

9

BRANCH, LOOP, AND NODE ANALYSES

CHAPTER OUTLINE

- 9-1 Simultaneous Equations in Circuit Analysis
- 9-2 Branch Current Method
- 9-3 Loop Current Method
- 9-4 Node Voltage Method
A Circuit Application

CHAPTER OBJECTIVES

- ◆ Discuss three methods to solve simultaneous equations
- ◆ Use the branch current method to find unknown quantities in a circuit
- ◆ Use loop analysis to find unknown quantities in a circuit
- ◆ Use node analysis to find unknown quantities in a circuit

KEY TERMS

- ◆ Simultaneous equations
- ◆ Matrix
- ◆ Determinant
- ◆ Loop
- ◆ Node
- ◆ Branch

A CIRCUIT APPLICATION PREVIEW

In the application, you will analyze a model of an amplifier using the methods covered in this chapter.

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INTRODUCTION

In the last chapter, you learned about the superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem, and several types of conversion methods. These theorems and conversion methods are useful in solving some types of circuit problems for both dc and ac.

In this chapter, three more circuit analysis methods are introduced. These methods, based on Ohm's law and Kirchhoff's laws, are particularly useful in the analysis of multiple-loop circuits having two or more voltage or current sources. The methods presented here can be used alone or in conjunction with the techniques covered in the previous chapters. With experience, you will learn which method is best for a particular problem or you may develop a preference for one of them.

In the branch current method, you will apply Kirchhoff's laws to solve for current in various branches of a multiple-loop circuit. A loop is a complete current path within a circuit. In the loop current method, you will solve for loop currents rather than branch currents. In the node voltage method, you will find the voltages at the independent nodes in a circuit. As you know, a node is the junction of two or more components.

9–1 SIMULTANEOUS EQUATIONS IN CIRCUIT ANALYSIS

The circuit analysis methods covered in this chapter allow you to find two or more unknown currents or voltages by solving simultaneous equations. These analysis methods, which include branch current, loop current, and node voltage methods, result in a number of equations equal to the number of unknowns. Coverage is limited to equations with two unknowns (2nd order) and equations with three unknowns (3rd order). These equations can then be solved simultaneously for the unknown quantities using one of the methods covered in this section.

After completing this section, you should be able to

- ♦ **Discuss three methods to solve simultaneous equations**
 - ♦ Write a set of simultaneous equations in standard form
 - ♦ Solve simultaneous equations using algebraic substitution
 - ♦ Solve simultaneous equations using determinants
 - ♦ Solve simultaneous equations using a calculator

Simultaneous equations consist of a set of n equations containing n unknowns, where n is a number with a value of 2 or more. The number of equations in the set must equal the number of unknowns. For example, to solve for two unknown variables, you must have two equations; to solve for three unknowns you must have three equations, and so on.

Second-Order Standard Form Equations

An equation with two variables is called a *second-order equation*. In circuit analysis, the variables can represent unknowns such as current or voltage. In order to solve for variables x_1 and x_2 , there must be a set of two equations containing those variables expressed in standard form.

In standard form, the x_1 variables are in the first position in each equation, and the x_2 variables are in the second position in each equation. The variables with their coefficients are on the left side of the equation, and the constants are on the right side.

The set of two simultaneous second-order equations written in standard form is

$$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 &= b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 &= b_2 \end{aligned}$$

In these simultaneous equations, the “a” is the coefficient of the variables x_1 and x_2 and can represent values of circuit components. Notice that the subscripts of the coefficients contain two numbers. For example, $a_{1,1}$ appears in the *first* equation as the coefficient of x_1 and $a_{2,1}$ appears in the *second* equation as the coefficient of x_1 . The “b” is the constant and can represent a voltage source. This notation will be useful when you use a calculator to solve the equations.

EXAMPLE 9–1

Assume the following two equations describe a particular circuit with two unknown currents I_1 and I_2 . The coefficients are resistance values and the constants are voltages in the circuit. Write the equations in standard form.

$$\begin{aligned} 2I_1 &= 8 - 5I_2 \\ 4I_2 - 5I_1 + 6 &= 0 \end{aligned}$$

Solution Rearrange the equations in standard form as follows:

$$2I_1 + 5I_2 = 8$$

$$5I_1 + 4I_2 = -6$$

Related Problem* Convert these two equations to standard form:

$$20x_1 + 15 = 11x_2$$

$$10 = 25x_2 + 18x_1$$

*Answers are at the end of the chapter.

A third-order equation contains three variables and a constant term. Just as in second-order equations, each variable has a coefficient. In order to solve for the variables x_1 , x_2 , and x_3 , there must be a set of three simultaneous equations containing those variables. The general form for three simultaneous third-order equations written in standard form is

$$a_{1,1}x_1 + a_{1,2}x_2 + a_{1,3}x_3 = b_1$$

$$a_{2,1}x_1 + a_{2,2}x_2 + a_{2,3}x_3 = b_2$$

$$a_{3,1}x_1 + a_{3,2}x_2 + a_{3,3}x_3 = b_3$$

EXAMPLE 9-2

Assume the following three equations describe a particular circuit with three unknown currents I_1 , I_2 , and I_3 . The coefficients are resistance values and the constants are known voltages in the circuit. Write the equations in standard form.

$$4I_3 + 2I_2 + 7I_1 = 0$$

$$5I_1 + 6I_2 + 9I_3 - 7 = 0$$

$$8 = 1I_1 + 2I_2 + 5I_3$$

Solution The equations are rearranged to put them in standard form as follows:

$$7I_1 + 2I_2 + 4I_3 = 0$$

$$5I_1 + 6I_2 + 9I_3 = 7$$

$$1I_1 + 2I_2 + 5I_3 = 8$$

Related Problem Convert these three equations to standard form:

$$10V_1 + 15 = 21V_2 + 50V_3$$

$$10 + 12V_3 = 25V_2 + 18V_1$$

$$12V_3 - 25V_2 + 18V_1 = 9$$

Solutions of Simultaneous Equations

Three ways for solving simultaneous equations are algebraic substitution, the determinant method, and using a calculator.

Solving by Substitution You can solve two or three simultaneous equations in standard form using algebraic substitution by first solving one of the variables in terms of the others.

However, because the process can become quite lengthy, we will restrict this method to second-order equations. Consider the following set of simultaneous equations:

$$2x_1 + 6x_2 = 8 \quad (\text{Eq. 1})$$

$$3x_1 + 6x_2 = 2 \quad (\text{Eq. 2})$$

Step 1. Solve for x_1 in terms of x_2 in Eq. 1.

$$2x_1 = 8 - 6x_2$$

$$x_1 = 4 - 3x_2$$

Step 2. Substitute the result for x_1 into Eq. 2 and solve for x_2 .

$$3x_1 + 6x_2 = 2$$

$$3(4 - 3x_2) + 6x_2 = 2$$

$$12 - 9x_2 + 6x_2 = 2$$

$$-3x_2 = -10$$

$$x_2 = \frac{-10}{-3} = 3.33$$

Step 3. Substitute the value for x_2 into the equation for x_1 in Step 1.

$$x_1 = 4 - 3x_2 = 4 - 3(3.33) = 4 - 9.99 = -5.99$$

Solving by Determinants The determinant method is a part of matrix algebra and provides a “cookbook” approach for solving simultaneous equations with two or three variables. A **matrix** is an array of numbers, and a **determinant** is effectively the solution to a matrix, resulting in a specific value. Second-order determinants are used for two variables and third-order determinants are used for three variables. The equations must be in standard form for a solution.

To illustrate the determinant method for second-order equations, let’s find the values of I_1 and I_2 in the following two equations expressed in standard form:

$$10I_1 + 5I_2 = 15$$

$$2I_1 + 4I_2 = 8$$

First, form the characteristic determinant from the matrix of the coefficients of the unknown currents. The first column in the determinant consists of the coefficients of I_1 , and the second column consists of the coefficients of I_2 . The resulting determinant is

$$\begin{array}{cc} \text{1st column} & \text{2nd column} \\ \begin{array}{|c|c|} \hline 10 & 5 \\ \hline 2 & 4 \\ \hline \end{array} \end{array}$$

An evaluation of this characteristic determinant requires three steps.

Step 1. Multiply the first number in the left column by the second number in the right column.

$$\left| \begin{array}{c} 10 \\ 2 \end{array} \times \begin{array}{c} 5 \\ 4 \end{array} \right| = 10 \times 4 = 40$$

Step 2. Multiply the second number in the left column by the first number in the right column.

$$\left| \begin{array}{c} 10 \\ 2 \end{array} \times \begin{array}{c} 5 \\ 4 \end{array} \right| = 2 \times 5 = 10$$

Step 3. Subtract the product in Step 2 from the product in Step 1.

$$40 - 10 = 30$$

This difference is the value of the characteristic determinant (30 in this case).

Next, replace the coefficients of I_1 in the first column of the characteristic determinant with the constants (fixed numbers) on the right side of the equations to form another determinant:

$$\begin{array}{c} \downarrow \\ \left| \begin{array}{cc} 15 & 5 \\ 8 & 4 \end{array} \right| \end{array} \quad \begin{array}{l} \text{Replace coefficients of } I_1 \\ \text{with constants from} \\ \text{right sides of equations.} \end{array}$$

Evaluate this I_1 determinant as follows:

$$\left| \begin{array}{cc} 15 & 5 \\ 8 & 4 \end{array} \right| \times 4 = 15 \times 4 = 60$$

$$\left| \begin{array}{cc} 15 & 5 \\ 8 & 4 \end{array} \right| \times 5 = 60 - (8 \times 5) = 60 - 40 = 20$$

The value of this determinant is 20.

Now solve for I_1 by dividing the I_1 determinant by the characteristic determinant as follows:

$$I_1 = \frac{\left| \begin{array}{cc} 15 & 5 \\ 8 & 4 \end{array} \right|}{\left| \begin{array}{cc} 10 & 5 \\ 2 & 4 \end{array} \right|} = \frac{20}{30} = 0.667 \text{ A}$$

To find I_2 , form another determinant by substituting the constants on the right side of the given equations for the coefficients of I_2 in the second column of the characteristic determinant.

$$\begin{array}{c} \downarrow \\ \left| \begin{array}{cc} 10 & 15 \\ 2 & 8 \end{array} \right| \end{array} \quad \begin{array}{l} \text{Replace coefficients of } I_2 \\ \text{with constants from} \\ \text{right sides of equations.} \end{array}$$

Solve for I_2 by dividing this determinant by the characteristic determinant previously found.

$$I_2 = \frac{\left| \begin{array}{cc} 10 & 15 \\ 2 & 8 \end{array} \right|}{30} = \frac{(10 \times 8) - (2 \times 15)}{30} = \frac{80 - 30}{30} = \frac{50}{30} = 1.67 \text{ A}$$

EXAMPLE 9-3

Solve the following set of equations for the unknown currents:

$$2I_1 - 5I_2 = 10$$

$$6I_1 + 10I_2 = 20$$

Solution Evaluate the characteristic determinant as follows:

$$\left| \begin{array}{cc} 2 & -5 \\ 6 & 10 \end{array} \right| = (2)(10) - (-5)(6) = 20 - (-30) = 20 + 30 = 50$$

Solving for I_1 yields

$$I_1 = \frac{\left| \begin{array}{cc} 10 & -5 \\ 20 & 10 \end{array} \right|}{50} = \frac{(10)(10) - (-5)(20)}{50} = \frac{100 - (-100)}{50} = \frac{200}{50} = 4 \text{ A}$$

Solving for I_2 yields

$$I_2 = \frac{\begin{vmatrix} 2 & 10 \\ 6 & 20 \end{vmatrix}}{50} = \frac{(2)(20) - (6)(10)}{50} = \frac{40 - 60}{50} = -0.4 \text{ A}$$

In a circuit problem, a result with a negative sign indicates that the direction of actual current is opposite to the assigned direction.

