Chapter # 8

Principle Stresses Under a Given Loading

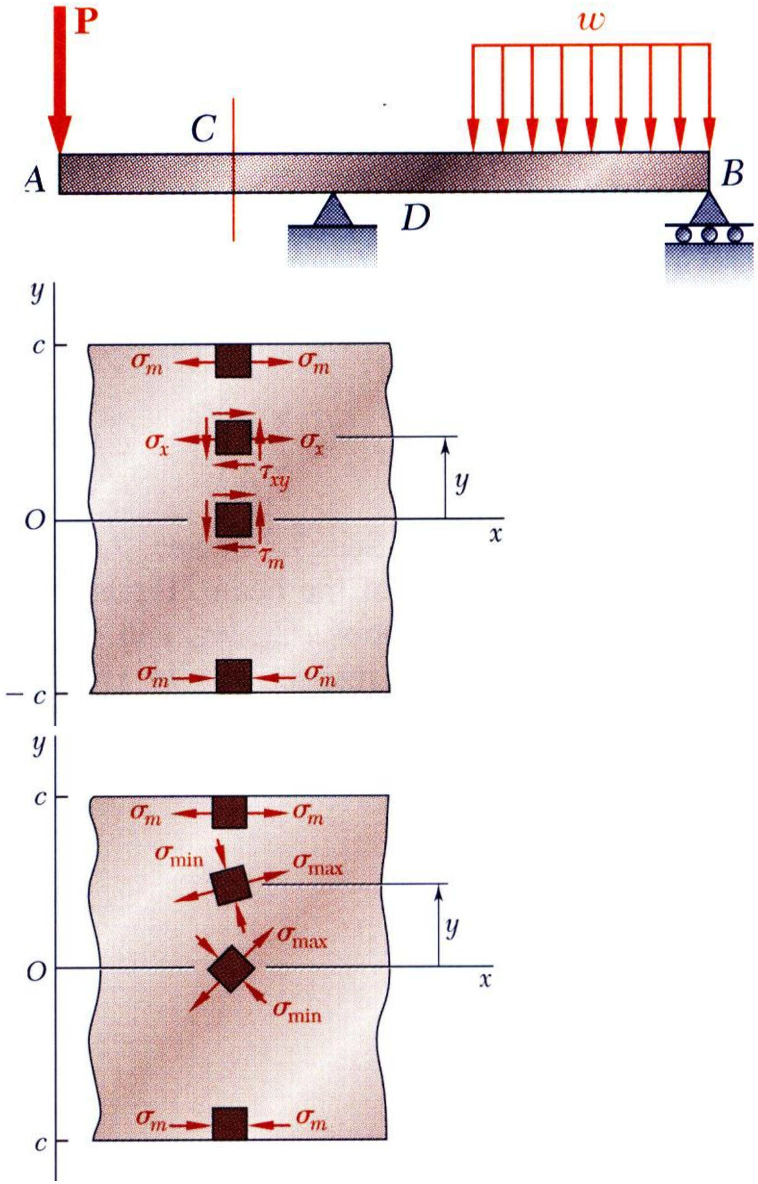
Principle Stresses Under a Given Loading

Introduction

- In Chaps. 1 and 2, you learned how to determine the normal stress due to centric loads
  In Chap. 3, you analyzed the distribution of shearing stresses in a circular member due to a twisting couple
  In Chap. 4, you determined the normal stresses caused by bending couples
  In Chaps. 5 and 6, you evaluated the shearing stresses due to transverse loads
  In Chap. 7, you learned how the components of stress are transformed by a rotation of the coordinate axes and how to determine the principal planes, principal stresses, and maximum shearing stress at a point.

- In Chapter 8, you will learn how to determine the stress in a structural member or machine element due to a combination of loads and how to find the corresponding principal stresses and maximum shearing stress

Principle Stresses in a Beam



- Prismatic beam subjected to transverse loading

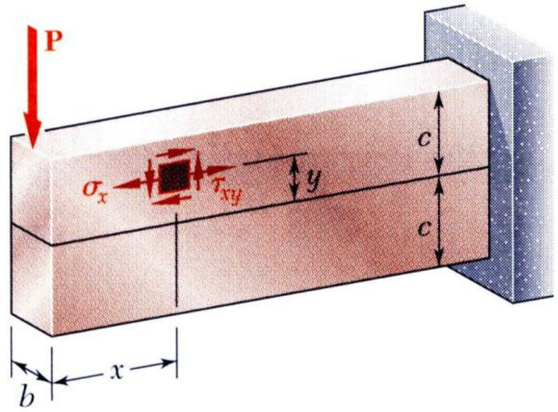
$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{Mc}{I}$$
$$\tau_{xy} = -\frac{VQ}{It} \qquad \tau_m = \frac{VQ}{It}$$

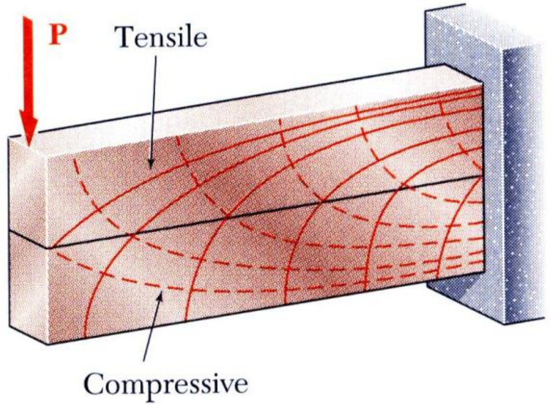
- Principal stresses determined from methods of Chapter 7

