Surveying-II
CE-207 (T)

Curves

Vertical curves
Lecture 3

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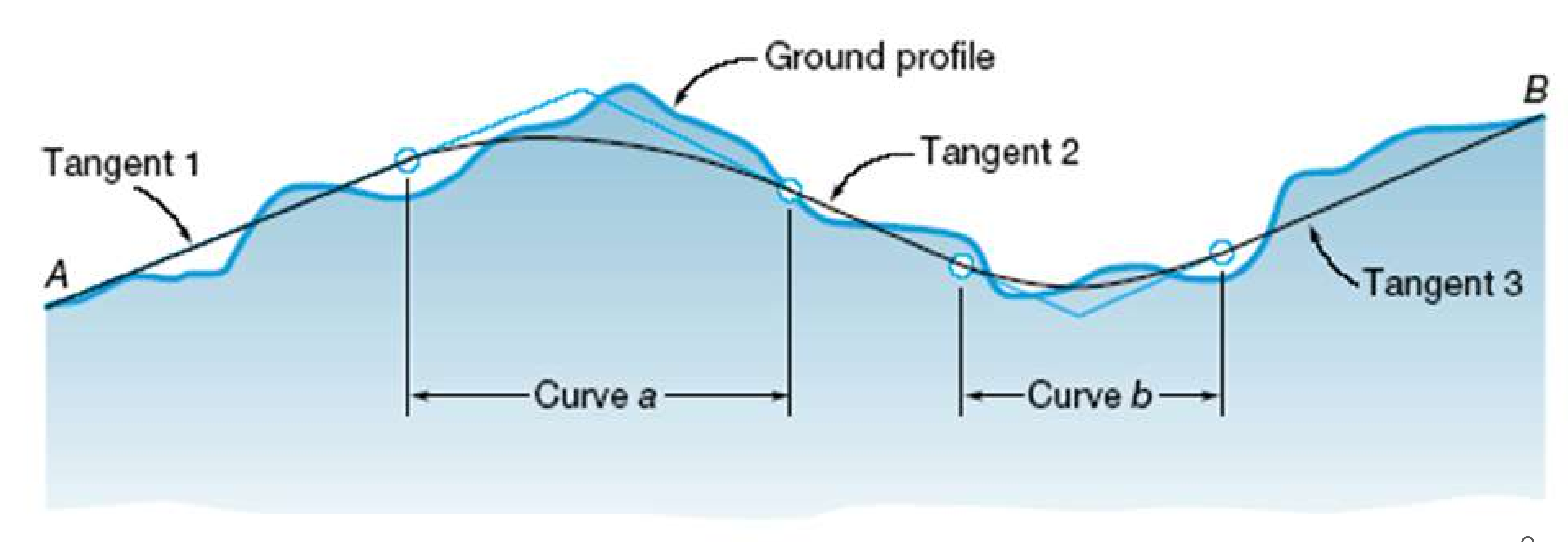
Curves

Vertical Curves

- In addition to horizontal curves that go to the right or left, roads also have vertical curves that go up or down.
- Vertical curves at a crest or the top of a hill are called summit curves, or oververticals.
- Vertical curves at the bottom of a hill or dip are called sag curves, or underverticals.

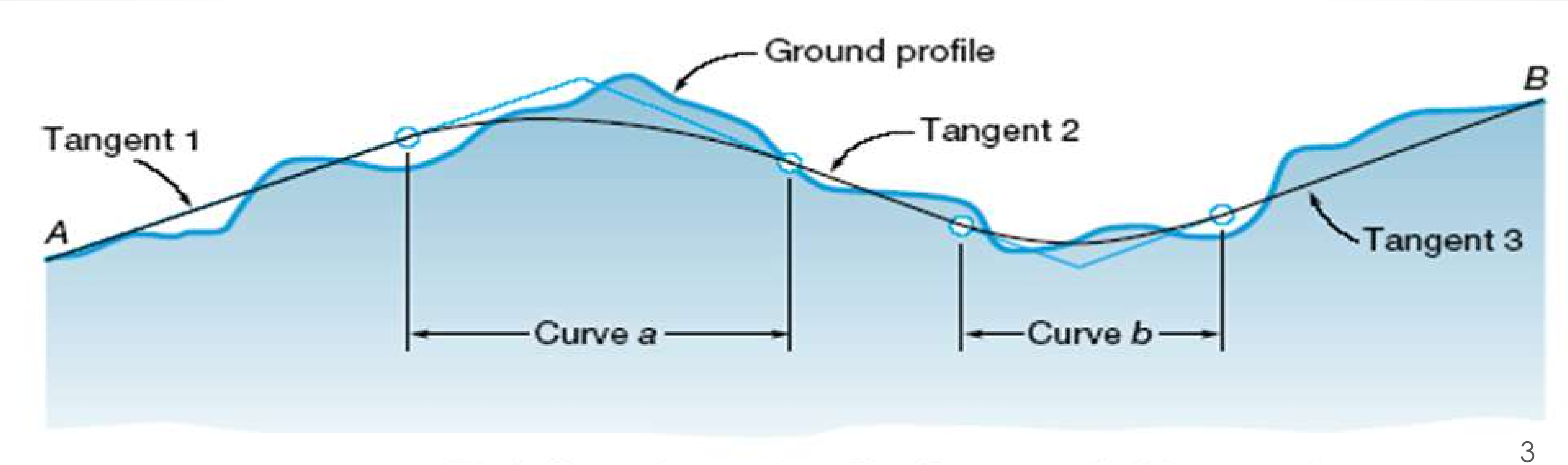


Grade line and ground profile of a proposed highway section.

Curves

Vertical Curves

- In the same way that horizontal curves are used to connect intersecting straights in the horizontal plane, vertical curves are used to connect intersecting straights in the vertical plane.
- These straights are usually called gradients and the combination of the of the gradients and vertical curve is known as vertical alignment.

