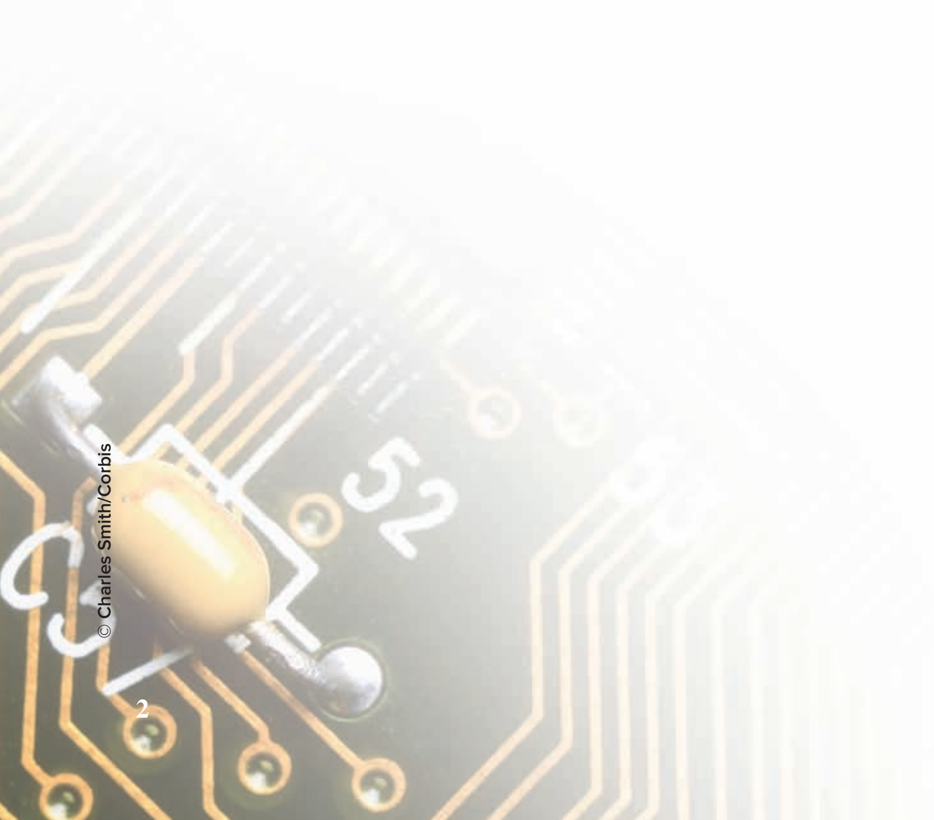
chapter 1

Introduction

This important chapter serves as a framework for the rest of the textbook. The topics in this chapter include formulas, voltage sources, current sources, two circuit theorems, and troubleshooting. Although some of the discussion will be review, you will find new ideas, such as circuit approximations, that can make it easier for you to understand semiconductor devices.



Chapter Outline

Objectives

After studying this chapter, you should be able to:

- Name the three types of formulas and explain why each is true.
- Explain why approximations are often used instead of exact formulas.
- Define an ideal voltage source and an ideal current source.
- Describe how to recognize a stiff voltage source and a stiff current source.
- State Thevenin's theorem and apply it to a circuit.
- State Norton's theorem and apply it to a circuit.
- List two facts about an open device and two facts about a shorted device.

Vocabulary

cold-solder joint
definition
derivation
duality principle
formula
ideal (first) approximation
law
Norton current
Norton resistance
open device
second approximation
shorted device
solder bridge
stiff current source
stiff voltage source
theorem
Thevenin resistance
Thevenin voltage
third approximation
troubleshooting

1-1 The Three Kinds of Formulas

A **formula** is a rule that relates quantities. The rule may be an equation, an inequality, or other mathematical description. You will see many formulas in this book. Unless you know why each one is true, you may become confused as they accumulate. Fortunately, there are only three ways formulas can come into existence. Knowing what they are will make your study of electronics more logical and satisfying.

GOOD TO KNOW

For all practical purposes, a formula is like a set of instructions written in mathematical shorthand. A formula describes how to go about calculating a particular quantity or parameter.

The Definition

When you study electricity and electronics, you have to memorize new words like *current, voltage,* and *resistance.* However, a verbal explanation of these words is not enough. Why? Because your idea of current must be mathematically identical to everyone else's. The only way to get this identity is with a **definition,** a formula invented for a new concept.

Here is an example of a definition. In your earlier course work, you learned that capacitance equals the charge on one plate divided by the voltage between plates. The formula looks like this:

$$C = \frac{Q}{V}$$

This formula is a definition. It tells you what capacitance C is and how to calculate it. Historically, some researcher made up this definition and it became widely accepted.

Here is an example of how to create a new definition out of thin air. Suppose we are doing research on reading skills and need some way to measure reading speed. Out of the blue, we might decide to define *reading speed* as the number of words read in a minute. If the number of words is W and the number of minutes is M, we could make up a formula like this:

$$S = \frac{W}{M}$$

In this equation, S is the speed measured in words per minute.

To be fancy, we could use Greek letters: ω for words, μ for minutes, and σ for speed. Our definition would then look like this:

$$\sigma = \frac{\omega}{\mu}$$

This equation still translates to speed equals words divided by minutes. When you see an equation like this and know that it is a definition, it is no longer as impressive and mysterious as it initially appears to be.

In summary, *definitions are formulas that a researcher creates.* They are based on scientific observation and form the basis for the study of electronics. They are simply accepted as facts. It's done all the time in science. A definition is true in the same sense that a word is true. Each represents something we want to talk about. When you know which formulas are definitions, electronics is easier to understand. Because definitions are starting points, all you need to do is understand and memorize them.

The Law

A **law** is different. It summarizes a relationship that already exists in nature. Here is an example of a law:

$$f = K\frac{Q_1Q_2}{d^2}$$

where f = force
K = a constant of proportionality, $9(10^9)$
Q_1 = first charge
Q_2 = second charge
d = distance between charges

This is Coulomb's law. It says that the force of attraction or repulsion between two charges is directly proportional to the charges and inversely proportional to the square of the distance between them.

This is an important equation, for it is the foundation of electricity. But where does it come from? And why is it true? To begin with, all the variables in this law existed before its discovery. Through experiments, Coulomb was able to prove that the force was directly proportional to each charge and inversely proportional to the square of the distance between the charges. Coulomb's law is an example of a relationship that exists in nature. Although earlier researchers could measure f, Q_1, Q_2, and d, Coulomb discovered the law relating the quantities and wrote a formula for it.

Before discovering a law, someone may have a hunch that such a relationship exists. After a number of experiments, the researcher writes a formula that summarizes the discovery. When enough people confirm the discovery through experiments, the formula becomes a law. *A law is true because you can verify it with an experiment.*

The Derivation

Given an equation like this:

$$y = 3x$$

we can add 5 to both sides to get:

$$y + 5 = 3x + 5$$

The new equation is true because both sides are still equal. There are many other operations like subtraction, multiplication, division, factoring, and substitution that preserve the equality of both sides of the equation. For this reason, we can derive many new formulas using mathematics.

