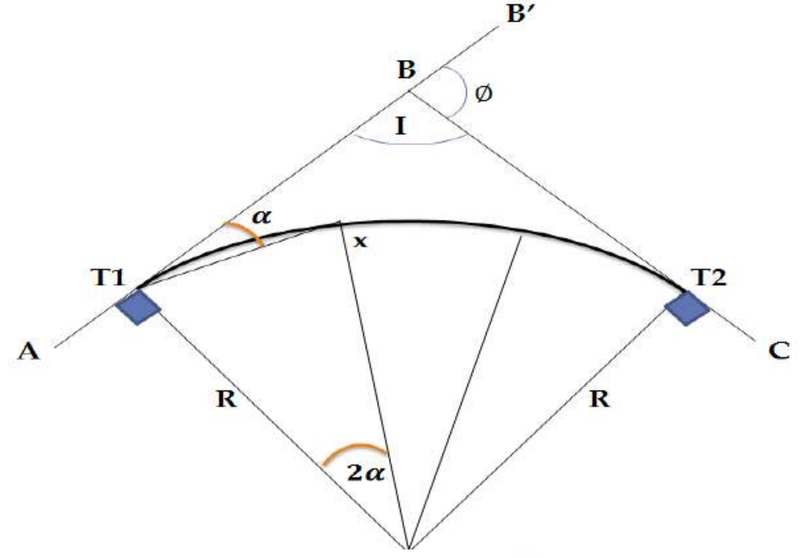
Important relationships for Circular Curves for Setting Out

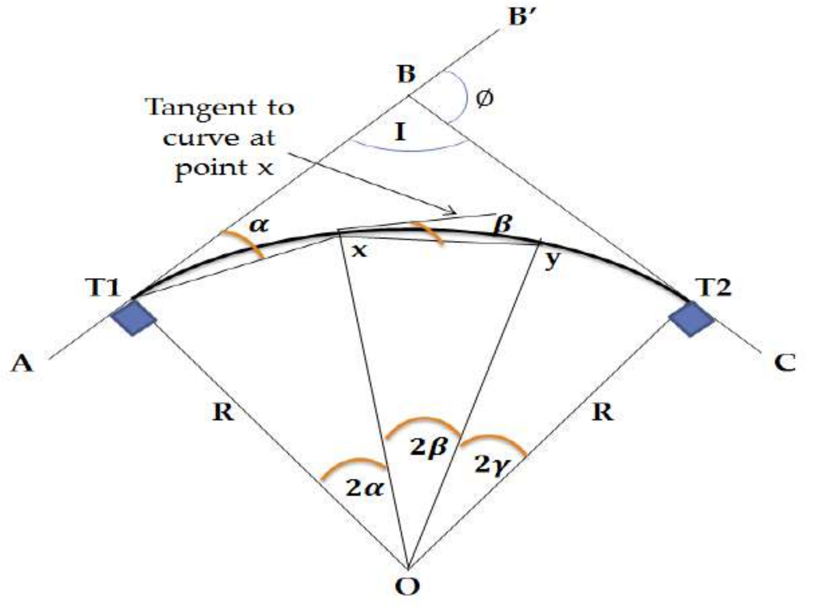
- The $\Delta\ BT_1T_2$ is an isosceles triangle and therefore the angle

$$\llcorner\ BT_1T_2 = \llcorner\ BT_2T_1 = \frac{\varnothing}{2}$$

The following definition can be given:

- The tangential angle α at T_1 to any point X on the curve T_1T_2 is equal to half the angle subtended at the centre of curvature O by the chord from T1 to that point.
- The tangential angle to any point on the curve is equal to the sum of the tangential angles from each chord up to that point.
- I.e. $T_1OT_2 = 2(\alpha + \beta + \gamma)$ and it follows
  - that $BT_1T_2 = (\alpha + \beta + \gamma)$.





Simple Curves

Problem 01: Two tangents intersect at chainage of 6 +26.57. it is proposed to insert a circular curve of radius 1000ft. The deflection angle being 16°38'. Calculate

a) chainage of tangents points

b) Lengths of long chord , Mid ordinate and External distance.

Solution:

Tangent length = $BT_1 = BT_2 = R\tan(\frac{\emptyset}{2})$

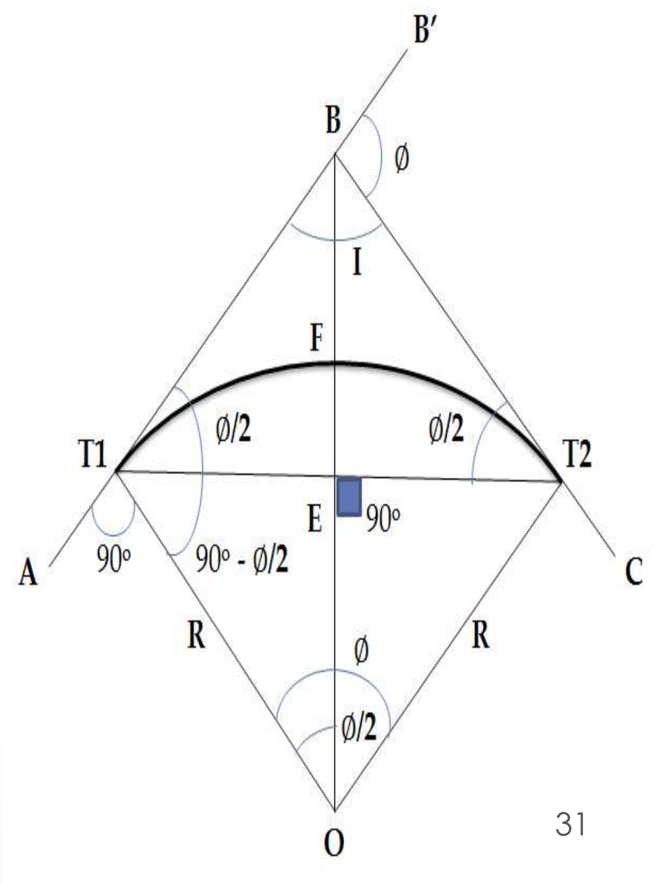
$BT_1 = BT_2 = 1000 \times \tan(16^o38`/2)$

$= 146.18$ ft

Length of curve = $L = \frac{\pi R \emptyset}{180^o}$

$L = \frac{\pi \times 1000 \times 16^o38`}{180^o} = 290.31$ft

Chainage of point of intersection	= 6 + 26.56
minus tangent length	= -1 + 46.18
chainage of T_1	= 4 + 80.39
plus L	= + 2 + 90.31
Chainage of T_2	= 7 + 70.70



Simple Curves

Problem 01:

Solution:

Length of chord = $\ell = 2\,R\,\sin(\frac{\emptyset}{2})$

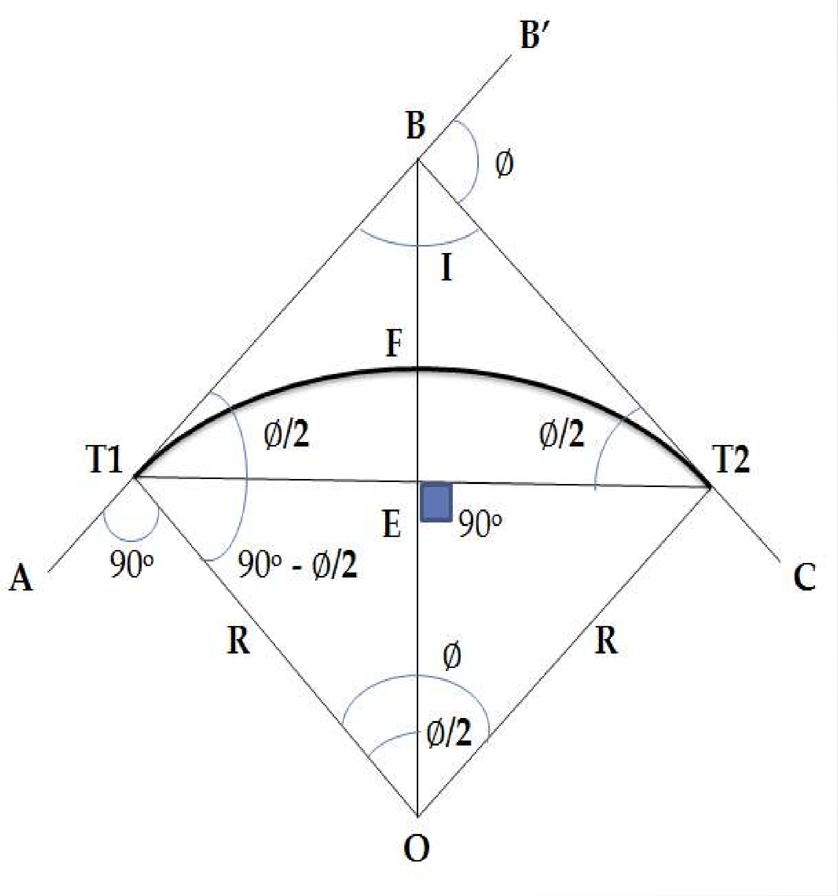
$\ell = 2 \times 1000 \times \sin(36^{o}38`/2) = 289.29$ ft

Mid ordinate = EF = $R\,(1 - \cos(\frac{\emptyset}{2}))$

EF= $1000 \times (\,1 - \cos(36^{o}38`/2)) = 10.52$ ft

Ex. distance = BF = $R\,(\,\sec\left(\frac{\emptyset}{2}\right) - 1)$

BF = $1000 \times ((1/\cos(\frac{\emptyset}{2}) - 1) = 10.63$ ft



Simple Curves

Problem 02: Two tangents intersect at chainage of 14 +87.33, with a deflection angle of 11º21`35``. Degree of curve is 6º. Calculate chainage of beginning and end of the curve.

Solution:

$D = 6^o$

$R = 5729.58 / D \text{ ft} = 954.93 \text{ ft}$

Tangent length $= BT_1 = BT_2 = R \tan(\frac{\emptyset}{2})$

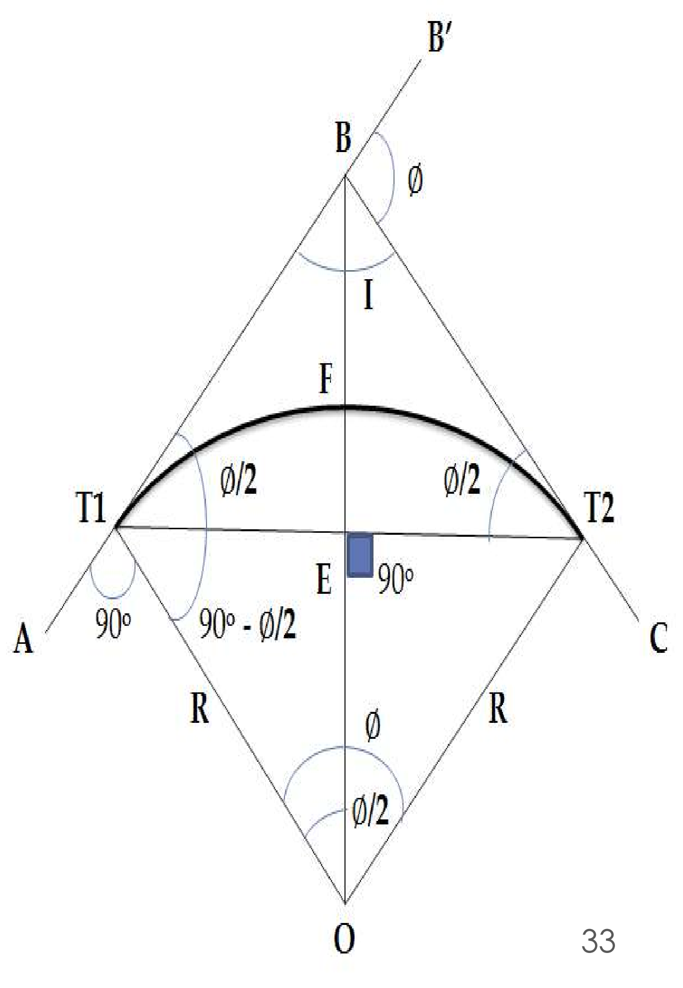
$BT_1 = BT_2 = 954.93 \times \tan(11^o21`35``/2)$

$BT_1 = BT_2 = 94.98 \text{ ft}$

Length of curve $= L = \frac{\pi R \emptyset}{180^o}$

$L = \frac{\pi \times 954.93 \times 11^o21`35``}{180^o} = 189.33 \text{ ft}$

Chainage of intersection point B	= 14 + 87.33
minus tangent length BT_1	= - 0 + 94.96
Chainage of T_1	= 13 + 92.35
plus L	= + 1 + 89.33
Chainage T_2	= 15 + 81.68



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

- These methods use the chain surveying tools only.
- These methods are used for the short curves which doesn't require high degree of accuracy.
- These methods are used for the clear situations on the road intersections.

a) By offset or ordinate from Long chord

b) By successive bisections of Arcs

c) By offset from the Tangents

d) By offset from the Chords produced

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

a) By offset or Ordinate from long chord

$ED = O_o$ = offset at mid point of T_1T_2

$PQ = O_x$ = offset at distance x from E , so that EP = x

$OT_1 = OT_2 = OD = R$ = Radius of the curve

Exact formula for offset at any point on the chord line may be derived as:

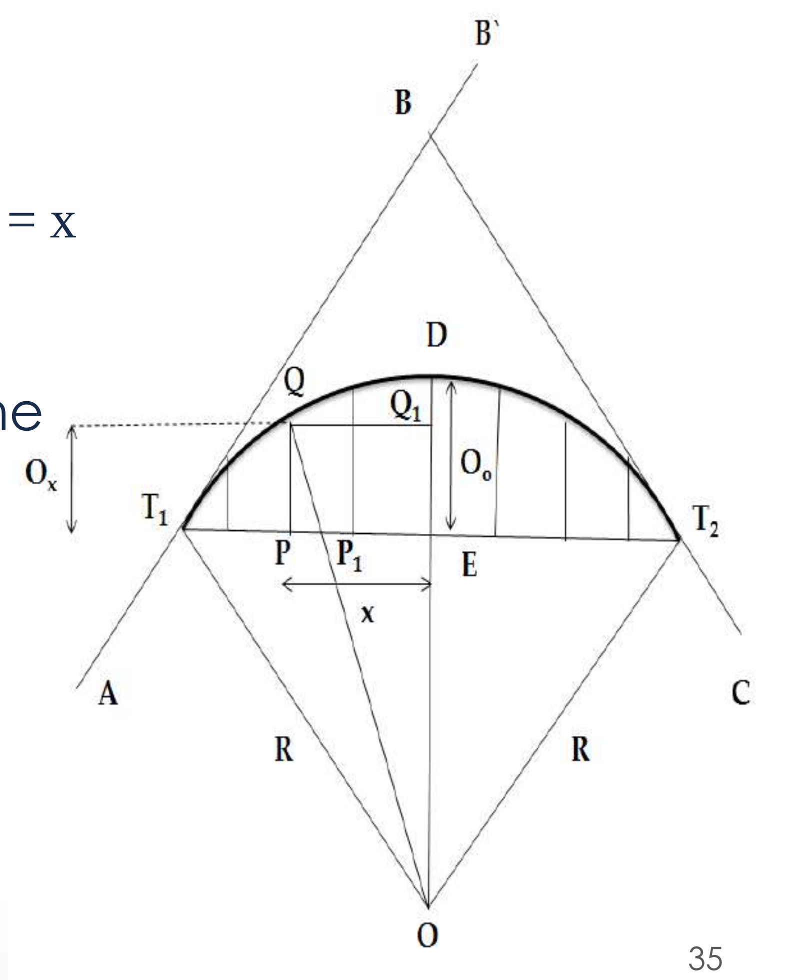
By Pathagoras theorem

$\Delta OT_1E,\ OT_1^2 = T_1E^2 + OE^2$

$OT_1 = R\ ,\ T_1E = \frac{\ell}{2}$

$OE = OD - DE = R - O_o$

$R^2 = (\ell/2)^2 + (R - O_o)^2$



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

a) By offset or Ordinate from long chord

$$DE = O_o = R - \sqrt{R^2 - \left(\frac{\ell}{2}\right)^2} \quad \text{-------} \; A$$

In eqn **A** two quantities are usually or must known.