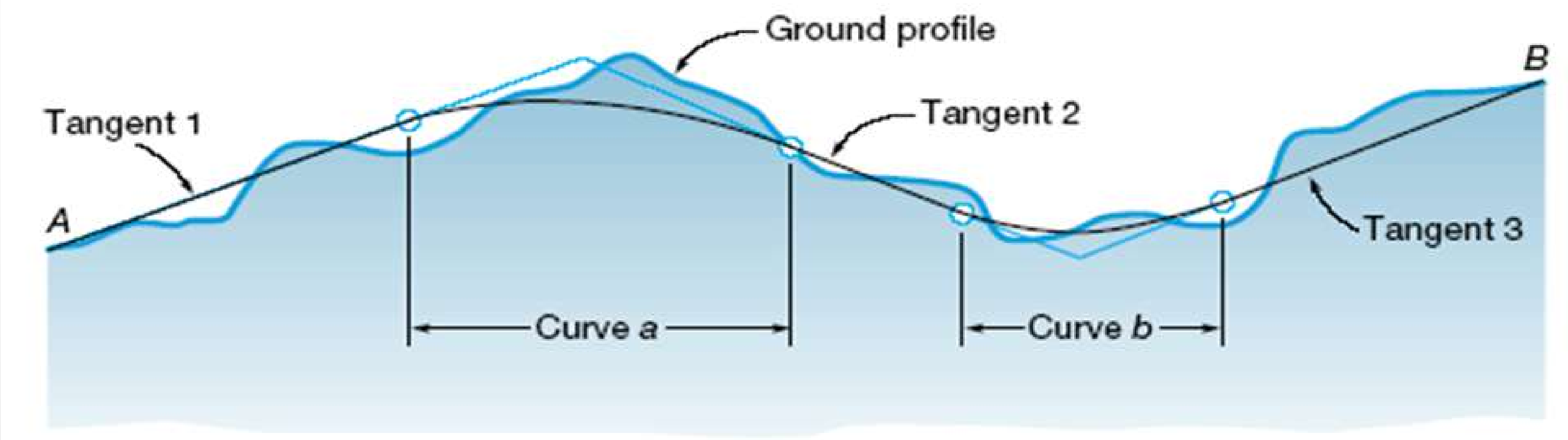


Grade line and ground profile of a proposed highway section.

Curves

Vertical Curves

- The vertical alignment is composed of a series of straight-line gradients connected by curves, normally parabolic in form.
- These vertical parabolic curves must therefore be provided at all changes in gradient.
- The curvature will be determined by the design speed, being sufficient to provide adequate driver comfort with appropriate stopping sight distances provided.



Grade line and ground profile of a proposed highway section.

Curves

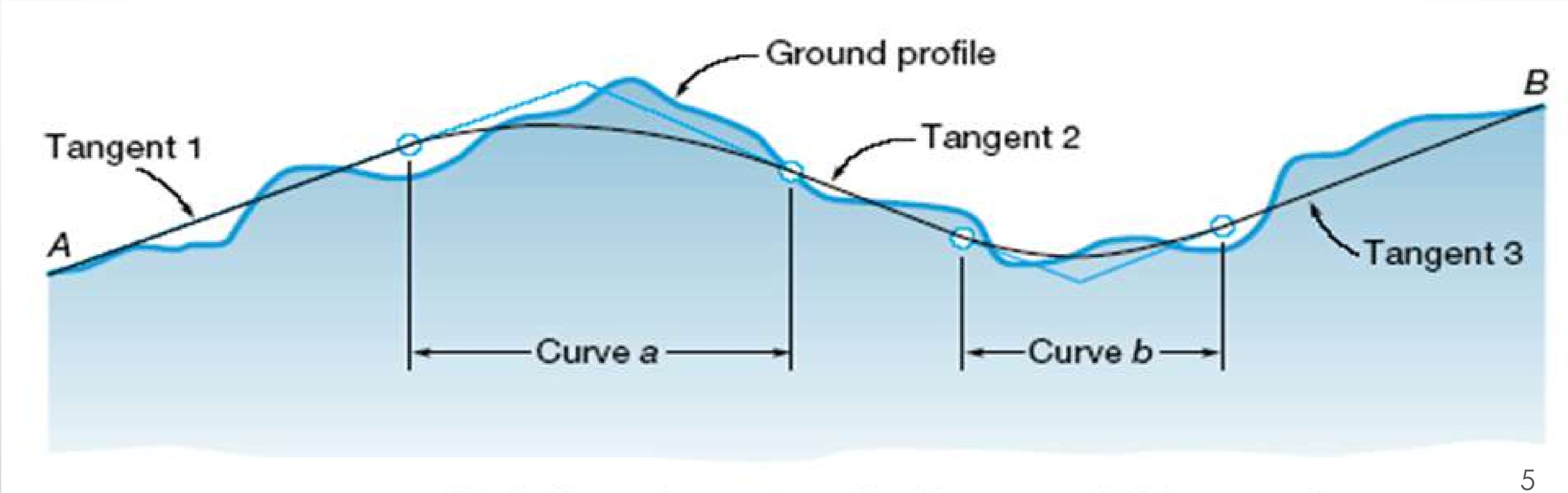
Vertical Curves

Purpose of Vertical Curves

- Vertical curves are similar to horizontal curves in that they are designed for a particular speed.

The main functions of a vertical curve are to provide:

- Safety and comfort travelling between gradients
- Adequate visibility to enable vehicles to stop or overtake safely.



Grade line and ground profile of a proposed highway section.

Curves

Vertical Curves

- When two different gradients meet they are connected by a curve in a vertical plane is called vertical curve. it is used in roads and railway.

Objective:

- To round off the angle at the apex and to obtain a gradual change in grade or gradient, so that abrupt change in grade is avoided at the apex.
- Vertical curves may be:

  a) Circular arc **b)** Arc of Parabola

- For simplicity of calculation work, the parabola is preferred.
- When the ratio of length of curve to radius is less than 1 to 10, i.e

$$\frac{L}{R} < \text{ 1 to 10}$$

- Than there is no practical difference b/w shapes of parabola, circle and an ellipse, the parabola will be used.

Curves

Vertical Curves

- In practice, road and railway gradients are comparatively flat and it is often unimportant what type of vertical curve is used.
- The usual curves are circular or parabolic. However, it is best to use a vertical curve having a constant rate of change of gradient, i.e., a parabola and as it turns out, parabolic vertical curves are very easy to calculate and use.

Curves

Vertical Curves

- Parabola has desirable characteristics of:

  1) Constant rate of change of slopes which contributes to sooth alignment transition.

  2) Ease of computation of vertical offsets which permits easily computed curve.

- The Grade for the gradient of a road is expressed in two ways:

  a) As a percentage i.e 2 % 1.5 %

  b) As 1 in n , where n is horizontal distance in meter or feet corresponding to 1m or 1ft rise or fall. E.g 1 in 80 & 1 in 100.