A **derivation** *is a formula that we can get from other formulas.* This means that we start with one or more formulas and, using mathematics, arrive at a new formula not in our original set of formulas. A derivation is true because mathematics preserves the equality of both sides of every equation between the starting formula and the derived formula.

For instance, Ohm was experimenting with conductors. He discovered that the ratio of voltage to current was a constant. He named this constant *resistance* and wrote the following formula for it:

$$R = \frac{V}{I}$$

This is the original form of Ohm's law. By rearranging it, we can get:

$$I = \frac{V}{R}$$

This is a derivation. It is the original form of Ohm's law converted to another equation.

Here is another example. The definition for capacitance is:

$$C = \frac{Q}{V}$$

We can multiply both sides by V to get the following new equation:

$$Q = CV$$

This is a derivation. It says that the charge on a capacitor equals its capacitance times the voltage across it.

What to Remember

Why is a formula true? There are three possible answers. To build your understanding of electronics on solid ground, classify each new formula in one of these three categories:

Definition: A formula invented for a new concept
Law: A formula for a relationship in nature
Derivation: A formula produced with mathematics

1-2 Approximations

We use approximations all the time in everyday life. If someone asks you how old you are, you might answer 21 (ideal). Or you might say 21 going on 22 (second approximation). Or, maybe, 21 years and 9 months (third approximation). Or, if you want to be more accurate, 21 years, 9 months, 2 days, 6 hours, 23 minutes, and 42 seconds (exact).

The foregoing illustrates different levels of approximation: an ideal approximation, a second approximation, a third approximation, and an exact answer. The approximation to use will depend on the situation. The same is true in electronics work. In circuit analysis, we need to choose an approximation that fits the situation.

The Ideal Approximation

Did you know that 1 foot of AWG 22 wire that is 1 inch from a chassis has a resistance of 0.016 Ω, an inductance of 0.24 μH, and a capacitance of 3.3 pF? If we had to include the effects of resistance, inductance, and capacitance in every calculation for current, we would spend too much time on calculations. This is why everybody ignores the resistance, inductance, and capacitance of connecting wires in most situations.

The **ideal approximation,** sometimes called the **first approximation,** is the simplest equivalent circuit for a device. For instance, the ideal approximation of a piece of wire is a conductor of zero resistance. This ideal approximation is adequate for everyday electronics work.

The exception occurs at higher frequencies, where you have to consider the inductance and capacitance of the wire. Suppose 1 inch of wire has an inductance of 0.24 μH and a capacitance of 3.3 pF. At 10 MHz, the inductive reactance is 15.1 Ω, and the capacitive reactance is 4.82 kΩ. As you see, a circuit designer can no longer idealize a piece of wire. Depending on the rest of the circuit, the inductance and capacitive reactances of a connecting wire may be important.

As a guideline, we can idealize a piece of wire at frequencies under 1 MHz. This is usually a safe rule of thumb. But it does not mean that you can be careless about wiring. In general, keep connecting wires as short as possible, because at some point on the frequency scale, those wires will begin to degrade circuit performance.

When you are troubleshooting, the ideal approximation is usually adequate because you are looking for large deviations from normal voltages and currents. In this book, we will idealize semiconductor devices by reducing them to simple equivalent circuits. With ideal approximations, it is easier to analyze and understand how semiconductor circuits work.

The Second Approximation

The ideal approximation of a flashlight battery is a voltage source of 1.5 V. The **second approximation** adds one or more components to the ideal approximation. For instance, the second approximation of a flashlight battery is a voltage source of 1.5 V and a series resistance of 1 Ω. This series resistance is called the *source* or *internal* resistance of the battery. If the load resistance is less than 10 Ω, the load voltage will be noticeably less than 1.5 V because of the voltage drop across the source resistance. In this case, accurate calculations must include the source resistance.

The Third Approximation and Beyond

The **third approximation** includes another component in the equivalent circuit of the device. An example of the third approximation will be examined when we discuss semiconductor diodes.

Even higher approximations are possible with many components in the equivalent circuit of a device. Hand calculations using these higher approximations can become difficult and time consuming. Because of this, computers using circuit simulation software are often used. For instance, Multisim by National Instruments (NI) and PSpice are commercially available computer programs that use higher approximations to analyze and simulate semiconductor circuits. Many of the circuits and examples in this book can be analyzed and demonstrated using this type of software.

Conclusion

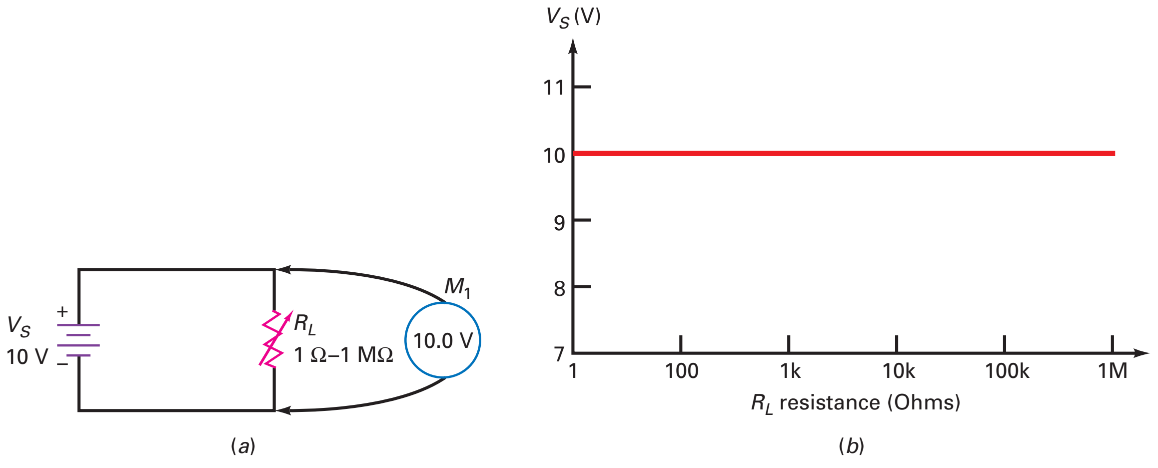
Which approximation to use depends on what you are trying to do. If you are troubleshooting, the ideal approximation is usually adequate. For many situations, the second approximation is the best choice because it is easy to use and does not require a computer. For higher approximations, you should use a computer and a program like Multisim. A Multisim tutorial can be found on the Instructor Resources section of *Connect for Electronic Principles*.

1-3 Voltage Sources

An *ideal dc voltage source* produces a load voltage that is constant. The simplest example of an ideal dc voltage source is a perfect battery, one whose internal resistance is zero. Figure 1-1*a* shows an ideal voltage source connected to a variable load resistance of 1 Ω to 10 MΩ. The voltmeter reads 10 V, exactly the same as the source voltage.

Figure 1-1*b* shows a graph of load voltage versus load resistance. As you can see, the load voltage remains fixed at 10 V when the load resistance changes from 1 Ω to 1 MΩ. In other words, an ideal dc voltage source produces a constant load voltage, regardless of how small or large the load resistance is. With an ideal voltage source, only the load current changes when the load resistance changes.

Figure 1-1 (*a*) Ideal voltage source and variable load resistance; (*b*) load voltage is constant for all load resistances.