In ΔOQQ_1, $\quad OQ^2 = QQ_1^2 + OQ_1^2$

$OQ_1 = OE + EQ_1 = OE + O_x$

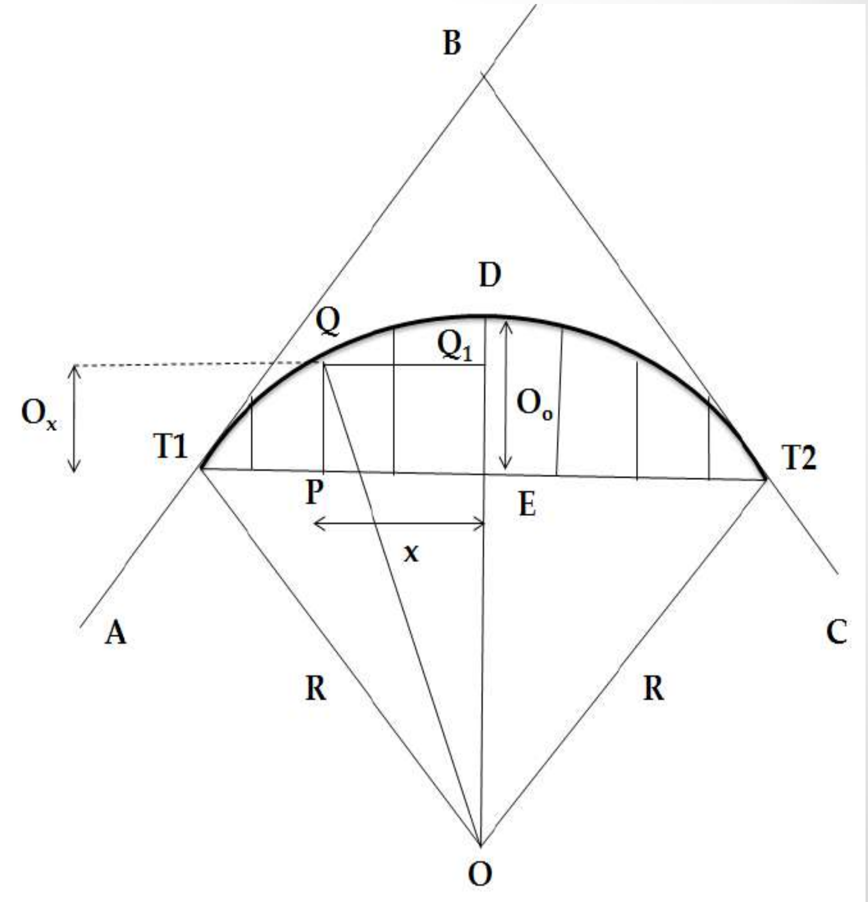
$OQ_1 = (R - O_o) + O_x$

$R^2 = x^2 + \{ (R - O_o) + O_x \}^2$

$O_x = \sqrt{(R^2 - x^2)} - (R - O_o)$

$O_x = \sqrt{(R^2 - x^2)} - (R - (R - \sqrt{(R^2 - (l/2)^2)}))$

- $O_x = \sqrt{(R^2 - x^2)} - (\sqrt{(R^2 - (\frac{\ell}{2})^2)})$ -- 1 exact formula



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

a) By offset or Ordinate from long chord

When the radius of the arc is larger as compare to the length of the chord, the offset may be calculated approximately by formula or

$$O_x = \frac{x\,(L - x)}{2R} \text{ -------- } 2 \text{ (Approximate formula)}$$

In eqn 1 the distance x is measured from the mid point of the long chord where as eqn 2 it is measured from the 1st tangent point T_1.

- This method is used for setting out short curves e.g curves for street kerbs.

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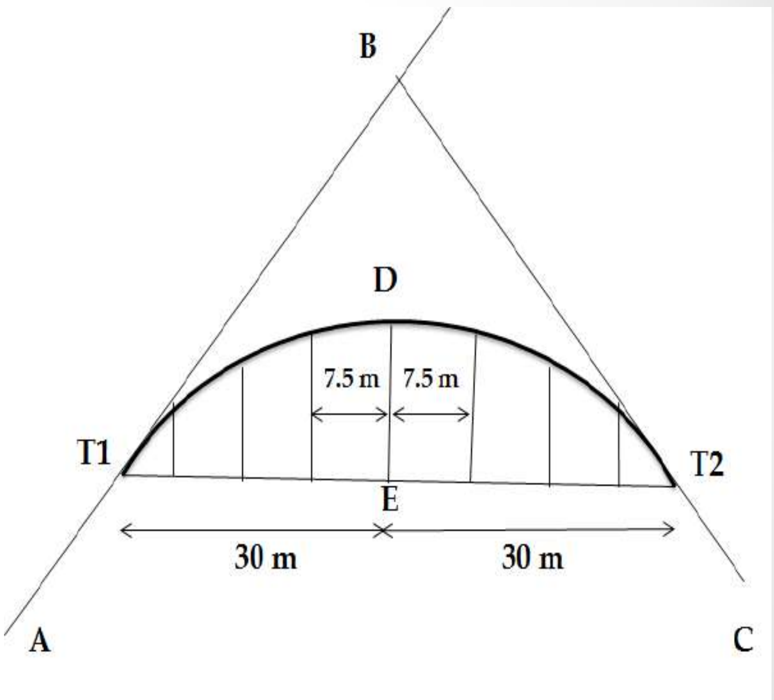
Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

a) By offset or Ordinate from long chord



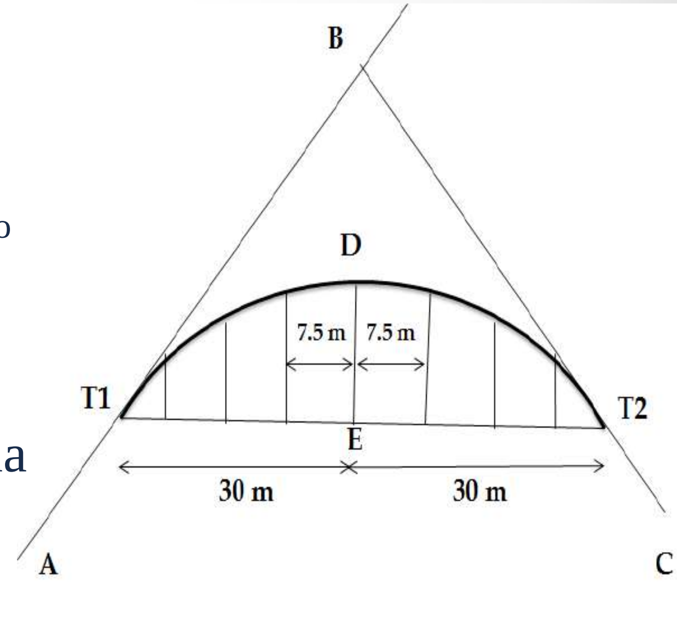
Working Method:

To set out the curve

- Divided the long chord into even number of equal parts.
- Set out offsets as calculated from the equation at each of the points of division. Thus obtaining the required points on the curve.
- Since the curve s symmetrical along ED, the offset for the right half of the curve will be same as those for the left half.

Simple Curves

Problem 03: calculate the ordinate at 7.5 m interval for a circular curve given that l = 60 m and R = 180 m, by offset or ordinate from long chord.



Solution:

Ordinate at middle of the long chord = verse sine = O_o

$O_o = R - \sqrt{(R^2 - (\frac{\ell}{2})^2)} = 180 - \sqrt{(180^2 - (\frac{60}{2})^2}$

$O_o = 2.52$ m

Various coo2.52 rdinates may be calculated by formula

$O_x = \sqrt{(R^2 - x^2)} - (\sqrt{(R^2 - (\frac{\ell}{2})^2)})$

x = distance measured from mid point of long chord.

$O_{7.5} = \sqrt{(180^2 - 7.5^2)} - (\sqrt{(180^2 - (\frac{60}{2})^2)}) = 2.34$ m

$O_{15} = \sqrt{(180^2 - 15^2)} - (\sqrt{(180^2 - (\frac{60}{2})^2)}) = 1.89$ m

$O_{22.5} = \sqrt{(180^2 - 22.5^2)} - (\sqrt{(180^2 - (\frac{60}{2})^2)}) = 1.14$ m

$O_{30} = \sqrt{(180^2 - 30^2)} - (\sqrt{(180^2 - (\frac{60}{2})^2)}) = 0$ m

X (m)	Ox (m)
0	2.52
7.5	2.34
15	1.89
22.5	1.14
30	0

Simple Curves
Method of Curve Ranging
1) Linear or Chain & Tape Method

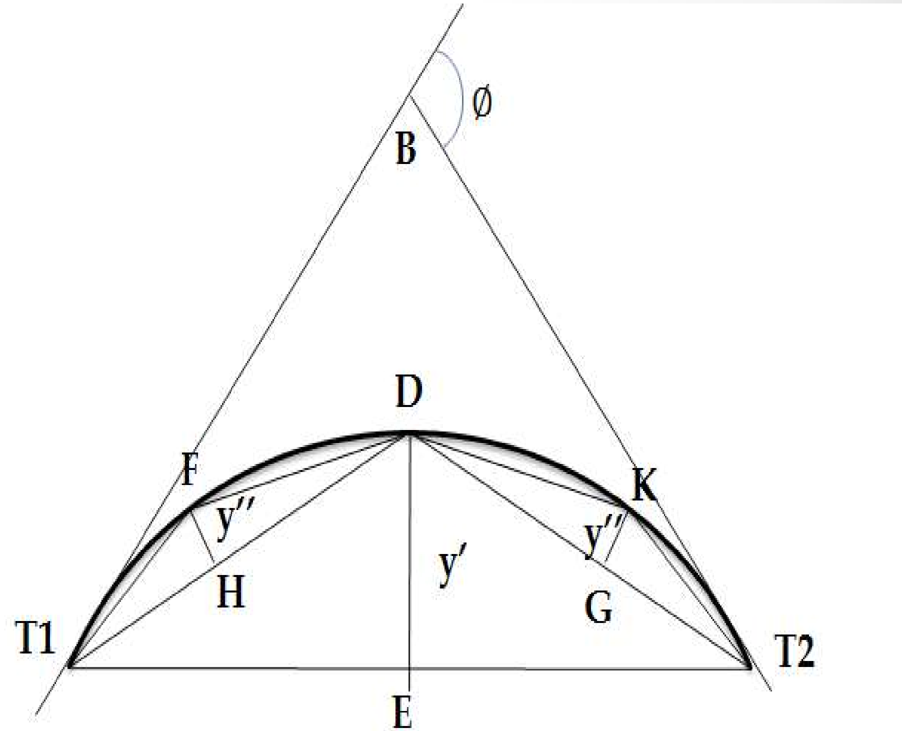
b) By Successive Bisection of Arcs

- Let T_1 and T_2 be the tangents points. Join T_1 and T_2 and bisect it at E.
- Setout offsets ED(y'), determined point **D** on the curve equal to

$$ED = y' = R\left(1 - \cos\left(\frac{\emptyset}{2}\right)\right)$$

- Join T_1D and DT_2 and bisect them at **F** and **G** respectively.
- Set out offset HF(y'') and GK(y'') each eqn be

$$FH = GK = y'' = R\left(1 - \cos\left(\frac{\emptyset}{4}\right)\right)$$



Obtain point **H** and **K** on a curve. By repeating the same process, obtain as many pints as required on the curve.

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

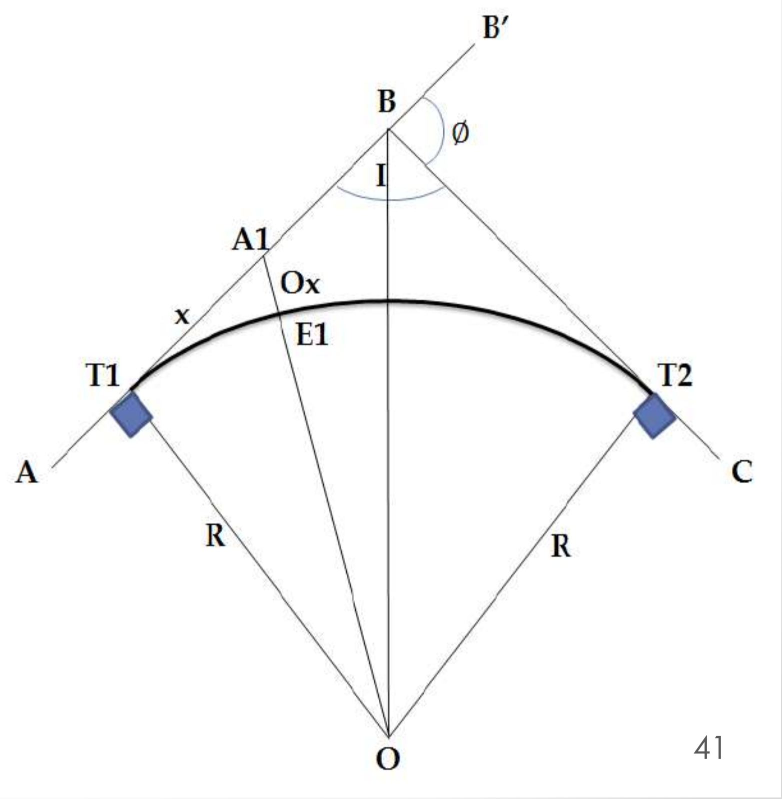
3) By Offsets from the Tangents

In this method the offsets are setout either radially or perpendicular to the tangents **BA** ad **BC** according to as the center **O** of the curve is accessible or inaccessible.

a) By Radial Offsets: (O is Accessible)

Working Method:

- Measure a distance x from T_1 on back tangent or from T_2 on the forward tangent.
- Measure a distance O_x along radial line A_1O.
- The resulting point E_1 lies on the curve.



Simple Curves

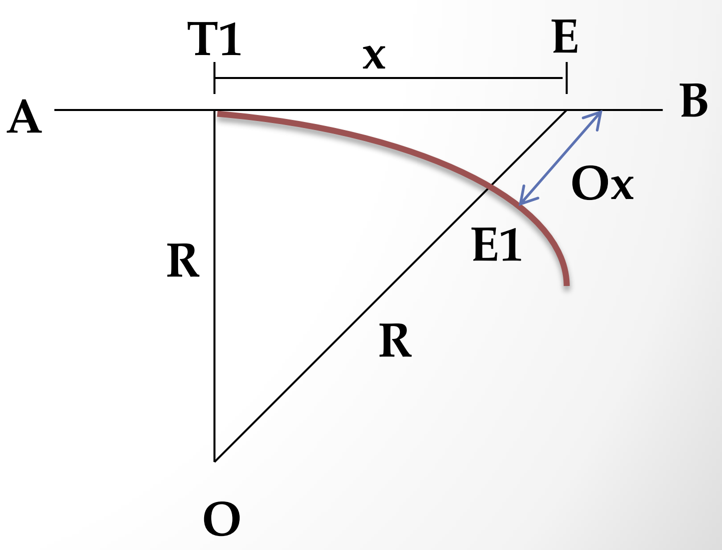
Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

In this method the offsets are setout either radially or perpendicular to the tangents **BA** ad **BC** according to as the center **O** of the curve is accessible or inaccessible.

a) By Radial Offsets:



$EE_1 = O_x$, offsets at distance x from T_1 along tangent AB.