Note that multiplication can be expressed either by the multiplication sign such as 2×10 or by parentheses such as $(2)(10)$.

Related Problem Solve the following set of equations for I_1 :

$$5I_1 + 3I_2 = 4$$

$$I_1 + 2I_2 = -6$$

Third-order determinants can be evaluated by the expansion method. We will illustrate this method by finding the unknown current values in the following three equations expressed in standard form:

$$1I_1 + 3I_2 - 2I_3 = 7$$

$$0I_1 + 4I_2 + 1I_3 = 8$$

$$-5I_1 + 1I_2 + 6I_3 = 9$$

The characteristic determinant for the matrix of coefficients for this set of equations is formed in a similar way to that used earlier for the second-order determinant. The first column consists of the coefficients of I_1 , the second column consists of the coefficients of I_2 , and the third column consists of the coefficients of I_3 , as shown below.

Coefficients of I_1	Coefficients of I_2	Coefficients of I_3	
↓	↓	↓	
	1	3	-2
	0	4	1
	-5	1	6

This third-order determinant is evaluated by the expansion method as follows:

Step 1. Rewrite the first two columns immediately to the right of the determinant.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 0 & 4 \\ -5 & 1 \end{vmatrix}$$

Step 2. Identify the three downward diagonal groups of three coefficients each.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 0 & 4 \\ -5 & 1 \end{vmatrix}$$

Step 3. Multiply the numbers in each diagonal and add the products.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 0 & 4 \\ -5 & 1 \end{vmatrix}$$

$(1)(4)(6) + (3)(1)(-5) + (-2)(0)(1) = 24 + (-15) + 0 = 9$

Step 4. Repeat Steps 2 and 3 for the three upward diagonal groups of three coefficients.

$$\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 0 & 4 \\ -5 & 1 \end{vmatrix}$$

$$(-5)(4)(-2) + (1)(1)(1) + (6)(0)(3) = 40 + 1 + 0 = 41$$

Step 5. Subtract the result in Step 4 from the result in Step 3 to get the value of the characteristic determinant.

$$9 - 41 = -32$$

Next, form another determinant by substituting the constants on the right of the equations for the coefficients of I_1 in the characteristic determinant.

$$\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}$$

Evaluate this determinant using the method described in the previous steps.

$$\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix} \begin{vmatrix} 7 & 3 \\ 8 & 4 \\ 9 & 1 \end{vmatrix}$$

$$\begin{aligned} &= [(7)(4)(6) + (3)(1)(9) + (-2)(8)(1)] - [(9)(4)(-2) + (1)(1)(7) + (6)(8)(3)] \\ &= (168 + 27 - 16) - (-72 + 7 + 144) = 179 - 79 = 100 \end{aligned}$$

Solve for I_1 by dividing this determinant by the characteristic determinant. The negative result indicates that the actual current is in a direction opposite to the original assumption.

$$I_1 = \frac{\begin{vmatrix} 7 & 3 & -2 \\ 8 & 4 & 1 \\ 9 & 1 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 0 & 4 & 1 \\ -5 & 1 & 6 \end{vmatrix}} = \frac{100}{-32} = -3.125 \text{ A}$$

You can find I_2 and I_3 in a similar way.

EXAMPLE 9-4

Determine the value of I_2 from the following set of equations:

$$2I_1 + 0.5I_2 + 1I_3 = 0$$

$$0.75I_1 + 0I_2 + 2I_3 = 1.5$$

$$3I_1 + 0.2I_2 + 0I_3 = -1$$

Solution Evaluate the characteristic determinant as follows:

$$\begin{vmatrix} 2 & 0.5 & 1 \\ 0.75 & 0 & 2 \\ 3 & 0.2 & 0 \end{vmatrix} \begin{vmatrix} 2 & 0.5 \\ 0.75 & 0 \\ 3 & 0.2 \end{vmatrix}$$

$$\begin{aligned} &= [(2)(0)(0) + (0.5)(2)(3) + (1)(0.75)(0.2)] - [(3)(0)(1) + (0.2)(2)(2) + (0)(0.75)(0.5)] \\ &= (0 + 3 + 0.15) - (0 + 0.8 + 0) = 3.15 - 0.8 = 2.35 \end{aligned}$$

Evaluate the determinant for I_2 is as follows:

$$\begin{vmatrix} 2 & 0 & 1 & 2 & 0 \\ 0.75 & 1.5 & 2 & 0.75 & 1.5 \\ 3 & -1 & 0 & 3 & -1 \end{vmatrix}$$

$$\begin{aligned} &= [(2)(1.5)(0) + (0)(2)(3) + (1)(0.75)(-1)] - [(3)(1.5)(1) + (-1)(2)(2) + (0)(0.75)(0)] \\ &= [0 + 0 + (-0.75)] - [4.5 + (-4) + 0] = -0.75 - 0.5 = -1.25 \end{aligned}$$

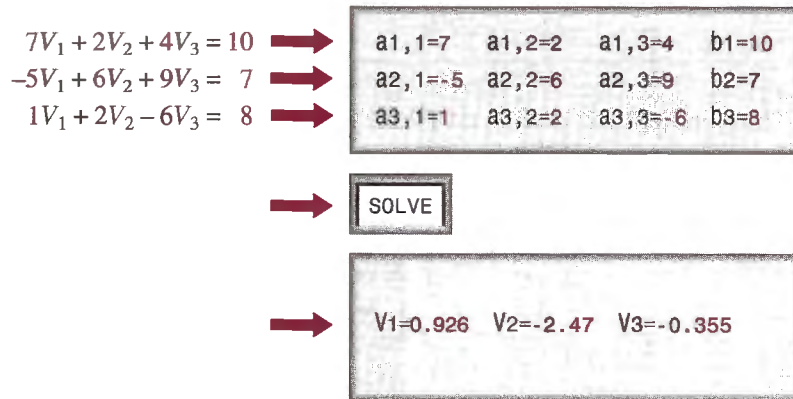
Finally, divide the two determinants.

$$I_2 = \frac{-1.25}{2.35} = -0.532 \text{ A} = -532 \text{ mA}$$

Related Problem Determine the value of I_1 in the set of equations used in this example.

Solving by Calculator Calculators generally employ matrix algorithms for simultaneous equation solution and make the results much easier to obtain. As with the two “manual” methods, it is important to first get the equations into the standard form before entering data on a calculator. Calculators that offer simultaneous equation solutions generally employ the notation discussed earlier in relation to the general form of equations. Variables are labeled as x_1, x_2 , etc, coefficients are designated as $a_{1,1}, a_{1,2}, a_{2,1}, a_{2,2}$, etc., and constants are designated as b_1, b_2 , etc.

A typical sequence for entering the data for a specific set of equations into a calculator is illustrated in a generic way for three simultaneous equations in Figure 9–1.



◀ FIGURE 9–1

We have selected the TI-86 and the TI-89 calculators to illustrate the procedure in the following two examples, although other scientific calculators can be used. If your calculator has simultaneous equation capability, consult the user’s manual for the correct procedure.

EXAMPLE 9–5

Use the TI-86 calculator to solve the following three simultaneous equations for the three unknowns.

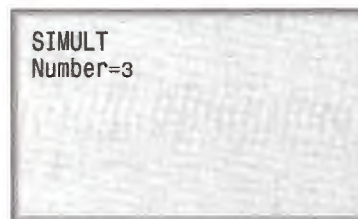
$$8I_1 + 4I_2 + 1I_3 = 7$$

$$2I_1 - 5I_2 + 6I_3 = 3$$

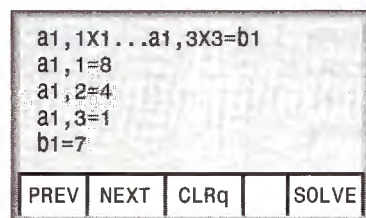
$$3I_1 + 3I_2 - 2I_3 = -5$$

Solution Press the 2nd key, then SIMULT to enter the number of equations as shown in Figure 9–2.

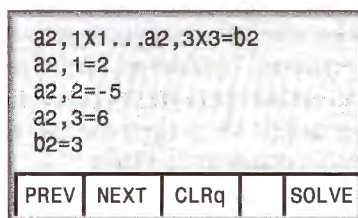
► FIGURE 9–2



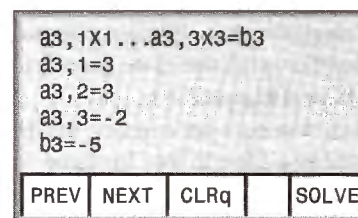
After you enter 3 and press ENTER, the first equation screen comes up. Enter the coefficients 8, 4, 1 and the constant 7 by pressing each number key followed by the ENTER key, which results in the screen shown in Figure 9–3(a). After you enter the last number and press ENTER, the second equation screen appears. Enter the coefficients 2, –5, 6 and the constant 3 as shown in Figure 9–3(b) (A negative value is entered by first pressing the (–) key.). Finally, enter the coefficients of the third equation (3, 3, –2) and the constant –5 as shown in Figure 9–3(c).



(a)



(b)

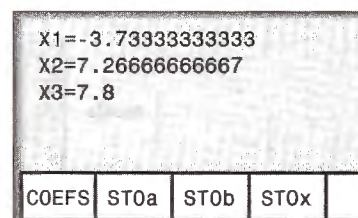


(c)

▲ FIGURE 9–3

Selecting SOLVE, which is the F5 key, produces the results displayed in Figure 9–4. X1 is I_1 , X2 is I_2 , and X3 is I_3 .

► FIGURE 9–4



Related Problem Edit the equations to change $a_{1,2}$ from 4 to –3, $a_{2,3}$ from 6 to 2.5, and b_3 from –5 to 8 and solve the modified equations.