Second Approximation

An ideal voltage source is a theoretical device; it cannot exist in nature. Why? When the load resistance approaches zero, the load current approaches infinity. No real voltage source can produce infinite current because a real voltage source always has some internal resistance. The second approximation of a dc voltage source includes this internal resistance.

Figure 1-2*a* illustrates the idea. A source resistance R_S of 1 Ω is now in series with the ideal battery. The voltmeter reads 5 V when R_L is 1 Ω. Why? Because the load current is 10 V divided by 2 Ω, or 5 A. When 5 A flows through the source resistance of 1 Ω, it produces an internal voltage drop of 5 V. This is why the load voltage is only half of the ideal value, with the other half being dropped across the internal resistance.

Figure 1-2*b* shows the graph of load voltage versus load resistance. In this case, the load voltage does not come close to the ideal value until the load resistance is much greater than the source resistance. But what does *much greater* mean? In other words, when can we ignore the source resistance?

Figure 1-2 (*a*) Second approximation includes source resistance; (*b*) load voltage is constant for large load resistances.

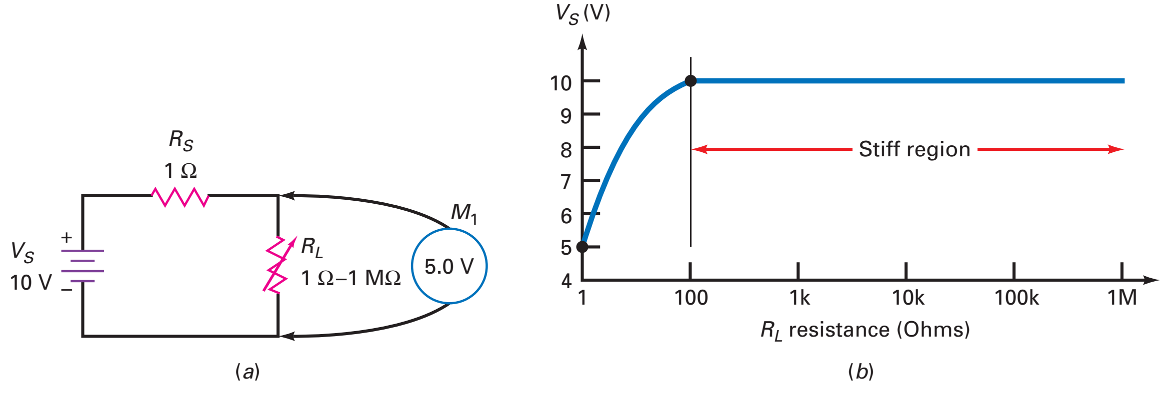
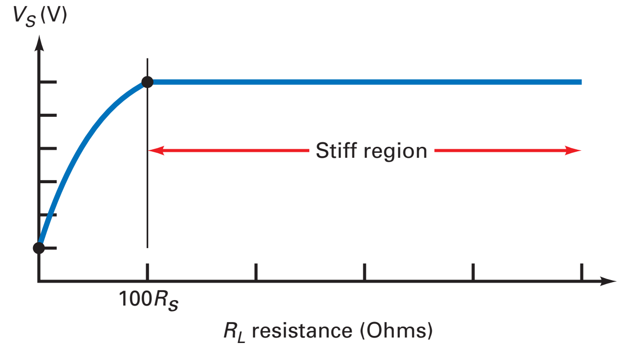


Figure 1-3 Stiff region occurs when load resistance is large enough.



Stiff Voltage Source

Now is the time when a new definition can be useful. So, let us invent one. We can ignore the source resistance when it is at least 100 times smaller than the load resistance. Any source that satisfies this condition is a **stiff voltage source.** As a definition,

Stiff voltage source: $R_S < 0.01R_L$ **(1-1)**

This formula defines what we mean by a *stiff voltage source*. The boundary of the inequality (where < is changed to =) gives us the following equation:

$$R_S = 0.01R_L$$

Solving for load resistance gives the minimum load resistance we can use and still have a stiff source:

$$R_{L(\min)} = 100R_S \quad \textbf{(1-2)}$$

In words, the minimum load resistance equals 100 times the source resistance.

Equation (1-2) is a derivation. We started with the definition of a stiff voltage source and rearranged it to get the minimum load resistance permitted with a stiff voltage source. As long as the load resistance is greater than $100R_S$, the voltage source is stiff. When the load resistance equals this worst-case value, the calculation error from ignoring the source resistance is 1 percent, small enough to ignore in a second approximation.

Figure 1-3 visually summarizes a stiff voltage source. The load resistance has to be greater than $100R_S$ for the voltage source to be stiff.

GOOD TO KNOW

A well-regulated power supply is a good example of a stiff voltage source.

Example 1-1

The definition of a stiff voltage source applies to ac sources as well as to dc sources. Suppose an ac voltage source has a source resistance of 50 Ω. For what load resistance is the source stiff?

SOLUTION Multiply by 100 to get the minimum load resistance:

$$R_L = 100R_S = 100(50\ \Omega) = 5\ \text{k}\Omega$$

As long as the load resistance is greater than 5 kΩ, the ac voltage source is stiff and we can ignore the internal resistance of the source.

A final point. Using the second approximation for an ac voltage source is valid only at low frequencies. *At high frequencies, additional factors such as lead inductance and stray capacitance come into play*. We will deal with these high-frequency effects in a later chapter.

PRACTICE PROBLEM 1-1 If the ac source resistance in Example 1-1 is 600 Ω, for what load resistance is the source stiff?

1-4 Current Sources

A dc voltage source produces a constant load voltage for different load resistances. A *dc current source* is different. It produces a constant load current for different load resistances. An example of a dc current source is a battery with a large source resistance (Fig. 1-4*a*). In this circuit, the source resistance is 1 MΩ and the load current is:

$$I_L = \frac{V_S}{R_S + R_L}$$

When R_L is 1 Ω in Fig. 1-4*a*, the load current is:

$$I_L = \frac{10\text{ V}}{1\text{ M}\Omega + 1\ \Omega} = 10\ \mu\text{A}$$

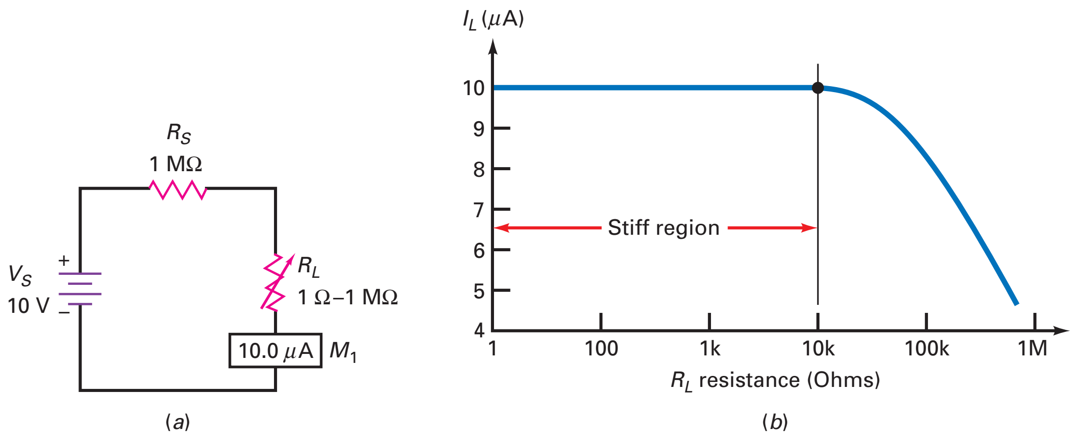
In this calculation, the small load resistance has an insignificant effect on the load current.