  1 in 50 = 2%, 1 in 25 = 4%,

Sign convention:

a) Gradient rising (ascending) to the right are taken +ve

b) Gradient falling (descending) to right are taken –ve

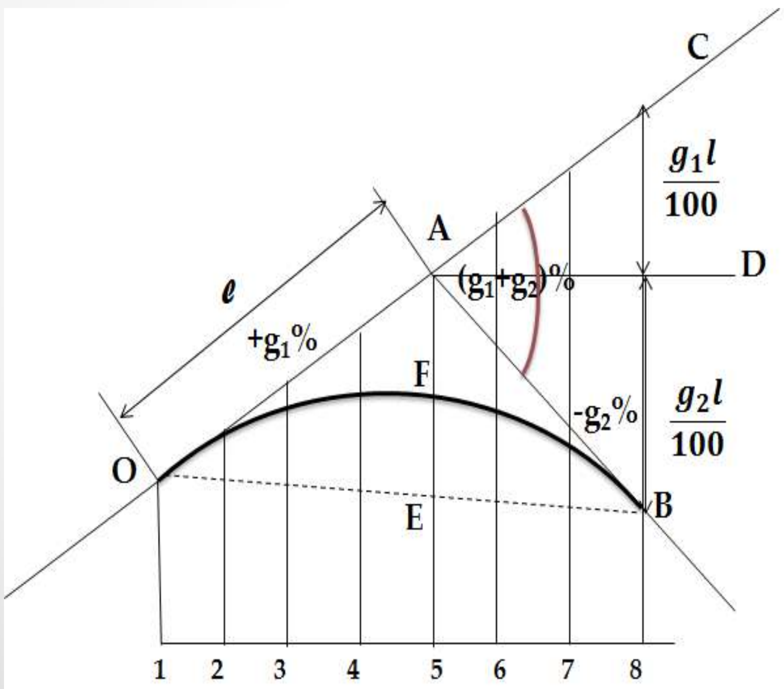
Curves

Vertical Curves

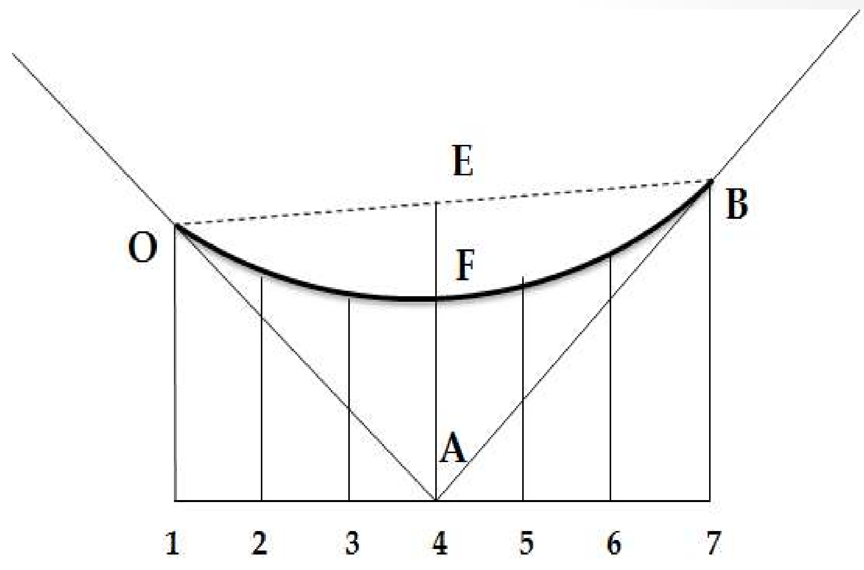
Types of vertical curve:

1) An up grade followed by a down grade

2) A down grade followed by an Up grade



Summit or Convex



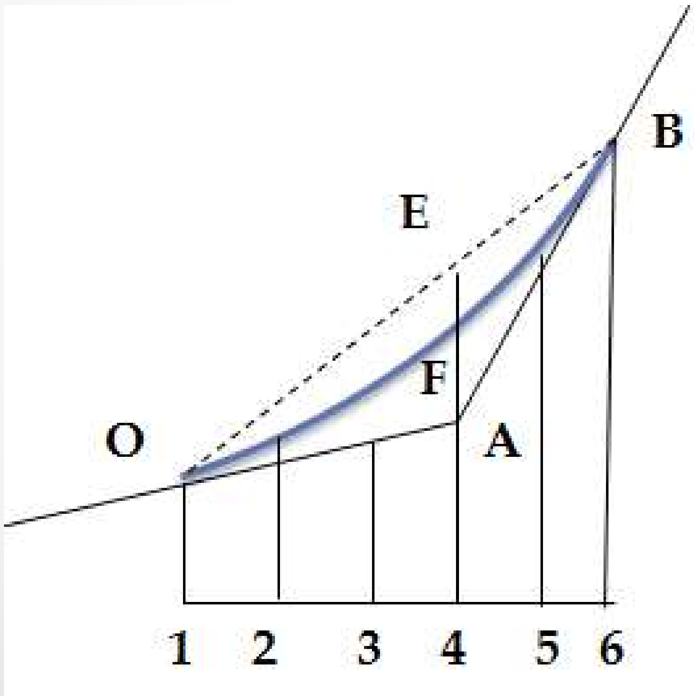
Sag or Concave

Curves

Vertical Curves

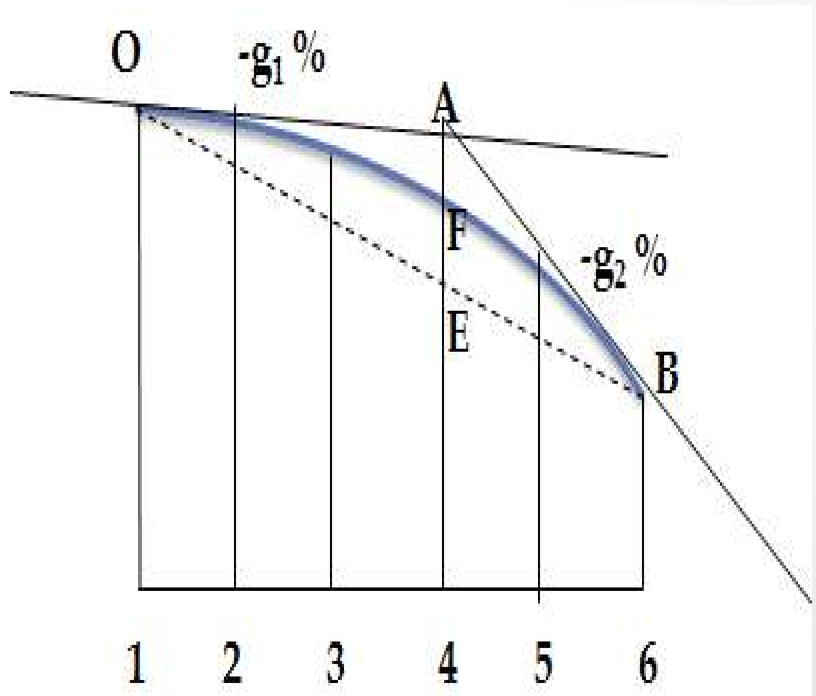
Types of vertical curve:

3) An up grade followed by another up grade



Sag or Concave

4) A down grade followed by another down grade



Summit or Convex

Curves

Vertical Curves

Types of parabola curves:

Two basic types

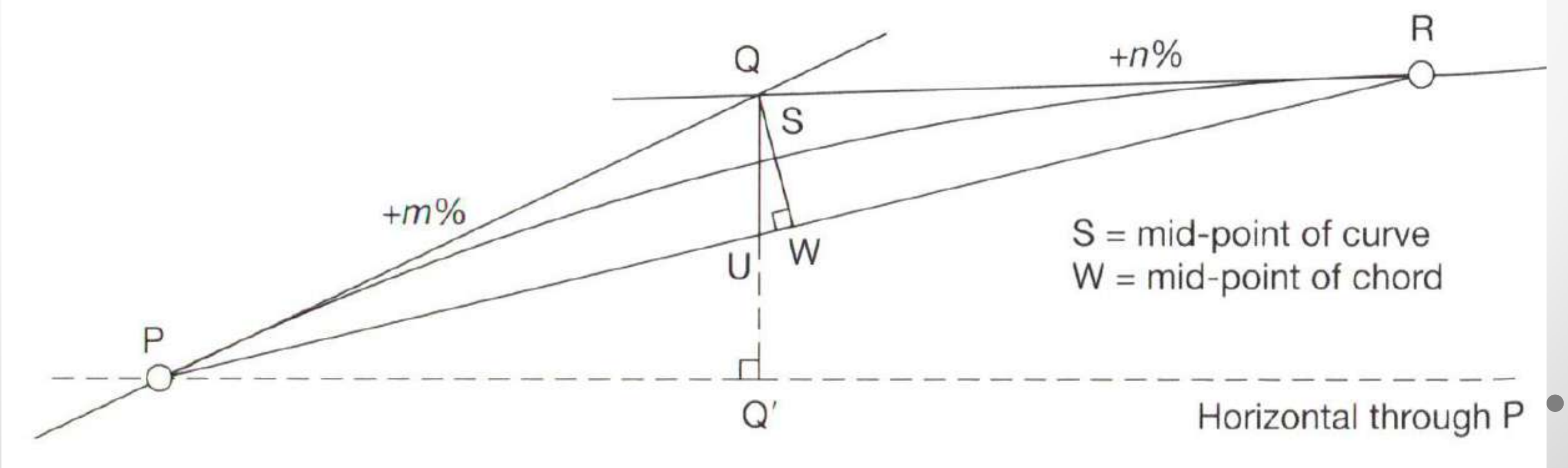
1) Cest 2) Sag

Curves

Vertical Curves

Assumptions in Vertical Curve Calculations

- Choosing a parabola simplifies the calculations of a vertical curve, further simplifications are also made if certain assumption are used.
- Vertical curves are generally designed with equal tangent lengths such that PQ = OR.
- Assumptions made are:
- Chord PWR = Arc PSR = PQ + QR
- Length along the tangents = horizontal length, i.e. PQ = PQ'
- QU = QW, i.e. there is no difference in dimensions measured either in the vertical plane or perpendicular to the entry tangent length



Curves

Vertical Curves

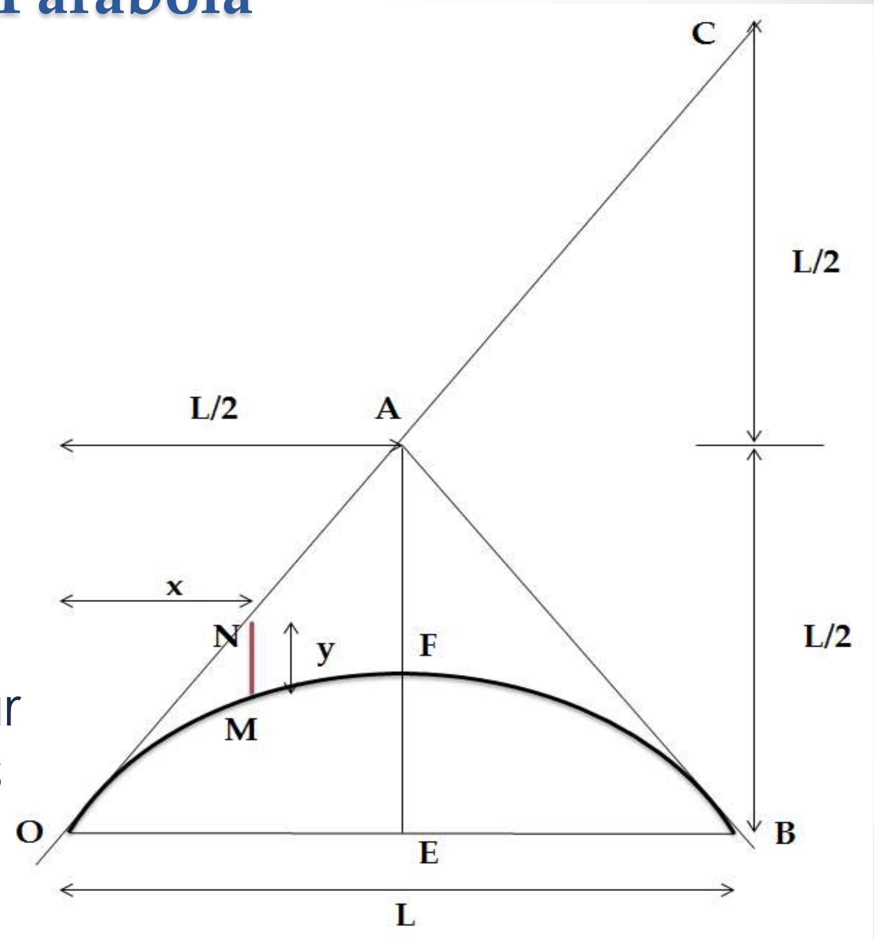
Properties of Parabola

a) The vertical through the intersection of tangent A, bisect OB.

b) OA = AB and FA = FE, where F is vertex of parabola

c) Offset y from tangent OA are proportional to the square of distance x from O.

The offsets should be perpendicular to the tangent but as flat gradients are usually involved it is used sufficiently accurate to take them vertically.



