Surveying-II
CE-207 (T)

Curves

Transition curves
Lecture 2

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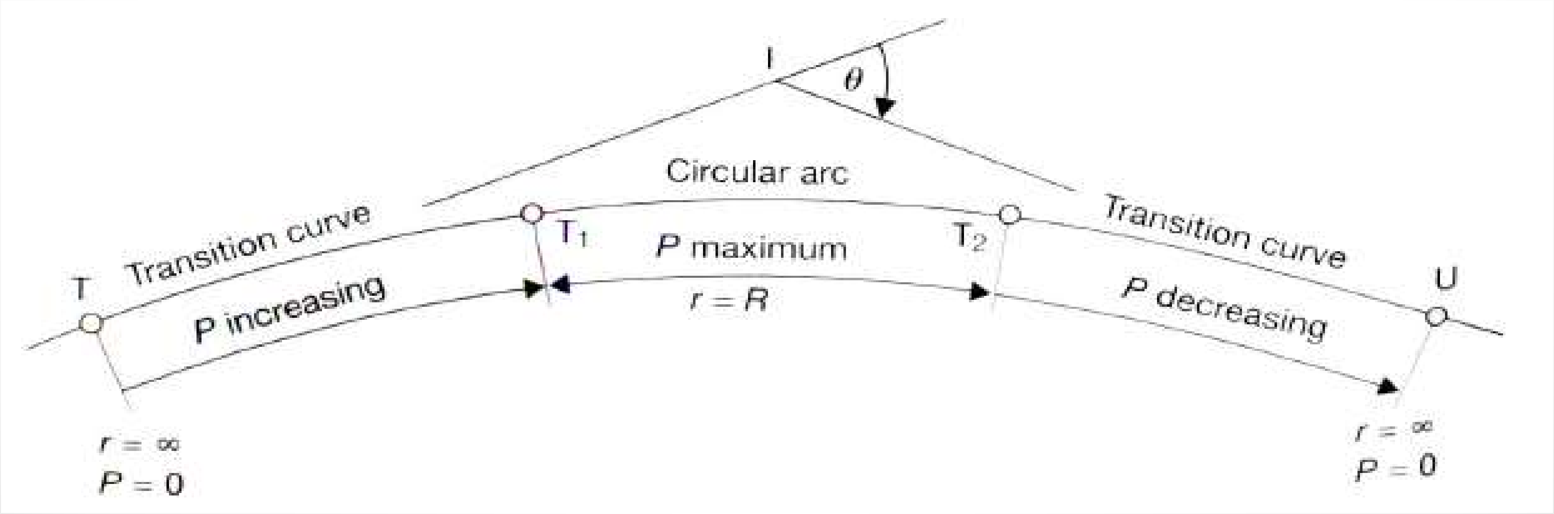
Curves

Transition Curves

- A curve of varying radius is called a transition curve. It is also called **Spiral Curve** or **Easement curve** .
- It is used on both highway & railway between tangent and a circular curve in order to have a smooth transition from tangent to the curve and from curve to the tangent.
- It is also inserted between two branches of compound curve.

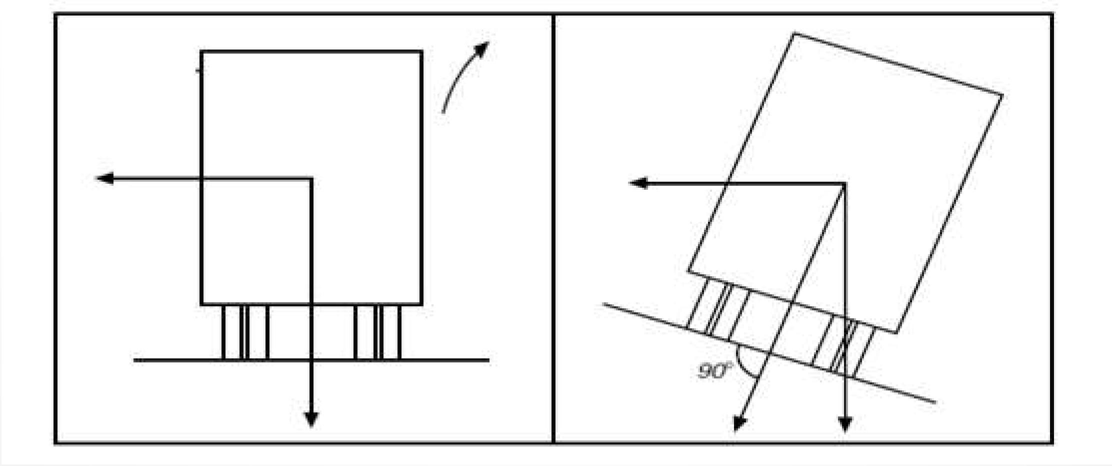


Curves

Transition Curves

The need for Transition Curves

- Circular curves are limited in road designs due to the forces which act on a vehicle as they travel around a bend.
- Transition curves are used to **built up** those forces gradually and uniformly thus ensuring the safety of passenger.
- Allows for gradual application of Super elevation or Cant.
- The Super elevation is designed such that the road surface is near perpendicular to the resultant force of gravity and centrifugal inertia.

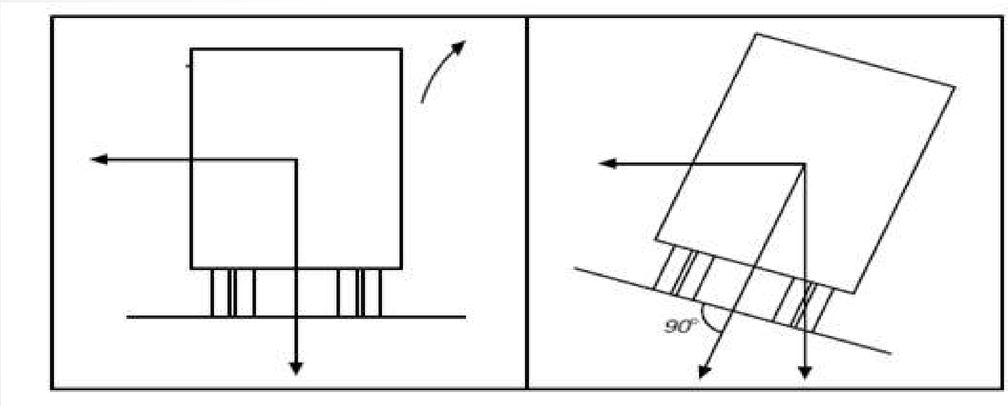


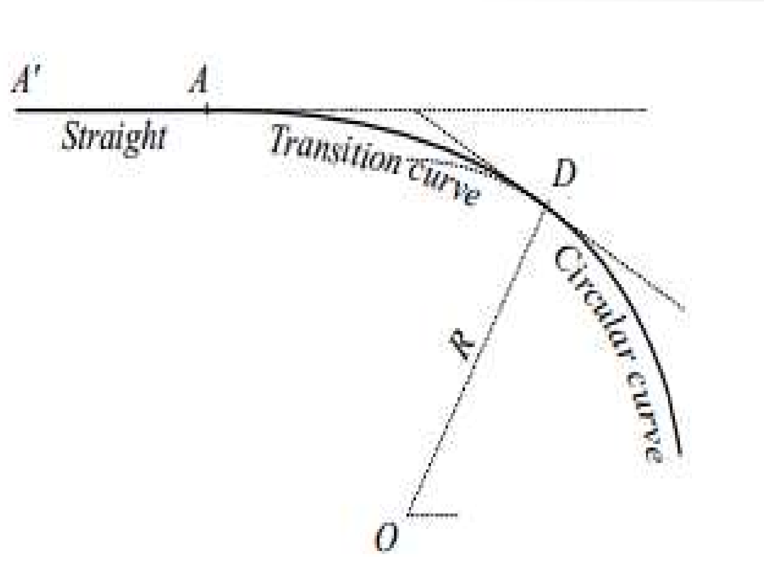
Curves

Transition Curves

The need for Transition Curves

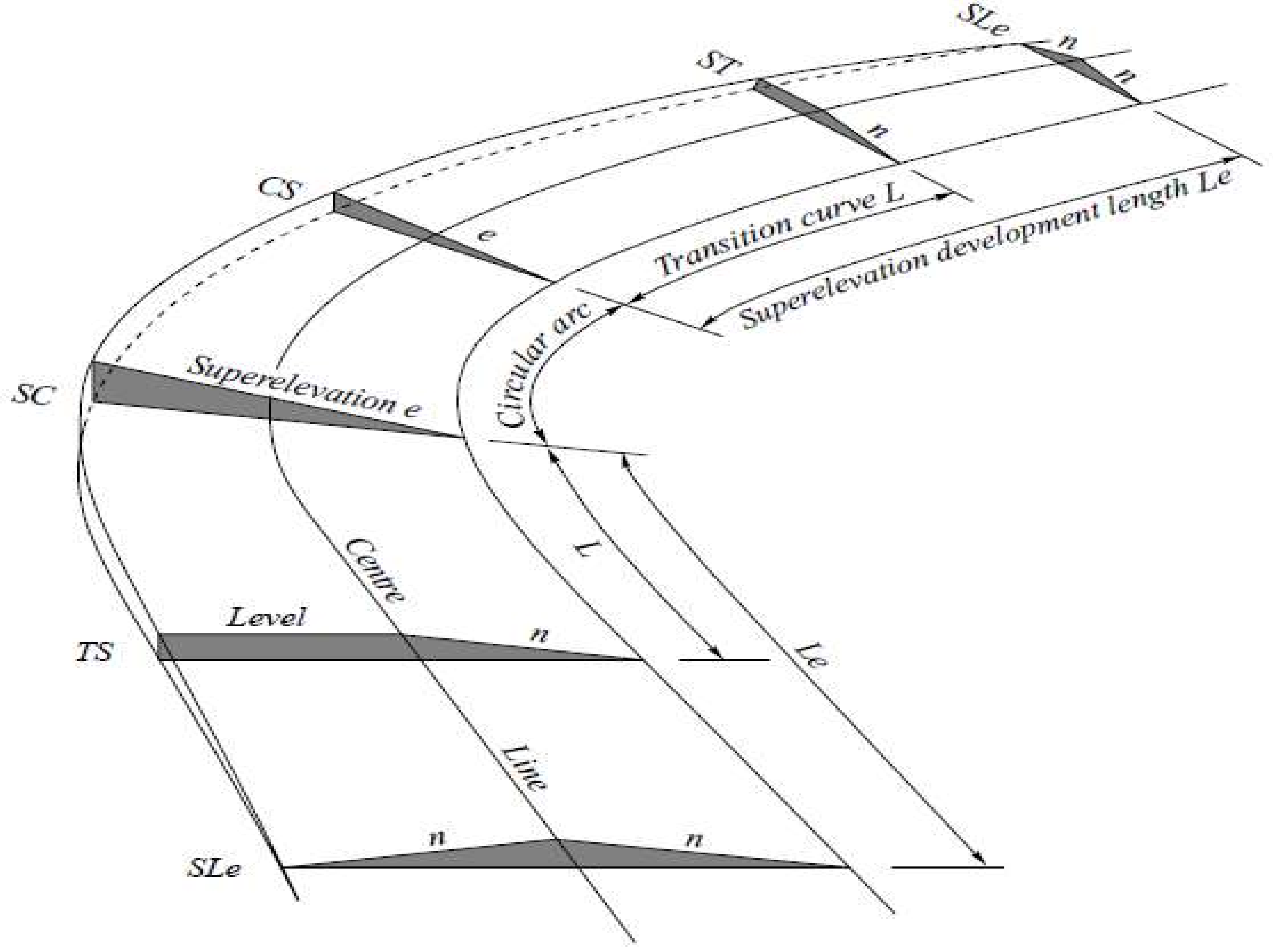
- In order to transition from a flat roadway to a fully super elevated section and still maintain the balance of forces, the degree or sharpness of the curve must begin at zero and increase steadily until maximum super elevation is reached. This is precisely what a Spiral Curve does.