Figure 1-4*b* shows the effect of varying the load resistance from 1 Ω to 1 MΩ. In this case, the load current remains constant at 10 μA over a large range. It is only when the load resistance is greater than 10 kΩ that a noticeable drop-off occurs in load current.

GOOD TO KNOW

At the output terminals of a constant current source, the load voltage V_L increases in direct proportion to the load resistance.

Figure 1-4 (*a*) Simulated current source with a dc voltage source and a large resistance; (*b*) load current is constant for small load resistances.



Stiff Current Source

Here is another definition that will be useful, especially with semiconductor circuits. We will ignore the source resistance of a current source when it is at least 100 times larger than the load resistance. Any source that satisfies this condition is a **stiff current source.** As a definition:

$$\textbf{Stiff current source: } R_S > 100R_L \qquad \textbf{(1-3)}$$

The upper boundary is the worst case. At this point:

$$R_S = 100R_L$$

Solving for load resistance gives the maximum load resistance we can use and still have a stiff current source:

$$R_{L(\max)} = 0.01R_S \qquad \textbf{(1-4)}$$

In words: The maximum load resistance equals 1⁄100 of the source resistance.

Equation (1-4) is a derivation because we started with the definition of a stiff current source and rearranged it to get the maximum load resistance. When the load resistance equals this worst-case value, the calculation error is 1 percent, small enough to ignore in a second approximation.

Figure 1-5 shows the stiff region. As long as the load resistance is less than $0.01R_S$, the current source is stiff.

Schematic Symbol

Figure 1-6*a* is the schematic symbol of an ideal current source, one whose source resistance is infinite. This ideal approximation cannot exist in nature, but it can exist mathematically. Therefore, we can use the ideal current source for fast circuit analysis, as in troubleshooting.

Figure 1-6*a* is a visual definition: It is the symbol for a current source. When you see this symbol, it means that the device produces a constant current I_S. It may help to think of a current source as a pump that pushes out a fixed number of coulombs per second. This is why you will hear expressions like "The current source pumps 5 mA through a load resistance of 1 kΩ."

Figure 1-6*b* shows the second approximation. The internal resistance is in parallel with the ideal current source, not in series as it was with an ideal voltage source. Later in this chapter we will discuss Norton's theorem. You will then see why the internal resistance must be in parallel with the current source. Summary Table 1-1 will help you understand the differences between a voltage source and a current source.

Figure 1-5 Stiff region occurs when load resistance is small enough.

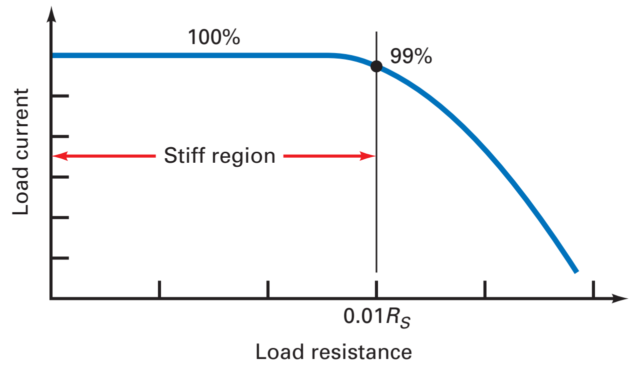
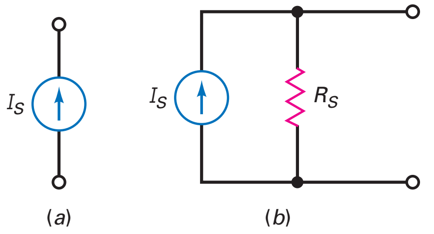


Figure 1-6 (*a*) Schematic symbol of a current source; (*b*) second approximation of a current source.



Summary Table 1-1	Properties of Voltage and Current Sources	
Quantity	**Voltage Source**	**Current Source**
R_S	Typically low	Typically high
R_L	Greater than $100R_S$	Less than $0.01R_S$
V_L	Constant	Depends on R_L
I_L	Depends on R_L	Constant

Example 1-2

A current source of 2 mA has an internal resistance of 10 MΩ. Over what range of load resistance is the current source stiff?

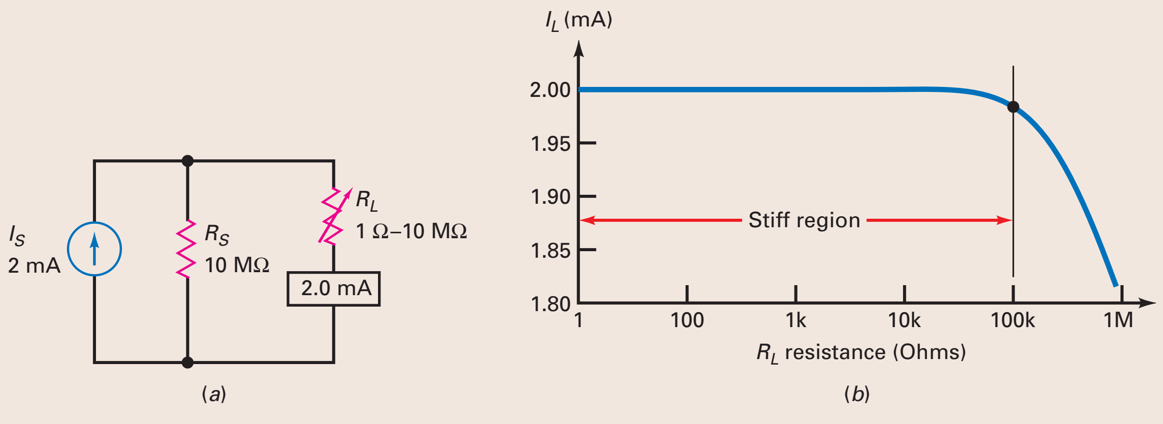
SOLUTION Since this is a current source, the load resistance has to be small compared to the source resistance. With the 100:1 rule, the maximum load resistance is:

$$R_{L(\max)} = 0.01(10\ \text{M}\Omega) = 100\ \text{k}\Omega$$

The stiff range for the current source is a load resistance from 0 to 100 kΩ.

Figure 1-7 summarizes the solution. In Fig. 1-7*a*, a current source of 2 mA is in parallel with 10 MΩ and a variable resistor set to 1 Ω. The ammeter measures a load current of 2 mA. When the load resistance changes from 1 Ω to 1 MΩ, as shown in Fig. 1-7*b*, the source remains stiff up to 100 kΩ. At this point, the load current is down about 1 percent from the ideal value. Stated another way, 99 percent of the source current passes through the load resistance. The other 1 percent passes through the source resistance. As the load resistance continues to increase, load current continues to decrease.

Figure 1-7 Solution.