In ΔOT_1E $\quad OT_1^2 + T_1E^2 = OE^2$

$OT_1 = R$, $T_1E = x$, $OE = R + O_x$

$R^2 + x^2 = (R + O_x)^2$

$R + Ox = \sqrt{(R^2 + O_x^2)}$

$O_x = \sqrt{(R^2 + X^2)} - R$ ---------A (Exact Formula)

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

a) By Radial Offsets:

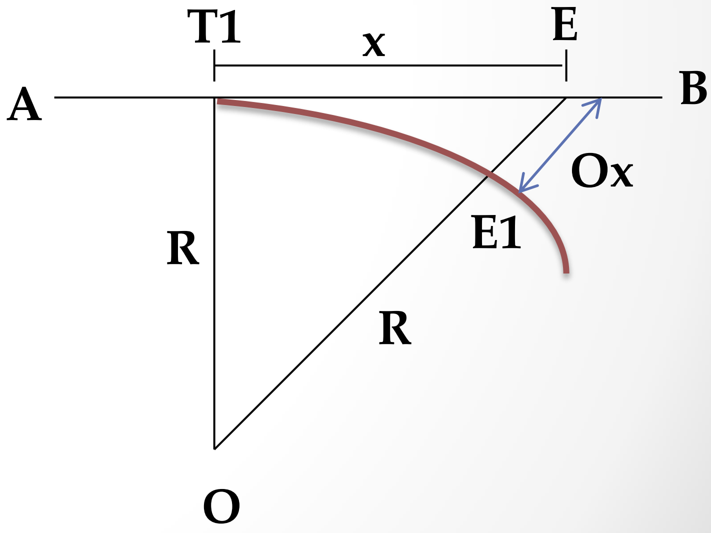
Expanding $\sqrt{(R^2 + x)}$, $O_x = R\left(1 + \frac{x^2}{2R^2} + \frac{x^4}{8R^4} + \ldots..\right) - R$

Taking any two terms

eqn (A) $O_x = R\left(1 + \frac{x^2}{2R^2}\right) - R$

$O_x = \frac{R\,x^2}{2\,R^2}$

$O_x = \frac{x^2}{2\,R}$ -------B (Approximate formula)



Used for short curve

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

a) By Radial Offsets:: Example

Required:

Set out a simple circular curve with R = 20m and $\emptyset = 45°$

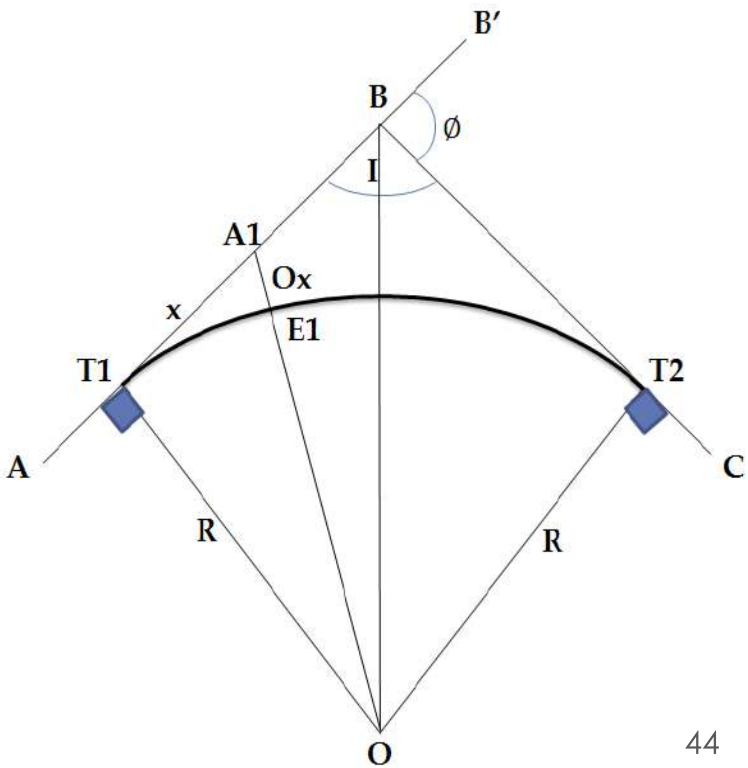
Calculations:

1. Calculate **T, L, *I***

2 $Ox = \sqrt{R^2 + x^2} - R$.

3. Prepare a table for x and Ox

X value	Ox value
0	
2	
4	
6	



4. Start setting out the curve.

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

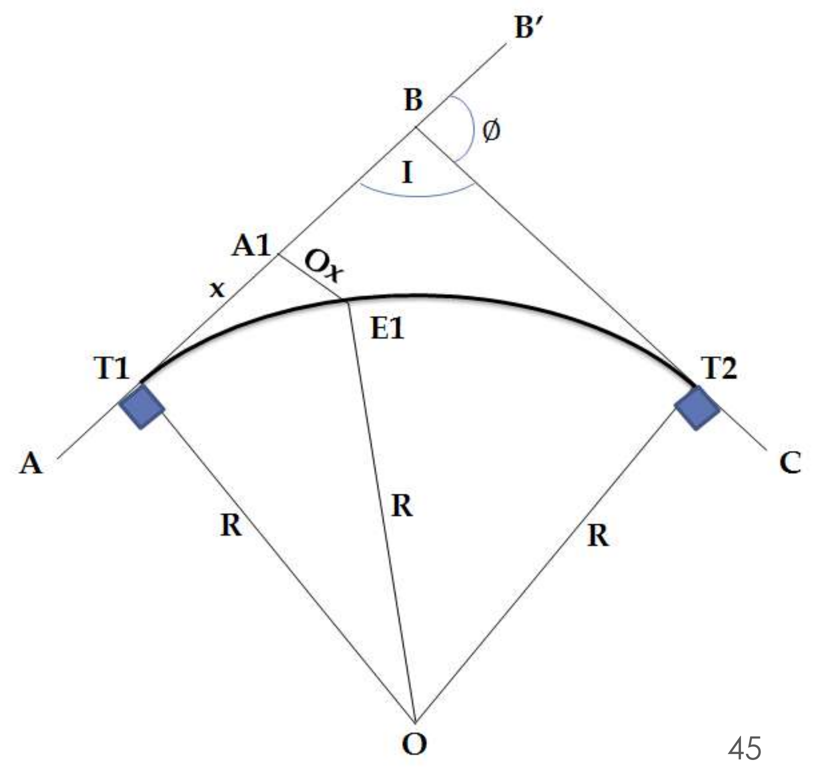
In this method the offsets are setout either radially or perpendicular to the tangents **BA** ad **BC** according to as the center **O** of the curve is accessible or inaccessible.

b) By Offsets Perpendicular to Tangents

(O is Inaccessible)

Working Method:

- Measure a distance x from T_1 on back tangent or from T_2 on the forward tangent.
- Erect a perpendicular of length O_x.
- The resulting point E_1 lies on the curve.



Simple Curves

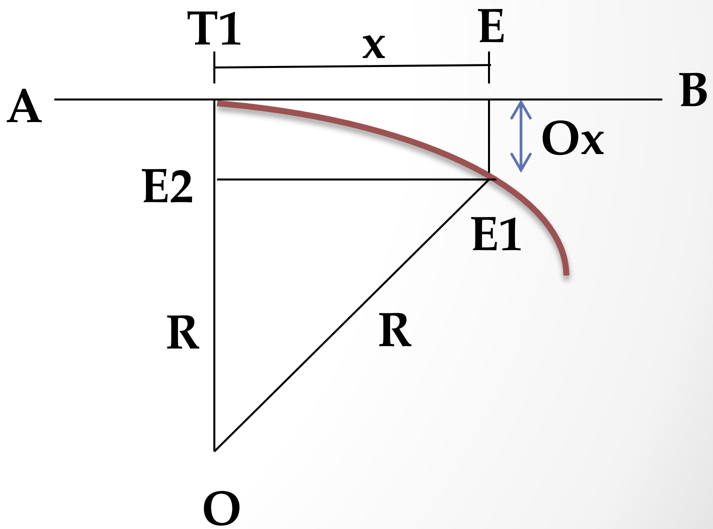
Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

b) By Offsets Perpendicular to Tangents

$EE_1 = O_x$ = T offset at a distance of x measured along tangent AB



ΔOE_1E_2, $OE_1^2 = OE_2^2 + E_1E_2^2$

$OE_1 = R$, $E_1E_2 = x$, $OE_2 = R - O_x$

$R^2 = (R - O_x)^2 + x^2$

$(R - O_x)^2 = R^2 - x^2$

$O_x = R - \sqrt{(R^2 - x^2)}$ ------- A (Exact Formula)

Expanding $\sqrt{(R^2 - x^2)}$ and neglecting higher power

$O_x = \dfrac{x^2}{2R}$ ------- Approximate Formula

Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

3) By Offsets from the Tangents

b) By Offsets Perpendicular to Tangents : Example

Required:

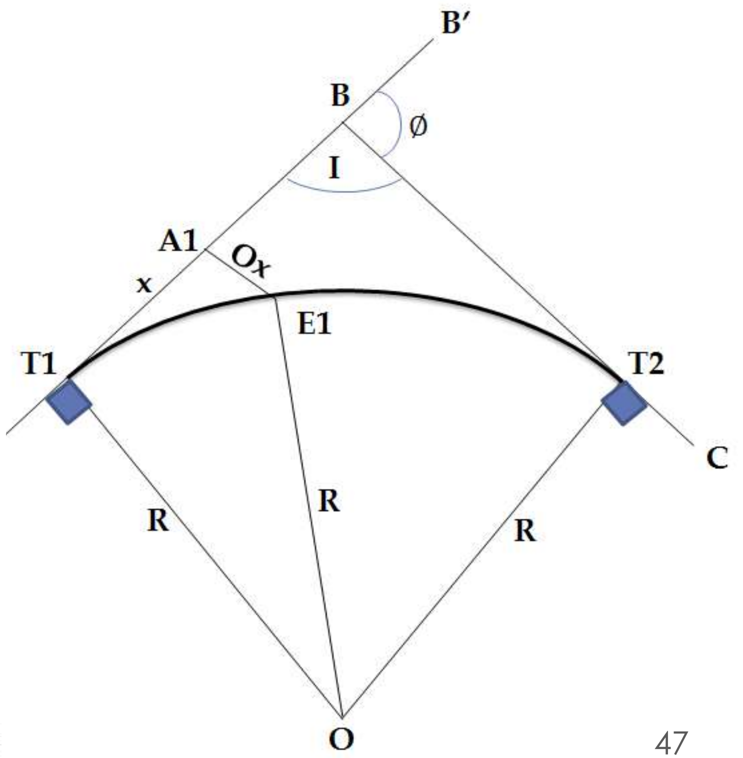
Set out a simple circular curve with R = 20m and $\phi = 45^{\circ}$

Calculations:

1. Calculate: T, I , L
2. $Ox = R - \sqrt{R^2 - x^2}$
3. Prepare table for x and Ox

$x\ (m)$	$Ox\ (m)$
0	0.000
2	0.100
4	0.404
6	0.921
8	1.670

4. Start setting out the curve.
5. Check the measured length of the curve by comparing it with the calculated one.



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

4) By Offsets from Chord Produced

$T_1E = T_1E_1 = b_o$ --- 1st chord of length "b_1"

EF, FG, etc = successive chords of length b_2 and b_3, each equal to length of unit chord.

$BT_1E = \alpha$ = angle b/w tangents T_1B and 1st chord T_1E

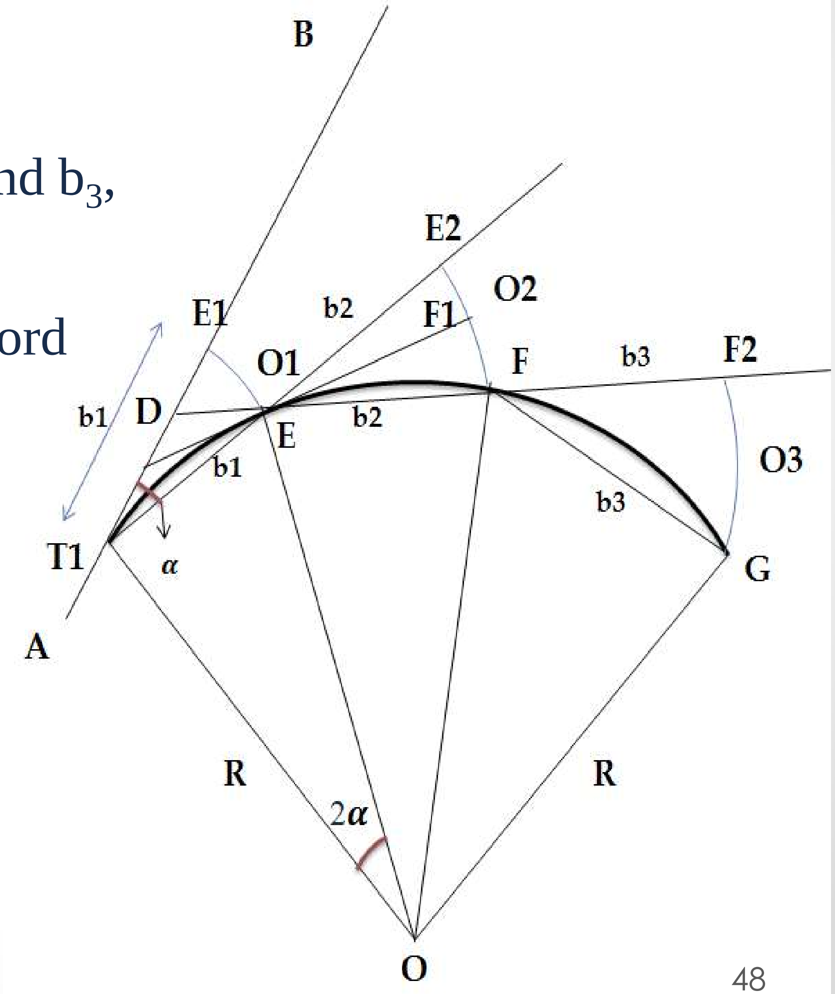
$E_1E = O_1$ = offset from tangent BT_1

$E_2F = O_2$ = offset from preceding chord T_1E produced.