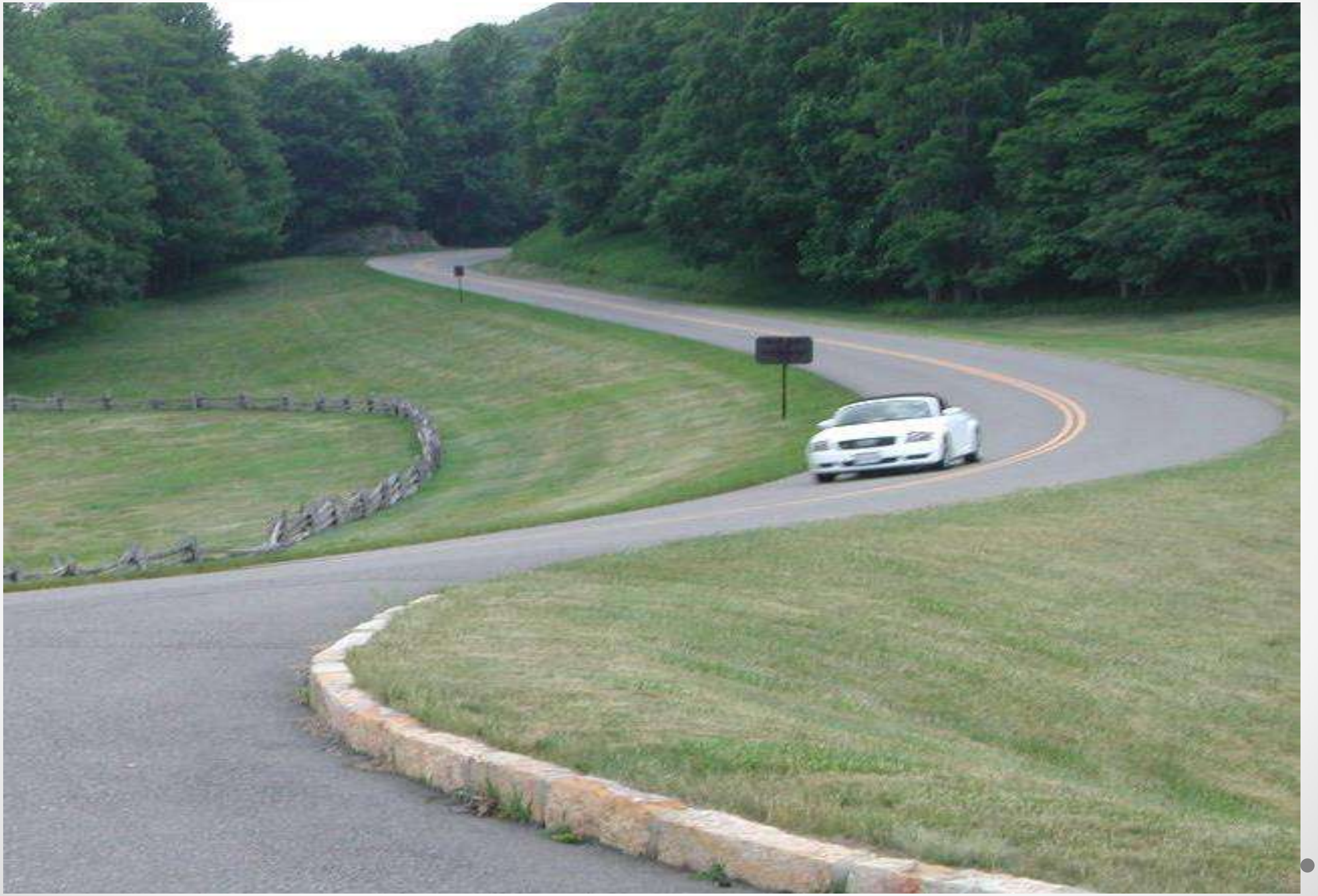
Curves

Transition Curves



Curves

Transition Curves

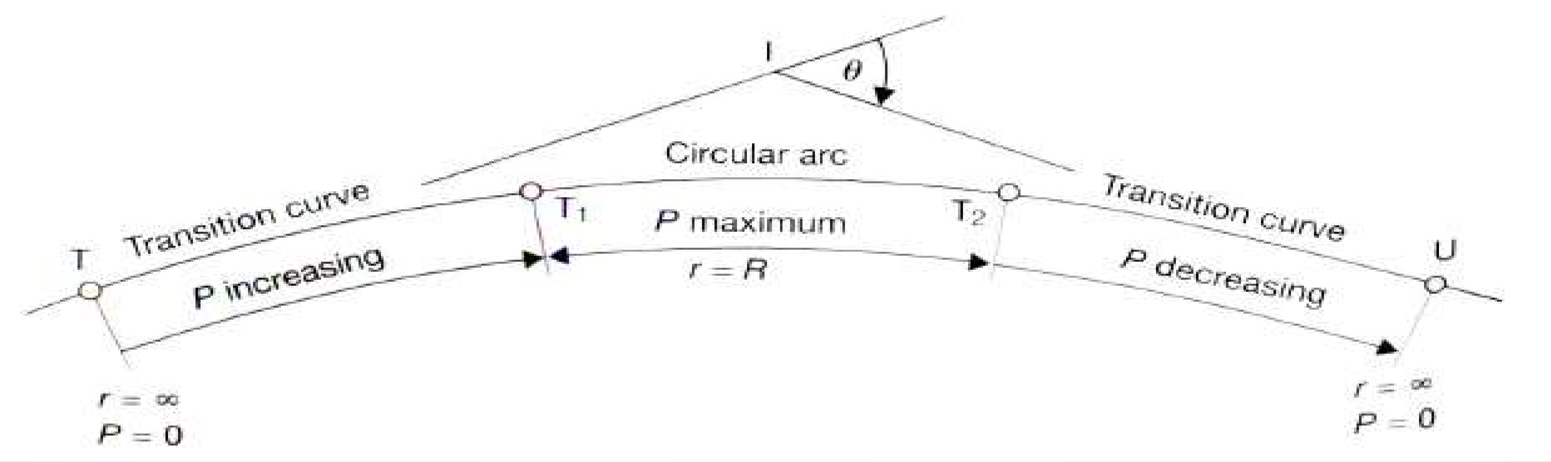


Curves

Transition Curves

Objects of providing a transition curve

- To accomplish gradually the transition from tangent to the circular curve or from circular curve to the tangent.
- To obtain a gradual increase of the curvature from zero at a tangent point to the specified quantity at junction of the transition curve with the circular curve.
- To Provide a satisfactory mean of obtaining gradual increase of super elevation from zero on the tangent to the specified amount on the main circular curve.



Curves

Transition Curves

Condition fulfill by a Transition Curve

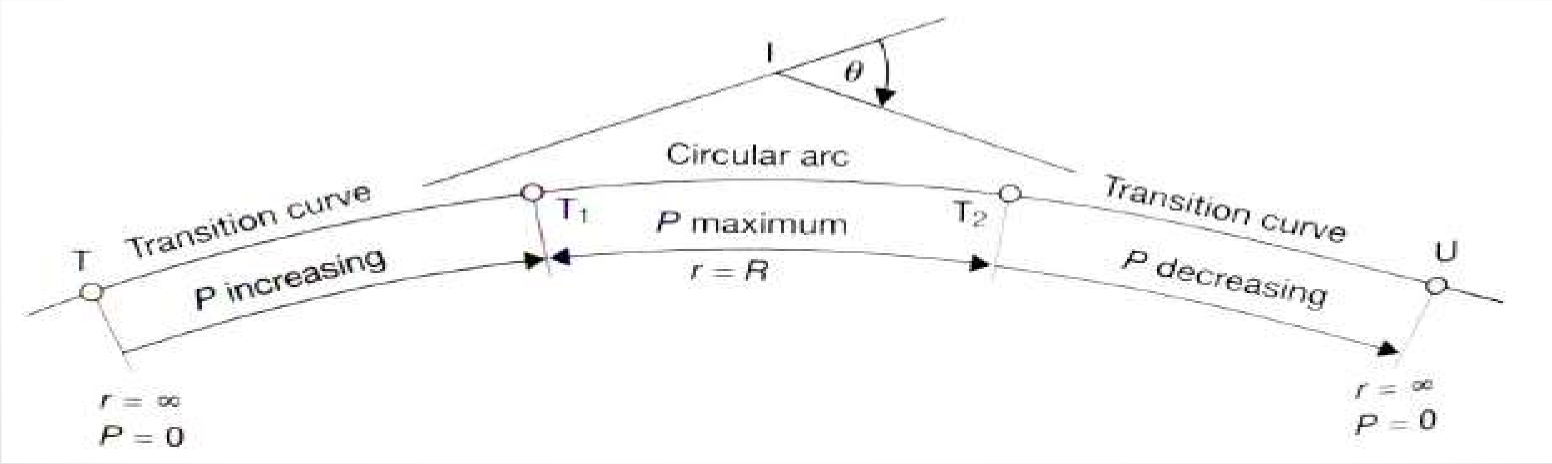
1.It should meet the original straight (tangent) tangentially .

2. It should meet circular curve tangentially.

3. Its radius at a junction with circular curve should be same as that of the circular curve .

4. The rate of increase of the curvature along the transition curve should be the same as that of the super elevation.

5.Its length should be such that full super elevation is attained at the junction with the circular curve.



Curves

Transition Curves

Types of the transition curve in common use are

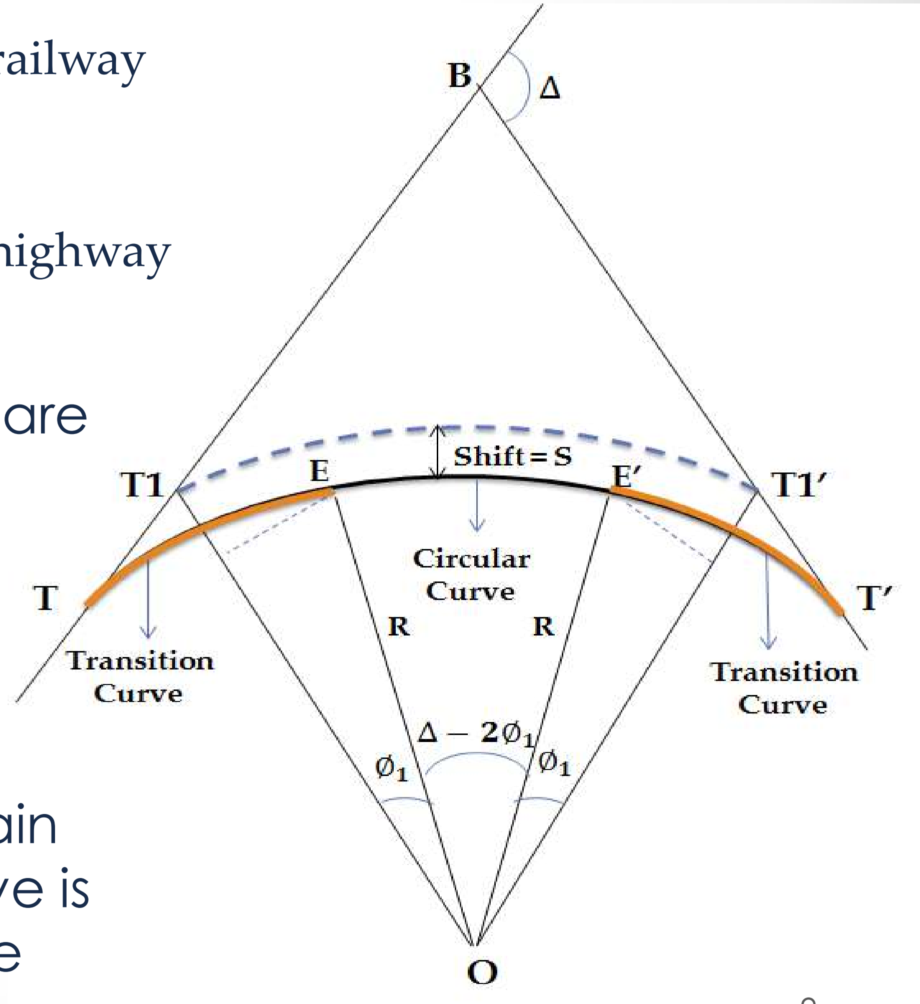
1) A clothoid or spiral
2) A cubic parabola

used in railway

3) A lemniscate

used in highway

- Only mathematical difference are here.
- In order to admit the transition curve , the main circular curve required to be shifted inward.
- When the transition curve is inserted at each end of the main circular curve the resulting curve is called combined or composite curve.