- Can the maximum normal stress within the cross-section be larger than

$$\sigma_m = \frac{Mc}{I}$$

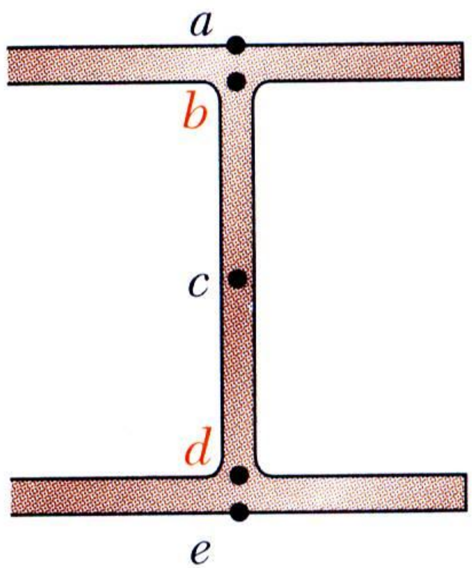
Principle Stresses in a Beam





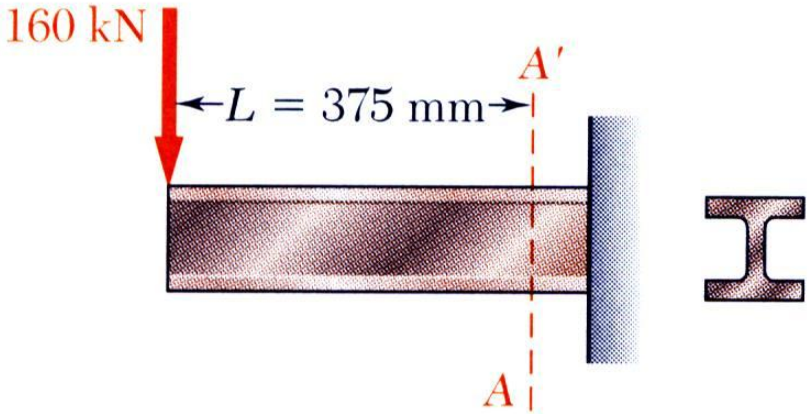
	$x = 2c$		$x = 8c$	
y/c	σ_{min}/σ_m	σ_{max}/σ_m	σ_{min}/σ_m	σ_{max}/σ_m
1.0	0	1.000	0	1.000
0.8	−0.010	0.810	−0.001	0.801
0.6	−0.040	0.640	−0.003	0.603
0.4	−0.090	0.490	−0.007	0.407
0.2	−0.160	0.360	−0.017	0.217
0	−0.250	0.250	−0.063	0.063
− 0.2	−0.360	0.160	−0.217	0.017
− 0.4	−0.490	0.090	−0.407	0.007
− 0.6	−0.640	0.040	−0.603	0.003
− 0.8	−0.810	0.010	−0.801	0.001
− 1.0	−1.000	0	−1.000	0

Principle Stresses in a Beam



- Cross-section shape results in large values of τ_{xy} near the surface where σ_x is also large.
- σ_{max} may be greater than σ_m

Sample Problem 8.1



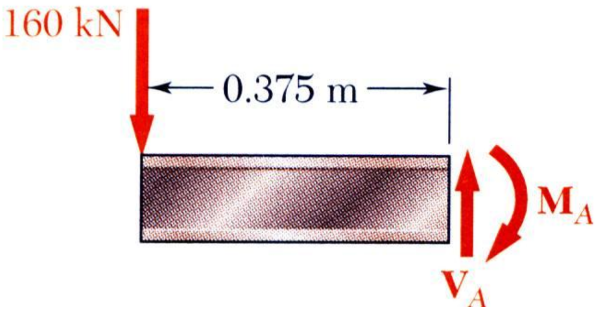
A 160-kN force is applied at the end of a W200x52 rolled-steel beam.

Neglecting the effects of fillets and of stress concentrations, determine whether the normal stresses satisfy a design specification that they be equal to or less than 150 MPa at section *A-A'*.

SOLUTION:

- Determine shear and bending moment in Section *A-A'*
- Calculate the normal stress at top surface and at flange-web junction.
- Evaluate the shear stress at flange-web junction.
- Calculate the principal stress at flange-web junction

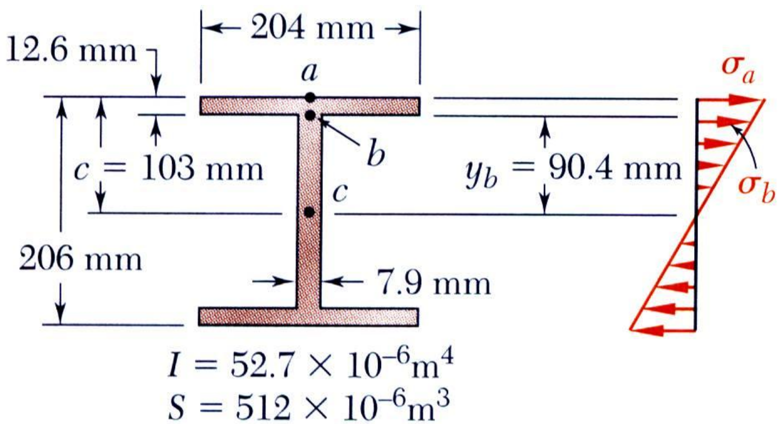
Sample Problem 8.1



SOLUTION:

- Determine shear and bending moment in Section *A-A'*

$$M_A = (160\,\text{kN})(0.375\,\text{m}) = 60\,\text{kN-m}$$
$$V_A = 160\,\text{kN}$$

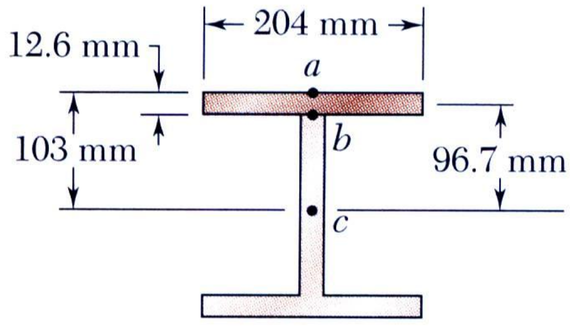


- Calculate the normal stress at top surface and at flange-web junction.

$$\sigma_a = \frac{M_A}{S} = \frac{60\,\text{kN}\cdot\text{m}}{512\times10^{-6}\,\text{m}^3}$$
$$= 117.2\,\text{MPa}$$

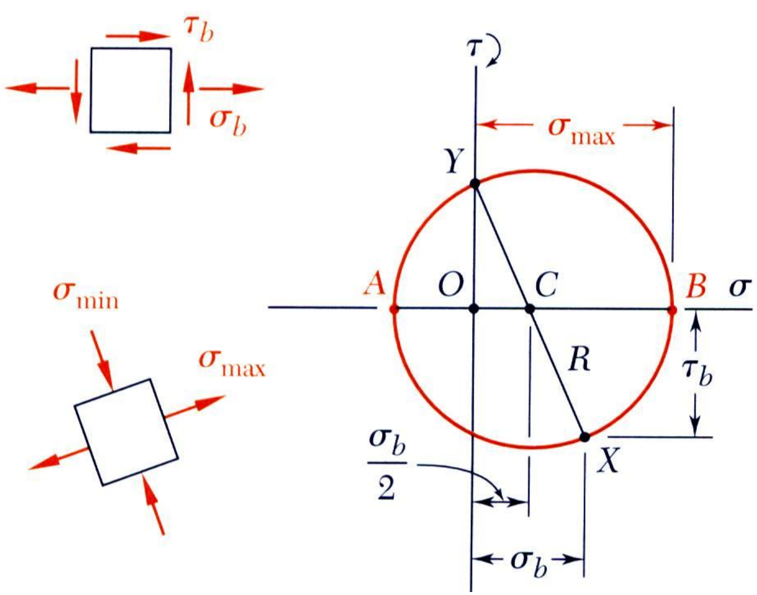
$$\sigma_b = \sigma_a \frac{y_b}{c} = (117.2\,\text{MPa})\frac{90.4\,\text{mm}}{103\,\text{mm}}$$
$$= 102.9\,\text{MPa}$$