Arc T_1E = chord T_1E

$T_1E = OT_1\ 2\alpha$

$T_1E = R\ 2\alpha$, $\qquad \alpha = \dfrac{T_1E}{2R}$ --------- 1



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

4) By Offsets from Chord Produced

Similarly chord EE_1 = arc E_1E

1^{st} offset $O_1 = E_1E = T_1E \times \alpha$

$O_1 = T_1E\ T_1E / 2R$

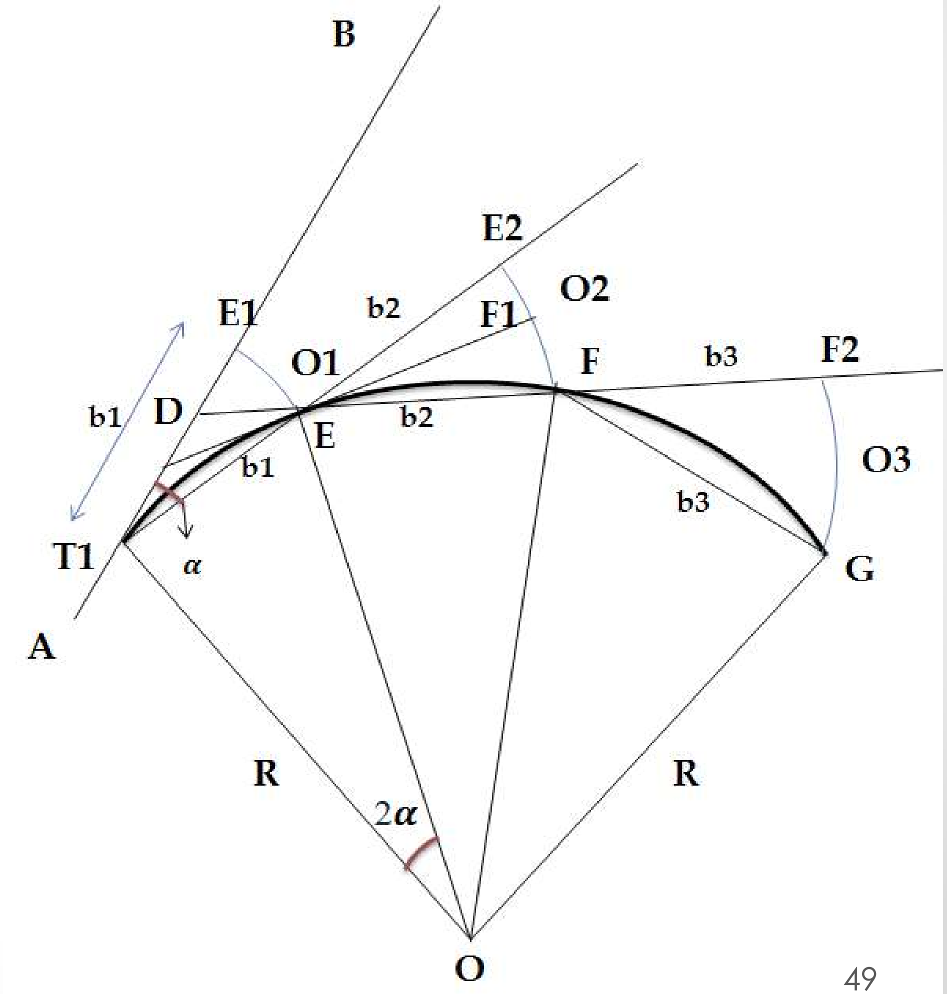
$O_1 = T_1E^2/2R = \frac{b_1^{\,2}}{2R}$ ----- 2

$\llcorner E_2EF_1 = \llcorner DET_1$ (vertically opposite)

$\llcorner DET_1 = \llcorner DT_1E$ since $DT_1 = DE$

::: $\llcorner E_2EF_1 = \llcorner DT_1E = \llcorner E_1T_1E$

The Δs E_1T_1E and E_2EF_2 being nearly isosceles may be considered similar



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

4) By Offsets from Chord Produced

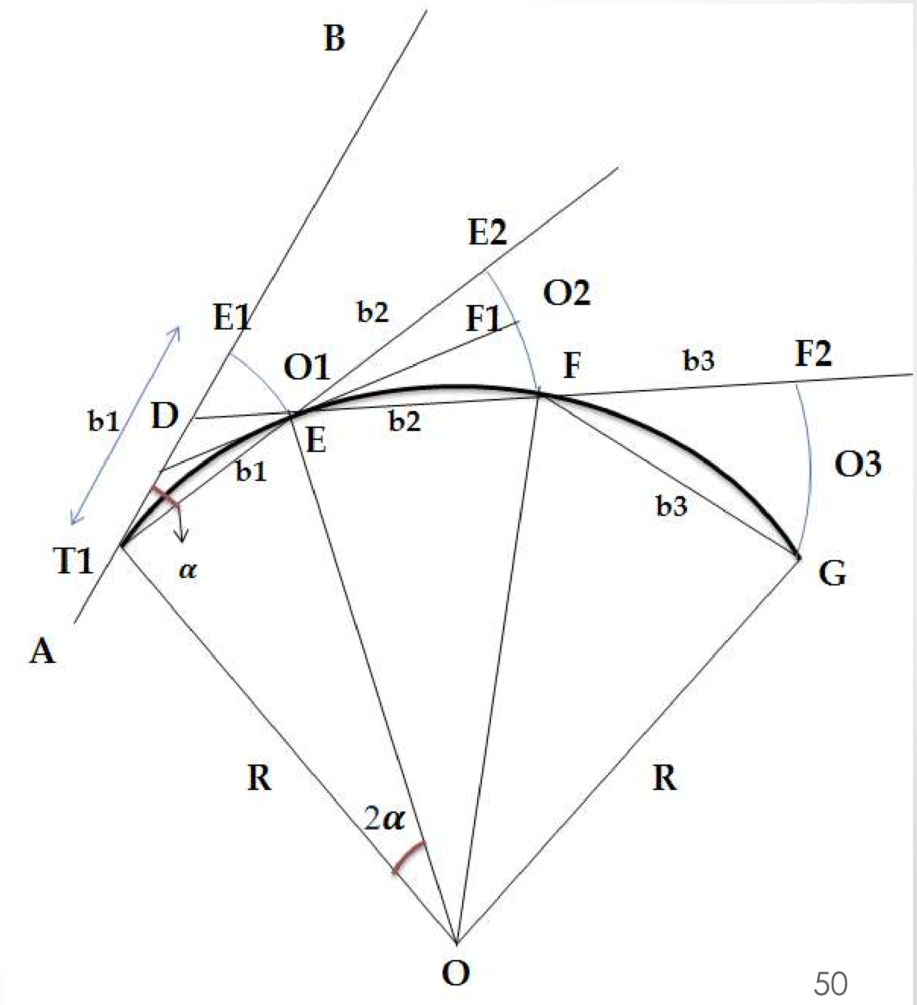
The Δs E_1T_1E and E_2EF_2 being nearly isosceles may be considered similar

$$\frac{E_2F_1}{EE_2} = \frac{E_1E}{T_1E} \quad \text{i.e} \quad \frac{E_2F_1}{b_2} = \frac{O_1}{b_1}$$

$$E_2F_1 = \frac{b_2O_1}{b_1} = \frac{b_2}{b_1} \text{ x } \frac{b_1{}^2}{2R} = \frac{b_2\, b_1}{2R}$$

F_1F being the offset from the tangent at E is equal to :

$$F_1F = \frac{EF^2}{2R} = \frac{b_2{}^2}{2R}$$



Simple Curves

Method of Curve Ranging

1) Linear or Chain & Tape Method

4) By Offsets from Chord Produced

Now 2nd offset O_2

$O_2 = E_2F = E_2F_1 + F_1F$

$$O_2 = E_2F = \frac{b_2 b_1}{2R} + \frac{b_2{}^2}{2R}$$

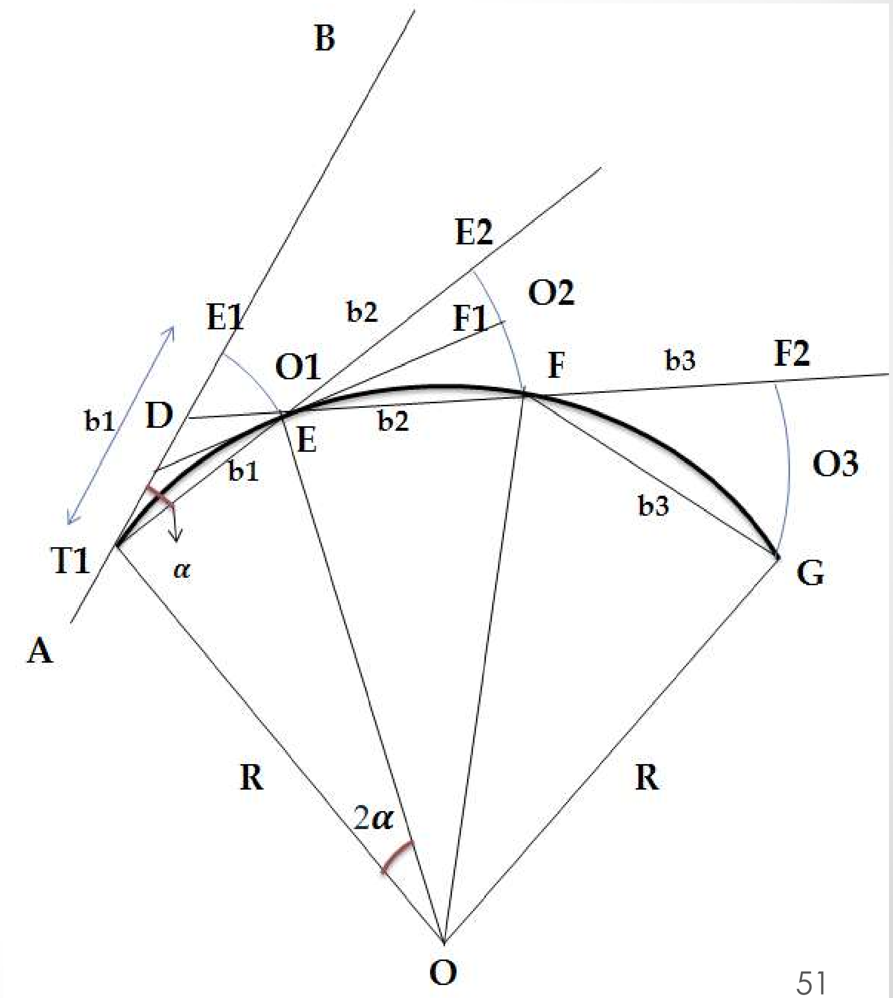
$$O_2 = E_2F = \frac{b_2\,(b_1 + b_2)}{2R}$$

Similarly 3rd offset

$$O_3 = \frac{b_3\,(b_2 + b_3)}{2R}$$, since $b_2=b_3=b_4 \ldots$

$$O_3 = \frac{b_2{}^2}{R}$$, so $O_3=O_4=O_5$ except for last offset.

$$O_n = \frac{b_n\,(bn-1 \ldots\ldots\ldots + b_n)}{2R}$$



Simple Curves

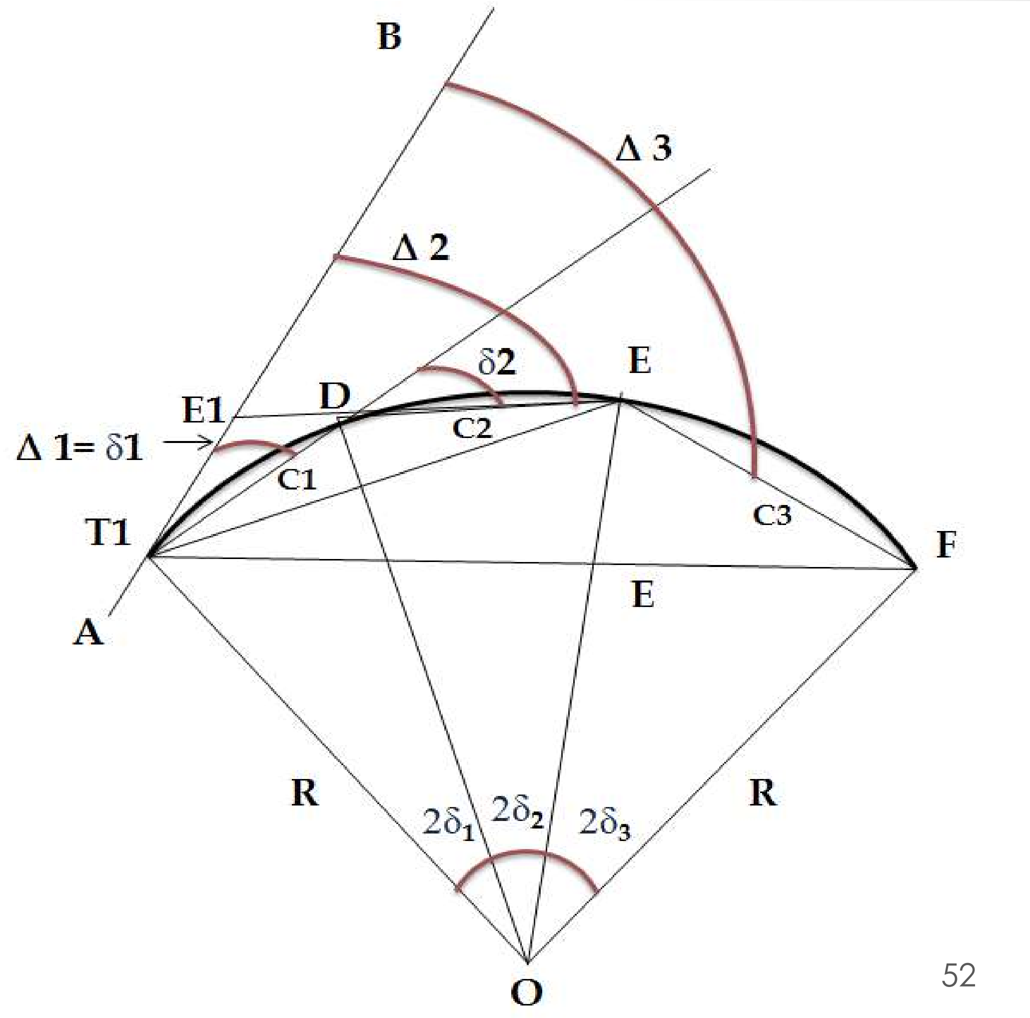
Method of Curve Ranging

2) Angular or Instrumental Methods

1) Rankine's Method of Tangential Angles
2) Two Theodolite Method

1) Rankine's Method of Tangential Angles

- In this method the curve is set out tangential angle often called deflection angles with a theodolite, chain or tape.



Simple Curves

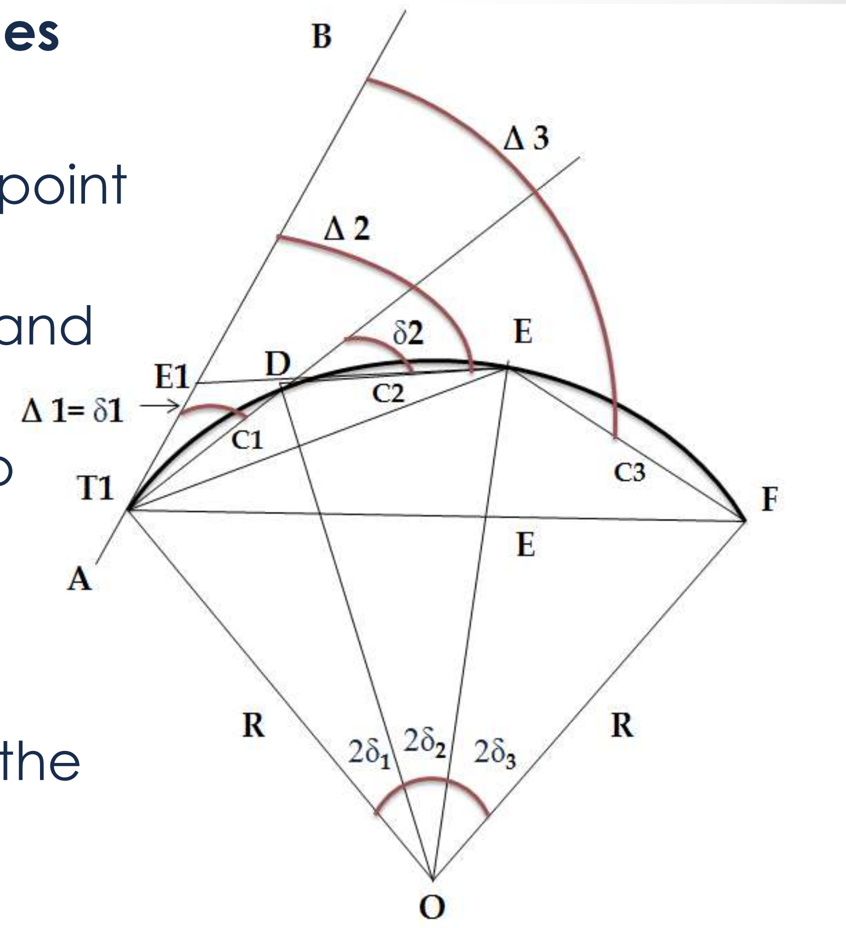
Method of Curve Ranging

2) Angular or Instrumental Methods

1) Rankine's Method of Tangential Angles

Working method:

1. Fix the theodolite device to be at point T_1 and directed at point B.
2. Measure the deflection angles δ_1 and the chords C_1.
3. Connect the ends of the chords to draw the curve.