Curves

Transition Curves

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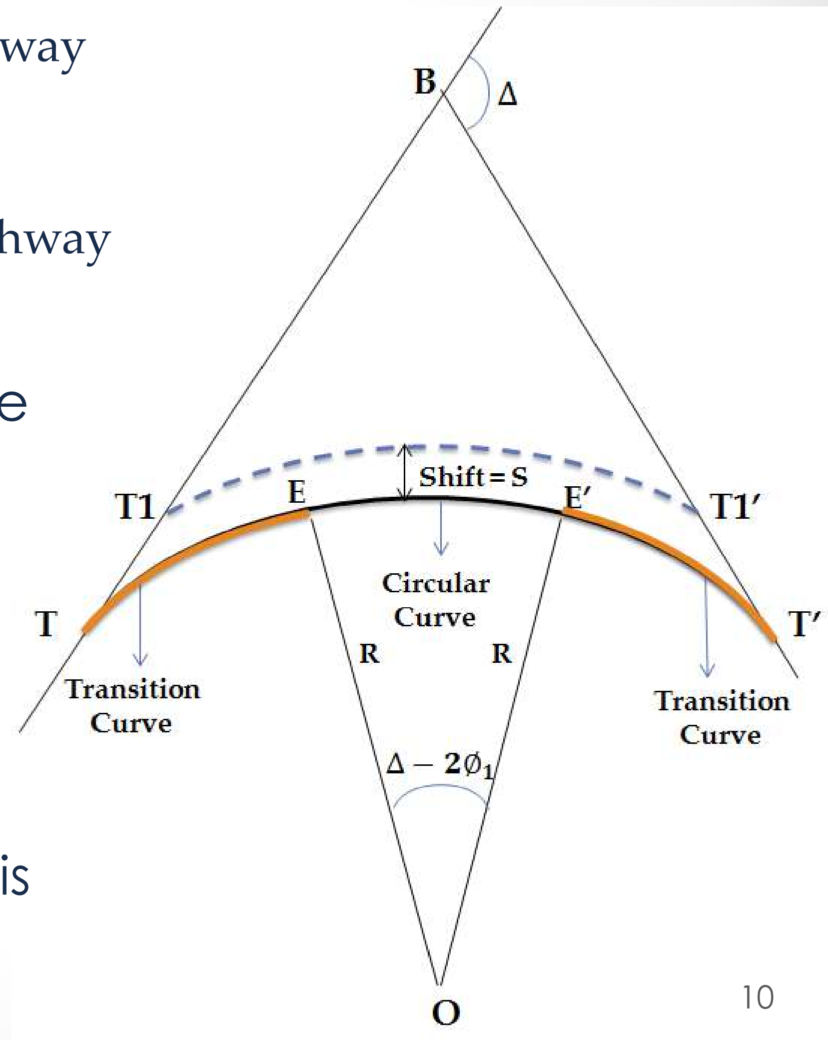


Curves

Transition Curves

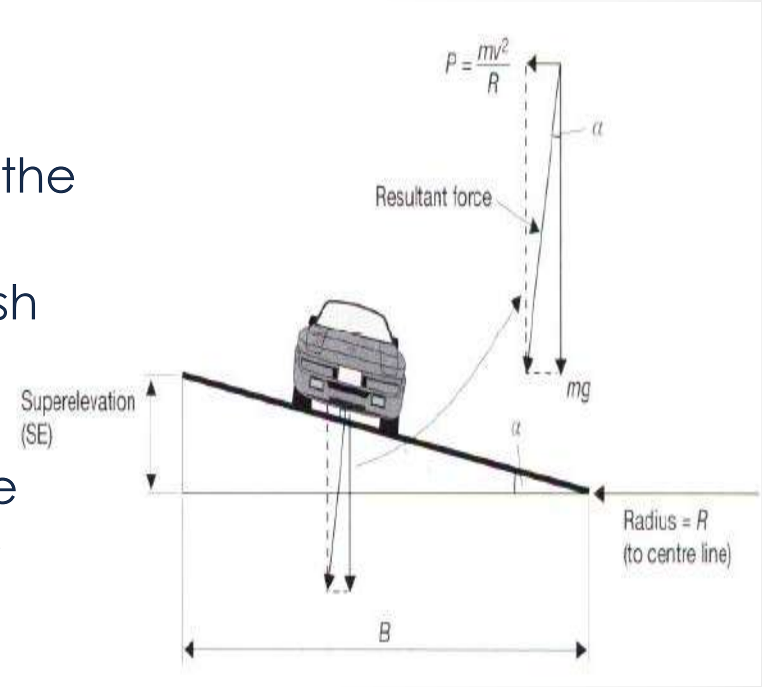
Super Elevation

When vehicle moves from tangent on to the curve the forces acting on it are

- Weight of the vehicle
- Centrifugal force, Both acting through the center of gravity of the vehicle.

The effect of the centrifugal force is to push the vehicle off the rail or road.

To counter act the action the outer rail or outer edge of the road is raised above the raising of outer edge of rail or road above the inner one is called **Super elevation** or **Cant**.



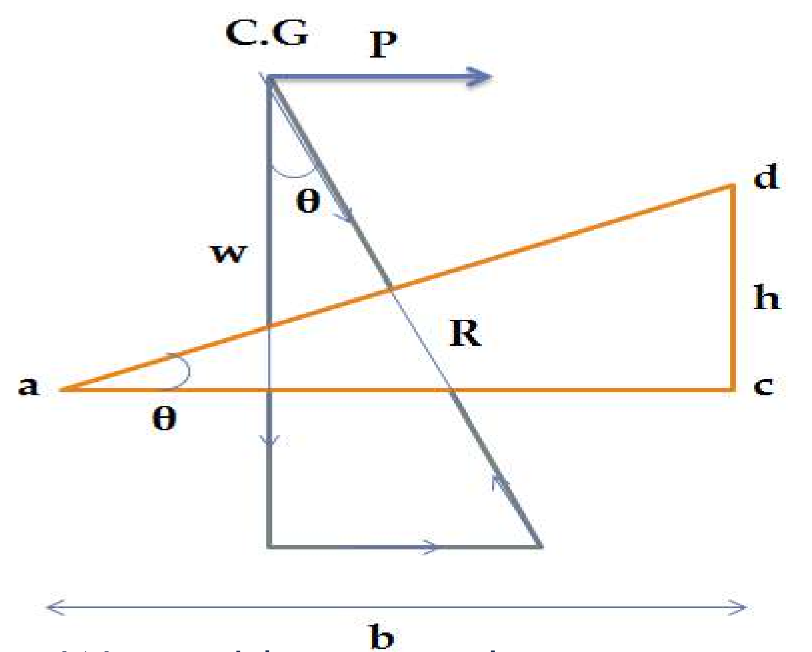
The amount of Super elevation depends upon

- Speed of the vehicle
- Radius of the curve

Curves

Transition Curves

Super Elevation



Let

W = weight of the vehicle

P = centrifugal Force

V = Speed of the vehicle, m/s

g = Acceleration due to gravity, m/s^2

R = Radius of the curve, m

h= Super elevation, m

b = Width of the road, m

For equilibrium the resultant R of the P & W must be equal & opposite to the reaction perpendicular to road or rail surface

$$P = \frac{mv^2}{R} = \frac{wv^2}{gR} \qquad :: w = mg$$

$$\frac{P}{w} = \frac{v^2}{gR}$$

Curves

Transition Curves

Super Elevation

$$\tan\theta = \frac{h}{b} = \frac{dc}{ac} = \frac{P}{W}$$

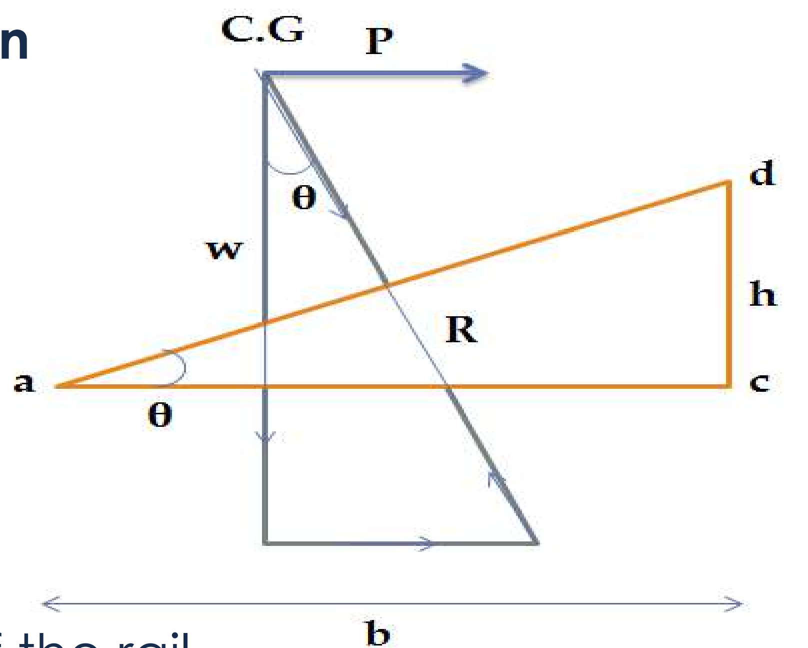
$$\tan\theta = \frac{h}{b} = \frac{P}{W} = \frac{v^2}{gR}$$

$$h = b\tan\theta$$

$h = b\,\dfrac{v^2}{gR}$ ………………… On highway

$h = b\,\dfrac{Gv^2}{gR}$ ………………… On railway

where G = Distance b/w the centre of the rail



Super elevation is gradually applied along a transition curve. Full super elevation is attained at junction of the transition curve with the circular curve .

Curves

Transition Curves

Length of the Transition curve

It may be determine in different ways:

1) As an Arbitrary:

Value from past experiment say 50m.

2) By an Arbitrary Gradient (slope) :

Length of the transition curve may be such that super elevation is applied at the uniform rate of 1 in n ,

Where n = 300 to 1200

i.e h feet rise in n feet length

1 feet rise in n feet length

Therefore **L = n h**

Where

L = Length of transition curve, m

h = super elevation, m

1 in n = Rate of canting

Curves

Transition Curves

Length of the Transition curve

3) By the Time Rate

Transition curve may be of such length that cant (super elevation) is applied at an arbitrary time rate of "**a**" cm/sec,

Where $\boldsymbol{a}$ varies from 2.5 cm/sec to 5 cm/sec

Time taken by vehicle in passing over the transition curve:

$$t = \frac{L}{v}\text{, sec}$$

Super elevation attain in this time:

$$h = a\,t$$

$$h = a\,\frac{L}{v}$$

$$L = \frac{h\,v}{a}$$

L = Length of the transition curve, m

h = amount of super elevation, cm

V = Speed of the vehicle, m/s

a = Time rate (cm/sec)

Curves

Transition Curves

Length of the transition curve

4) By the Rate of change of Radial Acceleration

This rate should be such that the passengers should not experience any sensation of discomfort when the train is travelling over the curve.