EXAMPLE 9–6

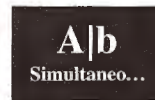
Use the TI-89 Titanium calculator to solve the same three simultaneous equations that were given in Example 9–5.

$$8I_1 + 4I_2 + 1I_3 = 7$$

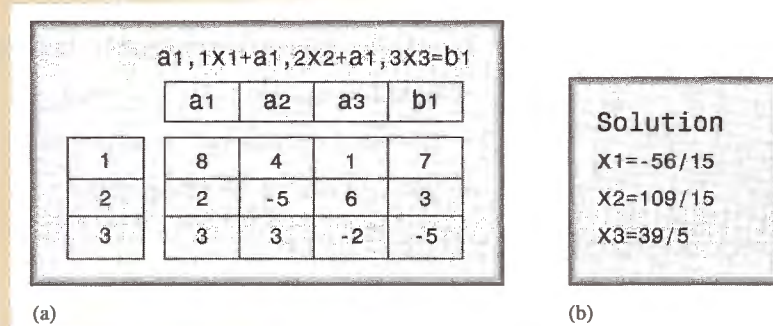
$$2I_1 - 5I_2 + 6I_3 = 3$$

$$3I_1 + 3I_2 - 2I_3 = -5$$

Solution From the Home screen select the simultaneous equation icon



Press ENTER. Select 3:New, then press ENTER again. Next specify the number of equations and the number of unknowns and press ENTER. On the simultaneous equation screen enter the coefficients and the constants as shown in Figure 9–5(a). Press ENTER after each number.



▲ FIGURE 9–5

After you have entered the coefficients and constants, press the F5 key to solve. As indicated by the screen in part (b), the results are returned as fractions. They agree with the TI-86 results, which were returned as decimal numbers.

Related Problem Repeat the Related Problem from Example 9–5 using the TI-89.

SECTION 9–1 REVIEW

1. Evaluate the following determinants:

(a) $\begin{vmatrix} 0 & -1 \\ 4 & 8 \end{vmatrix}$

(b) $\begin{vmatrix} 0.25 & 0.33 \\ -0.5 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 3 & 7 \\ 2 & -1 & 7 \\ -4 & 0 & -2 \end{vmatrix}$

2. Set up the characteristic determinant for the following set of simultaneous equations:

$$2I_1 + 3I_2 = 0$$

$$5I_1 + 4I_2 = 1$$

3. Find I_2 in Question 2.

4. Use your calculator to solve the following set of simultaneous equations for I_1 , I_2 , I_3 , and I_4 .

$$100I_1 + 220I_2 + 180I_3 + 330I_4 = 0$$

$$470I_1 + 390I_2 + 100I_3 + 100I_4 = 12$$

$$120I_1 - 270I_2 + 150I_3 - 180I_4 = -9$$

$$560I_1 + 680I_2 - 220I_3 + 390I_4 = 0$$

5. Modify the equations in Question 4 by changing the constant in the first equation to 8.5, the coefficient of I_3 in the second equation to 220, and the coefficient of I_1 in the fourth equation to 330. Solve the new set of equations for the currents.

9–2 BRANCH CURRENT METHOD

The branch current method is a circuit analysis method using Kirchhoff's voltage and current laws to find the current in each branch of a circuit by generating simultaneous equations. Once you know the branch currents, you can determine voltages.

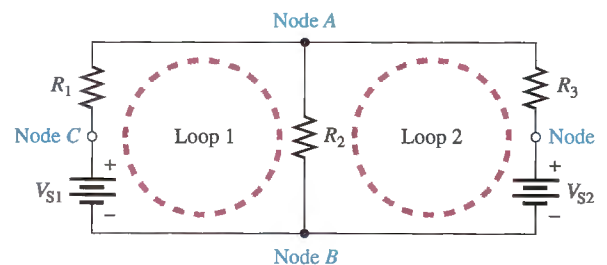
After completing this section, you should be able to

- ♦ Use the branch current method to find unknown quantities in a circuit
 - ♦ Identify loops and nodes in a circuit
 - ♦ Develop a set of branch current equations
 - ♦ Solve the branch current equations

Figure 9–6 shows a circuit that will be used as the basic model throughout the chapter to illustrate each of the three circuit analysis methods. In this circuit, there are only two nonredundant closed loops. A **loop** is a complete current path within a circuit, and you can view a set of nonredundant closed loops as a set of “windowpanes,” where each windowpane represents one nonredundant loop. Also, there are four nodes as indicated by the letters *A*, *B*, *C*, and *D*. A **node** is a point where two or more components are connected. A **branch** is a path that connects two nodes, and there are three branches in this circuit: one containing R_1 , one containing R_2 , and one containing R_3 .

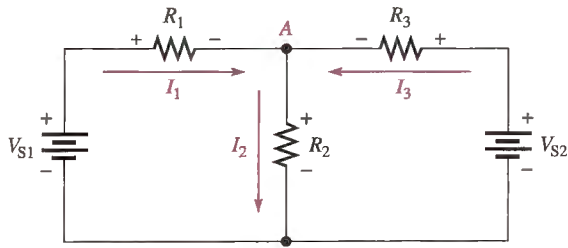
► FIGURE 9–6

Circuit showing loops, nodes, and branches.



The following are the general steps used in applying the branch current method.

- Step 1.** Assign a current in each circuit branch in an arbitrary direction.
- Step 2.** Show the polarities of the resistor voltages according to the assigned branch current directions.
- Step 3.** Apply Kirchhoff's voltage law around each closed loop (algebraic sum of voltages is equal to zero).
- Step 4.** Apply Kirchhoff's current law at the minimum number of nodes so that all branch currents are included (algebraic sum of currents at a node equals zero).
- Step 5.** Solve the equations resulting from Steps 3 and 4 for the branch current values.



◀ FIGURE 9-7

Circuit for demonstrating branch current analysis.

These steps are demonstrated with the aid of Figure 9-7. First, the **branch currents** I_1 , I_2 , and I_3 are assigned in the direction shown. Don't worry about the actual current directions at this point. Second, the polarities of the voltage drops across R_1 , R_2 , and R_3 are indicated in the figure according to the assigned current directions. Third, Kirchhoff's voltage law applied to the two loops gives the following equations where the resistance values are the coefficients for the unknown currents:

$$\text{Equation 1: } R_1 I_1 + R_2 I_2 - V_{S1} = 0 \quad \text{for loop 1}$$

$$\text{Equation 2: } R_2 I_2 + R_3 I_3 - V_{S2} = 0 \quad \text{for loop 2}$$

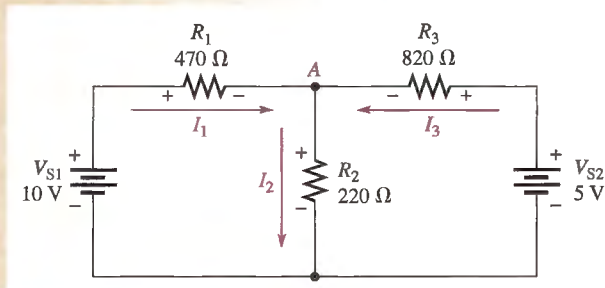
Fourth, Kirchhoff's current law is applied to node A, including all branch currents as follows:

$$\text{Equation 3: } I_1 - I_2 + I_3 = 0$$

The negative sign indicates that I_2 is out of the node. Fifth and last, the three equations must be solved for the three unknown currents, I_1 , I_2 , and I_3 . Example 9-7 shows how to solve equations by the *substitution* method.

EXAMPLE 9-7

Use the branch current method to find each branch current in Figure 9-8.



▲ FIGURE 9-8

Solution

Step 1: Assign branch currents as shown in Figure 9-8. Keep in mind that you can assume any current direction at this point and that the final solution will have a negative sign if the actual current is opposite to the assigned current.

Step 2: Mark the polarities of the resistor voltage drops in accordance with the assigned current directions as shown in the figure.

Step 3: Applying Kirchhoff's voltage law around the left loop gives

$$470I_1 + 220I_2 - 10 = 0$$

Around the right loop gives

$$220I_2 + 820I_3 - 5 = 0$$

where all resistance values are in ohms and voltage values are in volts. For simplicity, the units are not shown.

Step 4: At node A , the current equation is

$$I_1 - I_2 + I_3 = 0$$

Step 5: The equations are solved by substitution as follows. First, find I_1 in terms of I_2 and I_3 .

$$I_1 = I_2 - I_3$$

Now, substitute $I_2 - I_3$ for I_1 in the left loop equation.

$$\begin{aligned} 470(I_2 - I_3) + 220I_2 &= 10 \\ 470I_2 - 470I_3 + 220I_2 &= 10 \\ 690I_2 - 470I_3 &= 10 \end{aligned}$$

Next, take the right loop equation and solve for I_2 in terms of I_3 .

$$\begin{aligned} 220I_2 &= 5 - 820I_3 \\ I_2 &= \frac{5 - 820I_3}{220} \end{aligned}$$

Substituting this expression for I_2 into $690I_2 - 470I_3 = 10$ yields

$$\begin{aligned} 690\left(\frac{5 - 820I_3}{220}\right) - 470I_3 &= 10 \\ \frac{3450 - 565800I_3}{220} - 470I_3 &= 10 \\ 15.68 - 2571.8I_3 - 470I_3 &= 10 \\ -3041.8I_3 &= -5.68 \\ I_3 &= \frac{5.68}{3041.8} = 0.00187 \text{ A} = \mathbf{1.87 \text{ mA}} \end{aligned}$$

Now, substitute the value of I_3 in amps into the right loop equation.

$$220I_2 + 820(0.00187) = 5$$

Solve for I_2 .

$$I_2 = \frac{5 - 820(0.00187)}{220} = \frac{3.47}{220} = 0.0158 = \mathbf{15.8 \text{ mA}}$$

Substituting I_2 and I_3 values into the current equation at node A yields

$$\begin{aligned} I_1 - 0.0158 + 0.00187 &= 0 \\ I_1 &= 0.0158 - 0.00187 = 0.0139 \text{ A} = \mathbf{13.9 \text{ mA}} \end{aligned}$$

Related Problem



Determine the branch currents in Figure 9–8 with the polarity of the 5 V source reversed.

Use Multisim file E09-07 to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 9-2 REVIEW

1. What basic circuit laws are used in the branch current method?
2. When assigning branch currents, you should be careful that the assigned directions match the actual directions. (T or F)
3. What is a loop?
4. What is a node?

9-3 LOOP CURRENT METHOD

In the loop current method (also known as the mesh current method), you will work with loop currents instead of branch currents. An ammeter placed in a given branch will measure the branch current. Unlike branch currents, loop currents are mathematical quantities, rather than actual physical currents, that are used to make circuit analysis somewhat easier than with the branch current method.

After completing this section, you should be able to

- ♦ Use loop analysis to find unknown quantities in a circuit
 - ♦ Assign loop currents
 - ♦ Apply Kirchhoff's voltage law around each loop
 - ♦ Develop the loop equations
 - ♦ Solve the loop equations

A systematic method of loop analysis is given in the following steps and is illustrated in Figure 9-9, which is the same circuit configuration used in the branch current analysis. It demonstrates the basic principles well.