Curves

Vertical Curves

Properties of Parabola

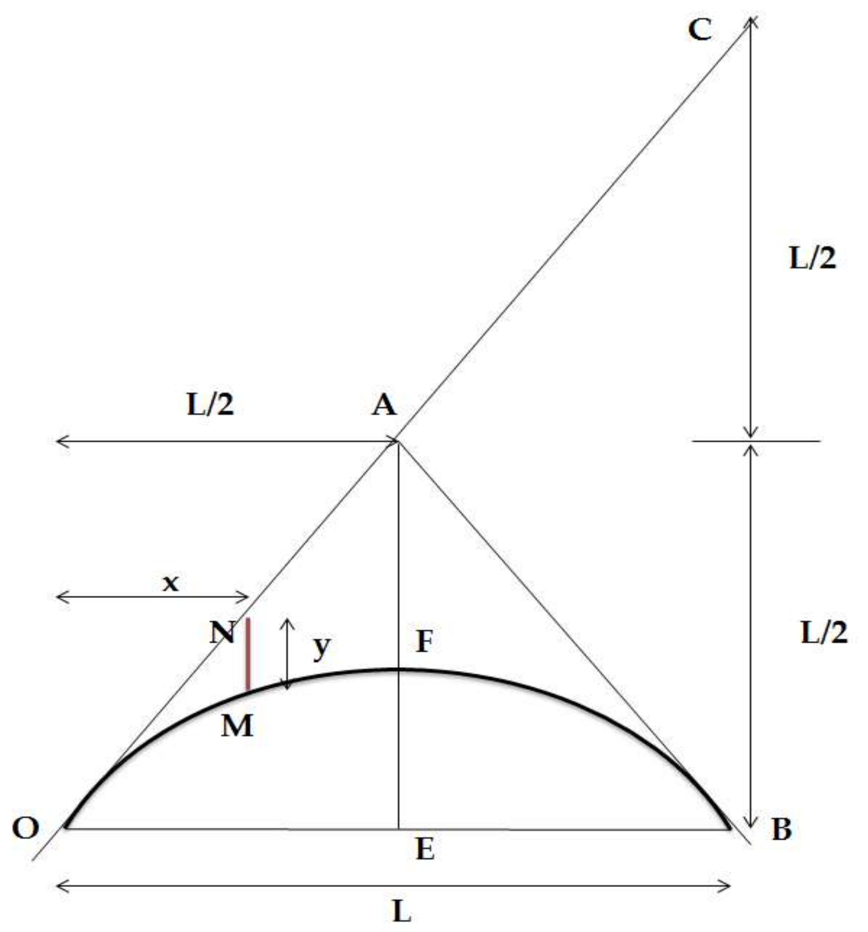
Let eqn of parabola is
$y = K\,x^2$

$\frac{d_y}{d_x}$ = 2 K x (gradient slope)

$\frac{d_{y}2}{d_{x}2}$ = 2 K (a constant)

i.e rate of change of slope is constant. Thus parabola gives an even rate of gradient.

For flat gradients it is usually accurate enough to treat length along tangent be equal to the horizontal projection of tangent.