It is taken as 30 cm/sec^2 , which is maximum that will pass unnoticed.

Centrifugal force = P = $\frac{mv^2}{R}$

Radial acceleration = a = $\frac{v^2}{R}$, m/sec^2

Rate of change of radial acceleration divide by the time

$$\alpha = \frac{a}{t} = \frac{\frac{v^2}{R}}{\frac{L}{v}} = \frac{v^3}{RL}$$

$$L = \frac{v^3}{\alpha R}$$

Out of these methods the 4th method is commonly used in determining length of the transition curve.

Curves

Transition Curves

Length of the transition curve

4) By the Rate of change of Radial Acceleration

By centrifugal ratio

The ratio of centrifugal force to weight is called centrifugal ratio.

$$P = \frac{mv^2}{R} x \frac{g}{g} = \frac{w\, v^2}{gR}$$

$$\frac{P}{W} = \frac{v^2}{gR}$$

Max value of C. Ratio on road is $\frac{1}{4}$

Max value of C. Ratio on railway is $\frac{1}{8}$

On Road: $\frac{v^2}{gR} = \frac{1}{4} \Rightarrow v = \sqrt{\frac{g\,R}{4}}$

$$L = \frac{v^3}{\alpha\, R} = \frac{\left(\sqrt{\frac{g\,R}{4}}\right)^3}{\alpha\, R} \quad \ldots \mathrm{I}$$

On Railway: $\frac{v^2}{gR} = \frac{1}{8} \Rightarrow v = \sqrt{\frac{g\,R}{8}}$

$$L = \frac{v^3}{\alpha\, R} = \frac{\left(\sqrt{\frac{g\,R}{4}}\right)^3}{\alpha\, R} \quad \ldots \mathrm{II}$$

Curves

Transition Curves

Ideal Transition curve

- The intrinsic equation of the ideal transition curve (clothoid spiral) may be deduced as:

TB = Initial tangent

T = Beginning of the transition curve

E = Junction point transition curve with circular curve

M= Any point of transition curve ,
l m along it from T.

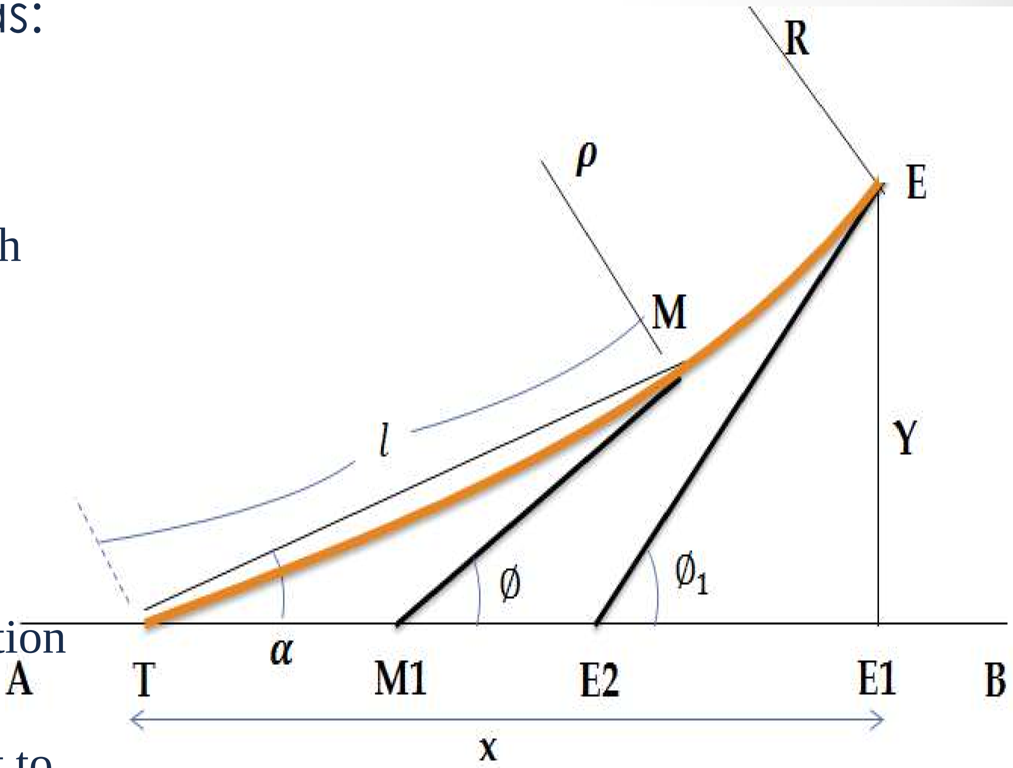
ρ =Radius of transition curve at M

R = Radius of the circular curve

$\emptyset$ = Inclination of tangent to the transition curve at M to initial tangent TB.

$\emptyset_1$ = angle b/w tangent TB and tangent to the T. Curve at junction E (spiral angle)

L = Length of the transition curve.



Curves

Transition Curves

Ideal Transition curve

The fundamental requirement of spiral curve is that its radius of curvature at any point shall vary inversely as a distance l from beginning of the curve.

$$\rho \propto \frac{1}{l} \text{ or } \rho = \frac{m}{l}, \text{ m = constant of proportionality}$$

$$\frac{1}{\rho} = m\, l$$

For all curves, $\frac{d\emptyset}{dl} = \text{Curvature} = \frac{1}{\rho}$

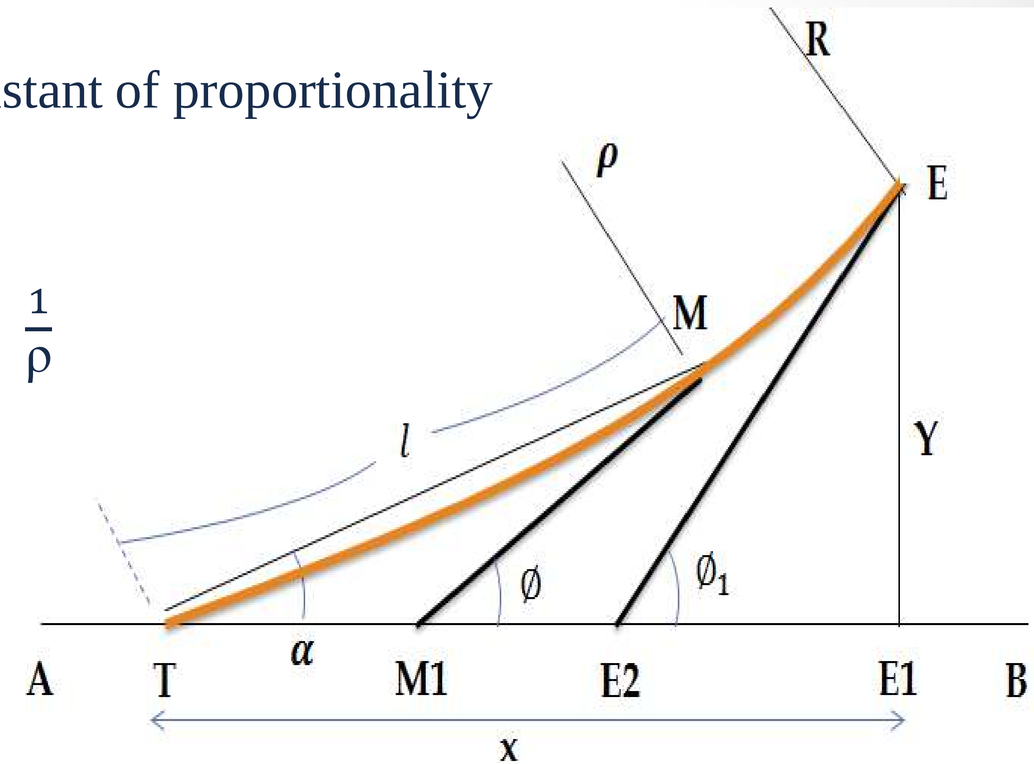
$$d\emptyset = \frac{dl}{\rho} = m\, l\, dl$$

Integrating, $\int d\emptyset = \int m\, l\, dl$

$$\emptyset = \frac{m\, l^2}{2} + C$$

When $l = 0$ then $\emptyset = 0$ then C = 0

$$\emptyset = \frac{\boldsymbol{m\, l^2}}{\boldsymbol{2}} \ldots (a)$$



Curves

Transition Curves

Ideal Transition curve

At point E:

l = L , Ø = Ø 1 (spiral has here max value)

$$\therefore \frac{1}{\rho} = \frac{1}{R} = m\,L \quad or\; m = \frac{1}{R\,L}$$

$$Ø1 = \frac{1}{R\,L} \text{ x } \frac{l^2}{2}$$

$$Ø = \frac{L}{2\,R} \;\ldots\ldots \text{(b)}$$

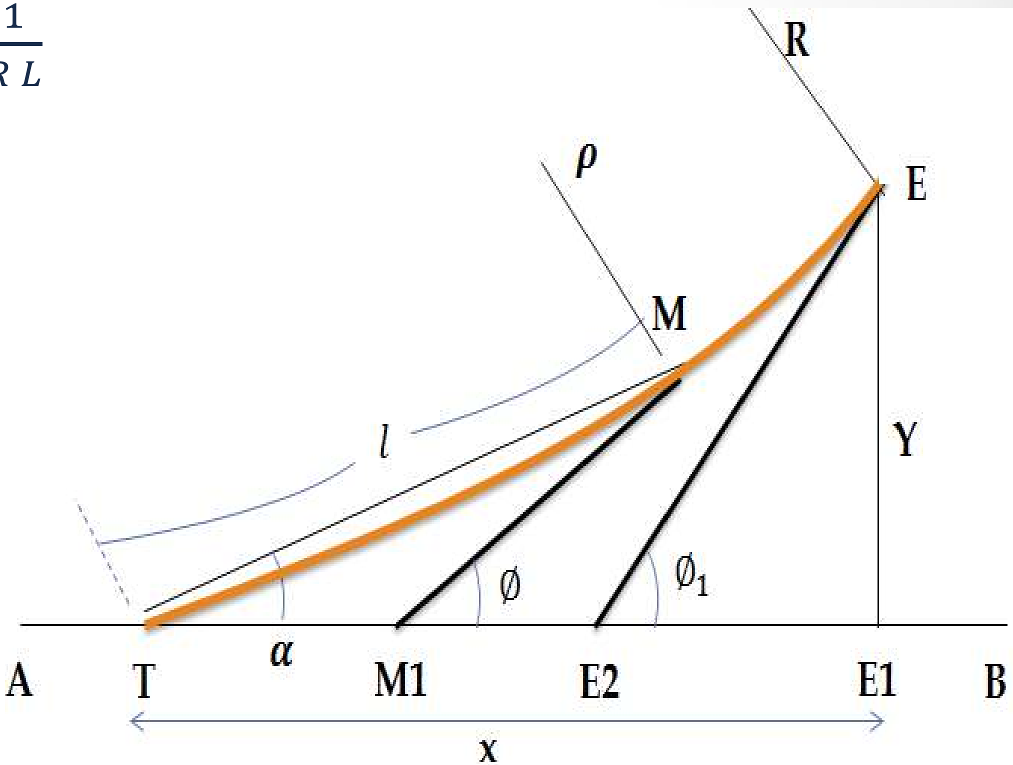
At point M:

$$Ø = Ø\,,\; m = \frac{1}{R\,L}$$

$$Ø = \frac{1}{R\,L}\; x\, \frac{l^2}{2}$$

$$Ø = \frac{l^2}{2\,R\,L} \;\ldots \text{(c)}$$

$$Ø = \frac{l^2}{2\,R\,L} = \frac{l^2}{2\,K}\,, \quad K = R\,L$$

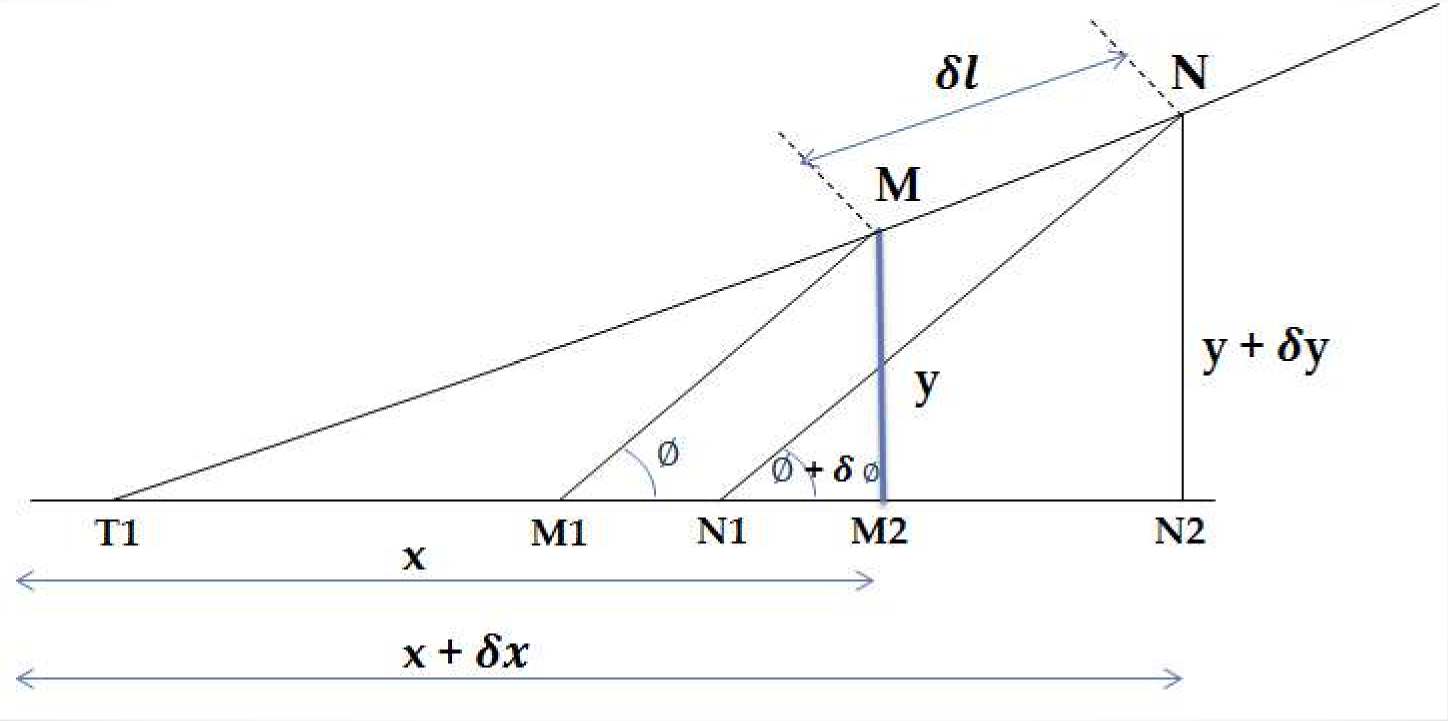


Curves

Transition Curves

Ideal Transition curve

If the curve is to be set out by offsets from the tangent at the commencement of the curve (T), it is necessary to calculate the rectangular (Cartesian) co-ordinates, the 'axes of co-ordinate' being the tangent at T as the x-axis and a line perpendicular to it as the y-axis.



Curves

Transition Curves

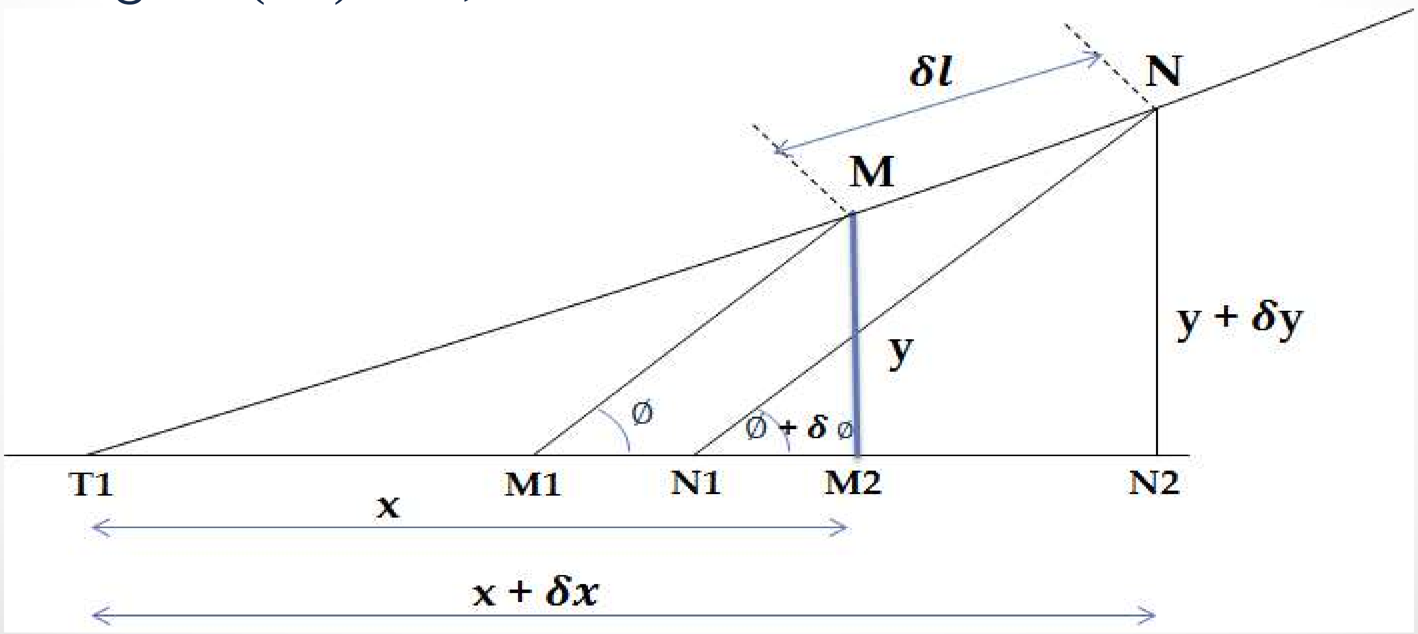
Ideal Transition curve

M and N be the two points at a distance δl apart on the curve. Let the co-ordinate of M and N be

M => (x , y)

N => (x + δx, y + δy)

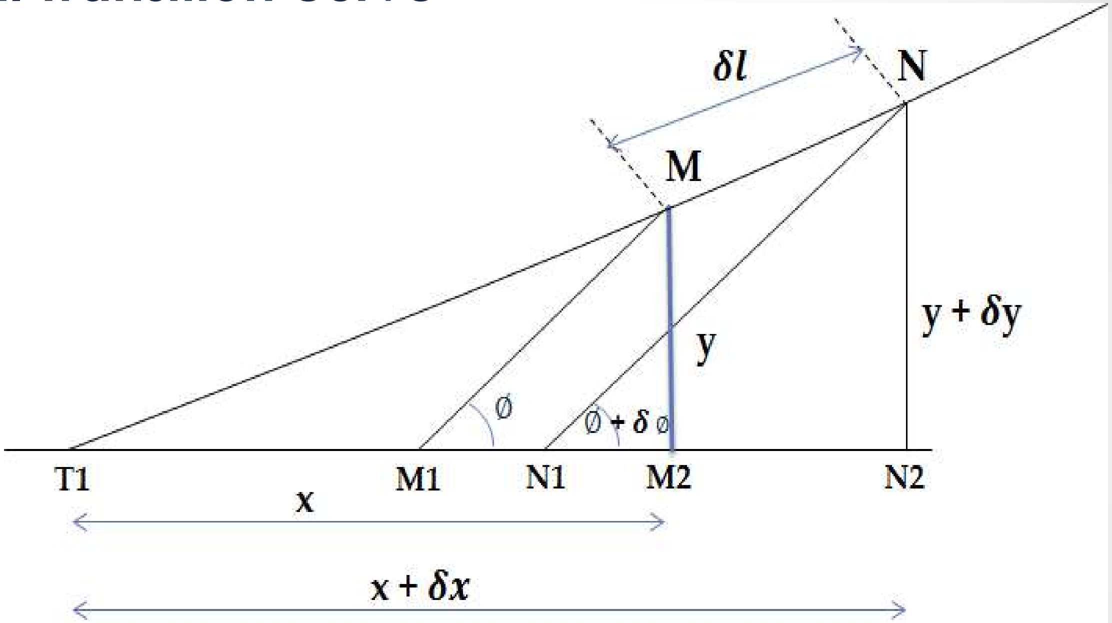
And respective inclinations of the tangents at M and N to the initial tangent (TB) at T, Ø and Ø + δØ



Curves

Transition Curves

Ideal Transition curve



$cos\emptyset = \frac{dx}{dl}$ and $sin\emptyset = \frac{dy}{dl}$

$dx = dl\ cos\emptyset$ and $dy = dl\ sin\emptyset$

$$cos\emptyset = (1 - \frac{\emptyset^2}{2!} + \frac{\emptyset^4}{4!} - \ etc)$$

$$sin\emptyset = (\emptyset - \frac{\emptyset^3}{3!} + \frac{\emptyset^5}{5!} - \ etc)$$

$$dx = \left(1 - \frac{\emptyset^2}{2!} + \frac{\emptyset^4}{4!} - \ ....\right) dl$$

$$dy = \left(\emptyset - \frac{\emptyset^3}{3!} + \frac{\emptyset^5}{5!} - \cdots.\right) dl$$

$$dx = (1 - (\frac{l^2}{2K})^2 * \frac{1}{2!} + (\frac{l^2}{2K})^4 * \frac{1}{4!} - \ldots) dl$$

Integrating dx

$$x = \left(l - \frac{l^5}{40\,k^2} + \frac{l^9}{3456\,k^4} \ldots\right).$$

$$\boldsymbol{x = l\left(1 - \frac{l^4}{40\,k^2} + \frac{l^8}{3456\,k^4} \ldots\right)} \ldots\textbf{(A)}$$

Curves

Transition Curves

Ideal Transition curve

At $\emptyset = \frac{l^2}{2\,k} = \frac{l^2}{2\,R\,L}$

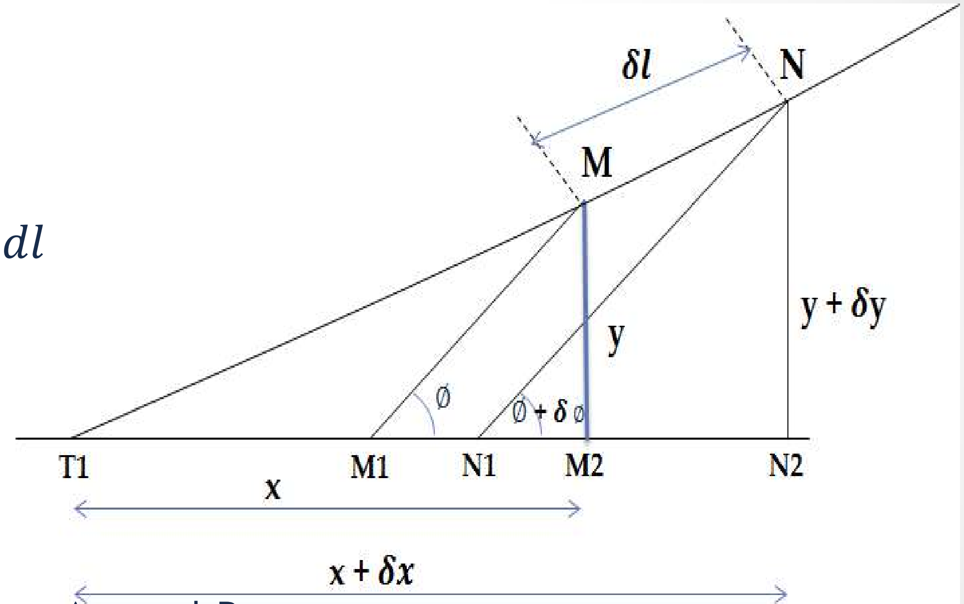
$$\boldsymbol{x = l\left(1 - \frac{\emptyset^2}{10} + \frac{\emptyset^4}{216} \dots\right)} \text{ ....(A 1)}$$

$$dy = \left(\frac{l^2}{2\,K} - \left(\frac{l^2}{2\,K}\right)^3 * \frac{1}{3!} + \left(\frac{l^2}{2\,K}\right)^5 * \frac{1}{5!} - \dots\right)dl$$