Deflection Angles:

The angles between the tangent and the ends of the chords from point T_1.

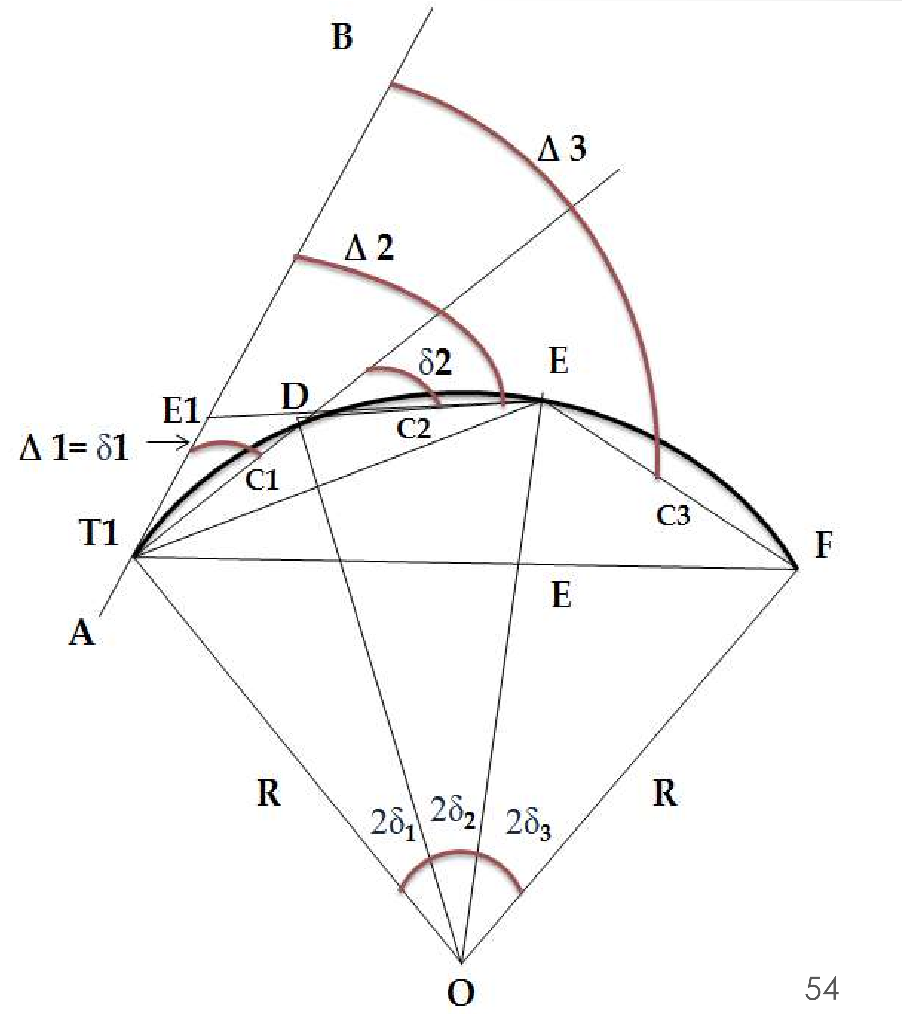
Simple Curves

Method of Curve Ranging

2) Angular or Instrumental Methods

1) Rankine's Method of Tangential Angles

- AB = Rear tangent to curve
- D,E,F = Successive point on the curve
- δ_1, δ_2, δ_3….. The tangential angles which each of successive chord …T_1D,DE,EF…… makes with the respective tangents at T_1, D, E.
- Δ_1 , Δ_2, Δ_3…… Total deflection angles
- C_1, C_2, C_3…….. Length of the chord. T_1D, DE, EF.



Simple Curves

Method of Curve Ranging

2) Angular or Instrumental Methods

1) Rankine's Method of Tangential Angles

Arc T_1D = chord T_1D = C_1

So, $T_1D = R \times 2\,\delta_1$

$$C_1 = T_1D = \frac{2\,\pi\,R\,\delta_1}{180}$$ And

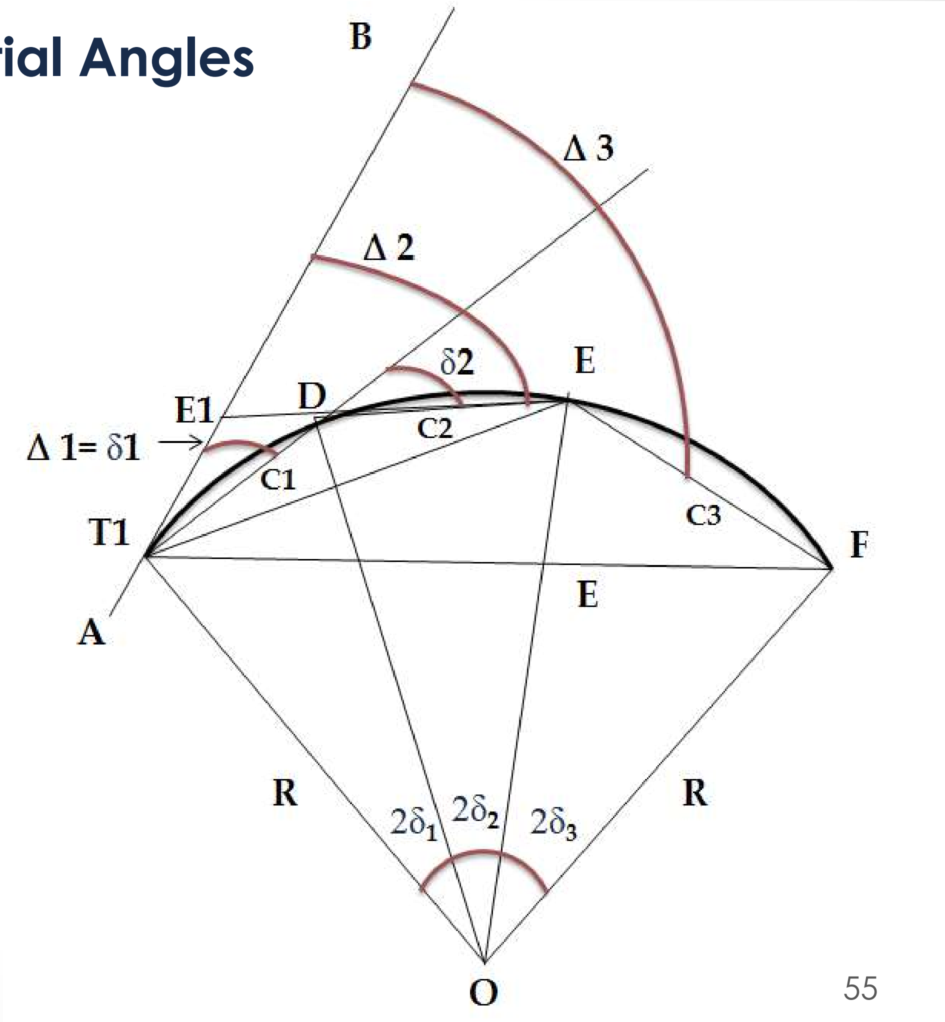
$$\delta_1 = \frac{C_1 * 180}{2\,\pi\,R} \text{ degree}$$

$$\delta_1 = \frac{C_1 * 180 * 60}{2\,\pi\,R} \text{ min}$$

$$\delta_1 = \frac{1718.9 * C_1}{R}$$

$$\delta_2 = \frac{1718.9 * C_2}{R}$$

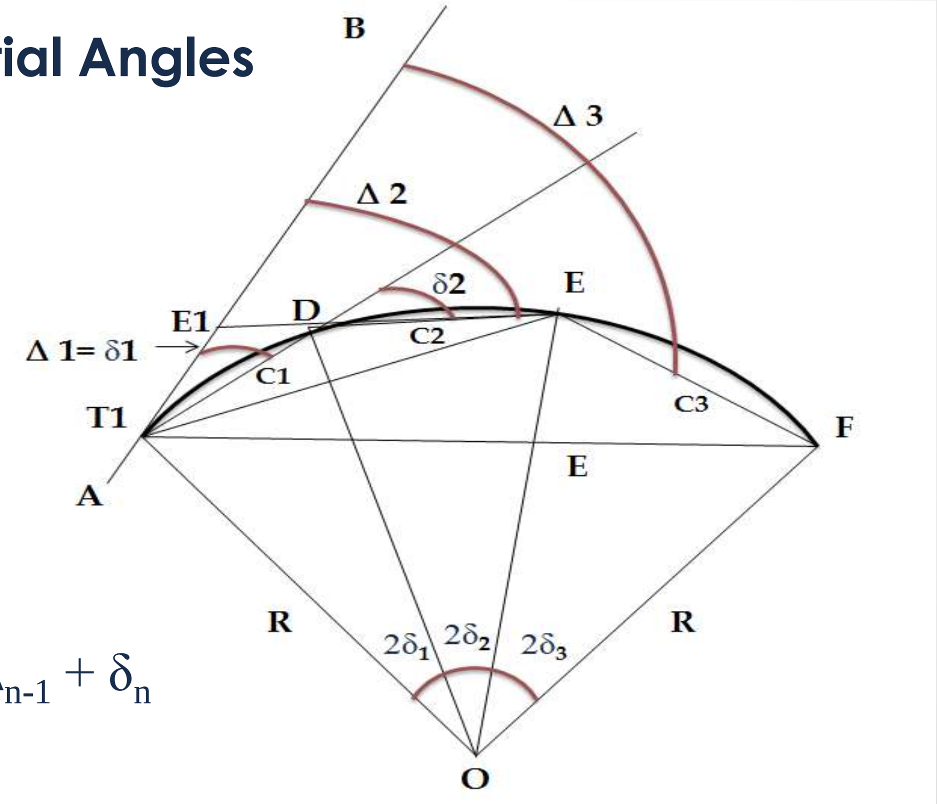
$$\delta_n = \frac{1718.9 * Cn}{R}$$



Simple Curves

Method of Curve Ranging

2) Angular or Instrumental Methods



1) Rankine's Method of Tangential Angles

Total deflection angle for the

1^{st} chord --$T_1D = BT_1D$ $\therefore \Delta_1 = \delta_1$

2^{nd} Chord --DE = $B T_1 E$

But $B T_1 E = B T_1 D + D T_1 E$

$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$

$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$

$\Delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4 = \Delta_3 + \delta_4$

$\Delta_n = \delta_1 + \delta_2 + \ldots\ldots\ldots\ldots + \delta_n = \Delta_{n-1} + \delta_n$

Check:

Total deflection angle $BT_1T_2 = \frac{\varphi}{2}$, φ = Deflection angle of the curve

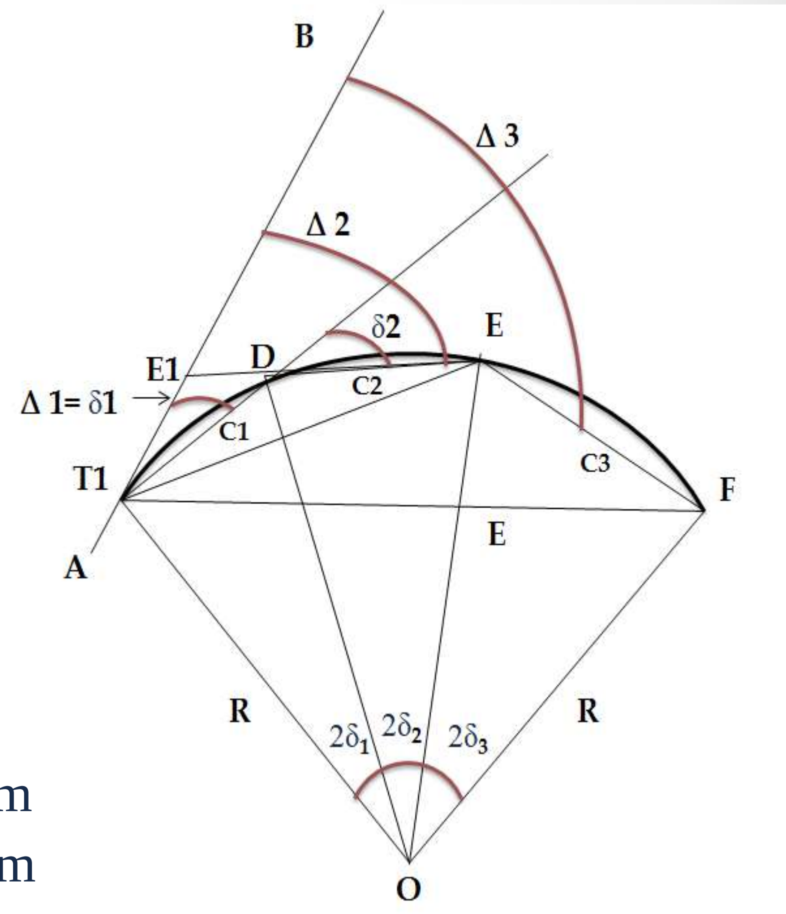
This Method give more accurate result and is used in railway & other important curve.

Simple Curves

Problem 04: Two tangents intersect at chainage 2140 m . ø= 18°24`. Calculate all the data necessary for setting out the curve, with R = 600 m and Peg interval being 20 m by:

1) By deflection angle ø

2) offsets from chords.