PRACTICE PROBLEM 1-2 What is the load voltage in Fig. 1-7*a* when the load resistance equals 10 kΩ?

Application Example 1-3

When you analyze transistor circuits, you will visualize a transistor as a current source. In a well-designed circuit, the transistor will act like a stiff current source, so you can ignore its internal resistance. Then you can calculate the load voltage. For instance, if a transistor is pumping 2 mA through a load resistance of 10 kΩ, the load voltage is 20 V.

1-5 Thevenin's Theorem

Every once in a while, somebody makes a big breakthrough in engineering and carries all of us to a new high. A French engineer, M. L. Thevenin, made one of these quantum leaps when he derived the circuit theorem named after him: Thevenin's theorem.

Definition of Thevenin Voltage and Resistance

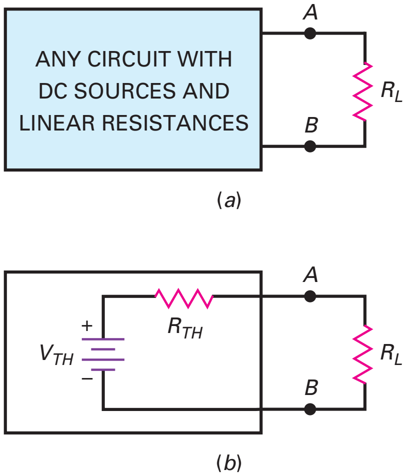
A **theorem** is a statement that we can prove mathematically. Because of this, it is not a definition or a law. So, we classify it as a derivation. Recall the following ideas about Thevenin's theorem from earlier courses. In Fig. 1-8*a*, the **Thevenin voltage** V_{TH} is defined as the voltage across the load terminals when the load resistor is open. Because of this, the Thevenin voltage is sometimes called the *open-circuit voltage*. As a definition:

$$\textbf{Thevenin voltage: } V_{TH} = V_{OC} \quad \textbf{(1-5)}$$

The **Thevenin resistance** is defined as the resistance that an ohmmeter measures across the load terminals of Fig. 1-8*a* when all sources are reduced to zero and the load resistor is open. As a definition:

$$\textbf{Thevenin resistance: } R_{TH} = R_{OC} \quad \textbf{(1-6)}$$

Figure 1-8 (*a*) Black box has a linear circuit inside of it; (*b*) Thevenin circuit.



With these two definitions, Thevenin was able to derive the famous theorem named after him.

There is a subtle point in finding the Thevenin resistance. Reducing a source to zero has different meanings for voltage and current sources. When you reduce a voltage source to zero, you are effectively replacing it with a short because that's the only way to guarantee zero voltage when a current flows through the voltage source. When you reduce a current source to zero, you are effectively replacing it with an open because that's the only way you can guarantee zero current when there is a voltage across the current source. To summarize:

To zero a voltage source, replace it with a short.
To zero a current source, replace it with an open.

The Derivation

What is Thevenin's theorem? Look at Fig. 1-8*a*. This black box can contain any circuit with dc sources and linear resistances. (A *linear resistance* does not change with increasing voltage.) Thevenin was able to prove that no matter how complicated the circuit inside the black box of Fig. 1-8*a* was, it would produce exactly the same load current as the simple circuit of Fig. 1-8*b*. As a derivation:

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} \tag{1-7}$$

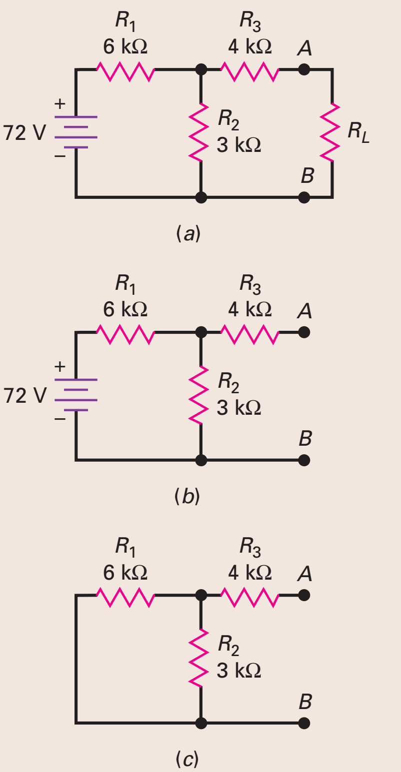
Let the idea sink in. Thevenin's theorem is a powerhouse tool. Engineers and technicians use the theorem constantly. Electronics could not possibly be where it is today without Thevenin's theorem. It not only simplifies calculations, it enables us to explain circuit operation that would be impossible to explain with only Kirchhoff equations.

Example 1-4

Multisim

What are the Thevenin voltage and resistance in Fig. 1-9*a*?

Figure 1-9 (*a*) Original circuit; (*b*) open-load resistor to get Thevenin voltage; (*c*) reduce source to zero to get Thevenin resistance.



SOLUTION First, calculate the Thevenin voltage. To do this, you have to open the load resistor. Opening the load resistance is equivalent to removing it from the circuit, as shown in Fig. 1-9*b*. Since 8 mA flows through 6 kΩ in series with 3 kΩ, 24 V will appear across the 3 kΩ. With no current through the 4 kΩ, 24 V will appear across the *AB* terminals. Therefore:

$$V_{TH} = 24 \text{ V}$$

Second, get the Thevenin resistance. Reducing a dc source to zero is equivalent to replacing it with a short, as shown in Fig. 1-9*c*. If we connect an ohmmeter across the *AB* terminals of Fig. 1-9*c*, what will it read?

It will read 6 kΩ. Why? Because looking back into the *AB* terminals with the battery shorted, the ohmmeter sees 4 kΩ in series with a parallel connection of 3 kΩ and 6 kΩ. We can write:

$$R_{TH} = 4\text{ k}\Omega + \frac{3\text{ k}\Omega \times 6\text{ k}\Omega}{3\text{ k}\Omega + 6\text{ k}\Omega} = 6\text{ k}\Omega$$

The product over sum of 3 kΩ and 6 kΩ is 2 kΩ, which, added to 4 kΩ, gives 6 kΩ.

Again, we need a new definition. Parallel connections occur so often in electronics that most people use a shorthand notation for them. From now on, we will use the following notation:

$$\| = \text{in parallel with}$$

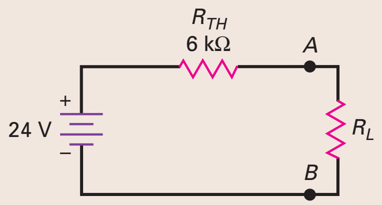
Whenever you see two vertical bars in an equation, it means *in parallel with.* In the electronics industry, you will see the foregoing equation for Thevenin resistance written like this:

$$R_{TH} = 4\text{ k}\Omega + (3\text{ k}\Omega \,\|\, 6\text{ k}\Omega) = 6\text{ k}\Omega$$

Most engineers and technicians know that the vertical bars mean *in parallel with,* so they automatically use product over sum or reciprocal method to calculate the equivalent resistance of 3 kΩ and 6 kΩ.