Integrating dy

$$\boldsymbol{y = \left(\frac{l^3}{6\,K} - \frac{l^7}{336\,k^3} + \frac{l^{11}}{42240\,k^5} \dots\right) \dots \text{(B)}}$$

$$\boldsymbol{y = \frac{l^3}{6\,K}\left(1 - \frac{\emptyset^2}{14} + \frac{\emptyset^4}{440} \dots\right)} \quad \dots .\,\textbf{(B1)}$$



1) Rejecting all the terms of equation A and B except 1st :

At $x = l$

$$y = \frac{l^3}{6\,R\,L} = \frac{x^3}{6\,R\,L} \quad \dots \text{ eqn of cubic parabola}$$

Curves

Transition Curves

Ideal Transition curve

2) If we take 1st term only of equation B

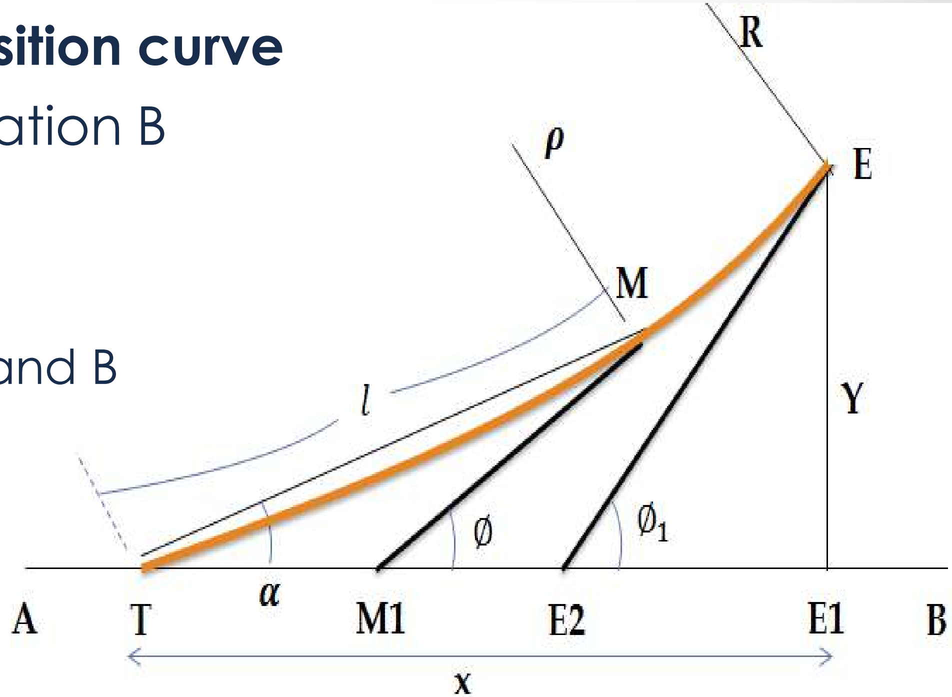
$y = \frac{l^3}{6\,K} = \frac{l^3}{6\,R\,L}$ … Eqn of Cubic Spiral

l--- along curve , y --- offset

3)Taking 1st and 2nd terms of eqns A and B

$x = l\left(1 - \frac{l^4}{40\,k^2}\right) = l\left(1 - \frac{l^4}{40\,(RL)2}\right)$

$y = \frac{l^3}{6\,K}\left(1 - \frac{l^4}{56\,k^3}\right) = \frac{l^3}{6\,K}\left(1 - \frac{l^4}{56\,(RL)3}\right)$



From which the co-ordinate of any point on the true or clothiod spiral nay be obtained, the length l measured along curve.

Curves

Transition Curves

Ideal Transition curve

Now $\tan\alpha = \frac{y}{x}$, where α = deflection angle i.e the angle MTB between the tangent and the line from T to any point M on the curve.

$$\tan\alpha = \frac{\frac{l^3}{6\,K}\left(1 - \frac{\emptyset^2}{14} + \frac{\emptyset^4}{440} \cdots\right)}{l\left(1 - \frac{\emptyset^2}{10} + \frac{\emptyset^4}{216} \cdots\right)}$$

$\tan\alpha = \frac{\emptyset}{3}\ (1 + \frac{\emptyset^2}{35} + ..)$, neglecting other terms

$$\tan\alpha = \frac{\emptyset}{3}$$

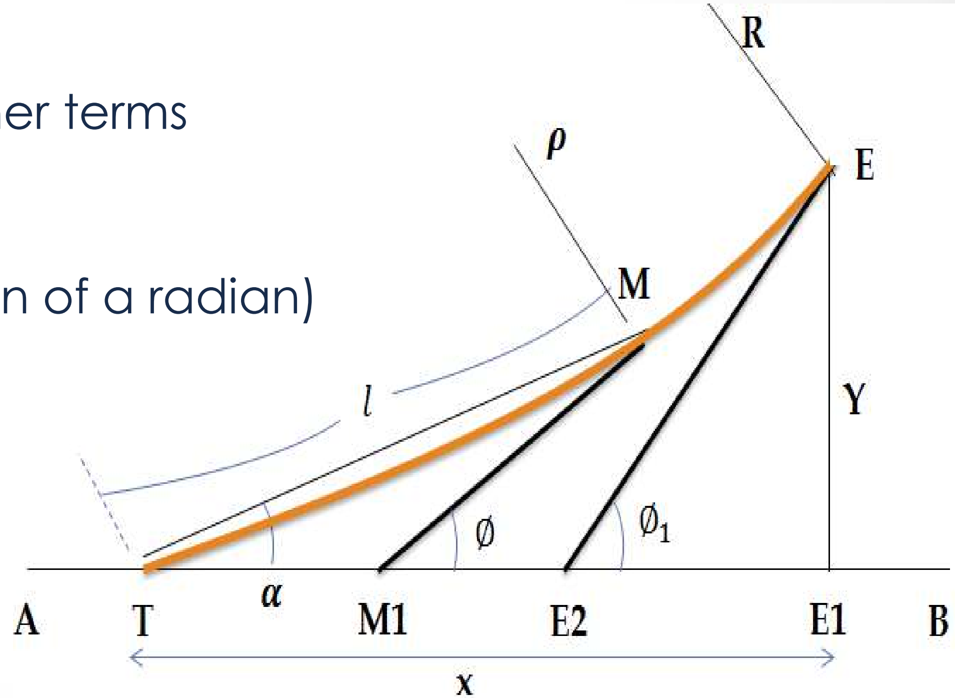
Since $\emptyset$ is usually small(a small fraction of a radian)

$\alpha = \frac{\emptyset}{3}$, But $\emptyset = \frac{l^2}{2\,R\,L}$

$\alpha = \frac{\frac{l^2}{2\,R\,L}}{3} = \frac{l^2}{6\,R\,L}$ radians

$\alpha = \frac{1800\ l^2}{\pi\,R\,L}$ minutes

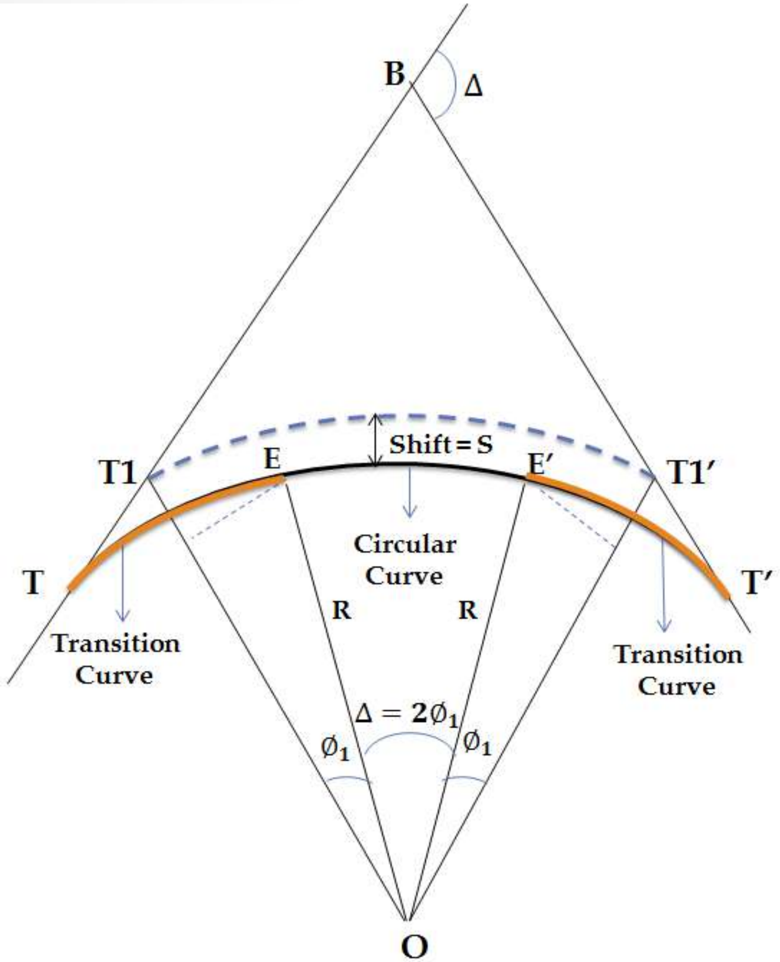
$\alpha = \frac{1800\ l^2}{60\,\pi\,R\,L}$ degrees