Solution:

$$BT_1 = BT_2 = R\tan\left(\frac{\emptyset}{2}\right) = 600\tan\frac{18^o24`}{2}$$

$$BT_1 = BT_2 = 97.18 \text{ m}$$

$$\text{Length of curve} = L = \left(\frac{\pi\, R\, \emptyset}{180^o}\right) = \left(\frac{\pi\, 600\; 18^o24`}{180^o}\right)$$

L = 192.68 m

Chainage of point of intersection	= 2140 m
Minus Tangent length	= - 97.18 m
Chainage of T_1	= 2042.82 m
Plus L	= + 192.68 m
Chainage of T_2	= 2235.50 m

Simple Curves

Problem 04:

Solution:

Length of 1st chord = C_1 = 2060 – 2042.82 = 17.18 m

$C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = 20$ m

C_{10} = 2235.50 – 2220 = 15.15 m

1) By deflection angle

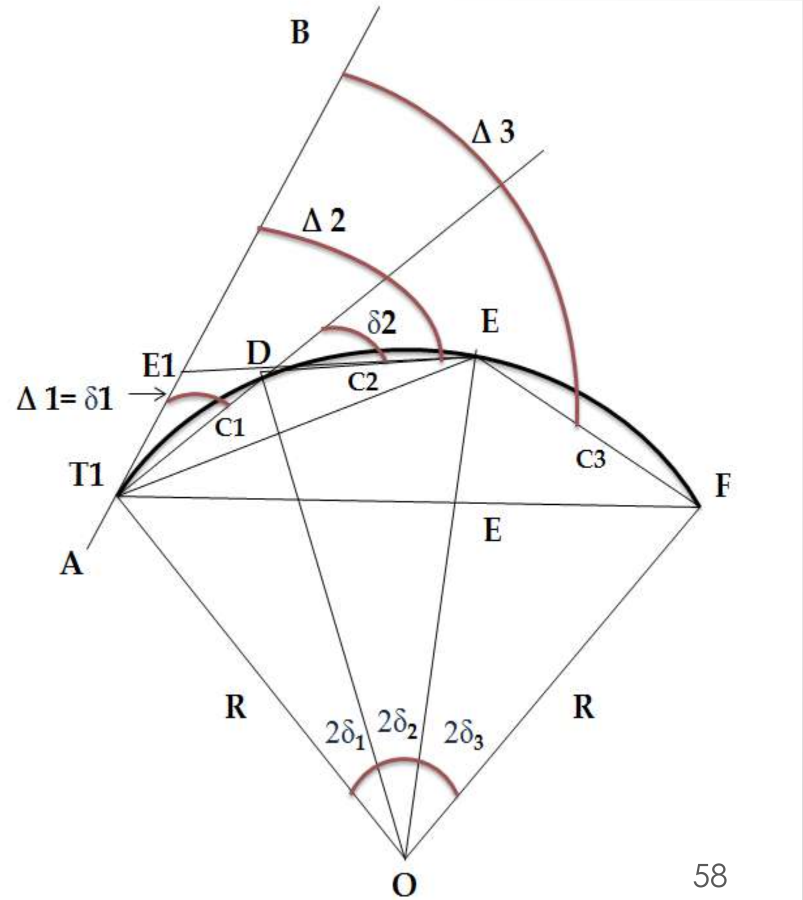
$$\delta_1 = \frac{1718.9\ C_1}{R}\ (\text{min}) = \frac{1718.9\ C_1}{60\ R}\ (\text{degree})$$

$$\delta_1 = \frac{1718.9 \times 17.18}{60 \times 600} = 0^o 49` 13.07``$$

$$\delta_2 = \frac{1718.9 \times 20}{60 \times 600} = 0^o 57` 17.8$$

$$\delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = \delta_9$$

$$\delta_{10} = \frac{1718.9 \times 15.15}{60 \times 600} = 0^o 44` 24.3``$$



Note: No of chords $= \frac{lenght\ of\ curve - C_1}{Interval}$

- = (192.68 – 17.18)/ 20 = 8.77=8

Simple Curves

Problem 04:

Solution:

1) By deflection angle

Total deflection (tangential) angle for the chords are:

$\Delta_1 = \delta_1 = 0^o49`13.07``$

$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2 = 1^o46`30.87``$

$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3 = 2^o43`48.67``$

$\Delta_4 = 3^o41`6.4$

$\Delta_5 = 4^o38`24.27``$

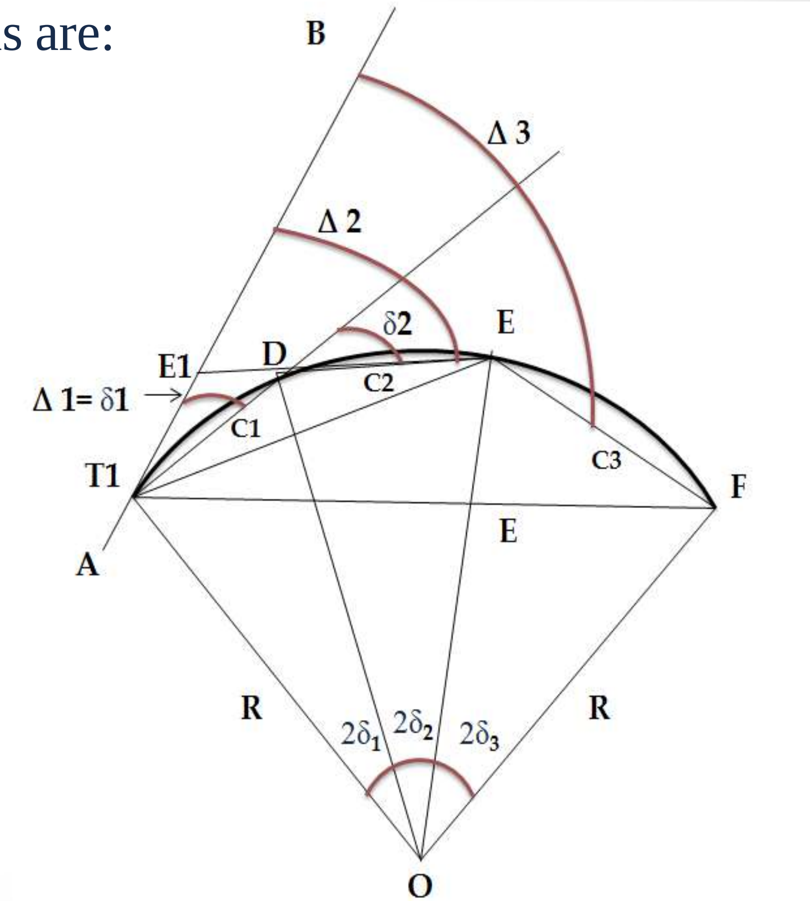
$\Delta_6 = 5^o35`42.07``$

$\Delta_7 = 6^o32``54.87``$

$\Delta_8 = 7^o30`17.67``$

$\Delta_9 = 8^o27`35.47``$

$\Delta_{10} = \Delta_9 + \delta_{10} = 9^o11`54.77``$



Check: $\Delta_{10} = \frac{\emptyset}{2} = \frac{18^o34`}{2} = 9^o12`0``$

Simple Curves

Problem 04:

Solution:

2) By Offsets from Chords

$$O_n = \frac{b_n\,(\,bn-1 \ldots\ldots\ldots\ldots + bn)}{2R}$$

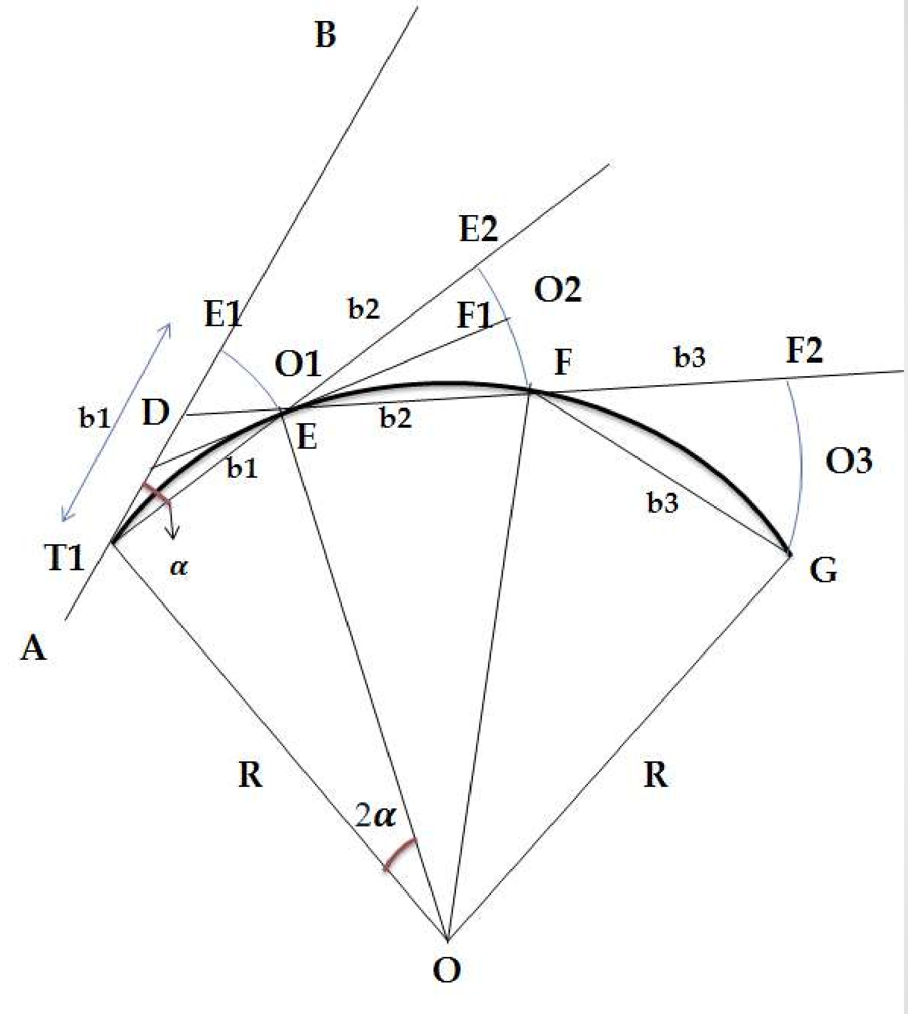
$$O_1 = \frac{b_n^{\;2}}{2R} = \frac{(17.18)2}{2\;x\;600} = 0.25 \text{ m}$$

$$O_2 = \frac{b_2\,(b_1 + b_2)}{2R} = \frac{20\;(17.18+20)}{2\;x\;600} = 0.62 \text{ m}$$

$$O_3 = \frac{b_3\,(b_2 + b_3)}{2R} = \frac{b_2^{\;2}}{R} = 0.67 \text{ m}$$

$$O_3 = O_4 = O_5 = O_6 = O_7 = O_8 = O_9$$

$$O_{10} = \frac{b_{10}\,(b_9 + b_{10})}{2R} = \frac{15.50\;(20+15.50)}{2\;x\;600} = 0.46 \text{ m}$$



Simple Curves

Method of Curve Ranging

2) Angular or Instrumental Methods

2) Two Theodolite Method

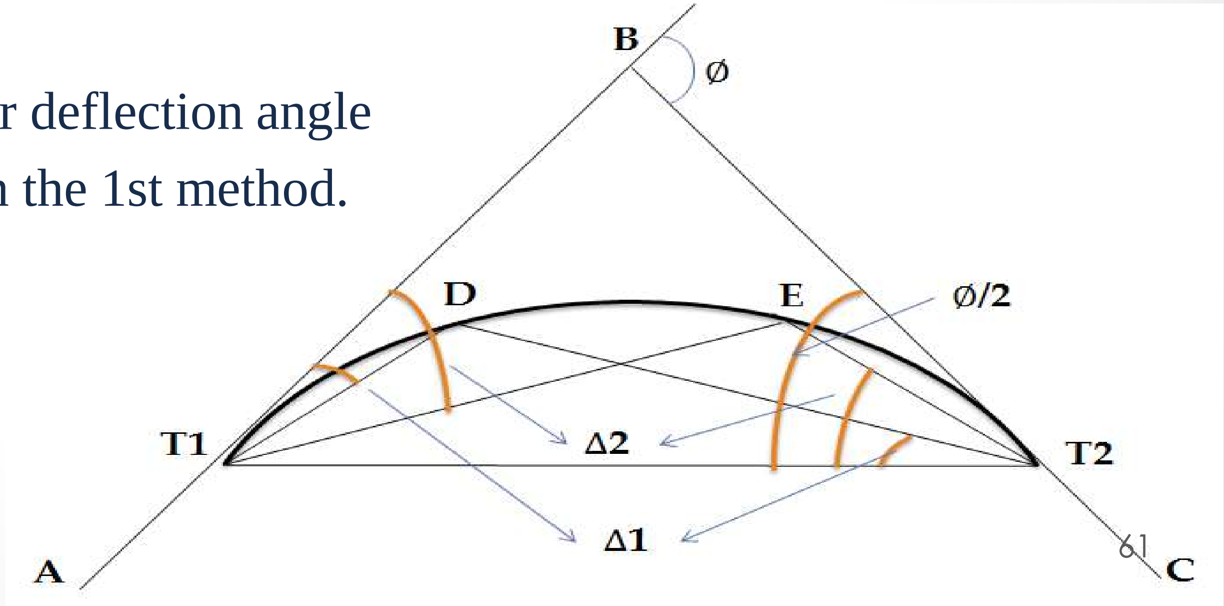
- This method is used when ground is not favorable for accurate chaining i.e rough ground , very steep slope or if the curve one water
- It is based on the fact that angle between tangent & chord is equal to the angle which that chord subtends in the opposite segments.

$\Delta 1$ is b/w tangent T1B & T_1D => BT1D $= T_1T_2D = \Delta_1$

$T_1E = \Delta_2 = T_1 T_2 E$

The total tangential angle or deflection angle

$\Delta_1, \Delta_2\ \Delta_3 \ldots$,As calculate in the 1st method.



Simple Curves

Obstacles in Setting Out Simple Curve

- The following obstacles occurring in common practice will be considered.

1) When the point of intersection of Tangent lines is inaccessible.

2) When the whole curve cannot be set out from the Tangent point, Vision being obstructed.

3) When the obstacle to chaining occurs.

Simple Curves

Obstacles in Setting Out Simple Curve

1) When the point of intersection of Tangent lines is inaccessible

- When intersection point falls in lake, river , wood or any other construction work

1) To determine the value of Ø

2) To locate the points T_1 & T_2

Calculate θ_1 & θ_2 by instrument (theodolite).