Figure 1-10 shows the Thevenin circuit with a load resistor. Compare this simple circuit with the original circuit of Fig. 1-9*a*. Can you see how much easier

Figure 1-10 Thevenin circuit for Fig. 1-9*a*.



it will be to calculate the load current for different load resistances? If not, the next example will drive the point home.

PRACTICE PROBLEM 1-4 Using Thevenin's theorem, what is the load current in Fig. 1-9*a* for the following values of R_L: 2 kΩ, 6 kΩ, and 18 kΩ?

If you really want to appreciate the power of Thevenin's theorem, try calculating the foregoing currents using the original circuit of Fig. 1-9*a* and any other method.

Application Example 1-5

Multisim

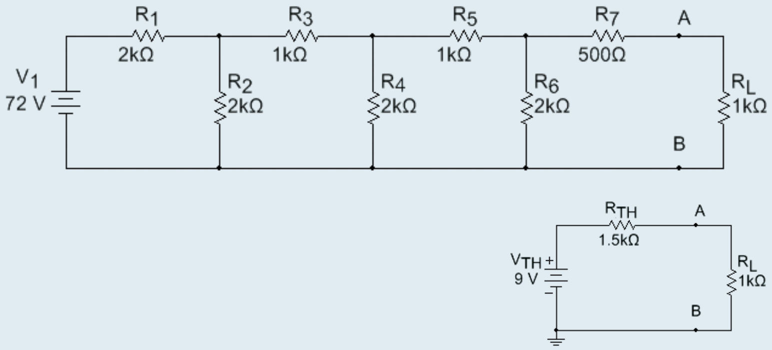
A *breadboard* is a circuit often built with solderless connections without regard to the final location of parts to prove the feasibility of a design. Suppose you have the circuit of Fig. 1-11*a* breadboarded on a lab bench. How would you measure the Thevenin voltage and resistance?

SOLUTION Start by replacing the load resistor with a multimeter, as shown in Fig. 1-11*b*. After you set the multimeter to read volts, it will indicate 9 V. This is the Thevenin voltage. Next, replace the dc source with a short (Fig. 1-11*c*). Set the multimeter to read ohms, and it will indicate 1.5 kΩ. This is the Thevenin resistance.

Are there any sources of error in the foregoing measurements? Yes: The one thing to watch out for is the input impedance of the multimeter when voltage is measured. Because this input impedance is across the measured terminals, a small current flows through the multimeter. For instance, if you use a moving-coil multimeter, the typical sensitivity is 20 kΩ per volt. On the 10-V range, the voltmeter has an input resistance of 200 kΩ. This will load the circuit down slightly and decrease the load voltage from 9 to 8.93 V.

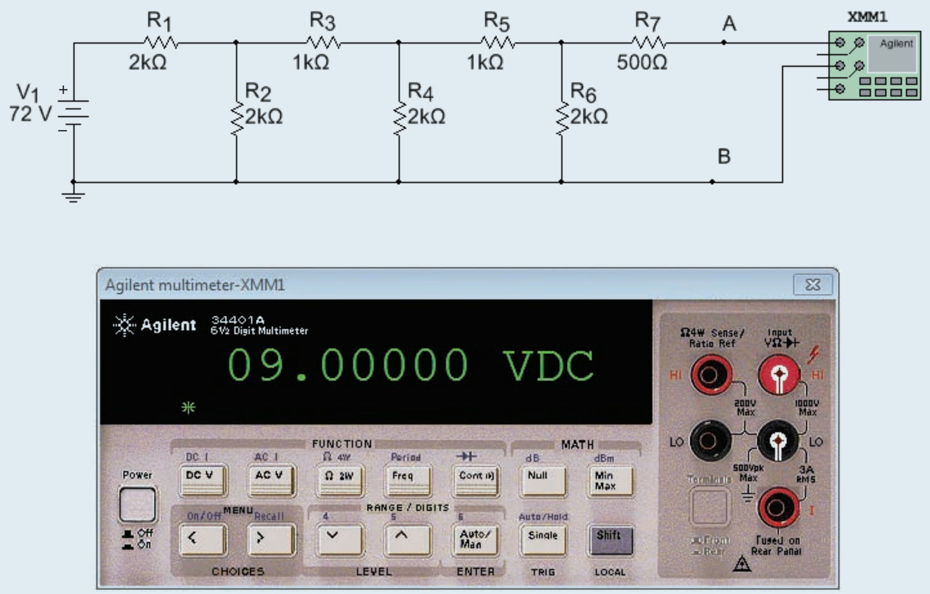
As a guideline, the input impedance of the voltmeter should be at least 100 times greater than the Thevenin resistance. Then, the loading error is less than 1 percent. *To avoid loading error, use a digital multimeter (DMM) instead of a moving-coil multimeter.* The input impedance of a DMM is at least 10 MΩ, which usually eliminates loading error. Loading error can also be produced when taking measurements with an oscilloscope. That is why in high-impedance circuits, a 10× probe should be used.

Figure 1-11 (*a*) Circuit on lab bench; (*b*) measuring Thevenin voltage; (*c*) measuring Thevenin resistance.

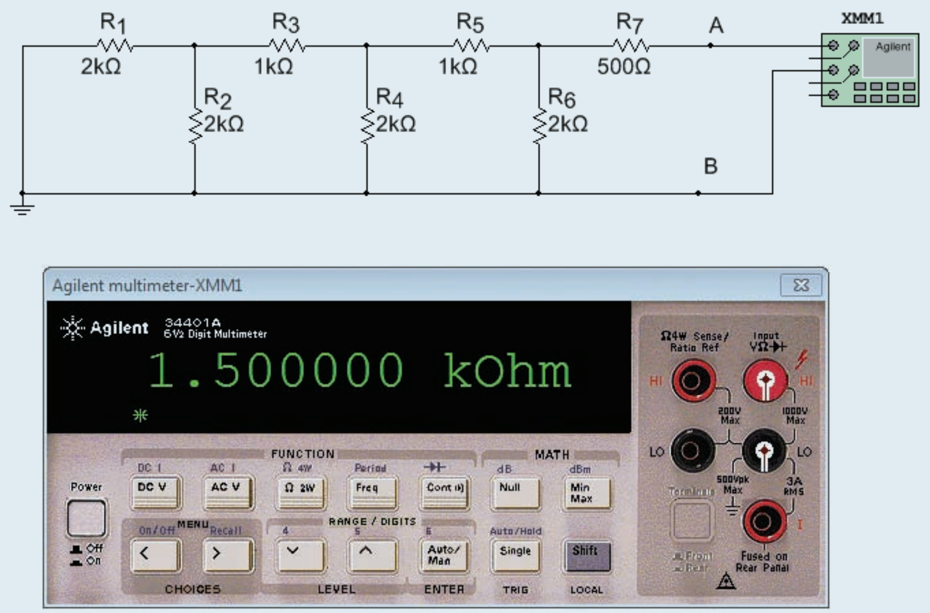


(*a*)

Figure 1-11 (continued)



(*b*)



(*c*)

1-6 Norton's Theorem

Recall the following ideas about Norton's theorem from earlier courses. In Fig. 1-12*a*, the Norton current I_N is defined as the load current when the load resistor is shorted. Because of this, the **Norton current** is sometimes called the *short-circuitc urrent.* As a definition:

$$\textbf{Norton current: } I_N = I_{SC} \quad \textbf{(1-8)}$$

The **Norton resistance** is the resistance that an ohmmeter measures across the load terminals when all sources are reduced to zero and the load resistor is open. As a definition:

$$\textbf{Norton resistance: } R_N = R_{OC} \quad \textbf{(1-9)}$$

Since Thevenin resistance also equals R_{OC}, we can write:

$$R_N = R_{TH} \quad \textbf{(1-10)}$$

This derivation says that Norton resistance equals Thevenin resistance. If you calculate a Thevenin resistance of 10 kΩ, you immediately know that the Norton resistance equals 10 kΩ.