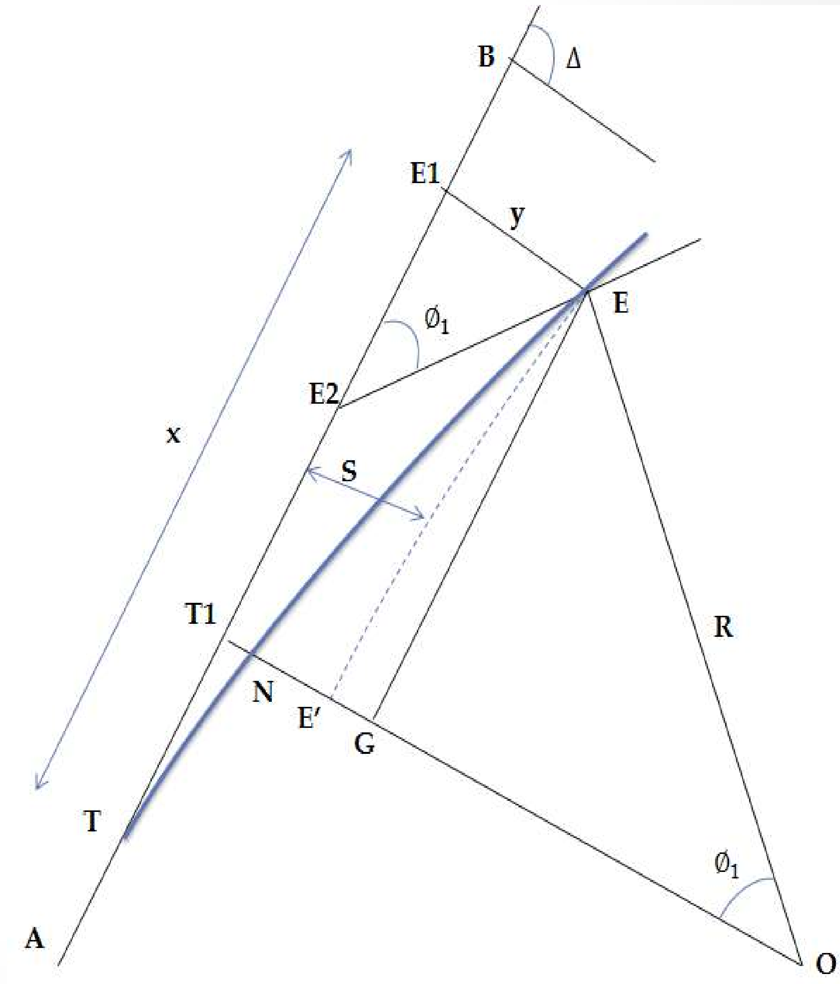


Curves

Transition Curves

Characteristics of a Transition curve





Curves

Transition Curves

Characteristics of a Transition curve

Let

TB = original tangent

T = commencement of the transition curve

E = end of the transition curve

EE_2 = tangent to both the transition and circular curve at E

Y = EE_1 = offset to junction point E of both curve

X = TE_1 = x co-ordinate of E

EE' = redundant circular curve

T_1 = point of intersection of line (OE`) T to tangent at the Cir. Curve at E` and original tangent TB

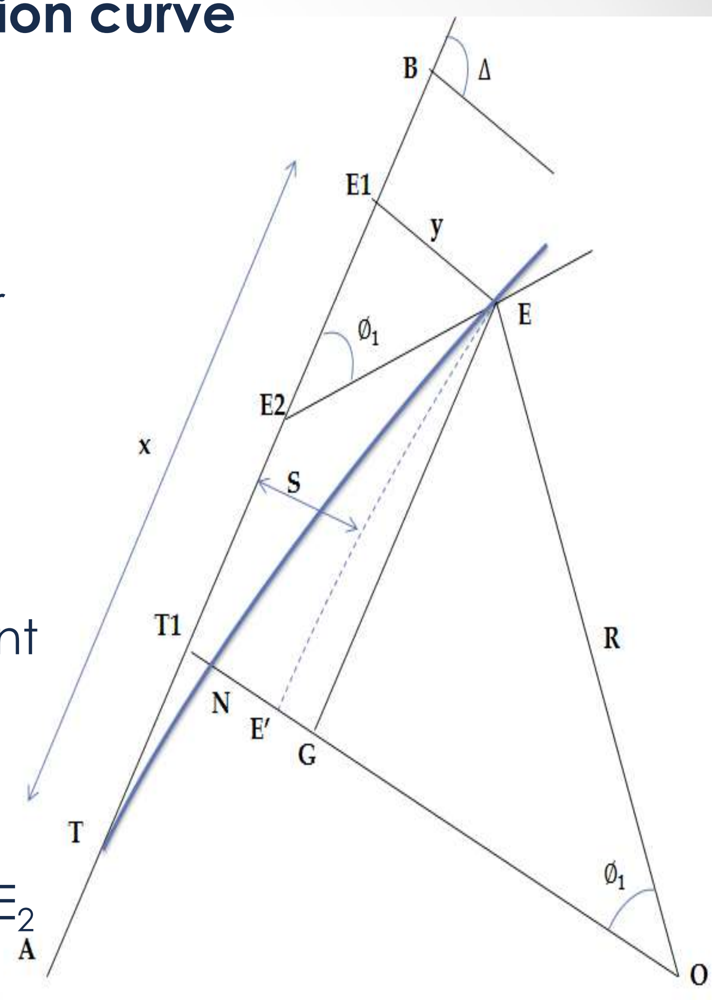
S = $E'T_1$ = shift of the circular curve

N = point in which OE' cuts the transition curve

$\emptyset_1$ = spiral angle (EE_2B) b/w common tangent EE_2 and original tangent TB

R = radius of circular curve

L = length of transition curve



Curves

Transition Curves

Characteristics of a Transition curve

a) $EE' = R\,\emptyset_1$; but $\emptyset_1 = \frac{L}{2\,R}$

$EE' = \frac{L}{2\,R} \times R = \frac{L}{2}$

But EN is very nearly equal to EE'

$EN = \frac{L}{2}$ … A

That is the shift $E`T_1$ bisect the transition Curve at N

Hence $TN = \frac{L}{2}$

b) Draw EG perpendicular to OE'

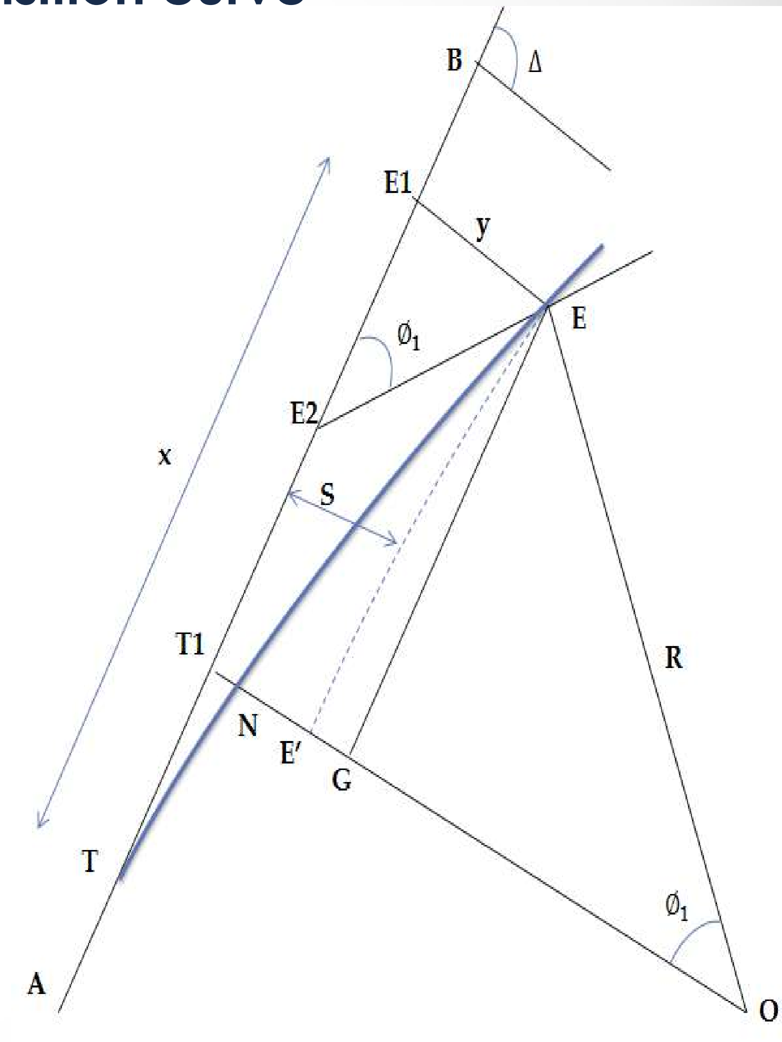
$S = E`T_1 = GT_1 - GE` = EE1 - GE`$

$S = Y - R(1 - \cos\emptyset_1)$ Or

$S = Y - 2\,R\sin^2\frac{\emptyset_1}{2}$

But $Y = \frac{L^3}{6\,R\,L} = \frac{L^2}{6\,R}$ and $\emptyset_1 = \frac{L}{2\,R}$

$S = \frac{L^2}{6\,R} - 2\,R\sin\left(\frac{\emptyset_1}{2}\right)^2 = \frac{L^2}{6\,R} - 2\,R\left(\frac{\emptyset_1}{2}\right)^2$



Curves

Transition Curves

Characteristics of a Transition curve

Neglecting higher power of $\emptyset_1$

$$S = \frac{L^2}{6\,R} - \frac{R}{2}\, x \left(\frac{L}{2\,R} \right)^2$$

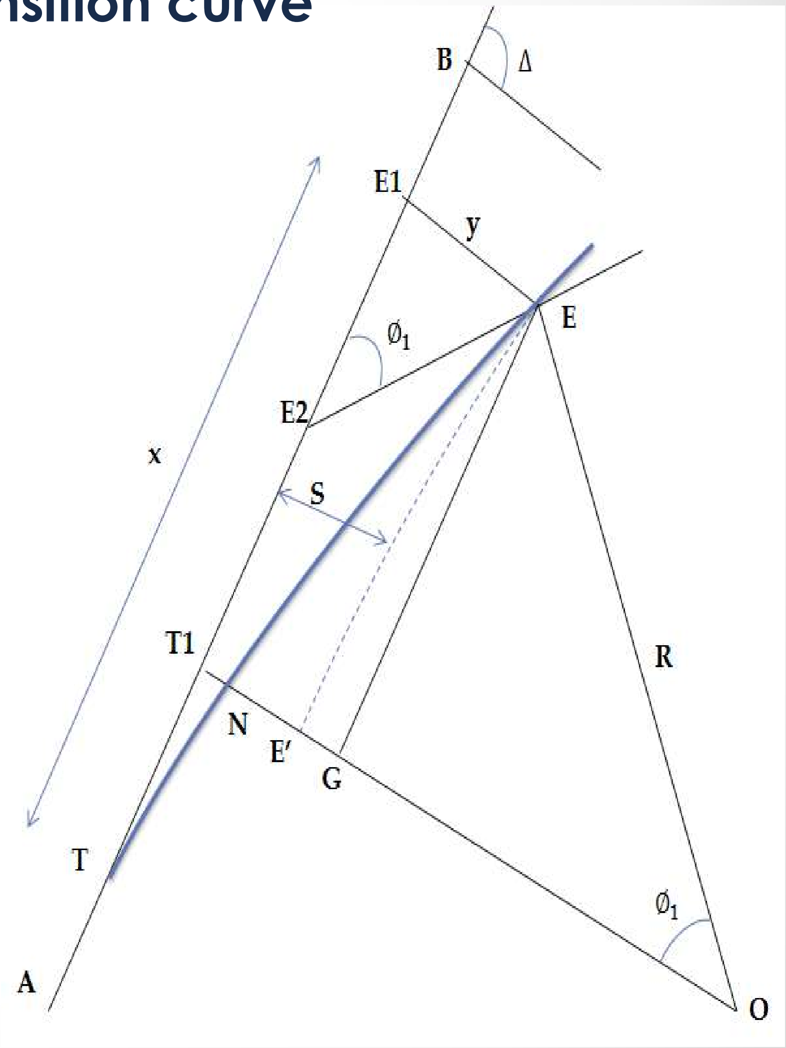
$$S = \frac{L^2}{6\,R} - \frac{L^2}{8\,R}$$

$$S = \frac{L^2}{24\,R} \;\ldots\ldots\; (B)$$

Also $NT_1 = \frac{TN^3}{6\,R\,L} = \frac{(L/2)^3}{6\,R\,L} = \frac{L^2}{48\,R} = \frac{S}{2}$

$NT_1 = \frac{S}{2}$

i.e Transition curve bisect the shift.