- Step 1.** Although direction of an assigned loop current is arbitrary, we will assign a current in the clockwise (CW) direction around each nonredundant closed loop, for consistency. This may not be the actual current direction, but it does not matter. The number of loop-current assignments must be sufficient to include current through all components in the circuit.
- Step 2.** Indicate the voltage drop polarities in each loop based on the assigned current directions.
- Step 3.** Apply Kirchhoff's voltage law around each closed loop. When more than one loop current passes through a component, include its voltage drop. This results in one equation for each loop.
- Step 4.** Using substitution or determinants, solve the resulting equations for the loop currents.

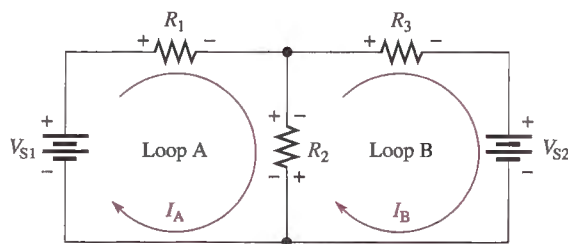


FIGURE 9-9

First, the loop currents I_A and I_B are assigned in the CW direction as shown in Figure 9-9. A loop current could be assigned around the outer perimeter of the circuit, but this would be redundant since I_A and I_B already pass through all of the components.

Second, the polarities of the voltage drops across R_1 , R_2 , and R_3 are shown based on the loop-current directions. Notice that I_A and I_B are in opposite directions through R_2 because R_2 is common to both loops. Therefore, two voltage polarities are indicated. In

reality, the R_2 current cannot be separated into two parts, but remember that the loop currents are basically mathematical quantities used for analysis purposes. The polarities of the voltage sources are fixed and are not affected by the current assignments.

Third, Kirchhoff's voltage law applied to the two loops results in the following two equations:

$$\begin{aligned} R_1 I_A + R_2(I_A - I_B) &= V_{S1} && \text{for loop A} \\ R_3 I_B + R_2(I_B - I_A) &= -V_{S2} && \text{for loop B} \end{aligned}$$

Notice that I_A is positive in loop A and I_B is positive in loop B.

Fourth, the like terms in the equations are combined and rearranged into standard form for convenient solution so that they have the same position in each equation, that is, the I_A term is first and the I_B term is second. The equations are rearranged into the following form. Once the loop currents are evaluated, all of the branch currents can be determined.

$$\begin{aligned} (R_1 + R_2)I_A - R_2 I_B &= V_{S1} && \text{for loop A} \\ -R_2 I_A + (R_2 + R_3)I_B &= -V_{S2} && \text{for loop B} \end{aligned}$$

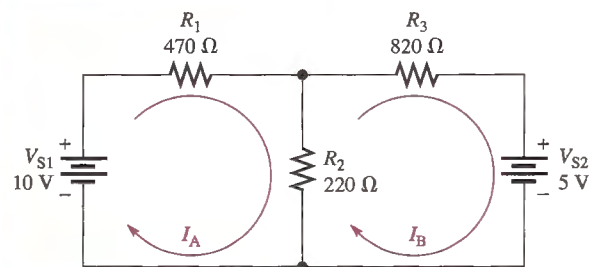
Notice that in the loop current method only two equations are required for the same circuit that required three equations in the branch current method. The last two equations (developed in the fourth step) follow a form to make loop analysis easier. Referring to these last two equations, notice that for loop A, the total resistance in the loop, $R_1 + R_2$, is multiplied by I_A (its loop current). Also in the loop A equation, the resistance common to both loops, R_2 , is multiplied by the other loop current, I_B , and subtracted from the first term. The same form is seen in the loop B equation except that the terms have been rearranged. From these observations, a concise rule for applying steps 1 to 4 is as follows:

(Sum of resistors in loop) times (loop current) minus (each resistor common to both loops) times (associated adjacent loop current) equals (source voltage in the loop).

Example 9–8 illustrates the application of this rule to the loop current analysis of a circuit.

EXAMPLE 9–8

Using the loop current method, find the branch currents in Figure 9–10.



▲ FIGURE 9–10

Solution Assign the loop currents (I_A and I_B) as shown in Figure 9–10; resistance values are in ohms and voltage values are in volts. Use the rule described to set up the two loop equations.

$$\begin{aligned} (470 + 220)I_A - 220I_B &= 10 \\ 690I_A - 220I_B &= 10 && \text{for loop A} \\ -220I_A + (220 + 820)I_B &= -5 \\ -220I_A + 1040I_B &= -5 && \text{for loop B} \end{aligned}$$

Use determinants to find I_A .

$$I_A = \frac{\begin{vmatrix} 10 & -220 \\ -5 & 1040 \end{vmatrix}}{\begin{vmatrix} 690 & -220 \\ -220 & 1040 \end{vmatrix}} = \frac{(10)(1040) - (-5)(-220)}{(690)(1040) - (-220)(-220)} = \frac{104000 - 1100}{717600 - 48400} = \frac{102900}{669200} = 13.9 \text{ mA}$$

Solving for I_B yields

$$I_B = \frac{\begin{vmatrix} 690 & 10 \\ -220 & -5 \end{vmatrix}}{669200} = \frac{(690)(-5) - (-220)(10)}{669200} = \frac{-3450 - (-2200)}{669200} = -1.87 \text{ mA}$$

The negative sign on I_B means that its assigned direction is opposite to the actual current.

Now find the actual branch currents. Since I_A is the only current through R_1 , it is also the branch current I_1 .

$$I_1 = I_A = 13.9 \text{ mA}$$

Since I_B is the only current through R_3 , it is also the branch current I_3 .

$$I_3 = I_B = -1.87 \text{ mA}$$

The negative sign indicates opposite direction of that originally assigned to I_B .

As originally assigned, both loop currents I_A and I_B are through R_2 in opposite directions. The branch current I_2 is the difference between I_A and I_B .

$$I_2 = I_A - I_B = 13.9 \text{ mA} - (-1.87 \text{ mA}) = 15.8 \text{ mA}$$

Keep in mind that once you know the branch currents, you can find the voltages by using Ohm's law. Notice that these results are the same as in Example 9-7 where the branch current method was used.

Related Problem Solve for the two loop currents using your calculator.



Use Multisim file E09-08 to verify the calculated results in this example and to verify your calculations for the related problem.

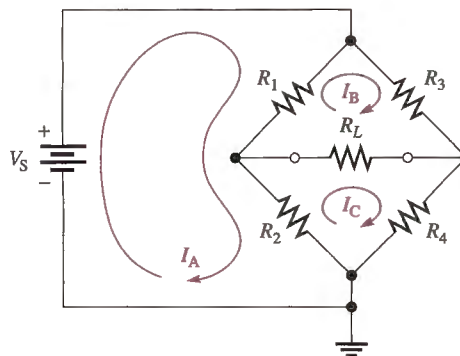
Circuits with More Than Two Loops

The loop current method can be systematically applied to circuits with any number of loops. Of course, the more loops there are, the more difficult is the solution, but calculators have greatly simplified solving simultaneous equations. Most circuits that you will encounter will not have more than three loops. Keep in mind that the loop currents are not the actual physical currents but are mathematical quantities assigned for analysis purposes.

A widely used circuit that you have already encountered is the Wheatstone bridge. The Wheatstone bridge was originally designed as a stand-alone measuring instrument but has largely been replaced with other instruments. However, the Wheatstone bridge circuit is incorporated in automated measuring instruments, and as explained previously, is widely used in the scale industry and in other measurement applications.

One method for solving the bridge parameters, which directly leads to finding the current in each arm of the bridge and the load current, is to write loop equations for the bridge. Figure 9-11 shows a Wheatstone bridge with three loops. Example 9-9 illustrates how to solve for all of the currents in the bridge.

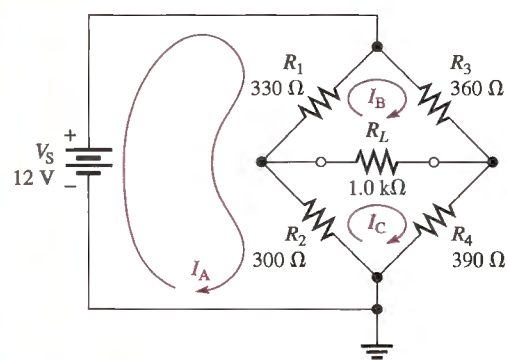
► **FIGURE 9–11**
Wheatstone bridge with three loops.



EXAMPLE 9–9

For the circuit in Figure 9–12, find the loop currents. Use the loop currents to solve for the current in each resistor (branch current).

► **FIGURE 9–12**



Solution Assign three clockwise loop currents (I_A , I_B , and I_C) as shown in Figure 9–12. Then write the loop equations. The equations for the loops are

$$\text{Loop A: } -12 + 330(I_A - I_B) + 330(I_A - I_C) = 0$$

$$\text{Loop B: } 330(I_B - I_A) + 360I_B + 1000(I_B - I_C) = 0$$

$$\text{Loop C: } 300(I_C - I_A) + 1000(I_C - I_B) + 390I_C = 0$$

Rearrange the equations into standard form:

$$\text{Loop A: } 630I_A - 330I_B - 300I_C = 12 \text{ V}$$

$$\text{Loop B: } -330I_A + 1690I_B - 1000I_C = 0$$

$$\text{Loop C: } -300I_A - 1000I_B + 1690I_C = 0$$

You can solve these equations with substitution, but this is tedious with three unknowns. The determinant method or directly solving with your calculator are simpler ways. Units are not shown until the end of the problem.

Evaluating the characteristic determinant using the expansion method,

$$\begin{vmatrix} 630 & -330 & -300 \\ -330 & 1690 & -1000 \\ -300 & -1000 & 1690 \end{vmatrix} \begin{vmatrix} 630 & -330 \\ -330 & 1690 \end{vmatrix}$$

$$= [(630)(1690)(1690) + (-330)(-1000)(-300) + (-300)(-330)(-1000)]$$

$$- [(-300)(1690)(-300) + (-1000)(-1000)(630) + (1690)(-330)(-330)]$$

$$= 635202000$$

Solving for I_A :

$$\frac{\begin{vmatrix} 12 & -330 & -300 \\ 0 & 1690 & -1000 \\ 0 & -1000 & 1690 \end{vmatrix}}{635202000} = \frac{(12)(1690)(1690) - (12)(-1000)(-1000)}{635202000} = 0.0351 \text{ A} = 35.1 \text{ mA}$$

Solving for I_B :

$$\frac{\begin{vmatrix} 630 & 12 & -300 \\ -330 & 0 & 1000 \\ -300 & 0 & 1690 \end{vmatrix}}{635202000} = \frac{(12)(-1000)(-300) - (-330)(12)(1690)}{635202000} = 0.0162 \text{ A} = 16.2 \text{ mA}$$

Solving for I_C :

$$\frac{\begin{vmatrix} 630 & -330 & 12 \\ -330 & 1690 & 0 \\ -300 & -1000 & 0 \end{vmatrix}}{635202000} = \frac{(12)(-330)(-1000) - (-300)(1690)(12)}{635202000} = 0.0158 \text{ A} = 15.8 \text{ mA}$$

The current in R_1 is the difference between I_A and I_B :

$$I_1 = (I_A - I_B) = 35.1 \text{ mA} - 16.2 \text{ mA} = \mathbf{18.9 \text{ mA}}$$

The current in R_2 is the difference between I_A and I_C :

$$I_2 = (I_A - I_C) = 35.1 \text{ mA} - 15.8 \text{ mA} = \mathbf{19.3 \text{ mA}}$$

The current in R_3 is I_B :

$$I_3 = I_B = \mathbf{16.2 \text{ mA}}$$

The current in R_4 is I_C :

$$I_4 = I_C = \mathbf{15.8 \text{ mA}}$$

The current in R_L is the difference between I_B and I_C :

$$I_L = (I_B - I_C) = 16.2 \text{ mA} - 15.8 \text{ mA} = \mathbf{0.4 \text{ mA}}$$

Related Problem Use a calculator to verify the loop currents in this example.