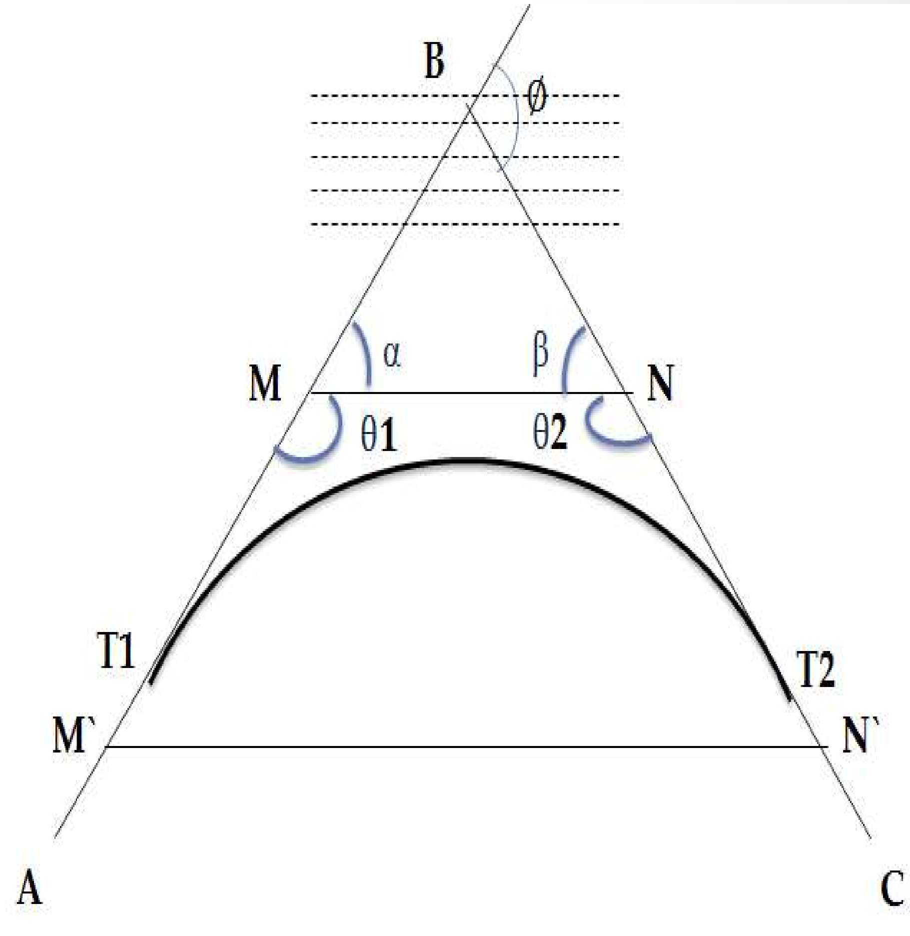
∟ BMN = $\alpha = 180 - \theta_1$

∟ BNM = $\beta = 180 - \theta_2$

Deflection angle = Ø = $\alpha + \beta$

Ø = 360^o – (sum of measured angles)

Ø = $360^o - (\theta_1 + \theta_2)$



Simple Curves

Obstacles in Setting Out Simple Curve

1) When the point of intersection of Tangent lines is inaccessible

Calculate the BM & BN from ΔBMN

By sine rule

$$\frac{BM}{\sin\beta} = \frac{MN}{\sin(180^o - (\alpha+\beta))}$$

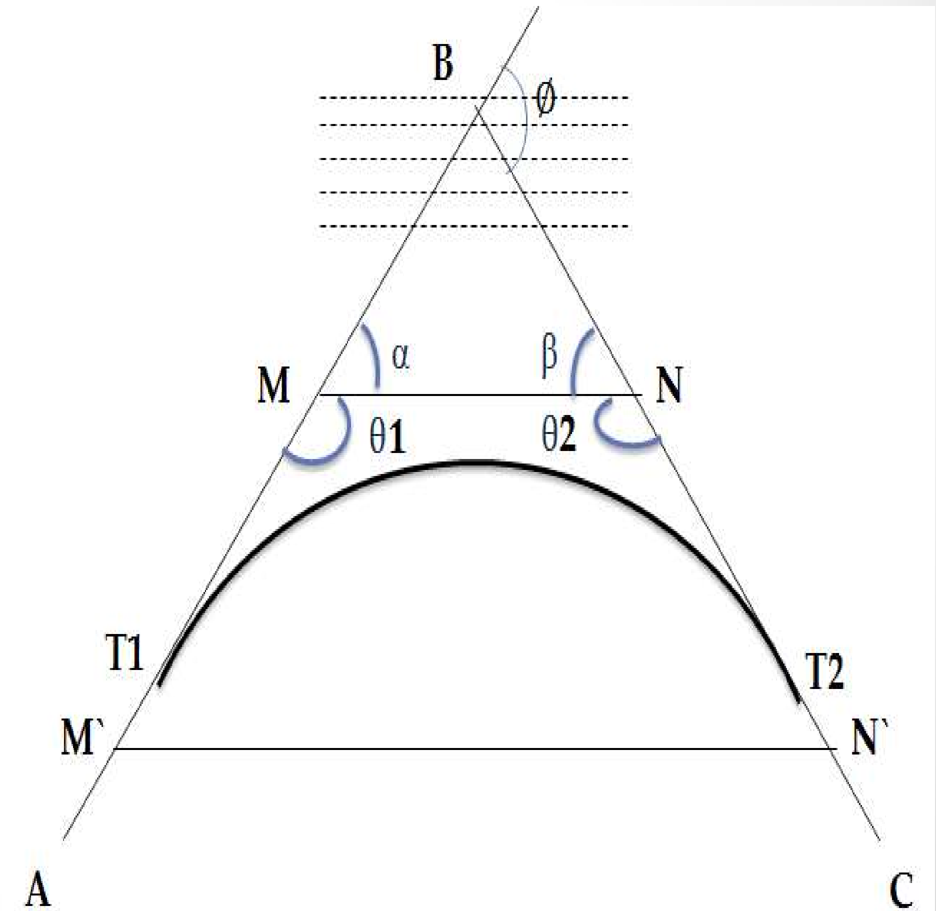
$$BM = \frac{MN \sin\beta}{\sin(180^o - (\alpha+\beta))}$$

$$BN = \frac{MN \sin\alpha}{\sin(180^o - (\alpha+\beta))}$$

$$BT_1 \;\&\; BT_2 = R * \tan\left(\frac{\emptyset}{2}\right)$$

$MT_1 = BT_1 - BM$

$NT_2 = BT_2 - BN$

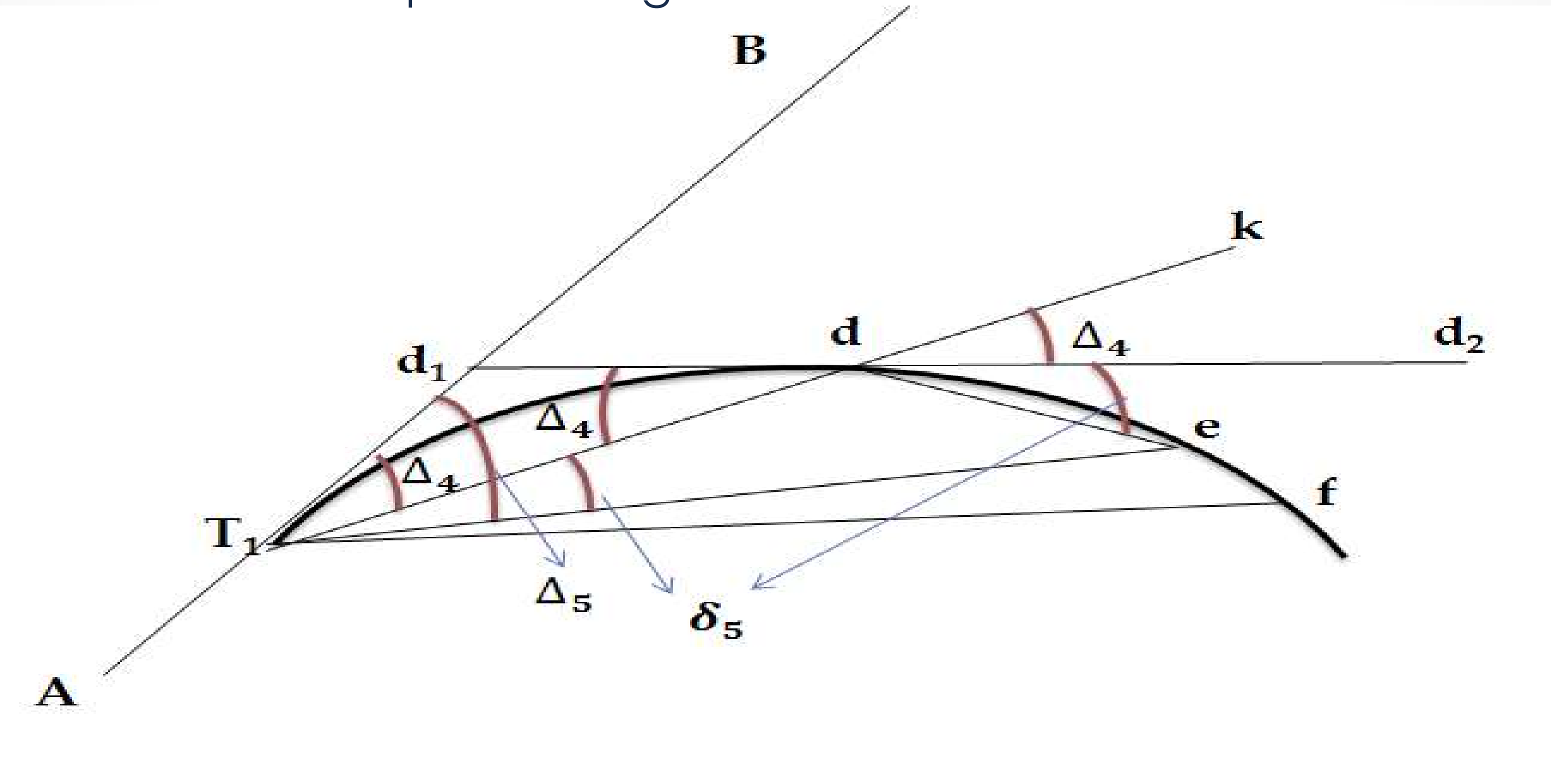


Simple Curves

Obstacles in Setting Out Simple Curve

2) When the whole curve cannot be set out from the Tangent point, Vision being obstructed

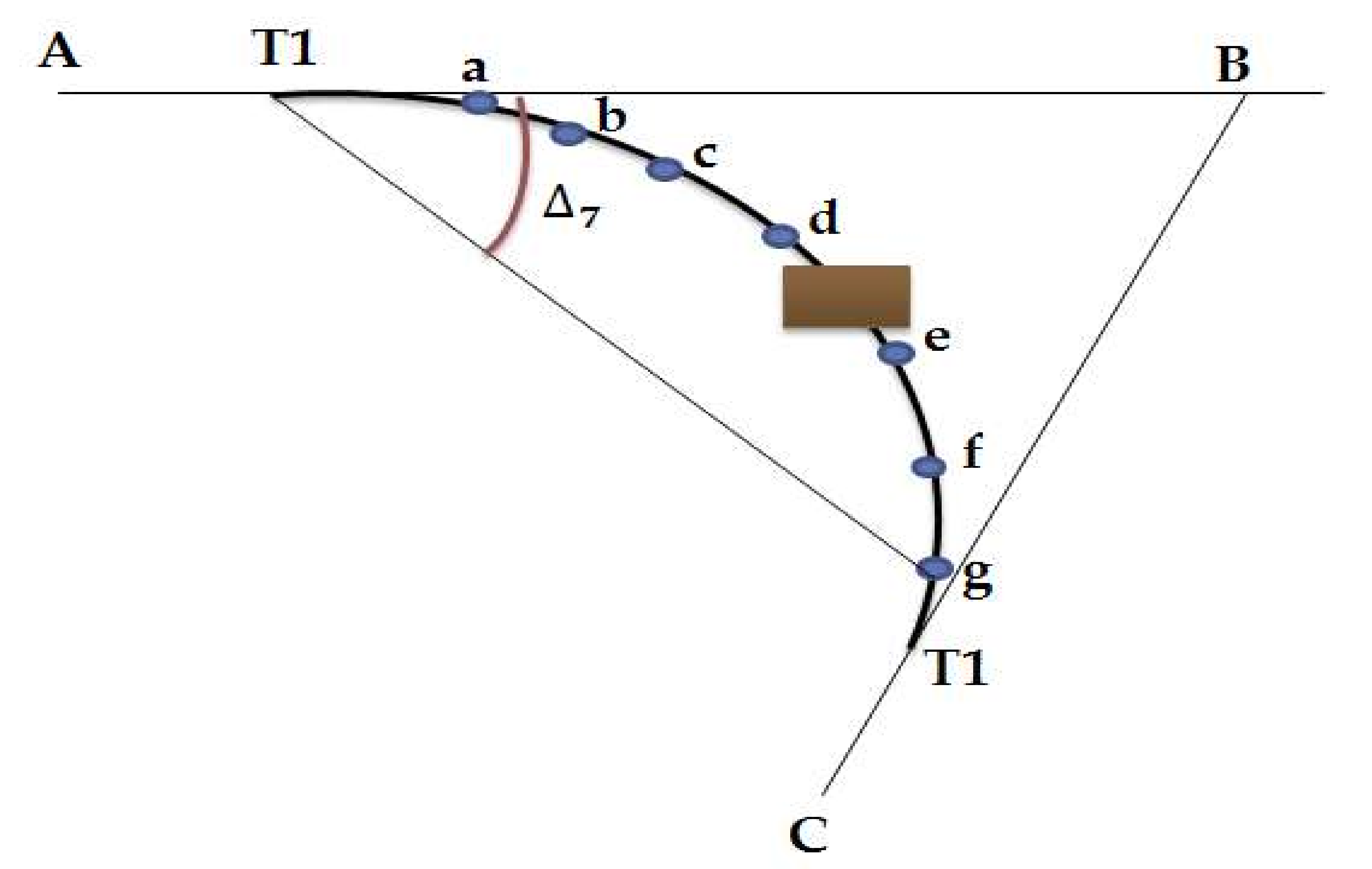
- As a rule the whole curve is to be set out from T_1 however obstructions intervening the line of sight i.e Building, cluster of tree, Plantation etc. In such a case the instrument required to be set up at one or more point along the curve.



Simple Curves

Obstacles in Setting Out Simple Curve

3) When the obstacle to chaining occurs



Simple Curves

Assignment

- Obstacles in Setting Out Simple Curve

  (Detail procedure)

  Page 130 Part II
- Example 1 (approximate method)
- Example 2
- Example 3

  Page 135 Part II

Curves
Compound Curves

- A compound curve consist of 2 arcs of different radii bending in the same direction and lying on the same side of their common tangent. Then the center being on the same side of the curve.