Sample Problem 8.1



- Evaluate shear stress at flange-web junction.

$$Q = (204 \times 12.6)96.7 = 248.6 \times 10^3\,\text{mm}^3$$
$$= 248.6 \times 10^{-6}\,\text{m}^3$$
$$\tau_b = \frac{V_A Q}{It} = \frac{(160\,\text{kN})(248.6 \times 10^{-6}\,\text{m}^3)}{(52.7 \times 10^{-6}\,\text{m}^4)(0.0079\,\text{m})}$$
$$= 95.5\,\text{MPa}$$

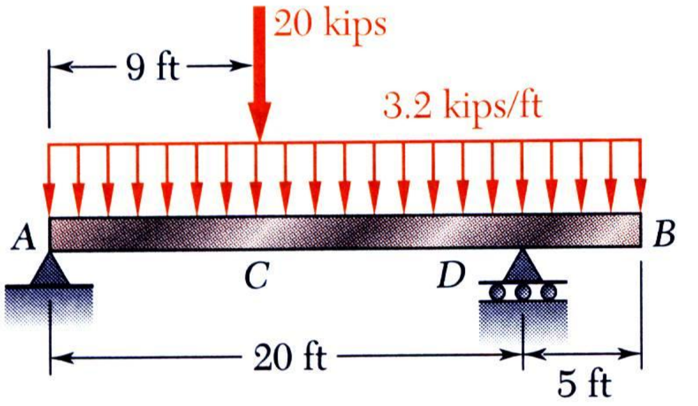


- Calculate the principal stress at flange-web junction

$$\sigma_{\max} = \tfrac{1}{2}\sigma_b + \sqrt{\left(\tfrac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$
$$= \frac{102.9}{2} + \sqrt{\left(\frac{102.9}{2}\right)^2 + (95.5)^2}$$
$$= 169.9\,\text{MPa} \quad (> 150\,\text{MPa})$$

Design specification is not satisfied.

Sample Problem 8.2

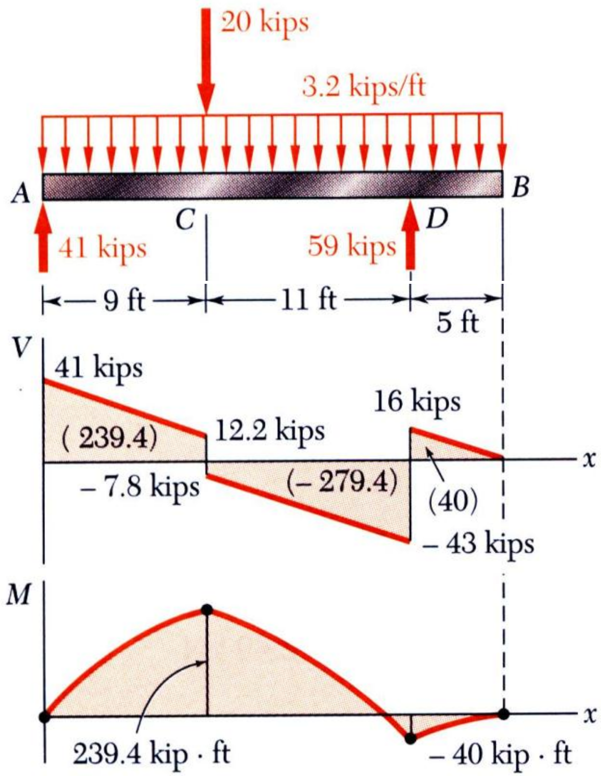


The overhanging beam supports a uniformly distributed load and a concentrated load. Knowing that for the grade of steel to used σ_{all} = 24 ksi and τ_{all} = 14.5 ksi, select the wide-flange beam which should be used.

SOLUTION:

- Determine reactions at *A* and *D*.
- Determine maximum shear and bending moment from shear and bending moment diagrams.
- Calculate required section modulus and select appropriate beam section.
- Find maximum normal stress.
- Find maximum shearing stress.

Sample Problem 8.2



Shape	S (in³)
W24 × 68	154
W21 × 62	127
W18 × 76	146
W16 × 77	134
W14 × 82	123
W12 × 96	131

SOLUTION:

- Determine reactions at A and D.

$$\sum M_A = 0 \quad \Rightarrow \quad R_D = 59\,\text{kips}$$
$$\sum M_D = 0 \quad \Rightarrow \quad R_A = 41\,\text{kips}$$

- Determine maximum shear and bending moment from shear and bending moment diagrams.

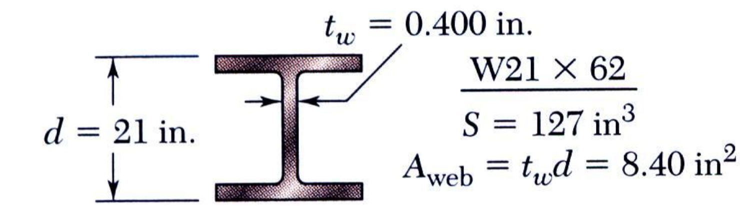
$$|M|_{\max} = 239.4\,\text{kip}\cdot\text{in} \quad \text{with} \quad V = 12.2\,\text{kips}$$
$$|V|_{\max} = 43\,\text{kips}$$

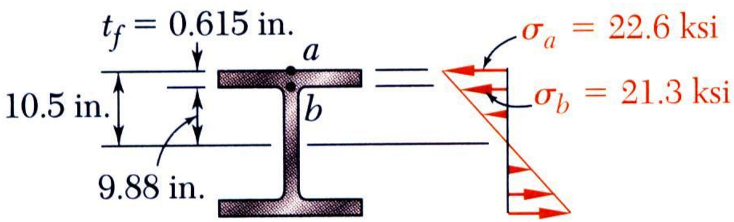
- Calculate required section modulus and select appropriate beam section.

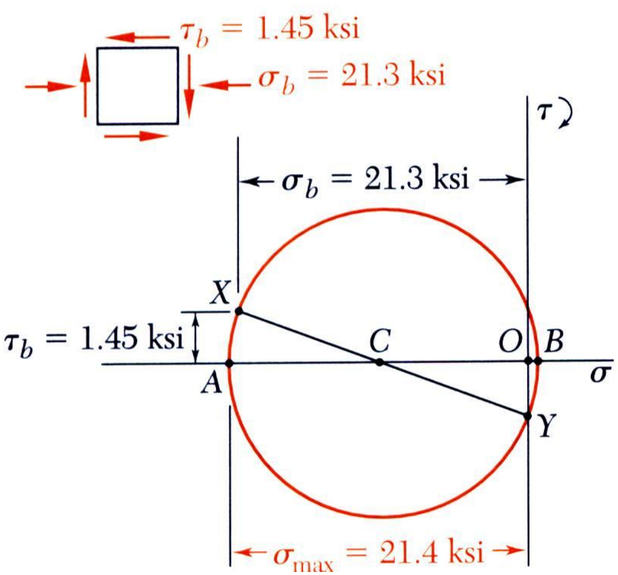
$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}} = \frac{24\,\text{kip}\cdot\text{in}}{24\,\text{ksi}} = 119.7\,\text{in}^3$$

select W21×62 beam section

Sample Problem 8.2







- Find maximum shearing stress.