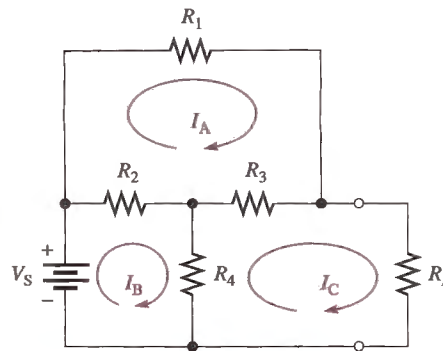


Use Multisim file E09-09 to verify the calculated results in this example and to confirm your calculations for the related problem.

Another useful three-loop circuit is the bridged-T circuit. While the circuit is primarily applied in ac filter circuits using reactive components, it is introduced here to illustrate the three-loop circuit solution. A loaded resistive bridged-T is shown in Figure 9-13.

Resistors will often be in $k\Omega$ (or even $M\Omega$) so the coefficients for simultaneous equations will become quite large if they are shown explicitly in solving equations. To simplify entering and solving equations with $k\Omega$, it is common practice to drop the $k\Omega$ in the equations and recognize that the unit for current is the mA if the voltage is volts. The following example of a bridged-T circuit illustrates this idea.

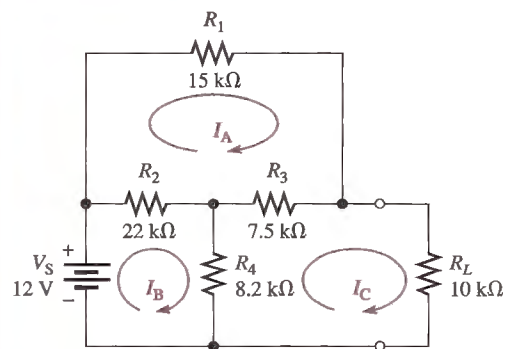
► FIGURE 9-13



EXAMPLE 9-10

Figure 9-14 shows a bridged-T circuit with three loops. Set up the standard form equations for the loop currents. Solve the equations with a calculator and find the current in each resistor.

► FIGURE 9-14



Solution Assign three clockwise loop currents (I_A , I_B , and I_C) as shown in Figure 9-14. Write the loop equations, but drop the k prefix from the resistances. Current will be in mA.

$$\text{Loop A: } 22(I_A - I_B) + 15I_A + 7.5(I_A - I_C) = 0$$

$$\text{Loop B: } -12 + 22(I_B - I_A) + 8.2(I_B - I_C) = 0$$

$$\text{Loop C: } 8.2(I_C - I_B) + 7.5(I_C - I_A) + 10I_C = 0$$

Rearrange the equations into standard form:

$$\text{Loop A: } 44.5I_A - 22I_B - 7.5I_C = 0$$

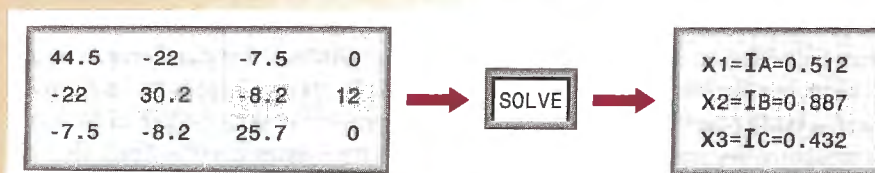
$$\text{Loop B: } -22I_A + 30.2I_B - 8.2I_C = 12$$

$$\text{Loop C: } -7.5I_A - 8.2I_B + 25.7I_C = 0$$

Calculator Solution: A calculator solution requires entry of the number of equations (3), the coefficients, and the constants. The calculator SOLVE function produces the results as shown in Figure 9-15. Because the resistors were in $\text{k}\Omega$, the unit for the loop currents is mA. Solve for the current in each resistor. The current in $R_1 = I_A$.

$$I_1 = 0.512\text{ mA}$$

► FIGURE 9-15



The current in R_2 is the difference between I_A and I_B .

$$I_2 = (I_A - I_B) = 0.512 \text{ mA} - 0.887 \text{ mA} = -0.375 \text{ mA}$$

The negative sign indicates that current is the opposite direction to I_A ; the positive side of the resistor is the right side.

The current in R_3 is $I_A - I_C$.

$$I_3 = 0.512 \text{ mA} - 0.432 \text{ mA} = 0.08 \text{ mA}$$

The current in R_4 is $I_B - I_C$.

$$I_4 = I_B - I_C = 0.887 \text{ mA} - 0.432 \text{ mA} = 0.455 \text{ mA}$$

The current in R_L is I_C .

$$I_L = 0.432 \text{ mA}$$

Related Problem Find the voltage across each resistor.



Use Multisim file E09-10 to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 9-3 REVIEW

1. Do the loop currents necessarily represent the actual currents in the branches?
2. When you solve for a current using the loop method and get a negative value, what does it mean?
3. What circuit law is used in the loop current method?

9-4 NODE VOLTAGE METHOD

Another method of analysis of multiple-loop circuits is called the node voltage method. It is based on finding the voltages at each node in the circuit using Kirchhoff's current law. Recall that a node is the junction of two or more components.

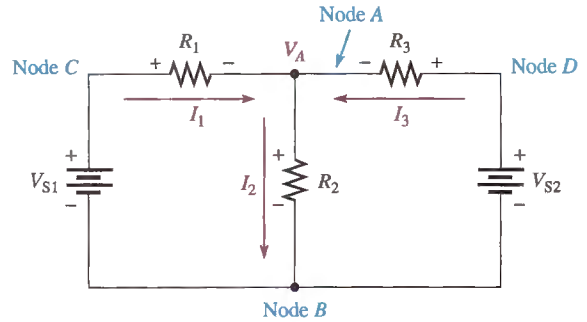
After completing this section, you should be able to

- ♦ Use node analysis to find unknown quantities in a circuit
 - ♦ Select the nodes at which the voltage is unknown and assign currents
 - ♦ Apply Kirchhoff's current law at each node
 - ♦ Develop and solve the node equations

The general steps for the node voltage method of circuit analysis are as follows:

- Step 1.** Determine the number of nodes.
- Step 2.** Select one node as a reference. All voltages will be relative to the reference node. Assign voltage designations to each node where the voltage is unknown.
- Step 3.** Assign currents at each node where the voltage is unknown, except at the reference node. The directions are arbitrary.
- Step 4.** Apply Kirchhoff's current law to each node where currents are assigned.
- Step 5.** Express the current equations in terms of voltages, and solve the equations for the unknown node voltages using Ohm's law.

► **FIGURE 9-16**
Circuit for node voltage analysis.



We will use Figure 9–16 to illustrate the general approach to node voltage analysis. First, establish the nodes. In this case, there are four nodes, as indicated in the figure. Second, let's use node *B* as the reference. Think of it as the circuit's reference ground. Node voltages *C* and *D* are already known to be the source voltages. The voltage at node *A* is the only unknown; it is designated as V_A . Third, arbitrarily assign the branch currents at node *A* as indicated in the figure. Fourth, the Kirchhoff current equation at node *A* is

$$I_1 - I_2 + I_3 = 0$$

Fifth, express the currents in terms of circuit voltages using Ohm's law.

$$I_1 = \frac{V_1}{R_1} = \frac{V_{S1} - V_A}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_A}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{S2} - V_A}{R_3}$$

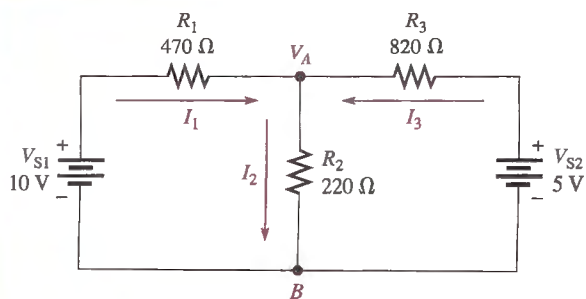
Substituting these terms into the current equation yields

$$\frac{V_{S1} - V_A}{R_1} - \frac{V_A}{R_2} + \frac{V_{S2} - V_A}{R_3} = 0$$

The only unknown is V_A ; so solve the single equation by combining and rearranging terms. Once you know the voltage, you can calculate all branch currents. Example 9–11 illustrates this method further.

EXAMPLE 9-11

Find the node voltage V_A in Figure 9–17 and determine the branch currents.



▲ **FIGURE 9-17**

Solution The reference node is chosen at B . The unknown node voltage is V_A , as indicated in Figure 9–17. This is the only unknown voltage. Branch currents are assigned at node A as shown. The current equation is

$$I_1 - I_2 + I_3 = 0$$

Substitution for currents using Ohm's law gives the equation in terms of voltages.

$$\frac{10 - V_A}{470} - \frac{V_A}{220} + \frac{5 - V_A}{820} = 0$$

Rearranging the terms yields

$$\begin{aligned} \frac{10}{470} - \frac{V_A}{470} - \frac{V_A}{220} + \frac{5}{820} - \frac{V_A}{820} &= 0 \\ -\frac{V_A}{470} - \frac{V_A}{220} - \frac{V_A}{820} &= -\frac{10}{470} - \frac{5}{820} \end{aligned}$$

To solve for V_A , combine the terms on each side of the equation and find the common denominator.

$$\begin{aligned} \frac{1804V_A + 3854V_A + 1034V_A}{847880} &= \frac{820 + 235}{38540} \\ \frac{6692V_A}{847880} &= \frac{1055}{38540} \\ V_A &= \frac{(1055)(847880)}{(6692)(38540)} = 3.47 \text{ V} \end{aligned}$$

You can now determine the branch currents.

$$I_1 = \frac{10 \text{ V} - 3.47 \text{ V}}{470 \ \Omega} = 13.9 \text{ mA}$$

$$I_2 = \frac{3.47 \text{ V}}{220 \ \Omega} = 15.8 \text{ mA}$$

$$I_3 = \frac{5 \text{ V} - 3.47 \text{ V}}{820 \ \Omega} = 1.87 \text{ mA}$$

These results agree with those for the same circuit in Example 9–7 and in Example 9–8 using the branch and loop current methods.