Curves

Transition Curves

Characteristics of a Transition curve

Tangent length for Transition Curve:

$BT = BT_1 + T_1T$

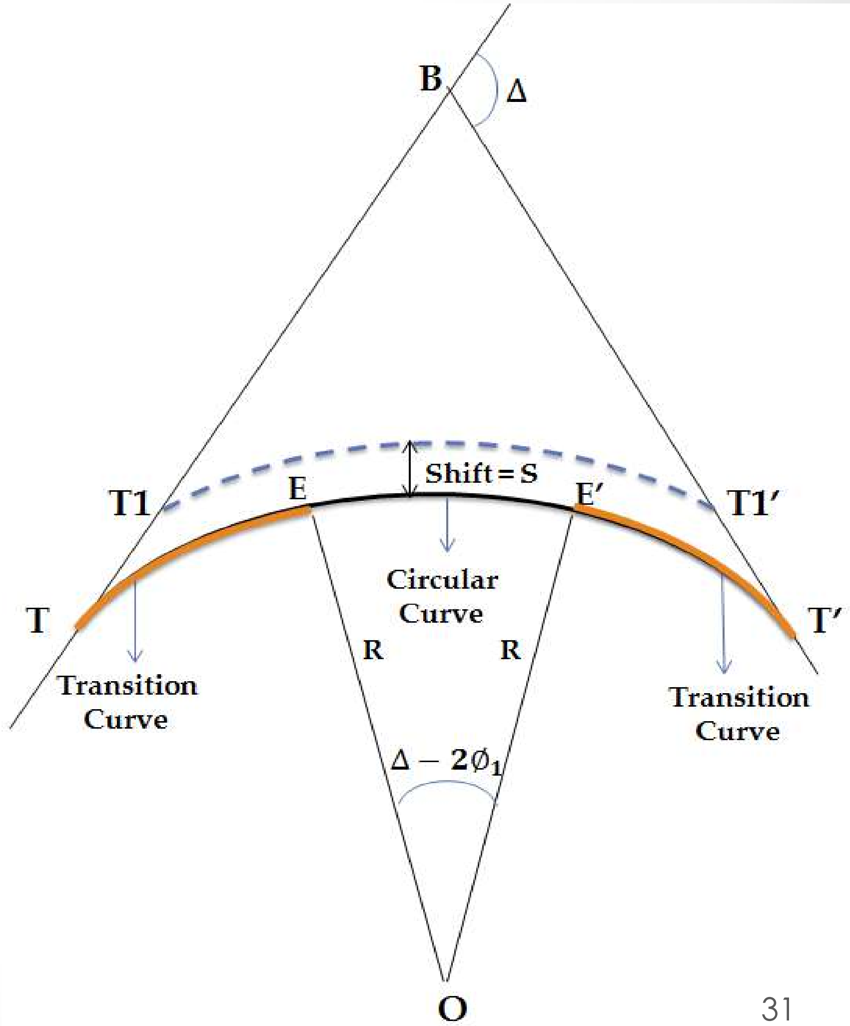
$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$

Length of Circular Curve:

Length of EE\` $= \dfrac{\pi R(\Delta - 2\emptyset_1)}{180^o}$

Length of Total Combined Curve:

$= \dfrac{\pi R(\Delta - 2\emptyset_1)}{180^o} + 2L$



Transition Curves

Problem 01: The full data refer to a composite curve.

Deflection angle (Δ) = 60°30’

Max speed 60 miles/hour

Centrifugal ratio ¼

Max ratio of radial acc. (α) 1feet/sec

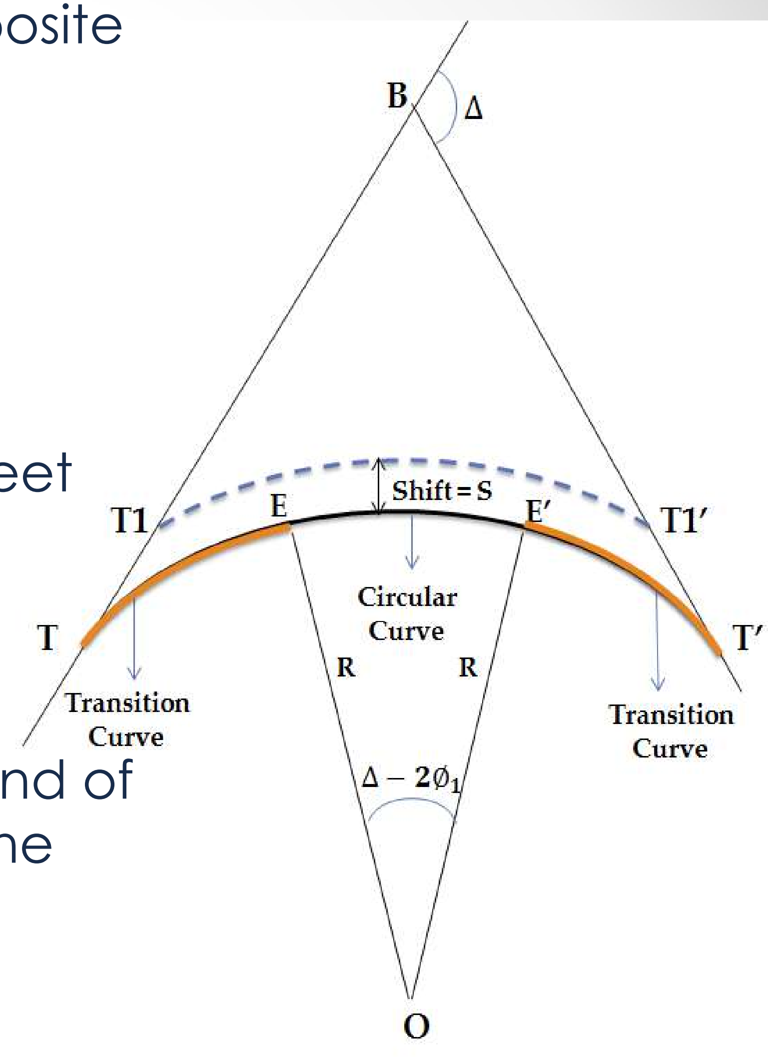
Chainage of intersection point at 8565 feet

Determine:

1) Radius of Circular curve

2) Length of transition curve

3) The chainage of the beginning and end of transition curve and at the junctions of the transition curve with the circular curve.



1 mile = 5280 feet

Transition Curves

Solution: Problem 01

1) Radius of Circular curve:

V = 60 mph

$$V = \frac{60\ x\ 5280}{60\ x\ 60}\frac{ft}{sec} = 88\frac{ft}{sec}$$

$$C.R = \frac{v^2}{g\ R} = \frac{1}{4}$$

$$R = \frac{4\ v^2}{g} = \frac{4\ (88)^2}{32.2} = 961.99\ feet$$

2) Length of Transition curve:

$$\alpha = \frac{v^3}{R\ L}$$

$$1 = \frac{(88)^3}{961.99\ L}$$

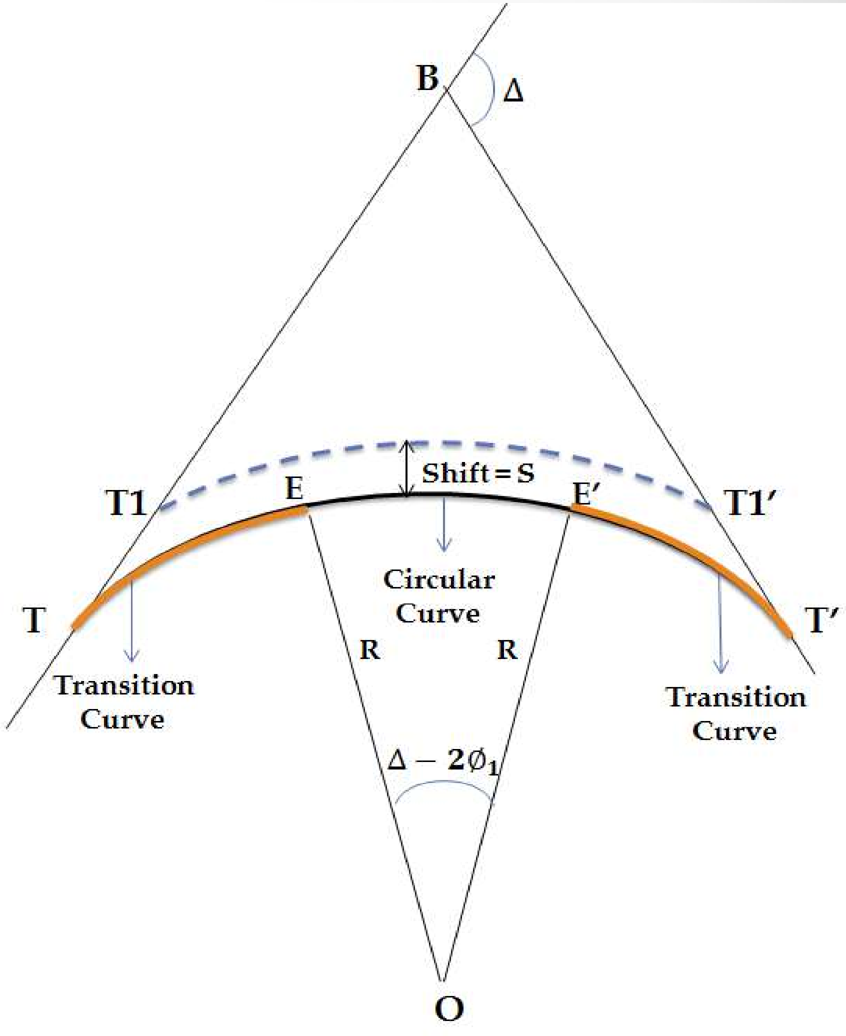
L = 798.40 feet

3) Tangent Length:

$$BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}$$

$$\text{Shift} = S = \frac{L^2}{24\ R} = 21.74\ feet$$

$$BT = (961.99 + 21.74)\tan\frac{\Delta}{2} + 798.40 = 927.89\ feet$$



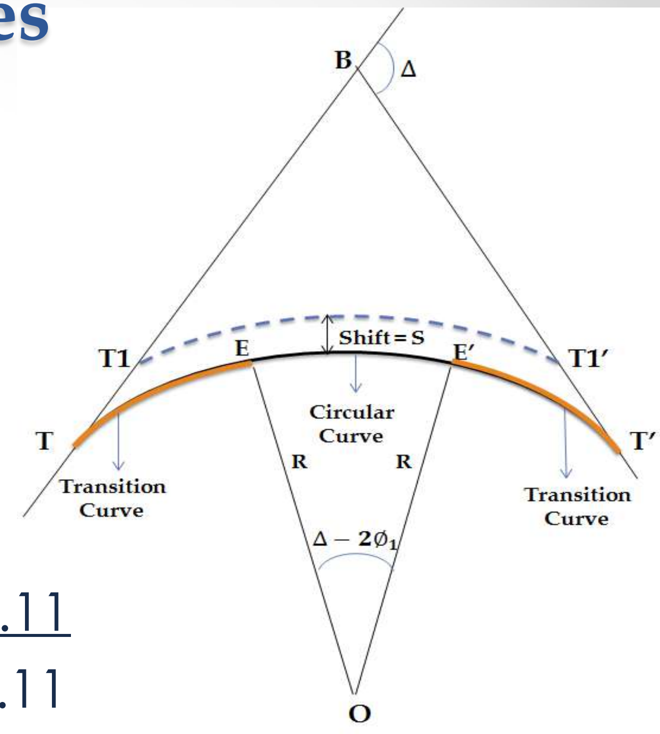
Transition Curves

Solution: Problem 01

4) EE' = length of Cir. Curve = $\frac{\pi\, R\,(\Delta - 2\emptyset)}{180o}$

$\emptyset = \frac{L}{2\,R}\, x\, \frac{180^o}{\pi}$ = 21°5'45.69''

EE' = 307.39 feet (length of Cir. Curve)



Chainage of intersection point B	= 8565
Minus tangent length	= - 927.11
Chainage of T	= 7637.11
Plus length of T. Curve	= + 708.40
Chainage of E	= 8345.51
Plus length of Cir. curve	= + 307.39
Chainage of E'	= 8652.90
Plus length of T. Curve	= + 708.40
Chainage of T'	= 9361.30 feet

Transition Curves

Assignment No 2

Example 1, 2, 3and 4 page 193