  Assuming uniform shearing stress in web,

$$\tau_{max} = \frac{V_{max}}{A_{web}} = \frac{43\,\text{kips}}{8.40\,\text{in}^2} = 5.12\,\text{ksi} < 14.5\,\text{ksi}$$

- Find maximum normal stress.

$$\sigma_a = \frac{M_{max}}{S} = \frac{2873\,60\,\text{kip}\cdot\text{in}}{127\,\text{in}^3} = 22.6\,\text{ksi}$$

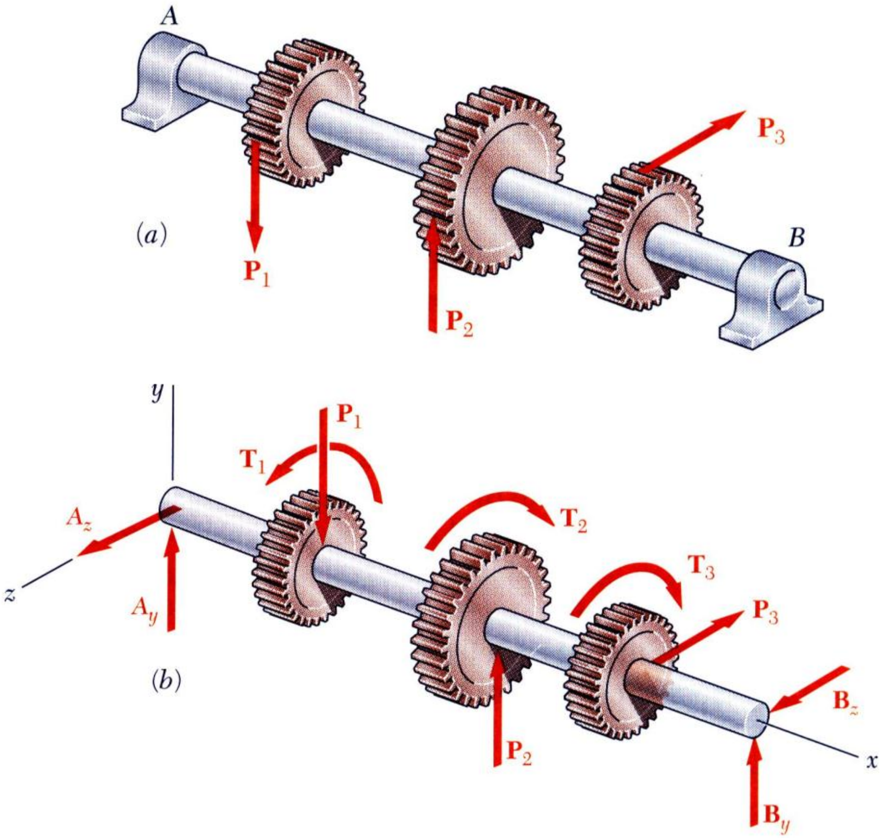
$$\sigma_b = \sigma_a \frac{y_b}{c} = (22.6\,\text{ksi})\frac{9.88}{10.5} = 21.3\,\text{ksi}$$

$$\tau_b = \frac{V}{A_{web}} = \frac{12.2\,\text{kips}}{8.40\,\text{in}^2} = 1.45\,\text{ksii}$$

$$\sigma_{max} = \frac{21.3\,\text{ksi}}{2} + \sqrt{\left(\frac{21.3\,\text{ksi}}{2}\right)^2 + (1.45\,\text{ksi})^2}$$

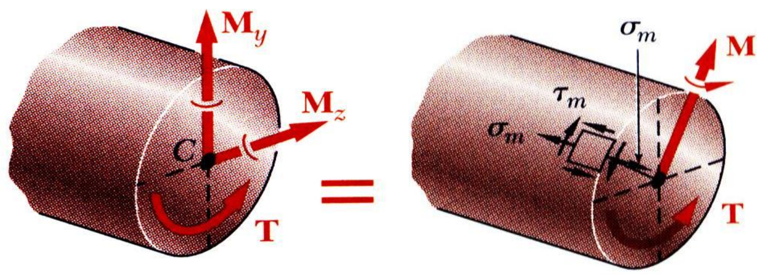
$$= 21.4\,\text{ksi} < 24\,\text{ksi}$$

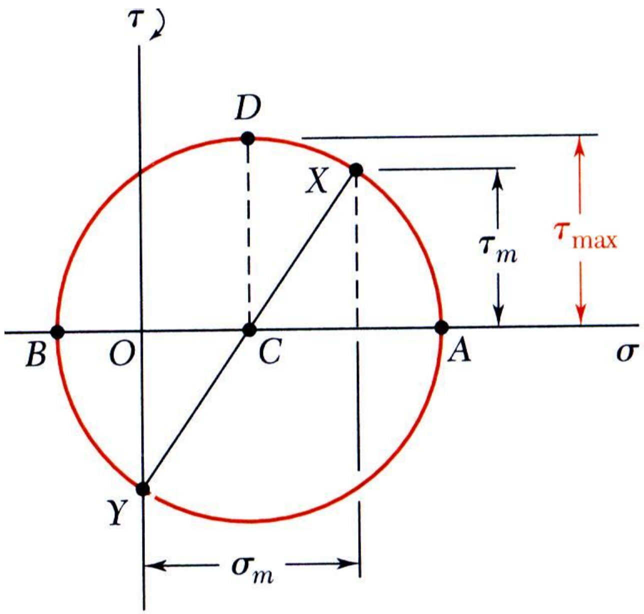
Design of a Transmission Shaft



- If power is transferred to and from the shaft by gears or sprocket wheels, the shaft is subjected to transverse loading as well as shear loading.
- Normal stresses due to transverse loads may be large and should be included in determination of maximum shearing stress.
- Shearing stresses due to transverse loads are usually small and contribution to maximum shear stress may be neglected.