Related Problem Find V_A in Figure 9–17 if the 5 V source is reversed.



Use Multisim file E09-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

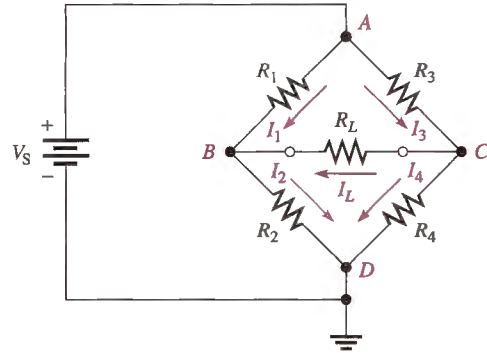
Example 9–11 illustrated an obvious advantage to the node method. The branch current method required three equations for the three unknown currents. The loop current method reduced the number of simultaneous equations but required the extra step of converting the fictitious loop currents to the actual currents in the resistors. The node method for the circuit in Figure 9–17 reduced the equations to one, in which all of the currents were written in terms of one unknown node voltage. The node voltage method also has the advantage of finding unknown voltages, which are easier to directly measure than current.

Node Voltage Method for a Wheatstone Bridge

The node voltage method can be applied to a Wheatstone bridge. The Wheatstone bridge is shown with nodes identified in Figure 9–18 with currents shown. Node D is usually selected as the reference node, and node A has the same potential as the source voltage.

► FIGURE 9-18

Wheatstone bridge with node assignments.



When setting up the equations for the two unknown node voltages (B and C), it is necessary to specify a current direction as described in the general steps. The direction of current in R_L is dependent on the bridge resistances; if the assigned direction is incorrect, it will show up as a negative current in the solution.

Kirchhoff's current law is then written for each of the unknown nodes. Each current is then expressed in terms of node voltages using Ohm's law as follows:

Node B :

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

Node C :

$$I_3 = I_L + I_4$$

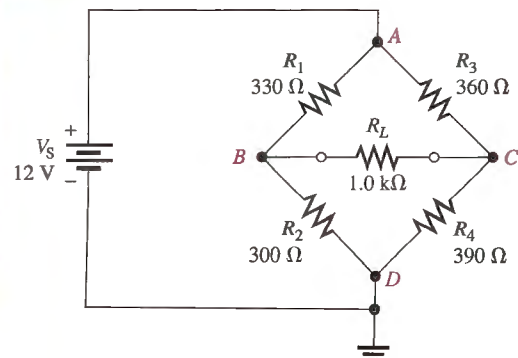
$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

The equations are put in standard form and can be solved with any of the methods you have learned. The following example illustrates this for the Wheatstone bridge that was solved by loop equations in Example 9-9.

EXAMPLE 9-12

For the circuit in Figure 9-19, find the node voltages at node B and node C . Node D is the reference, and node A has the same voltage as the source. Use the results to calculate the current in each resistor. Compare the result to the loop current method in Example 9-9.

► FIGURE 9-19



Solution Apply Kirchhoff's current law at node B and node C in terms of node voltages. To keep coefficients more manageable, all resistances are shown in $k\Omega$; current will be in mA.

Node B :

$$I_1 + I_L = I_2$$

$$\frac{V_A - V_B}{R_1} + \frac{V_C - V_B}{R_L} = \frac{V_B}{R_2}$$

$$\frac{12 - V_B}{0.330 \text{ k}\Omega} + \frac{V_C - V_B}{1.0 \text{ k}\Omega} = \frac{V_B}{0.300 \text{ k}\Omega}$$

Node C :

$$I_3 = I_L + I_4$$

$$\frac{V_A - V_C}{R_3} = \frac{V_C - V_B}{R_L} + \frac{V_C}{R_4}$$

$$\frac{12 \text{ V} - V_C}{0.360 \text{ k}\Omega} = \frac{V_C - V_B}{1.0 \text{ k}\Omega} + \frac{V_C}{0.390 \text{ k}\Omega}$$

Rearrange the equations for each node into standard form. For convenience, units are not shown until the end of the problem.

Node B : Multiply each term in the expression for node B by $R_1R_2R_L$ and combine like terms to get the standard form.

$$R_2R_L(V_A - V_B) + R_1R_2(V_C - V_B) = R_1R_LV_B$$

$$(1.0)(0.30)(12 - V_B) + (0.33)(0.30)(V_C - V_B) = (0.33)(1.0)V_B$$

$$0.729V_B - 0.099V_C = 3.6$$

Node C : Multiply each term in the expression for node C by $R_3R_4R_L$ and combine like terms to get the standard form.

$$R_4R_L(V_A - V_C) = R_3R_4(V_C - V_B) + R_3R_LV_C$$

$$(1.0)(0.39)(12 - V_C) = (0.36)(0.39)(V_C - V_B) + (0.36)(1.0)V_C$$

$$0.1404V_B - 0.8904V_C = -4.68$$

You can solve the two simultaneous equations using substitution, determinants, or the calculator. Solving by determinants,

$$0.729V_B - 0.099V_C = 3.6$$

$$0.1404V_B - 0.8904V_C = -4.68$$

$$V_B = \frac{\begin{vmatrix} 3.6 & -0.099 \\ -4.68 & -0.8904 \end{vmatrix}}{\begin{vmatrix} 0.729 & -0.099 \\ 0.1404 & -0.8904 \end{vmatrix}} = \frac{(3.6)(-0.8904) - (-0.099)(-4.68)}{(0.729)(-0.8904) - (0.1404)(-0.099)} = 5.78 \text{ V}$$

$$V_C = \frac{\begin{vmatrix} 0.729 & 3.6 \\ 0.1404 & -4.68 \end{vmatrix}}{\begin{vmatrix} 0.729 & -0.099 \\ 0.1404 & -0.8904 \end{vmatrix}} = \frac{(0.729)(-4.68) - (0.1404)(3.6)}{(0.729)(-0.8904) - (0.1404)(-0.099)} = 6.17 \text{ V}$$

Related Problem Using Ohm's law, determine the current in each resistor.



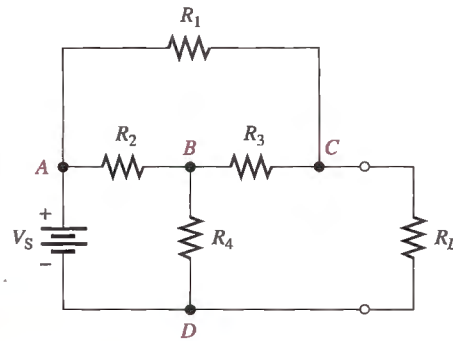
Use Multisim file E09-12 to verify the calculated results in this example and to confirm your calculations for the related problem.

Node Voltage Method for the Bridged-T Circuit

Applying the node voltage method to the bridged-T circuit also results in two equations with two unknowns. As in the case of the Wheatstone bridge, there are four nodes as shown in Figure 9–20. Node D is the reference and node A is the source voltage, so the two unknown voltages are at nodes C and D . The effect of a load resistor on the circuit is usually the most important question, so the voltage at node C is the focus. A calculator solution of the simultaneous equations is simplified for analyzing the effect of various loads because only the equation for node C is affected when the load changes. Example 9–13 illustrates this idea.

► **FIGURE 9–20**

The bridged-T circuit with node assignments.

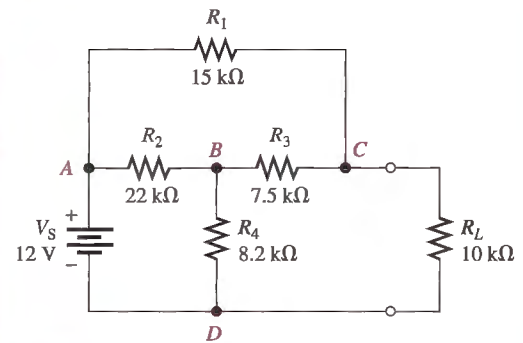


EXAMPLE 9–13

The circuit in Figure 9–21 is the same as in Example 9–10.

- Solve for the voltage across R_L using node analysis and a calculator.
- Find the effect on the load voltage when the load resistor is changed to $15\text{ k}\Omega$.

► **FIGURE 9–21**



Solution (a) Apply Kirchhoff's current law at nodes B and C in terms of node voltages. To keep coefficients more manageable, all resistances are shown in $\text{k}\Omega$. The current will be in mA .

Node B :

$$I_2 = I_3 + I_4$$

$$\frac{V_A - V_B}{R_2} = \frac{V_B - V_C}{R_3} + \frac{V_B}{R_4}$$

$$\frac{12 - V_B}{22\text{ k}\Omega} = \frac{V_B - V_C}{7.5\text{ k}\Omega} + \frac{V_B}{8.2\text{ k}\Omega}$$

Node C:

$$I_1 + I_3 = I_L$$

$$\frac{V_A - V_C}{R_1} + \frac{V_B - V_C}{R_3} = \frac{V_C}{R_L}$$

$$\frac{12\text{ V} - V_C}{15\text{ k}\Omega} + \frac{V_B - V_C}{7.5\text{ k}\Omega} = \frac{V_C}{10\text{ k}\Omega}$$

Rearrange the equations for each node into standard form. For convenience, units are not shown until the end of the problem.

Node B: Multiply each term in the equation for node B by $R_2R_3R_4$ to cancel the denominator. Combine like terms to get the standard form.

$$R_3R_4(V_A - V_B) = R_2R_4(V_B - V_C) + R_2R_3V_B$$

$$(7.5)(8.2)(12 - V_B) = (22)(8.2)(V_B - V_C) + (22)(7.5)V_B$$

$$406.9V_B - 180.4V_C = 738$$

Node C: Multiply each term in the equation for node C by $R_1R_3R_L$ and combine like terms to get the standard form.

$$R_3R_L(V_A - V_C) + R_1R_L(V_B - V_C) = R_1R_3V_C$$

$$(7.5)(10)(12 - V_C) + (15)(10)(V_B - V_C) = (15)(7.5)V_C$$

$$150V_B - 337.5V_C = -900$$

Calculator Solution: The two equations in standard form are

$$406.9V_B - 180.4V_C = 738$$

$$150V_B - 337.5V_C = -900$$

Enter the number of equations (2), the coefficients, and the constants into a calculator to solve for V_B and V_C as shown in Figure 9–22. As a check, notice that this voltage implies that the load current is 0.432 mA, which is in agreement with the result found by the loop current method in Example 9–10.