R_S = Smaller radius

R_L = Larger radius

T_S = smaller tangent length = BT_1

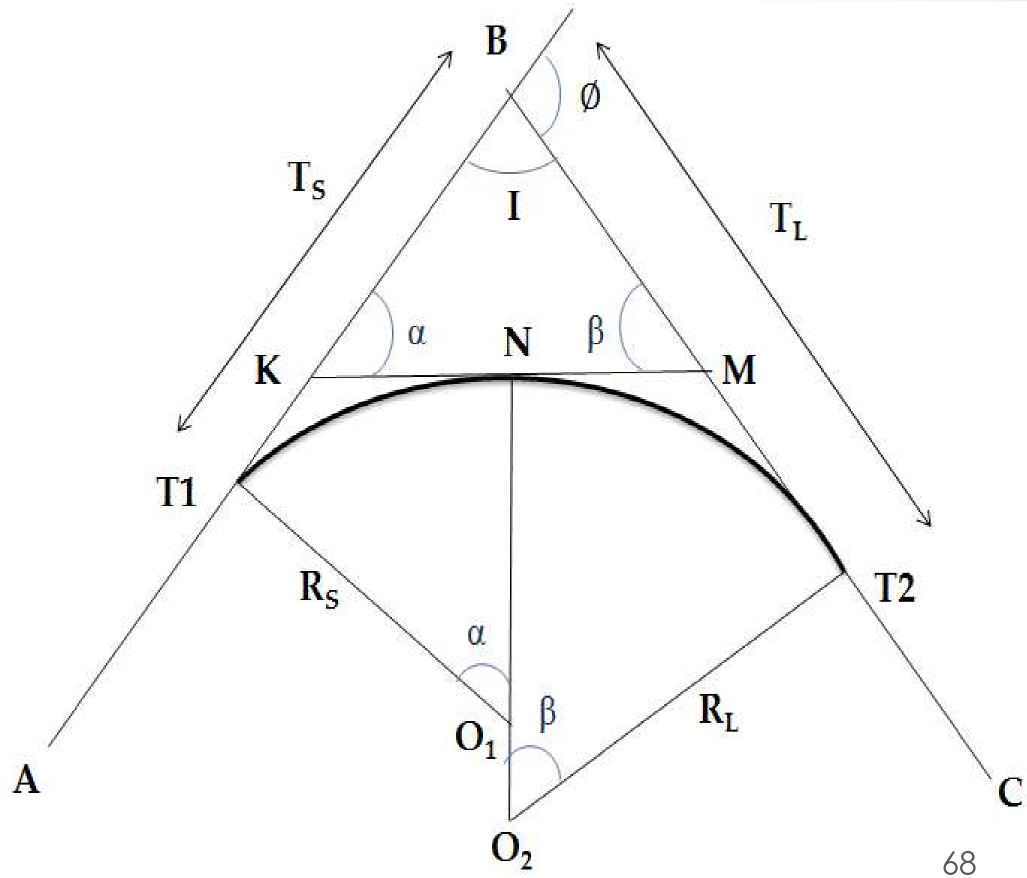
T_L = larger tangent length = BT_2

α = deflection angle b/w common tangent and rear tangent

β = angle of deflection b/w common tangent and forward tangent

N = point of compound curvature

KM = common tangent



Curves

Compound Curves

Elements of Compound Curve

$\emptyset = \alpha + \beta$

$KT_1 = KN = R_S \tan(\frac{\alpha}{2})$

$MN = MT_2 = R_L \tan(\frac{\beta}{2})$

From ΔBKN, by sine rule

$$\frac{BK}{\sin \beta} = \frac{MK}{\sin I}$$

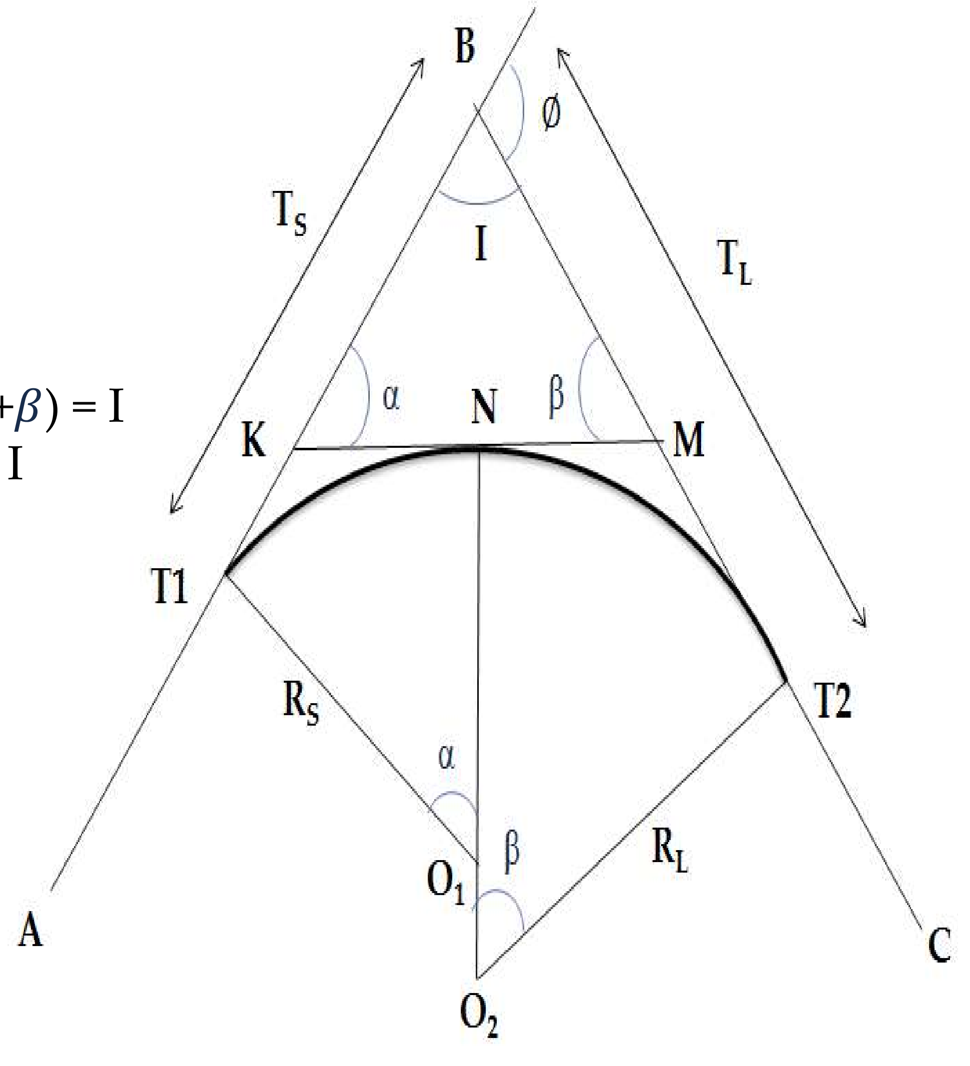
$$\frac{BK}{\sin \beta} = \frac{MK}{\sin(180^o - (\alpha + \beta))}$$

$$BK = \frac{MK \sin \beta}{\sin(180^o - (\alpha + \beta))}$$

$$BM = \frac{MK \sin \alpha}{\sin(180^o - (\alpha + \beta))}$$

$180^o - (\alpha + \beta) = I$

$180^o - \emptyset = I$



Curves

Compound Curves

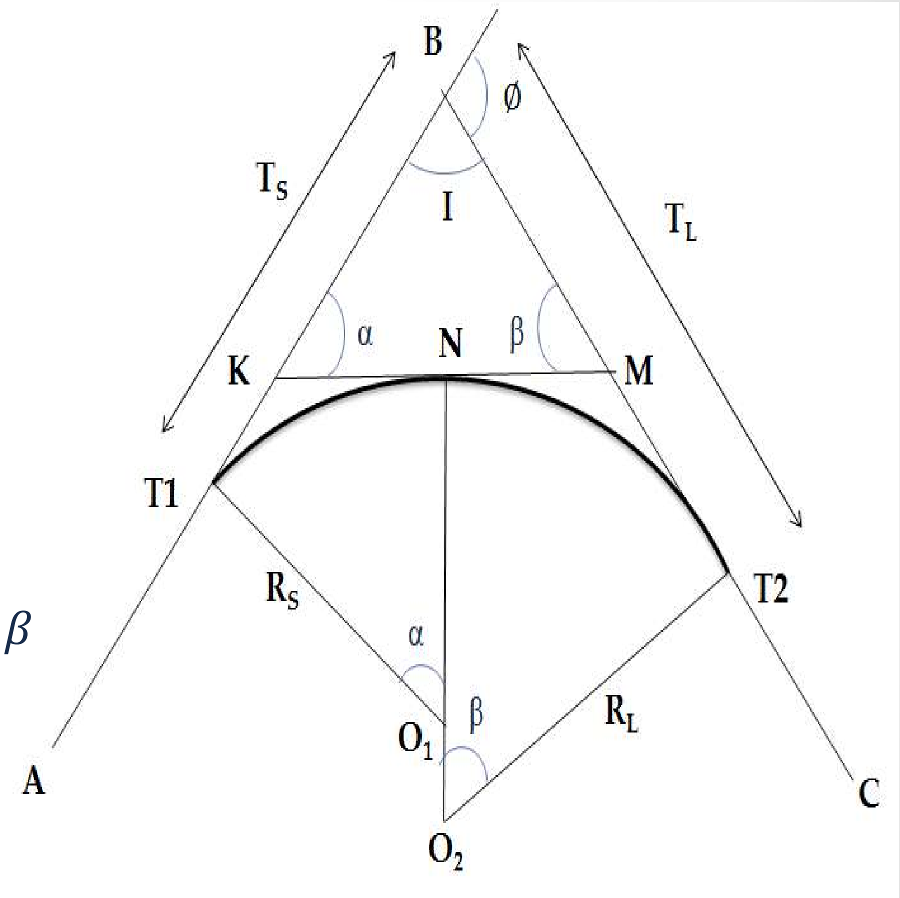
Elements of Compound Curve

$T_S = BT_1 = BK + KT_1$

$$T_S = BT_1 = \frac{MK \sin \beta}{\sin(180^o - (\alpha + \beta))} + R_S \tan\left(\frac{\alpha}{2}\right)$$

$$T_L = BT_2 = \frac{MK \sin \alpha}{\sin(180^o - (\alpha + \beta))} + R_L \tan\left(\frac{\beta}{2}\right)$$

Of the seven quantities R_S, R_L, TS, T_L, $\emptyset$, α, β four must be known.



Curves

Compound Curves

Problem: Two tangents AB & BC are intersected by a line KM. the angles AKM and KMC are 140° & 145° respectively. The radius of 1st arc is 600m and of 2nd arc is 400m. Find the chainage of tangent points and the point of compound curvature given that the chainage of intersection point is 3415 m.

Solution:

$\alpha = 180^o - 140^o = 40^o$

$\beta = 180^o - 145^o = 35^o$

$\emptyset = \alpha + \beta = 75^o$

$I = 180^o - 75^o = 105^o$

$KT_1 = KN = R_L \tan(\frac{\alpha}{2}) = 600 \tan(40^o/2)$

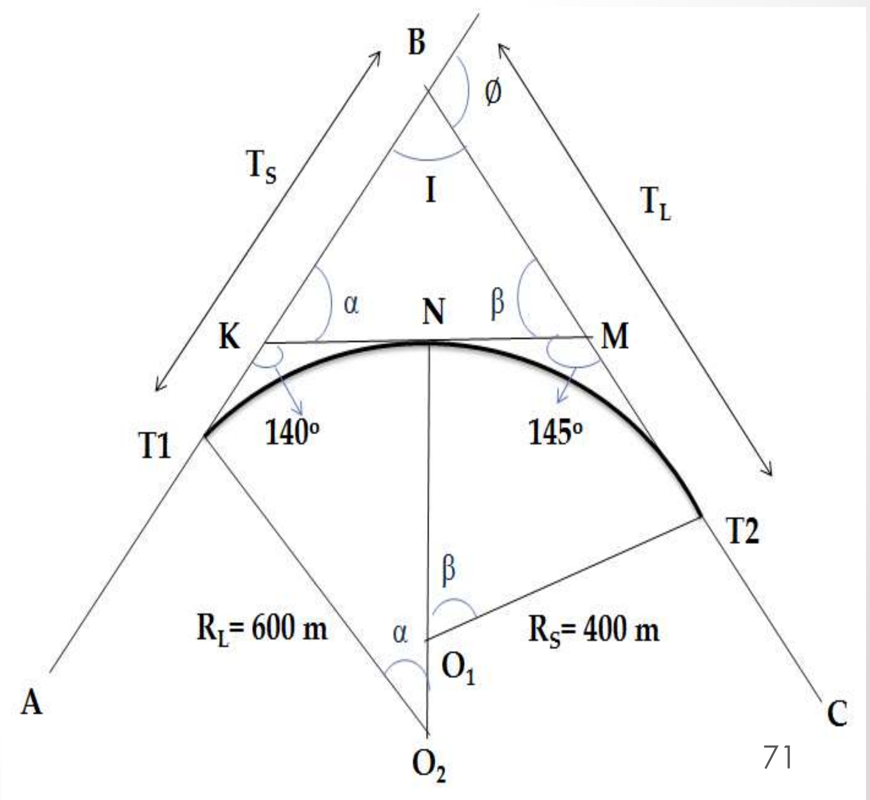
$KT_1 = KN = 218.38$ m

$MN = MT_2 = R_S \tan(\frac{\beta}{2}) = 400 \tan(35^o/2)$

$MN = MT_2 = 126.12$ m

$KM = MT_2 + MN = 218.38 + 126.12$

$KM = 344.50$ m



Curves

Compound Curves

Solution:

Fin ΔBKM, by sin rule

$$\frac{BK}{\sin\,\beta} = \frac{MK}{\sin(I)}$$

$$BK = \frac{MK\sin\,\beta}{\sin(I)} = \frac{344.50\ x\ sin35^o}{\sin\,105^o} = 204.57\text{ m}$$

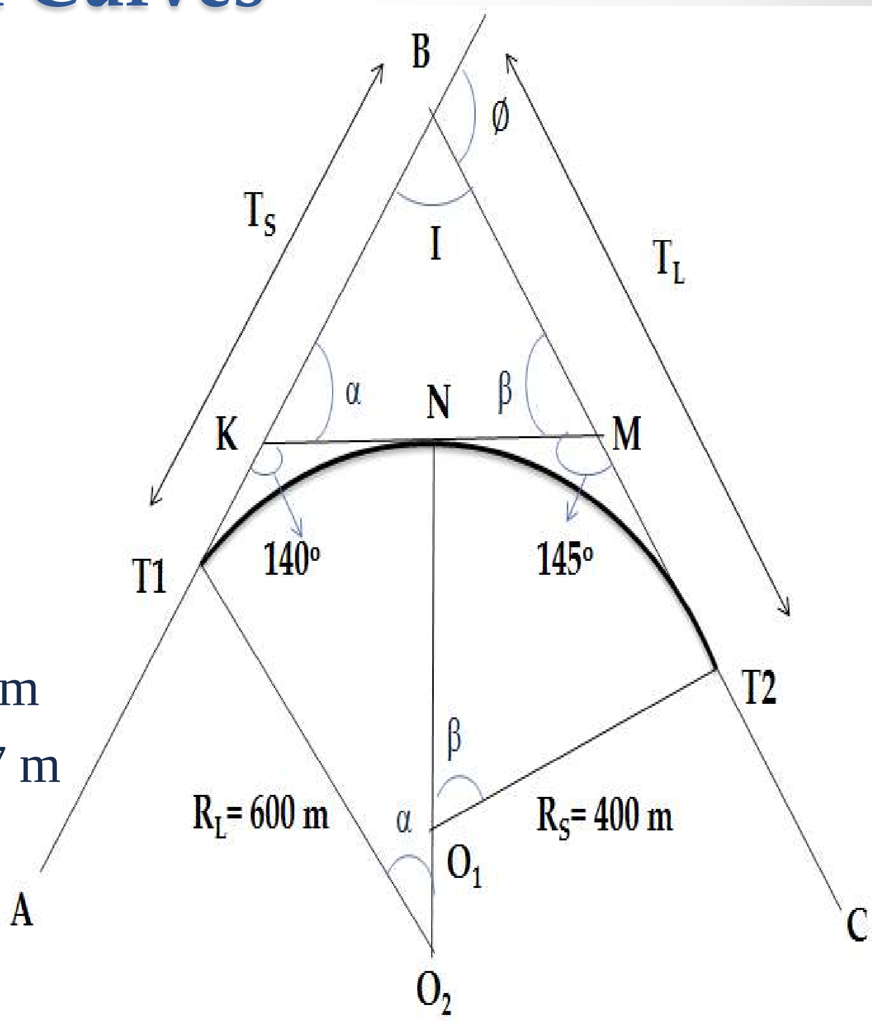
$$BM = \frac{MK\sin\alpha}{\sin(I)} = \frac{344.50\ x\ sin40^o}{\sin\,105^o} = 229.25\text{ m}$$

$T_L = KT_1 + BK = 218.38 + 204.57 = 422.95$ m

$T_S = MT_2 + BM = 126.12 + 229.25 = 355.37$ m

$$L_L = \frac{\pi\ RL\ \alpha}{180^o} = \frac{\pi\ x\ 600\ x4\ 0}{180^o} = 418.88\text{ m}$$

$$L_S = \frac{\pi\ R_S\beta}{180^o} = \frac{\pi\ x\ 400\ x\ 30}{180^o} = 244.35\text{ m}$$



Curves

Compound Curves

Solution:

Chainage of intersection point	= 3415 m
Minus T_L	= - 422.95 m
Chainage of T_2	= 2992.05 m
Plus L	= + 418.88 m
Chainage of compound curvature (N)	= 3410.93 m
Plus L_S	= + 244.35 m
Chainage of T_2	= 3655.25 m