Design of a Transmission Shaft





- At any section,

$$\sigma_m = \frac{Mc}{I} \quad \text{where} \quad M^2 = M_y^2 + M_z^2$$

$$\tau_m = \frac{Tc}{J}$$

- Maximum shearing stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + (\tau_m)^2} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

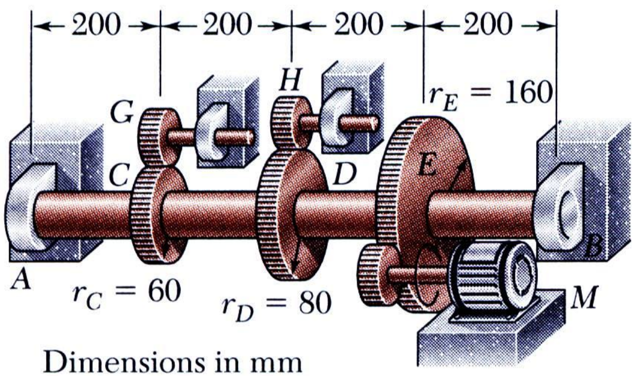
for a circular or annular cross-section, $2I = J$

$$\tau_{max} = \frac{c}{J}\sqrt{M^2 + T^2}$$

- Shaft section requirement,

$$\left(\frac{J}{c}\right)_{min} = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

Sample Problem 8.3

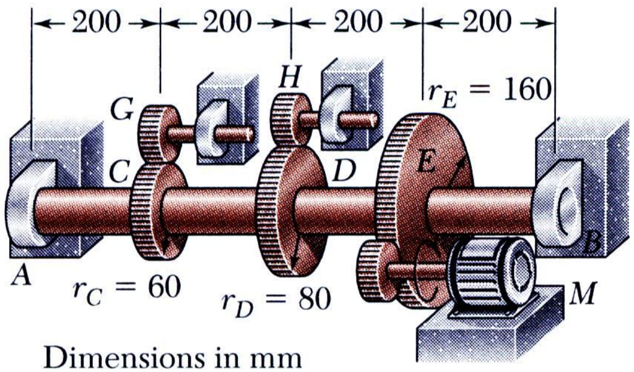


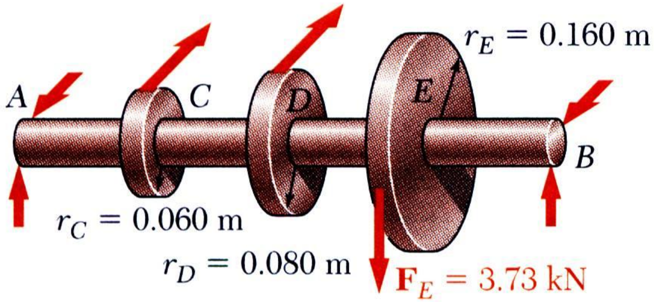
Solid shaft rotates at 480 rpm and transmits 30 kW from the motor to gears *G* and *H*; 20 kW is taken off at gear *G* and 10 kW at gear *H*. Knowing that σ_{all} = 50 MPa, determine the smallest permissible diameter for the shaft.

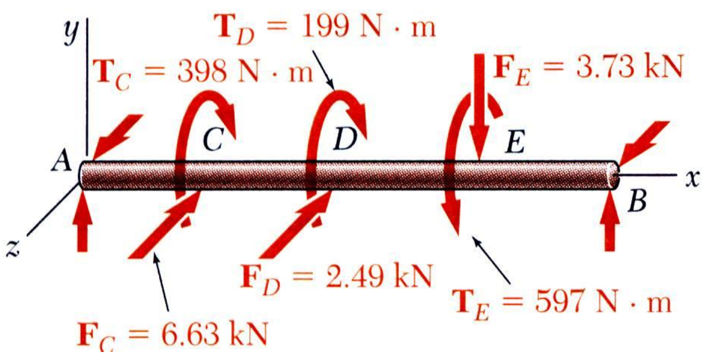
SOLUTION:

- Determine the gear torques and corresponding tangential forces.
- Find reactions at *A* and *B*.
- Identify critical shaft section from torque and bending moment diagrams.
- Calculate minimum allowable shaft diameter.

Sample Problem 8.3







SOLUTION:

- Determine the gear torques and corresponding tangential forces.

$$T_E = \frac{P}{2\pi f} = \frac{30\,\text{kW}}{2\pi(80\,\text{Hz})} = 597\,\text{N}\cdot\text{m}$$

$$F_E = \frac{T_E}{r_E} = \frac{597\,\text{N}\cdot\text{m}}{0.16\,\text{m}} = 3.73\,\text{kN}$$

$$T_C = \frac{20\,\text{kW}}{2\pi(80\,\text{Hz})} = 398\,\text{N}\cdot\text{m} \qquad F_C = 6.63\,\text{kN}$$

$$T_D = \frac{10\,\text{kW}}{2\pi(80\,\text{Hz})} = 199\,\text{N}\cdot\text{m} \qquad F_D = 2.49\,\text{kN}$$

- Find reactions at *A* and *B*.

$$A_y = 0.932\,\text{kN} \qquad A_z = 6.22\,\text{kN}$$

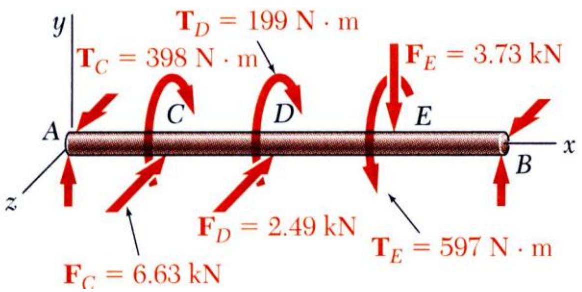
$$B_y = 2.80\,\text{kN} \qquad B_z = 2.90\,\text{kN}$$