FIGURE 9–22



(b) To calculate the load voltage with a 15 k Ω load resistor, notice that the equation for node B is unaffected. The node C equation is modified as follows:

$$\frac{12\text{ V} - V_C}{15\text{ k}\Omega} + \frac{V_B - V_C}{7.5\text{ k}\Omega} = \frac{V_C}{15\text{ k}\Omega}$$

$$(7.5)(15)(12 - V_C) + (15)(15)(V_B - V_C) = (15)(7.5)V_C$$

$$225V_B - 450V_C = -1350$$

Change the parameters for the node C equation and press solve. The result is

$$V_C = V_L = 5.02\text{ V}$$

Related Problem

For the 15 k Ω load, what is the voltage at node B?



Use Multisim file E09-13 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 9-5
REVIEW

1. What circuit law is the basis for the node voltage method?
2. What is the reference node?



A Circuit Application

Dependent sources were covered in Chapter 8 and applied to the modeling of transistors and amplifiers. In this circuit application, you will see how a particular type of amplifier can be modeled and analyzed using the methods introduced in this chapter. The point is not to learn how amplifiers work because that is beyond the scope of this text and will be covered in a later course. The focus is on the application of circuit analysis methods to circuit models. The amplifier is simply used as an example to illustrate how you can apply analysis methods to a practical circuit.

Operational amplifiers are integrated circuit devices that are widely used in analog applications for signal processing. An operational amplifier symbol is shown in Figure 9-23(a). The equivalent dependent source model is shown in part (b). The gain (A) of the dependent source can be positive or negative, depending on how it is configured.

Assume you need to calculate in detail the effect of an operational amplifier circuit on a transducer that serves as an input. Some transducers, such as pH meters, appear as a small source voltage with a high series resistance. The transducer that is shown here is modeled as a small Thevenin dc voltage source in series with a Thevenin resistance of $10\text{ k}\Omega$.

Practical amplifiers are created using an operational amplifier with external components. Figure 9-24(a) shows one type of amplifier configuration that includes the Thevenin resistance of the source along with two other external resistors. R_S represents the Thevenin source resistance. R_L is connected from the operational amplifier output to ground as a load, and R_F is connected from the output to the input as a feedback resistor. Feedback is used in most operational amplifier circuits and is simply a path from the output back to the input. It has many advantages as you will learn in a later course.

FIGURE 9-23

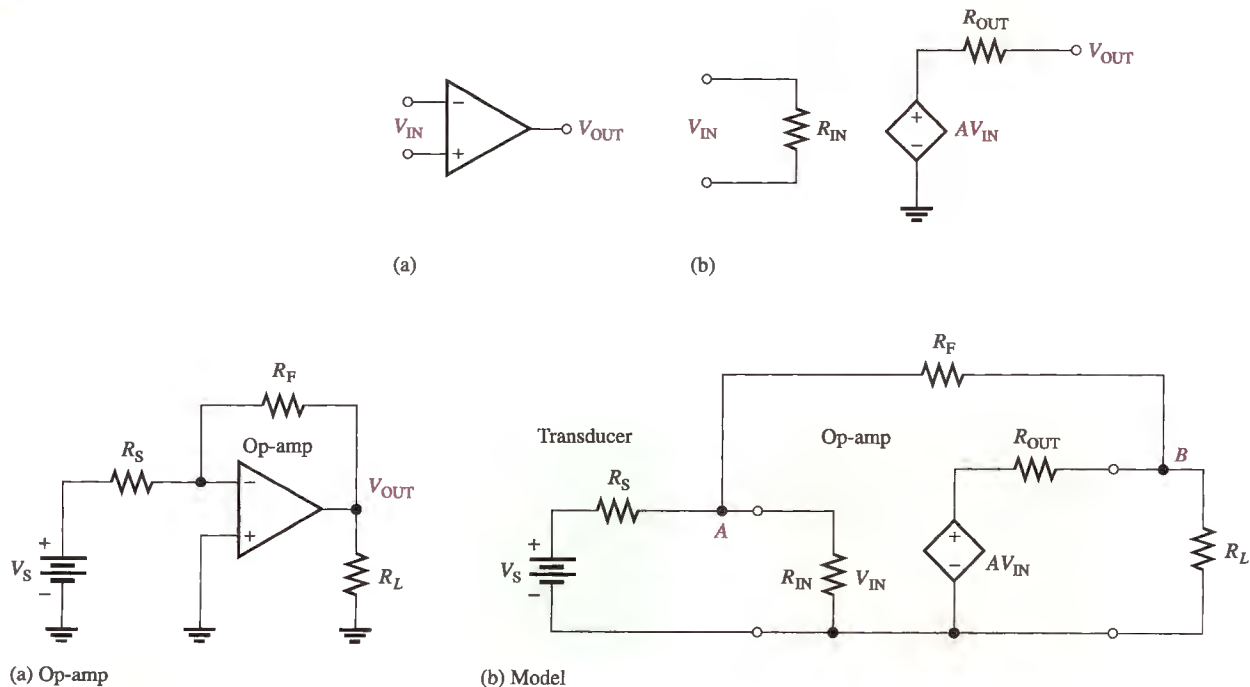


FIGURE 9-24

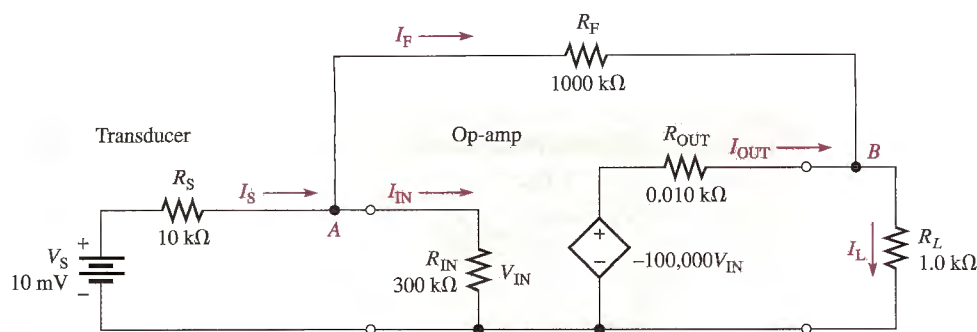


FIGURE 9-25

Figure 9-24(b) is the equivalent circuit model of the source, the op-amp, and the load that we will use for analysis purposes. The internal gain of the dependent source, shown with the letter A on the op-amp block, is negative because it is an inverting amplifier (the output has the opposite sign of the input). This internal gain is generally extremely high. Although it is a very large number, the actual gain of the circuit with the external components is much lower because it is controlled by the external components rather than the internal gain.

The specific values for the circuit in this application are given in Figure 9-25 along with assigned currents. All values are shown in $k\Omega$ to simplify entering the coefficients in the equations that you will use. Although op-amp circuits have excellent simple approximations for determining the output voltage, there are times you may want to know the exact output. You can apply your knowledge from this chapter to the circuit to find the exact output voltage.

The amplifier model in Figure 9-25 can be analyzed easiest with the node voltage method because there are only two nodes, A and B , that have unknown voltages. At node A , the voltage is designated V_A and is the same as the input to the op-amp (V_{IN}). At node B , the voltage is designated V_B and also represents the output (or load voltage), V_L . Current names and directions are assigned as shown.

Analysis

Apply Kirchhoff's current law at each of the unknown nodes to write the node equations.

$$\text{Node } A: I_S = I_F + I_{IN}$$

$$\text{Node } B: I_{OUT} + I_F = I_L$$

Next, apply Ohm's law and let $V_{IN} = V_A$. The internal source voltage for the op-amp is AV_{IN} , so this is written as AV_A to express the unknowns in terms of V_A and V_B .

$$\text{Node } A: \frac{V_S - V_A}{R_S} = \frac{V_A}{R_{IN}} + \frac{V_A - V_B}{R_F}$$

$$\text{Node } B: \frac{AV_A - V_B}{R_{OUT}} + \frac{V_A - V_B}{R_F} = \frac{V_B}{R_L}$$

Expressing the equations in standard form,

$$\text{Node } A: -\left(\frac{1}{R_S} + \frac{1}{R_{IN}} + \frac{1}{R_F}\right)V_A + \left(\frac{1}{R_F}\right)V_B = -\left(\frac{1}{R_S}\right)V_S$$

$$\text{Node } B: -\left(\frac{A}{R_{OUT}} + \frac{1}{R_F}\right)V_A + \left(\frac{1}{R_L} + \frac{1}{R_{OUT}} + \frac{1}{R_F}\right)V_B = 0$$

- ◆ Substitute the values that were given in Figure 9-25 into the standard form equations. Solve the equations to find V_{IN} and V_L . (Resistance can be entered in $k\Omega$.)
- ◆ Calculate the input current, I_{IN} , and the current in the feedback resistor, I_F .

Review

1. Does the output voltage change if the load resistor R_L is doubled?
2. Does the output voltage change if the feedback resistor R_F is doubled?

SUMMARY

- ◆ Simultaneous equations can be solved by substitution, by determinants, or by a graphics calculator.
- ◆ The number of equations must be equal to the number of unknowns.
- ◆ Second-order determinants are evaluated by adding the signed cross-products.
- ◆ Third-order determinants are evaluated by the expansion method.
- ◆ The branch current method is based on Kirchhoff's voltage law and Kirchhoff's current law.

- ♦ The loop current method is based on Kirchhoff's voltage law.
- ♦ A loop current is not necessarily the actual current in a branch.
- ♦ The node voltage method is based on Kirchhoff's current law.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Branch One current path that connects two nodes.

Determinant The solution of a matrix consisting of an array of coefficients and constants for a set of simultaneous equations.

Loop A closed current path in a circuit.

Matrix An array of numbers.

Node The junction of two or more components.

Simultaneous equations A set of n equations containing n unknowns, where n is a number with a value of 2 or more.

SELF-TEST

Answers are at the end of the chapter.

1. Assuming the voltage source values in Figure 9–6 are known, there is/are
 - (a) 3 nonredundant loops
 - (b) 1 unknown node
 - (c) 2 nonredundant loops
 - (d) 2 unknown nodes
 - (e) both answers (b) and (c)
2. In assigning the direction of branch currents,
 - (a) the directions are critical
 - (b) they must all be in the same direction
 - (c) they must all point into a node
 - (d) the directions are not critical
3. The branch current method uses
 - (a) Ohm's law and Kirchhoff's voltage law
 - (b) Kirchhoff's voltage and current laws
 - (c) the superposition theorem and Kirchhoff's current law
 - (d) Thevenin's theorem and Kirchhoff's voltage law
4. A characteristic determinant for two simultaneous equations will have
 - (a) 2 rows and 1 column
 - (b) 1 row and 2 columns
 - (c) 2 rows and 2 columns
5. The first row of a certain determinant has the numbers 2 and 4. The second row has the numbers 6 and 1. The value of this determinant is
 - (a) 22
 - (b) 2
 - (c) -22
 - (d) 8
6. The expansion method for evaluating determinants is
 - (a) good only for second-order determinants
 - (b) good only for both second and third-order determinants
 - (c) good for any determinant
 - (d) easier than using a calculator
7. The loop current method is based on
 - (a) Kirchhoff's current law
 - (b) Ohm's law
 - (c) the superposition theorem
 - (d) Kirchhoff's voltage law
8. The node voltage method is based on
 - (a) Kirchhoff's current law
 - (b) Ohm's law
 - (c) the superposition theorem
 - (d) Kirchhoff's voltage law
9. In the node voltage method,
 - (a) currents are assigned at each node
 - (b) currents are assigned at the reference node
 - (c) the current directions are arbitrary

- (d) currents are assigned only at the nodes where the voltage is unknown
 (e) both answers (c) and (d)
10. Generally, the node voltage method results in
 (a) more equations than the loop current method
 (b) fewer equations than the loop current method
 (c) the same number of equations as the loop current method

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 9–26.