Basic Idea

What is Norton's theorem? Look at Fig. 1-12*a*. This black box can contain any circuit with dc sources and linear resistances. Norton proved that the circuit inside the black box of Fig. 1-12*a* would produce exactly the same load voltage as the simple circuit of Fig. 1-12*b*. As a derivation, Norton's theorem looks like this:

$$V_L = I_N(R_N \| R_L) \quad \textbf{(1-11)}$$

In words: The load voltage equals the Norton current times the Norton resistance in parallel with the load resistance.

Earlier we saw that Norton resistance equals Thevenin resistance. But notice the difference in the location of the resistors: Thevenin resistance is always in series with a voltage source; Norton resistance is always in parallel with a current source.

Note: If you are using electron flow, keep the following in mind. In the electronics industry, the arrow inside the current source is almost always drawn in the direction of conventional current. The exception is a current source drawn with a dashed arrow instead of a solid arrow. In this case, the source pumps electrons in the direction of the dashed arrow.

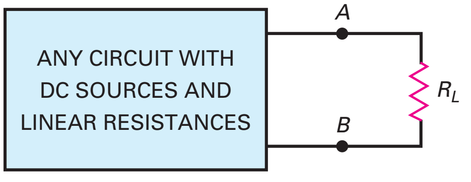
GOOD TO KNOW

Like Thevenin's theorem, Norton's theorem can be applied to ac circuits containing inductors, capacitors, and resistors. For ac circuits, the Norton current I_N is usually stated as a complex number in polar form, whereas the Norton impedance Z_N is usually expressed as a complex number in rectangular form.

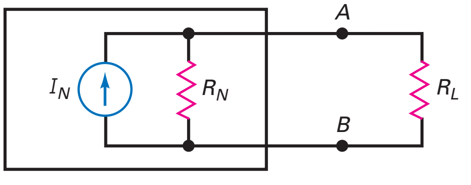
The Derivation

Norton's theorem can be derived from the **duality principle.** It states that for any theorem in electrical circuit analysis, there is a dual (opposite) theorem in which

Figure 1-12 (*a*) Black box has a linear circuit inside of it; (*b*) Norton circuit.

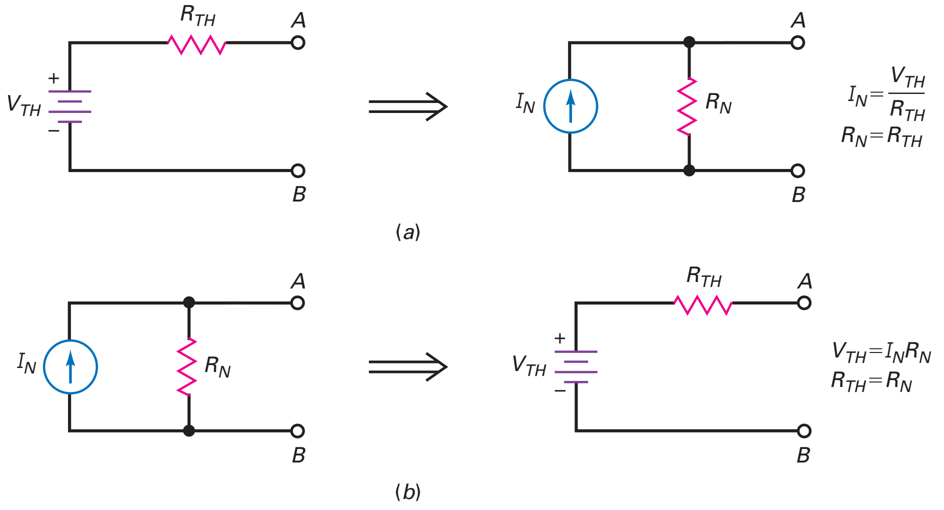


(*a*)



(*b*)

Figure 1-13 Duality principle: Thevenin's theorem implies Norton's theorem and vice versa. (*a*) Converting Thevenin to Norton; (*b*) converting Norton to Thevenin.



one replaces the original quantities with dual quantities. Here is a brief list of dual quantities:

Voltage ⟷ Current
Voltage source ⟷ Current source
Series ⟷ Parallel
Series resistance ⟷ Parallel resistance

Figure 1-13 summarizes the duality principle as it applies to Thevenin and Norton circuits. It means that we can use either circuit in our calculations. As you will see later, both equivalent circuits are useful. Sometimes, it is easier to use Thevenin. At other times, we use Norton. It depends on the specific problem. Summary Table 1-2 shows the steps for getting the Thevenin and Norton quantities.

Summary Table 1-2	**Thevenin and Norton Values**	
Process	**Thevenin**	**Norton**
Step 1	Open the load resistor.	Short the load resistor.
Step 2	Calculate or measure the open-circuit voltage. This is the Thevenin voltage.	Calculate or measure the short-circuit current. This is the Norton current.
Step 3	Short voltage sources and open current sources.	Short voltage sources, open current sources, and open load resistor.
Step 4	Calculate or measure the open-circuit resistance. This is the Thevenin resistance.	Calculate or measure the open-circuit resistance. This is the Norton resistance.

Relationships Between Thevenin and Norton Circuits

We already know that the Thevenin and Norton resistances are equal in value but different in location: Thevenin resistance is in series with a voltage source, and Norton resistance is in parallel with a current source.

We can derive two more relationships, as follows. We can convert any Thevenin circuit to a Norton circuit, as shown in Fig. 1-13*a*. The proof is straightforward. Short the *AB* terminals of the Thevenin circuit, and you get the Norton current:

$$I_N = \frac{V_{TH}}{R_{TH}} \tag{1-12}$$

This derivation says that the Norton current equals the Thevenin voltage divided by the Thevenin resistance.

Similarly, we can convert any Norton circuit to a Thevenin circuit, as shown in Fig. 1-13*b*. The open-circuit voltage is:

$$V_{TH} = I_N R_N \tag{1-13}$$

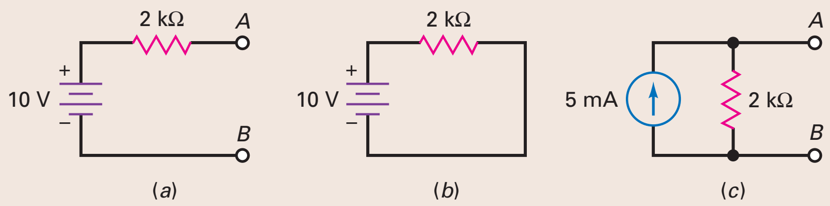
This derivation says that the Thevenin voltage equals the Norton current times the Norton resistance.