Curves

Vertical Curves

Properties of Parabola

a) To find K, at x = L

$$y = BC = \frac{g_1}{100} x \frac{L}{2} + (-\frac{g_2}{100} x \frac{L}{2})$$

$$y = \frac{(g_1 - g_2)\, L}{200}$$

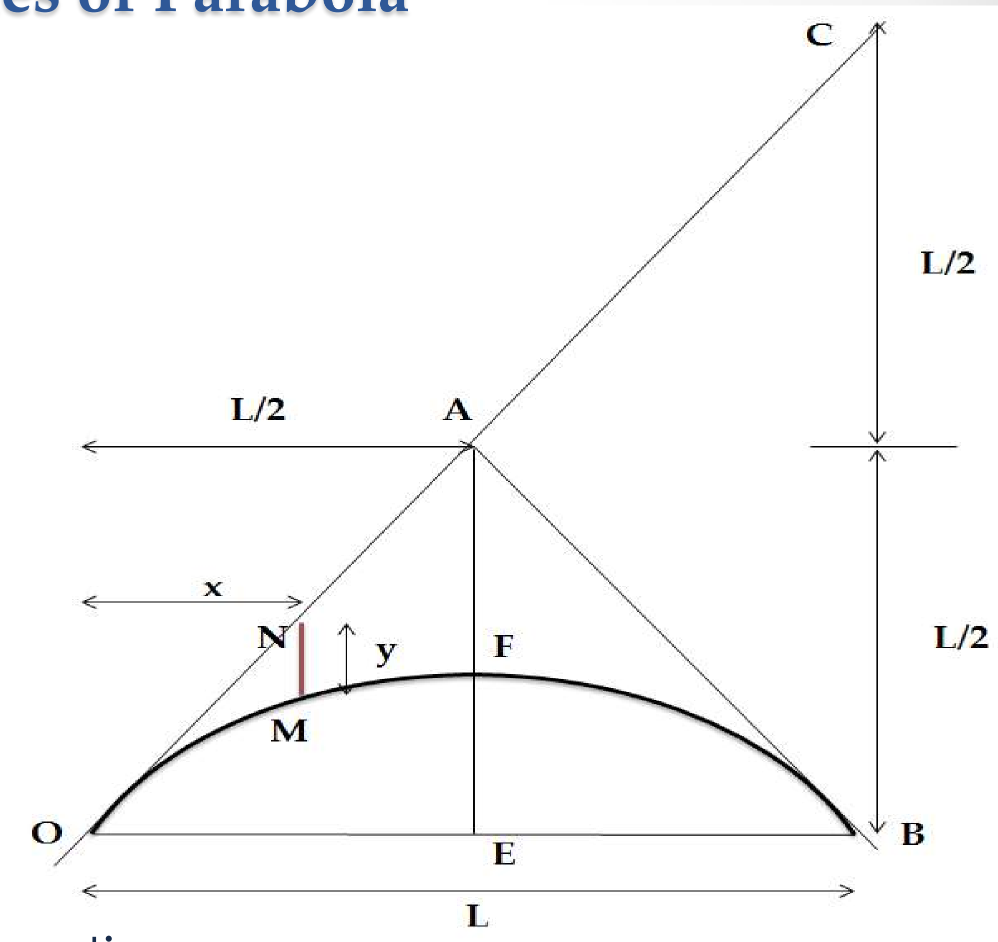
From eqn y = k x^2

$$\frac{(g_1 - g_2)\, L}{200} = K\ x\ L^2$$

$$K = \frac{(g_1 - g_2)}{200L}$$

$$y = \frac{(g_1 - g_2)}{200L}\, x^2$$

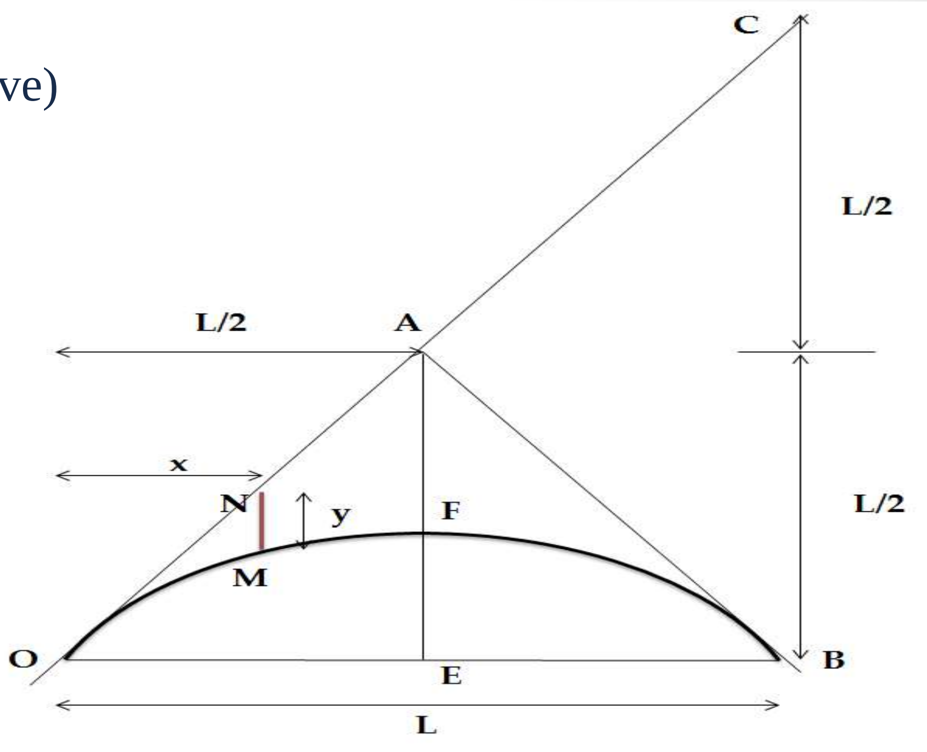
General form of Parabolic equation



Curves

Vertical Curves

Properties of Parabola



b) At $x = \frac{L}{2}$, y= AF (from tangent to curve)

$$y = \frac{(g_1 - g_2)}{200L} \left(\frac{L}{2}\right)^2$$

$$y = \frac{(g_1 - g_2)}{800} L$$

From similar Δs OAE ,OCB

$$\frac{EA}{L/2} = \frac{BC}{L}$$

$$EA = \frac{BC}{2}, \quad y = BC$$

$$EA = \frac{1}{2} x \frac{(g_1 - g_2) L}{(200)} = \frac{(g_1 - g_2) L}{(400)} = 2\,AF$$

i.e AF = FE= $\frac{(g_1 - g_2)}{800}$ L In vertical curve we have horizontal distance and offsets only.

Curves

Vertical Curves

Location of Highest and Lowest Point

Taking **O** as datum any point on the curve will have height

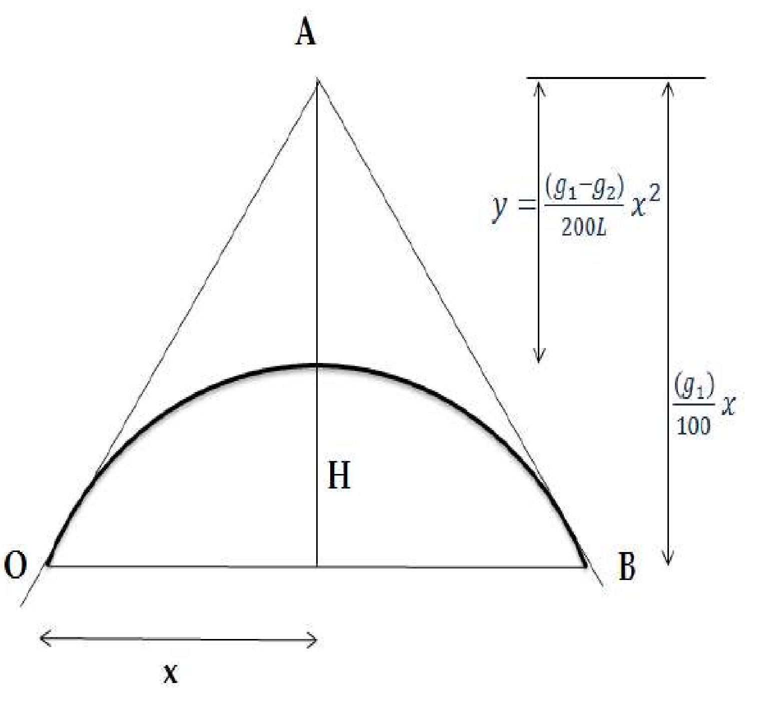
$$H = y = \frac{(g_1\, x)}{100} - \frac{(g_1 - g_2)}{200L}\, x^2$$

For max value of H

$$\frac{dH}{dx} = 0$$

$$\frac{dH}{dx} = \frac{(g_1)}{100} - \frac{(g_1 - g_2)}{100L}\, x \quad = 0$$

$x = \dfrac{g_1 L}{(g_1 - g_2)}$, gives distance at point where H is maximum , applicable for both crest as well as sag.



Length of curve $= L = \dfrac{(g_1\% \;-\; g_2\%)}{r\%}$

r = rate of change of gradients = 0.1 per 100 ft or 30m.

If r is given then $x = \dfrac{g_1 L}{r}$

Curves

Vertical Curves

Length of Vertical Curve

Length of vertical curve is influenced by

1) Centrifugal effect

2) Visibility

At sags and at summit, formed by flat gradients, centrifugal effect is the chief consideration.

while at summit where the algebraic change of gradient is large, visibility is main concern.

1) Centrifugal effect:

A minimum radius of 1000 m should be used at sags and brows. This gives a centrifugal acceleration of 0.75 $^{m}/_{sec2}$ at 100 $^{km}/_{h}$.

Curves

Vertical Curves

Length of Vertical Curve

1) Centrifugal effect:

Parabola can be approximate to circle,

$$\frac{d_y 2}{d_x 2} = 2\,\mathrm{K} = \frac{1}{R}, \qquad K = \frac{(g_1 - g_2)}{200L}$$

$$\frac{1}{R} = \frac{2\,(g_1 - g_2)}{200\,L}$$

$$R = \frac{100\,L}{(g_1 - g_2)}, \qquad \mathrm{L} = \frac{R\,(g_1 - g_2)}{100}$$

$$L_{min}\ (\text{at R} = 1000\ \text{m}) = \frac{1000\,(g_1 - g_2)}{100},$$

$$L_{min} = 10\,(g_1 - g_2)$$

Curves

Vertical Curves

Length of Vertical Curve

1) Centrifugal effect:

Parabola can be approximate to circle,

$$\frac{d_y 2}{d_x 2} = 2\text{ K } = \frac{1}{R}, \qquad K = \frac{(g_1 - g_2)}{200L}$$

$$\frac{1}{R} = \frac{2\,(g_1 - g_2)}{200\,L}$$

$$R = \frac{100\,L}{(g_1 - g_2)}, \qquad \text{L} = \frac{R\,(g_1 - g_2)}{100}$$

$$L_{min}\text{ (at R = 1000 m)} = \frac{1000\,(g_1 - g_2)}{100},$$

$$L_{min} = 10\,(g_1 - g_2)$$

Thus if two 1 in 25 gradients meet in a sag, the minimum length of the curve should be L = 10 (4 +4) = 80 m

Curves

Vertical Curves

Length of Vertical Curve

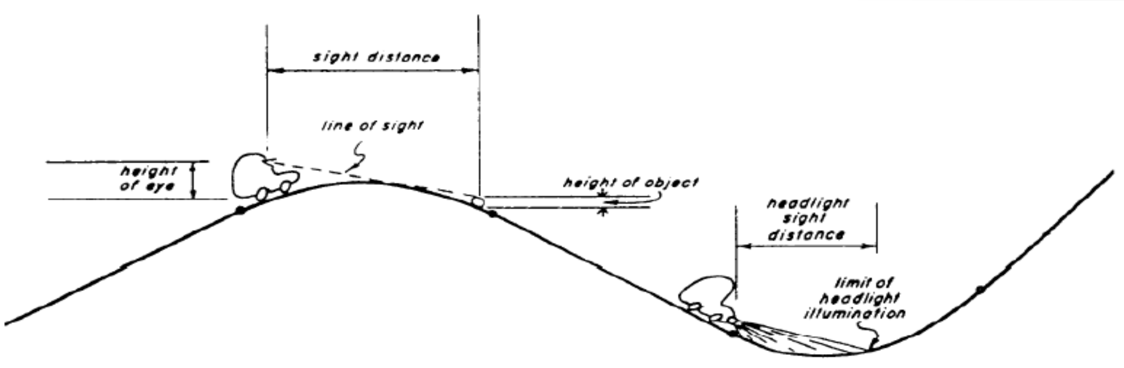
2) Sight distance:

At summits where speed of 100 km/ hr are contemplated, the requirement of visibility 1.e the sight line will lead to longer curves than one obtained by above formula.

In the design of summit or convex curve it is required to calculate the minimum length of curve which will give the required sight distance.

“ The length of road visible to the driver clear of object while driving is called sight distance”.

On highways



$$L = \frac{(g_1 - g_2)\ S^2}{800\ h}$$

Where S = sight distance,

h =height of drivers eye above road ,

h = 1.50 (AASHTO), h= 1.1 m (book)

Curves

Vertical Curves

Length of Vertical Curve

2) Sight distance:

Three cases will be considered:

1) The length of curve is grater than the sight distance (L > S)

2) The length of curve is smaller than the sight distance (L < S)

3) The length of curve is equal to required sight distance (L = S)

Assignment

Curves

Vertical Curves

Procedure for Computing a Vertical Curve

Two methods

1) Tangent corrections or tangent offset Method

2) By Chord Gradients

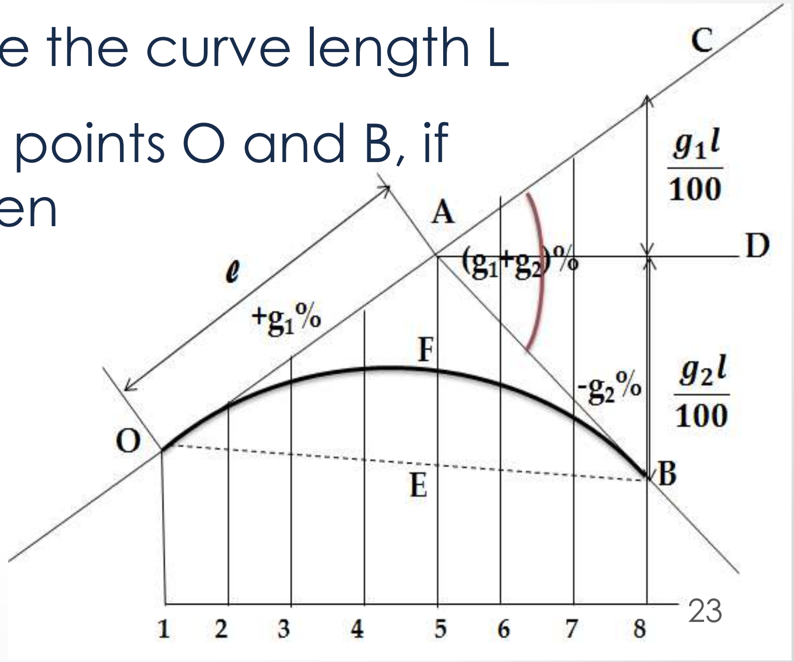
1) Tangent corrections or tangent offset Method

a) Determined gradients and compute the curve length L

b) Compute the chainage of tangent points O and B, if chainage of intersection point A is given

Chainage of O = chainage of A - $\frac{L}{2}$

Chainage of B = chainage of A + $\frac{L}{2}$



Curves

Vertical Curves

Procedure for Computing a Vertical Curve

1) Tangent corrections or tangent offset Method

c) Compute the elevation of tangent point O and B from the elevation of intersection point A

$$\text{Elevation of O} = \text{elevation of A} - \frac{g_1 L}{200}$$

$$\text{Elevation of B} = \text{elevation of A} - \frac{g_2 L}{200}$$

d) Compute the tangent offset for the stations or pegs on the curve

$$\text{Tangent offset} = \frac{(g_1 - g_2)\, x^2}{200\, L}$$

e) Find the elevation of stations on the curve by eqn

Elevation of station on curve at a distance x = elevation of station on tangent ∓ tangent offset or correction

-ve sing in case of crest curve, +ve sign in case of sag curve.