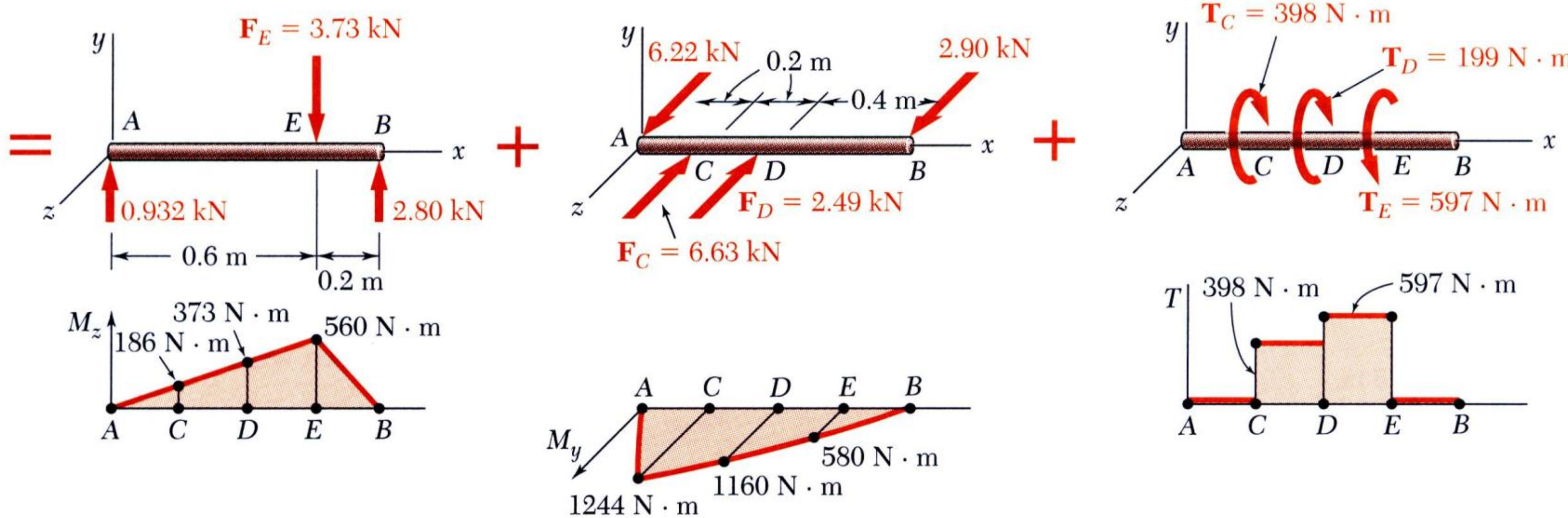
Sample Problem 8.3

- Identify critical shaft section from torque and bending moment diagrams.

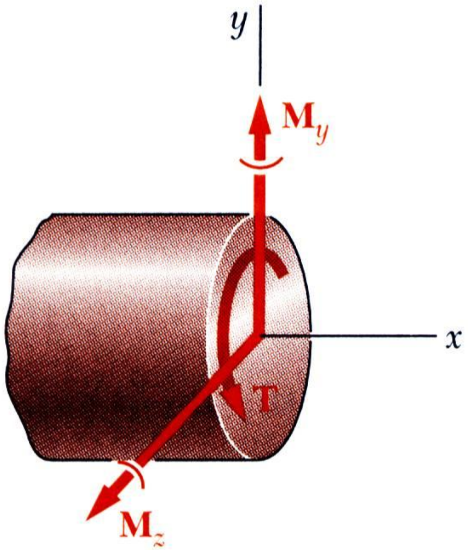
$$\left(\sqrt{M^2+T^2}\right)_{max} = \sqrt{\left(1160^2+373^2\right)+597^2}$$

$$= 1357\,\mathrm{N\cdot m}$$





Sample Problem 8.3



- Calculate minimum allowable shaft diameter.

$$\frac{J}{c} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{1357\ \text{N}\cdot\text{m}}{50\,\text{MPa}} = 27.14\times10^{-6}\,\text{m}^3$$

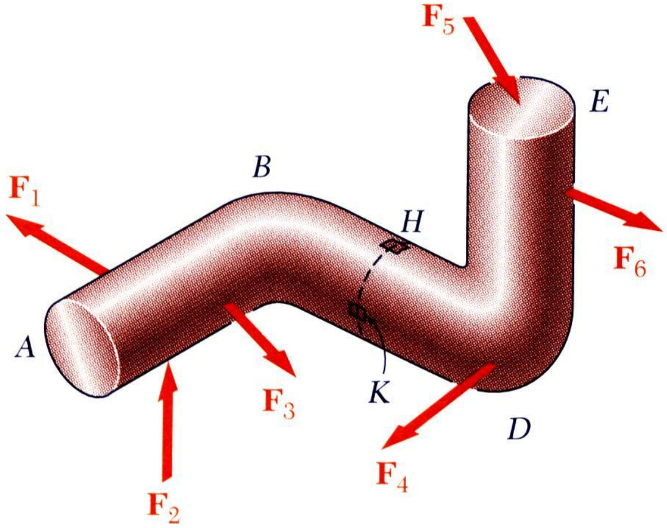
For a solid circular shaft,

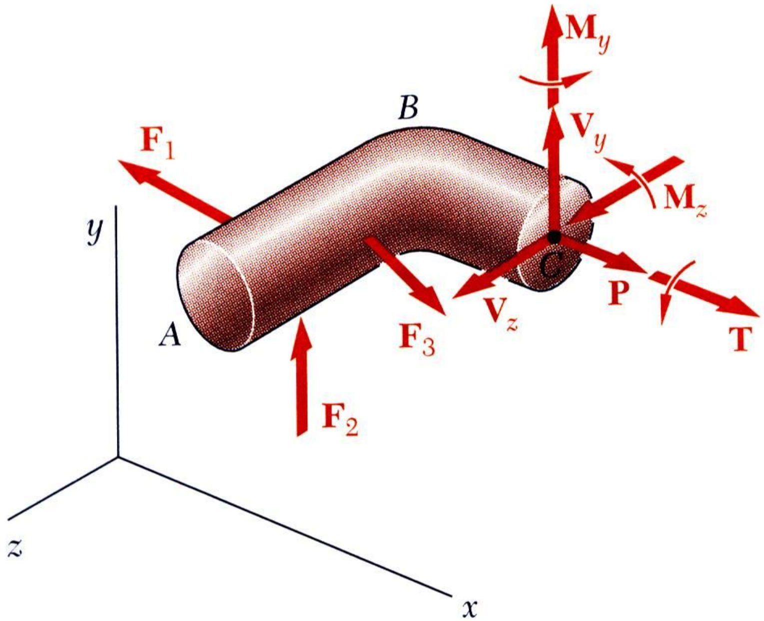
$$\frac{J}{c} = \frac{\pi}{2}c^3 = 27.14\times10^{-6}\,\text{m}^3$$

$$c = 0.02585\,\text{m} = 25.85\,\text{m}$$

$$\boxed{d = 2c = 51.7\ \text{mm}}$$

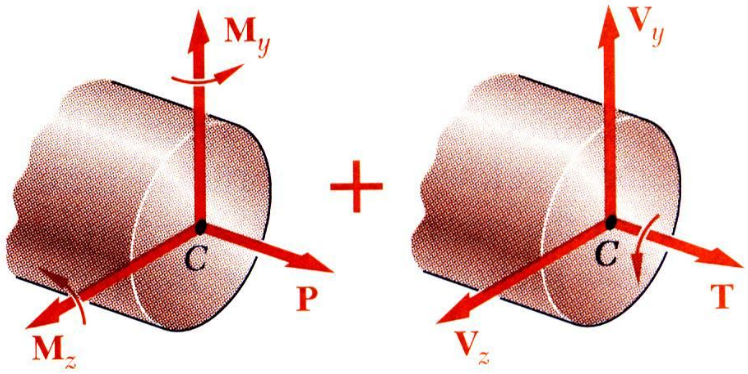
Stresses Under Combined Loadings





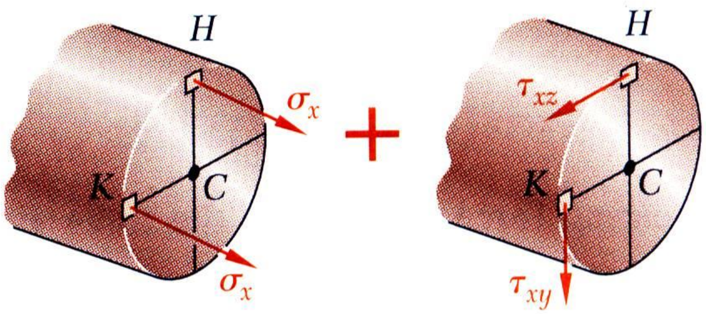
- Wish to determine stresses in slender structural members subjected to arbitrary loadings.
- Pass section through points of interest. Determine force-couple system at centroid of section required to maintain equilibrium.
- System of internal forces consist of three force components and three couple vectors.
- Determine stress distribution by applying the superposition principle.