Figure 1-13 summarizes the equations for converting either circuit into the other.

Example 1-6

Suppose that we have reduced a complicated circuit to the Thevenin circuit shown in Fig. 1-14*a*. How can we convert this to a Norton circuit?

Figure 1-14 Calculating Norton current.



SOLUTION Use Eq. (1-12) to get:

$$I_N = \frac{10\text{ V}}{2\text{ k}\Omega} = 5\text{ mA}$$

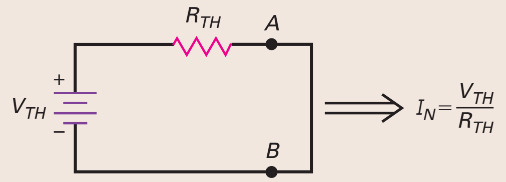
Figure 1-14*c* shows the Norton circuit.

Most engineers and technicians forget Eq. (1-12) soon after they leave school. But they always remember how to solve the same problem using Ohm's law. Here is what they do. Look at Fig. 1-14*a*. Visualize a short across the *AB* terminals, as shown in Fig. 1-14*b*. The short-circuit current equals the Norton current:

$$I_N = \frac{10\text{ V}}{2\text{ k}\Omega} = 5\text{ mA}$$

This is the same result, but calculated with Ohm's law applied to the Thevenin circuit. Figure 1-15 summarizes the idea. This memory aid will help you calculate the Norton current, given the Thevenin circuit.

Figure 1-15 A memory aid for Norton current.



PRACTICE PROBLEM 1-6 If the Thevenin resistance of Fig. 1-14*a* is 5 kΩ, determine the Norton current value.

1-7 Troubleshooting

Troubleshooting means finding out why a circuit is not doing what it is supposed to do. The most common troubles are opens and shorts. Devices like transistors can become open or shorted in a number of ways. One way to destroy any transistor is by exceeding its maximum-power rating.

Resistors become open when their power dissipation is excessive. But you can get a shorted resistor indirectly as follows. During the stuffing and soldering of printed-circuit boards, an undesirable splash of solder may connect two nearby conducting lines. Known as a **solder bridge,** this effectively shorts any device between the two conducting lines. On the other hand, a poor solder connection usually means no connection at all. This is known as a **cold-solder joint** and means that the device is open.

Besides opens and shorts, anything is possible. For instance, temporarily applying too much heat to a resistor may permanently change the resistance by several percent. If the value of resistance is critical, the circuit may not work properly after the heat shock.

And then there is the troubleshooter's nightmare: the intermittent trouble. This kind of trouble is difficult to isolate because it appears and disappears. It may be a cold-solder joint that alternately makes and breaks a contact, or a loose cable connector, or any similar trouble that causes on-again, off-again operation.

An Open Device

Always remember these two facts about an **open device:**

The current through an open device is zero.
The voltage across it is unknown.

The first statement is true because an open device has infinite resistance. No current can exist in an infinite resistance. The second statement is true because of Ohm's law:

$$V = IR = (0)(\infty)$$

In this equation, zero times infinity is mathematically indeterminate. You have to figure out what the voltage is by looking at the rest of the circuit.

A Shorted Device

A shorted device is exactly the opposite. Always remember these two statements about a **shorted device:**

> ***The voltage across a shorted device is zero.***
> ***The current through it is unknown.***

The first statement is true because a shorted device has zero resistance. No voltage can exist across zero resistance. The second statement is true because of Ohm's law:

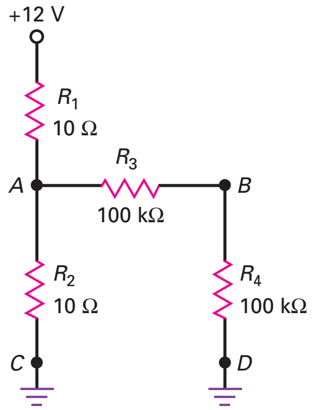
$$I = \frac{V}{R} = \frac{0}{0}$$

Zero divided by zero is mathematically meaningless. You have to figure out what the current is by looking at the rest of the circuit.

Procedure

Normally, you measure voltages with respect to ground. From these measurements and your knowledge of basic electricity, you can usually deduce the trouble. After you have isolated a component as the top suspect, you can unsolder or disconnect the component and use an ohmmeter or other instrument for confirmation.

Figure 1-16 Voltage divider and load used in troubleshooting discussion.



Normal Values

In Fig. 1-16, a stiff voltage divider consisting of R_1 and R_2 drives resistors R_3 and R_4 in series. Before you can troubleshoot this circuit, you have to know what the normal voltages are. The first thing to do, therefore, is to work out the values of V_A and V_B. The first is the voltage between A and ground. The second is the voltage between B and ground. Because R_1 and R_2 are much smaller than R_3 and R_4 (10 Ω versus 100 kΩ), the stiff voltage at A is approximately +6 V. Furthermore, since R_3 and R_4 are equal, the voltage at B is approximately +3 V. When this circuit is trouble free, you will measure 6 V between A and ground, and 3 V between B and ground. These two voltages are the first entry of Summary Table 1-3.

R_1 Open

When R_1 is open, what do you think happens to the voltages? Since no current can flow through the open R_1, no current can flow through R_2. Ohm's law tells us the voltage across R_2 is zero. Therefore, $V_A = 0$ and $V_B = 0$, as shown in Summary Table 1-3 for R_1 open.

R_2 Open

When R_2 is open, what happens to the voltages? Since no current can flow through the open R_2, the voltage at A is pulled up toward the supply voltage. Since R_1 is much smaller than R_3 and R_4, the voltage at A is approximately 12 V. Since R_3 and R_4 are equal, the voltage at B becomes 6 V. This is why $V_A = 12$ V and $V_B = 6$ V, as shown in Summary Table 1-3 for an R_2 open.

Summary Table 1-3	Troubles and Clues	
Trouble	V_A	V_B
Circuit OK	6 V	3 V
R_1 open	0	0
R_2 open	12 V	6 V
R_3 open	6 V	0
R_4 open	6 V	6 V
C open	12 V	6 V
D open	6 V	6 V
R_1 shorted	12 V	6 V
R_2 shorted	0	0
R_3 shorted	6 V	6 V
R_4 shorted	6 V	0

Remaining Troubles

If ground *C* is open, no current can pass through R_2. This is equivalent to an open R_2. This is why the trouble *C* open has $V_A = 12$ V and $V_B = 6$ V in Summary Table 1-3.

You should work out all of the remaining entries in Summary Table 1-3, making sure that you understand why each voltage exists for the given trouble.

Example 1-7

In Fig. 1-16, you measure $V_A = 0$ and $V_B = 0$. What is the trouble?

SOLUTION Look at Summary Table 1-3. As you can see, two troubles are possible: R_1 open or R_2 shorted. Both of these produce zero voltage at points *A* and *B*. To isolate the trouble, you can disconnect R_1 and measure it. If it measures open, you have found the trouble. If it measures OK, then R_2 is the trouble.

PRACTICE PROBLEM 1-7 What could the possible troubles be if you measure $V_A = 12$ V and $V_B = 6$ V in Fig. 1-16?