Curves
Vertical Curves

Example 01: Find the length of vertical curve connecting two uniform grades from the following data:

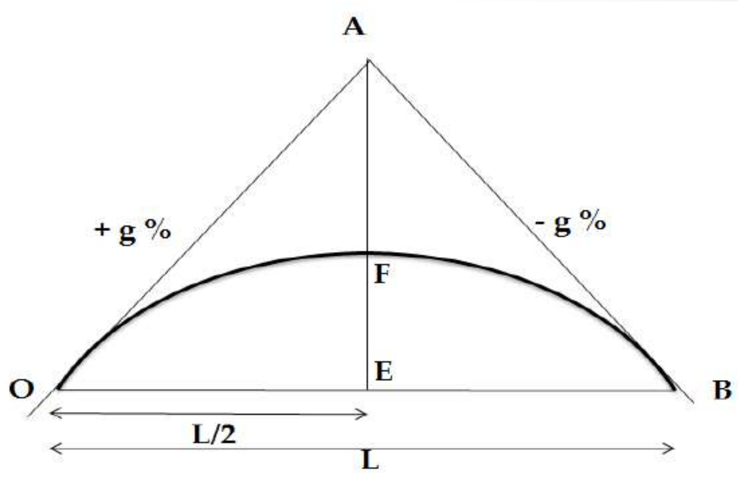
a) + .8% and - 0.6%, rate of change of grade is 0.1 per 30m

b) – 0.5% and +1%, rate of change of grade is 0.05 per 30m

Solution:

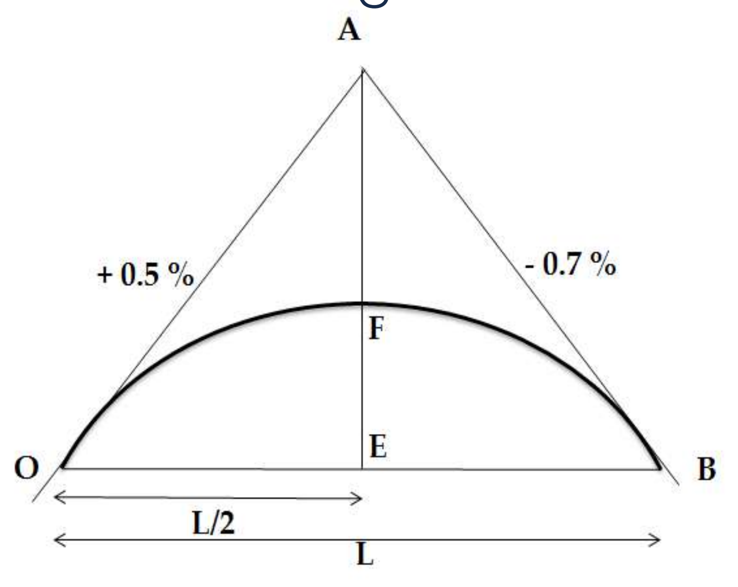
a) $L = \dfrac{(g_1\% - g_2\%)}{r\%} = \dfrac{(0.8 - (-.6))}{0.1}\ x\ 30 = 420\text{ m}$

b) $L = \dfrac{(-0.5 - (+1))}{0.05}\ x\ 30 = 900\text{ m}$



Vertical Curves

Example 02: Calculate the R.L of various stations pegs on a vertical curve connecting two uniform grades of + 0.5% and -0.7%. The chainage and R.L of point of intersection are 500 m and 350.750 m respectively. Take the rate of change of grade as 0.1% per 30m.



Solution:

1) Length of vertical curve L

$$L = \frac{(g1\% - g2\%)}{r\%} \times 30 = \frac{(0.5 - (-.7))}{0.1} \times 30 = 360 \text{ m}$$

$$\frac{L}{2} = 180\ m$$

2) Chainage of A = 500 m

Chainage of O = chainage of A - $\frac{L}{2}$ = 500 – 180 = 320 m

Chainage of B = chainage of A + $\frac{L}{2}$ = 500 + 180 = 680 m

Solution: 02

Vertical Curves

3) R.L of Point of intersection A = 330.75 m

R.L of O = 330.75 - $\frac{g_1 L}{200}$ = 330.75 - $\frac{(0.5)(360)}{200}$ = 329.85 m

R.L of B = 330.75 - $\frac{g_2 L}{200}$ = 330.75 - $\frac{(0.7)(360)}{200}$ = 329.49 m

R.L of mid point E of chord OB = ½ (R.L of O + R.L of B) = ½ (329.85 +329.49)

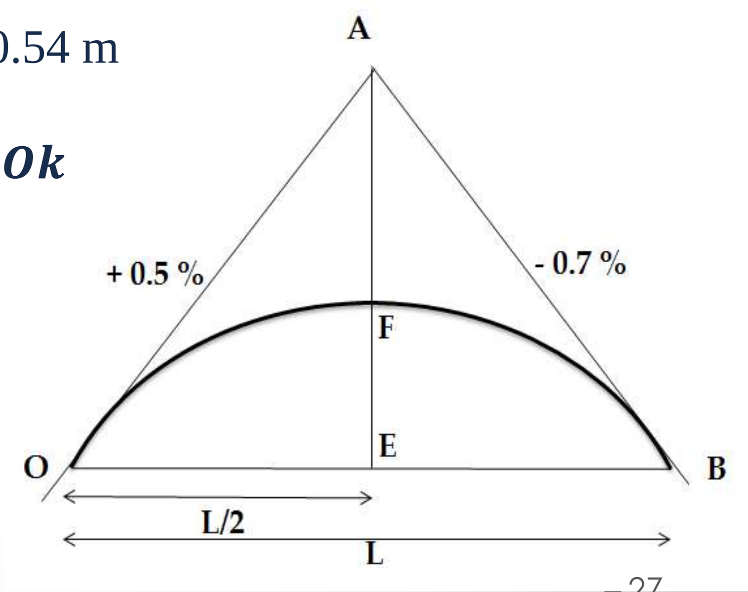
= 329.67 m

R.L of F (Vertex of curve) = ½ (R.L of B + R.L of A) = ½ (330.75 + 329.67)

= 330.21 m

The difference AF b/w A and F = 330.75 – 330.21 = 0.54 m

Check: AF = $\boldsymbol{\frac{(g1 - g2)\, L}{800} = \frac{(0.5 - (-0.7))\, 360}{800} = 0.54\, m\ Ok}$



4) 1st point on the curve chainage = 350 m , x= 30m

R.L of 1st point on tangent = 329.85 + $\frac{g1\, x}{100}$ = 330 m

Tangent correction =y = $\frac{(g_1 - g_2)\, x^2}{200\, L}$ = 0.015 m

R.L of 1st point on the curve = 330 m – 0.015 = 329.985 m

Vertical Curves

Solution: 02

Station	Chainage (m)	Grade Elevation (m)	Tangent Correction (m)	Curve Elevation (m)	
0	320	329.85	0	329.85	Start of V.C
1	350	330.00	– 0.015	329.985	
2	380	330.15	– 0.06	330.090	
3	410	330.30	– 0.135	330.165	
4	440	330.45	– 0.240	330.210	
5	470	330.60	– 0.375	330.225	
6 (F)	500	330.75	– 0.54	330.210	Vertex of V.C
7	530	330.54	– 0.375	330.165	
8	560	330.33	– 0.240	330.090	
9	590	330.12	– 0.135	329.985	
10	620	330.91	– 0.06	329.850	
11	650	330.70	– 0.015	329.685	
12 (B)	680	330.49	0	329.490	End od V.C

Vertical Curves

Assignment

Three cases of Sight distance:

Example 3 and 4 , page 220