Stresses Under Combined Loadings

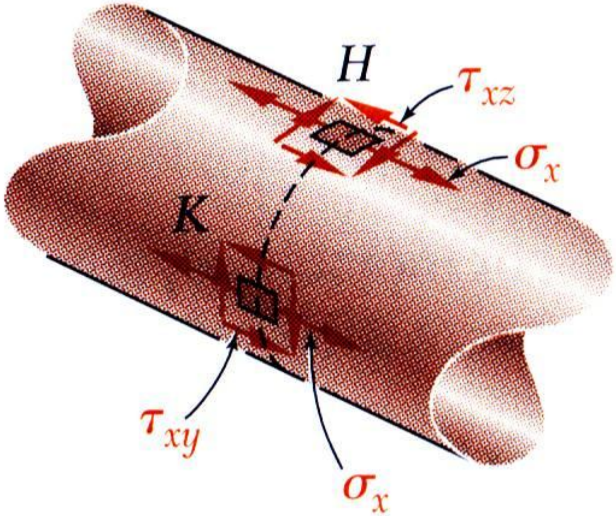


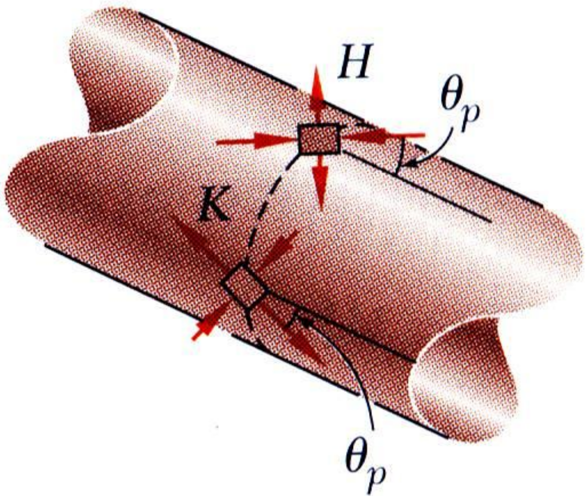
- Axial force and in-plane couple vectors contribute to normal stress distribution in the section.

- Shear force components and twisting couple contribute to shearing stress distribution in the section.



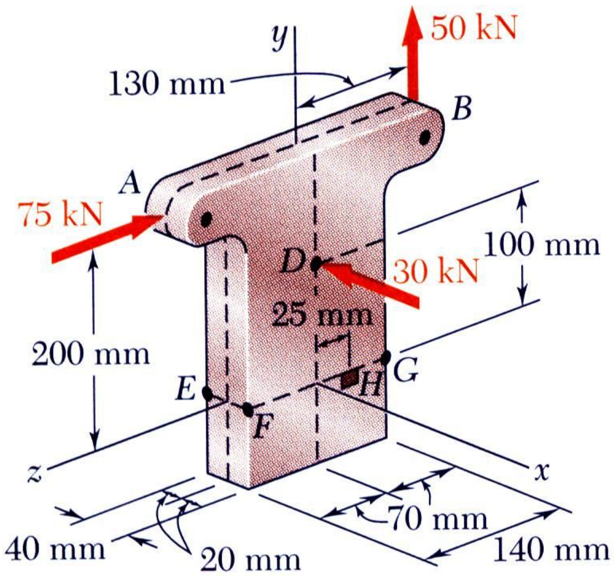
Stresses Under Combined Loadings





- Normal and shearing stresses are used to determine principal stresses, maximum shearing stress and orientation of principal planes.
- Analysis is valid only to extent that conditions of applicability of superposition principle and Saint-Venant's principle are met.

Sample Problem 8.5

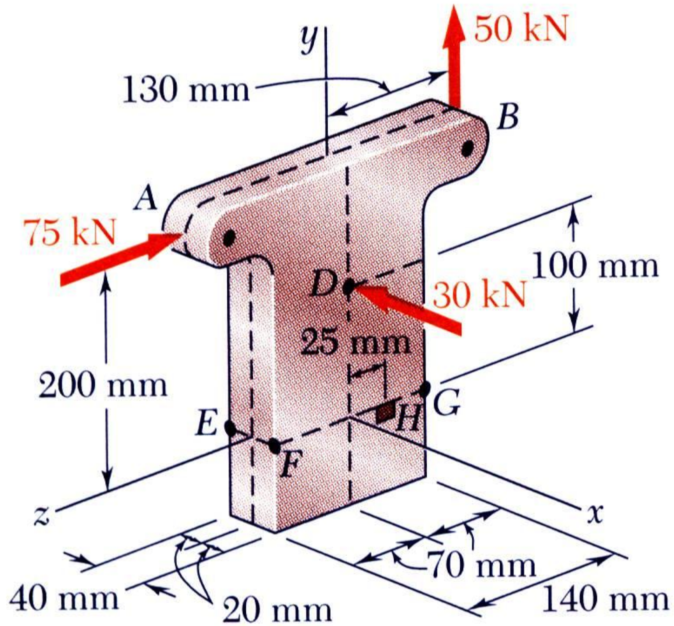


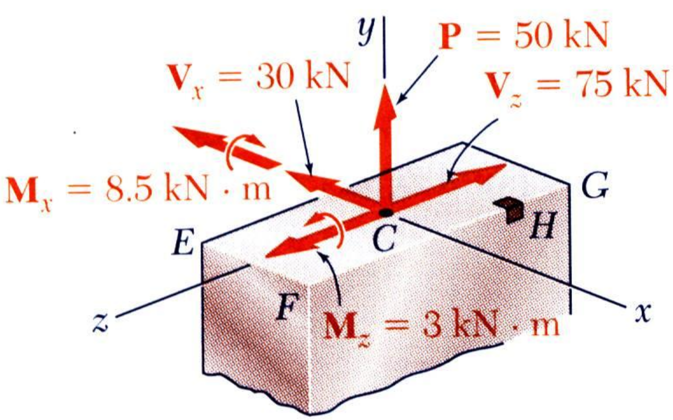
Three forces are applied to a short steel post as shown. Determine the principle stresses, principal planes and maximum shearing stress at point *H*.

SOLUTION:

- Determine internal forces in Section *EFG*.
- Evaluate normal stress at *H*.
- Evaluate shearing stress at *H*.
- Calculate principal stresses and maximum shearing stress. Determine principal planes.

Sample Problem 8.5





SOLUTION:

- Determine internal forces in Section *EFG*.

$$V_x = -30\,\text{kN} \quad P = 50\,\text{kN} \quad V_z = -75\,\text{kN}$$

$$M_x = (50\,\text{kN})(0.130\,\text{m}) - (75\,\text{kN})(0.200\,\text{m})$$

$$= -8.5\,\text{kN}\cdot\text{m}$$

$$M_y = 0 \quad M_z = (30\,\text{kN})(0.100\,\text{m}) = 3\,\text{kN}\cdot\text{m}$$

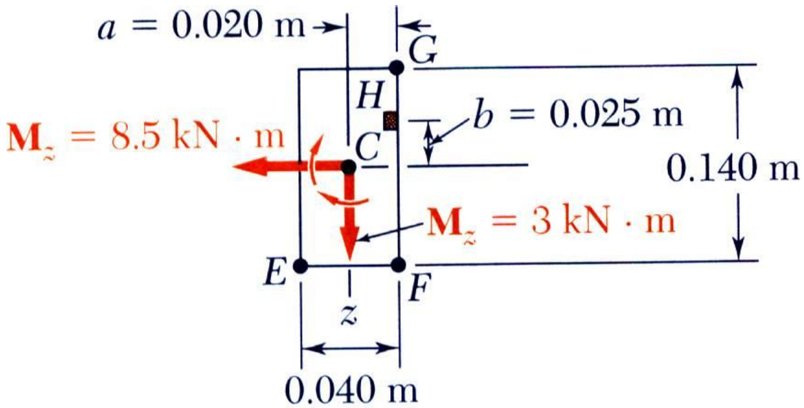
Note: Section properties,

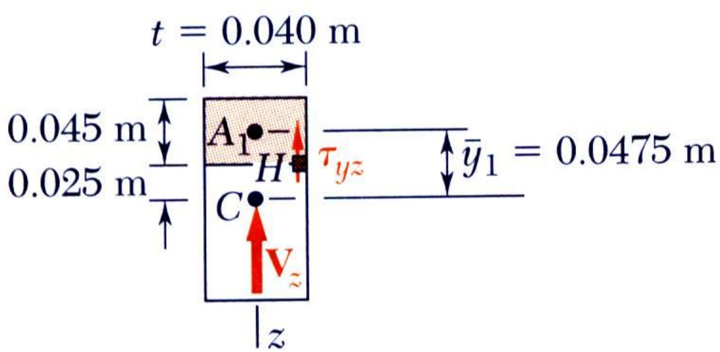
$$A = (0.040\,\text{m})(0.140\,\text{m}) = 5.6\times10^{-3}\,\text{m}^2$$

$$I_x = \tfrac{1}{12}(0.040\,\text{m})(0.140\,\text{m})^3 = 9.15\times10^{-6}\,\text{m}^4$$

$$I_z = \tfrac{1}{12}(0.140\,\text{m})(0.040\,\text{m})^3 = 0.747\times10^{-6}\,\text{m}^4$$

Sample Problem 8.5





- Evaluate normal stress at *H*.

$$\sigma_y = +\frac{P}{A} + \frac{|M_z|a}{I_z} - \frac{|M_x|b}{I_x}$$

$$= \frac{50\,\text{kN}}{5.6\times10^{-3}\,\text{m}^2} + \frac{(3\,\text{kN}\cdot\text{m})(0.020\,\text{m})}{0.747\times10^{-6}\,\text{m}^4} - \frac{(8.5\,\text{kN}\cdot\text{m})(0.025\,\text{m})}{9.15\times10^{-6}\,\text{m}^4}$$

$$= (8.93 + 80.3 - 23.2)\,\text{MPa} = 66.0\,\text{MPa}$$

- Evaluate shearing stress at *H*.

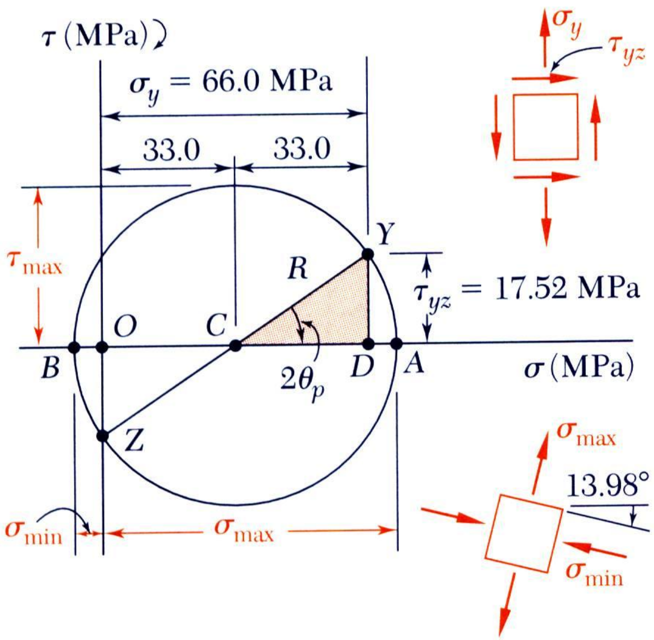
$$Q = A_1\bar{y}_1 = [(0.040\,\text{m})(0.045\,\text{m})](0.0475\,\text{m})$$

$$= 85.5\times10^{-6}\,\text{m}^3$$

$$\tau_{yz} = \frac{V_z Q}{I_x t} = \frac{(75\,\text{kN})(85.5\times10^{-6}\,\text{m}^3)}{(9.15\times10^{-6}\,\text{m}^4)(0.040\,\text{m})}$$

$$= 17.52\,\text{MPa}$$

Sample Problem 8.5



- Calculate principal stresses and maximum shearing stress.
  Determine principal planes.

$$\tau_{\max} = R = \sqrt{33.0^2 + 17.52^2} = 37.4\,\text{MPa}$$

$$\sigma_{\max} = OC + R = 33.0 + 37.4 = 70.4\,\text{MPa}$$

$$\sigma_{\min} = OC - R = 33.0 - 37.4 = -7.4\,\text{MPa}$$

$$\tan 2\theta_p = \frac{CY}{CD} = \frac{17.52}{33.0} \qquad 2\theta_p = 27.96^\circ$$

$$\theta_p = 13.98^\circ$$

$$\tau_{\max} = 37.4\,\text{MPa}$$

$$\sigma_{\max} = 70.4\,\text{MPa}$$

$$\sigma_{\min} = -7.4\,\text{MPa}$$

$$\theta_p = 13.98^\circ$$