- If R_2 opens, the current through R_3
 (a) increases (b) decreases (c) stays the same
- If the 6 V source shorts out, the voltage at point A with respect to ground
 (a) increases (b) decreases (c) stays the same
- If R_2 becomes disconnected from ground, the voltage at point A with respect to ground
 (a) increases (b) decreases (c) stays the same

Refer to Figure 9–27.

- If the current source fails open, the current through R_2
 (a) increases (b) decreases (c) stays the same
- If R_2 opens, the current through R_3
 (a) increases (b) decreases (c) stays the same

Refer to Figure 9–30.

- If R_1 opens, the magnitude of the voltage between the A and B terminals
 (a) increases (b) decreases (c) stays the same
- If R_3 is replaced by a $10\ \Omega$ resistor, V_{AB}
 (a) increases (b) decreases (c) stays the same
- If point B shorts to the negative side of the source, V_{AB}
 (a) increases (b) decreases (c) stays the same
- If the negative side of the source is grounded, V_{AB}
 (a) increases (b) decreases (c) stays the same

Refer to Figure 9–32.

- If a voltage source V_{S2} fails open, the voltage at A with respect to ground
 (a) increases (b) decreases (c) stays the same
- If a short develops from point A to ground, the current through R_3
 (a) increases (b) decreases (c) stays the same
- If R_2 opens, the voltage across R_3
 (a) increases (b) decreases (c) stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
 Answers to odd-numbered problems are at the end of the book.

SECTION 9–1 Simultaneous Equations in Circuit Analysis

- Using the substitution method, solve the following set of equations for I_{R1} and I_{R2} .

$$100I_1 + 50I_2 = 30$$

$$75I_1 + 90I_2 = 15$$

2. Evaluate each determinant:

$$(a) \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} \quad (b) \begin{vmatrix} 9 & -1 \\ 0 & 5 \end{vmatrix} \quad (c) \begin{vmatrix} 12 & 15 \\ -2 & -1 \end{vmatrix} \quad (d) \begin{vmatrix} 100 & 50 \\ 30 & -20 \end{vmatrix}$$

3. Using determinants, solve the following set of equations for both currents:

$$\begin{aligned} -I_1 + 2I_2 &= 4 \\ 7I_1 + 3I_2 &= 6 \end{aligned}$$

4. Evaluate each of the determinants:

$$(a) \begin{vmatrix} 1 & 0 & -2 \\ 5 & 4 & 1 \\ 2 & 10 & 0 \end{vmatrix} \quad (b) \begin{vmatrix} 0.5 & 1 & -0.8 \\ 0.1 & 1.2 & 1.5 \\ -0.1 & -0.3 & 5 \end{vmatrix}$$

5. Evaluate each of the determinants:

$$(a) \begin{vmatrix} 25 & 0 & -20 \\ 10 & 12 & 5 \\ -8 & 30 & -16 \end{vmatrix} \quad (b) \begin{vmatrix} 1.08 & 1.75 & 0.55 \\ 0 & 2.12 & -0.98 \\ 1 & 3.49 & -1.05 \end{vmatrix}$$

6. Find I_3 in Example 9-4.

7. Solve for I_1, I_2, I_3 in the following set of equations using determinants:

$$\begin{aligned} 2I_1 - 6I_2 + 10I_3 &= 9 \\ 3I_1 + 7I_2 - 8I_3 &= 3 \\ 10I_1 + 5I_2 - 12I_3 &= 0 \end{aligned}$$

*8. Find $V_1, V_2, V_3,$ and V_4 from the following set of equations using the calculator:

$$\begin{aligned} 16V_1 + 10V_2 - 8V_3 - 3V_4 &= 15 \\ 2V_1 + 0V_2 + 5V_3 + 2V_4 &= 0 \\ -7V_1 - 12V_2 + 0V_3 + 0V_4 &= 9 \\ -1V_1 + 20V_2 - 18V_3 + 0V_4 &= 10 \end{aligned}$$

9. Solve the two simultaneous equations in Problem 1 using your calculator.

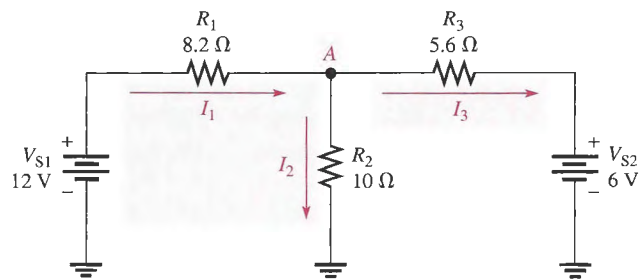
10. Solve the three simultaneous equations in Problem 7 using your calculator.

SECTION 9-2 Branch Current Method

11. Write the Kirchhoff current equation for the current assignment shown at node A in Figure 9-26.

12. Solve for each of the branch currents in Figure 9-26.

13. Find the voltage drop across each resistor in Figure 9-26 and indicate its actual polarity.

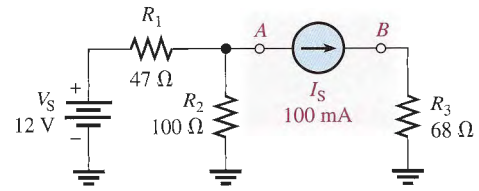


▲ FIGURE 9-26

*14. Find the current through each resistor in Figure 9-27.

15. In Figure 9-27, determine the voltage across the current source (points A and B).

▶ FIGURE 9-27



SECTION 9-3 Loop Current Method

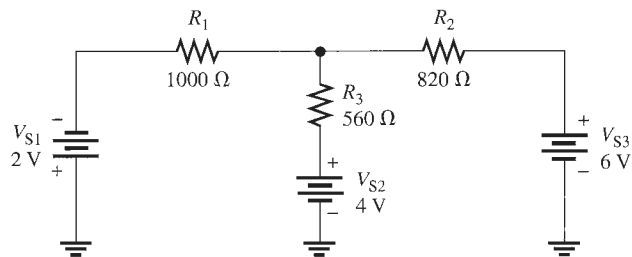
16. Write the characteristic determinant for the equations:

$$\begin{aligned} 0.045I_A + 0.130I_B + 0.066I_C &= 0 \\ 0.177I_A + 0.0420I_B + 0.109I_C &= 12 \\ 0.078I_A + 0.196I_B + 0.029I_C &= 3.0 \end{aligned}$$

17. Using the loop current method, find the loop currents in Figure 9-28.

18. Find the branch currents in Figure 9-28.

19. Determine the voltages and their proper polarities for each resistor in Figure 9-28.

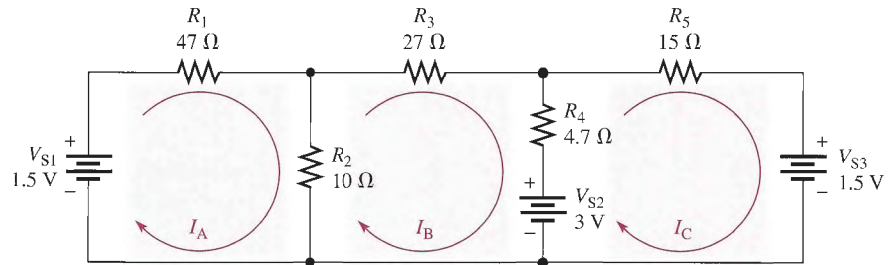


▲ FIGURE 9-28

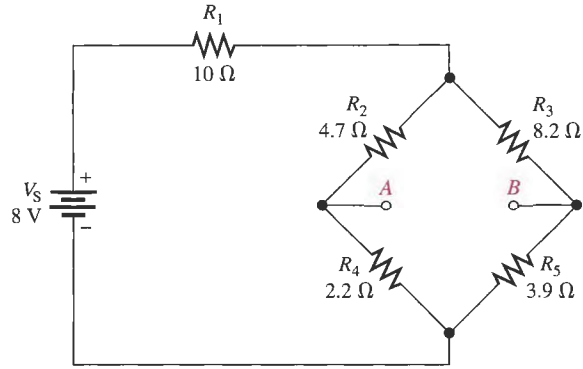
20. Write the loop equations for the circuit in Figure 9-29.

21. Solve for the loop currents in Figure 9-29 using your calculator.

22. Find the current through each resistor in Figure 9-29.

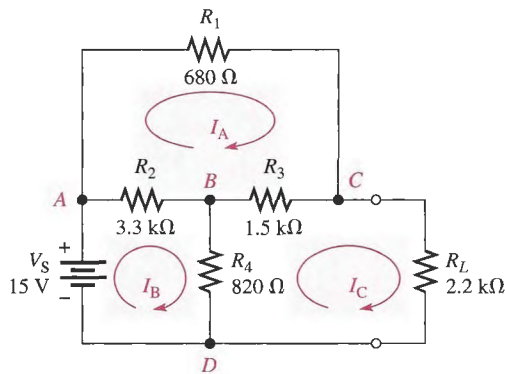


▲ FIGURE 9-29



▲ FIGURE 9-30

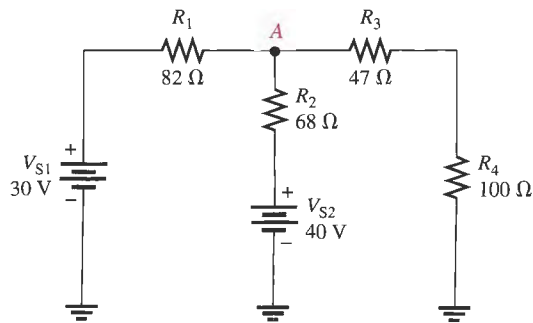
23. Determine the voltage across the open bridge terminals, *A* and *B*, in Figure 9-30.
24. When a $10\ \Omega$ resistor is connected from terminal *A* to terminal *B* in Figure 9-30, what is the current through it?
25. Write the loop equations in standard form for the bridged-T circuit in Figure 9-31.



▲ FIGURE 9-31

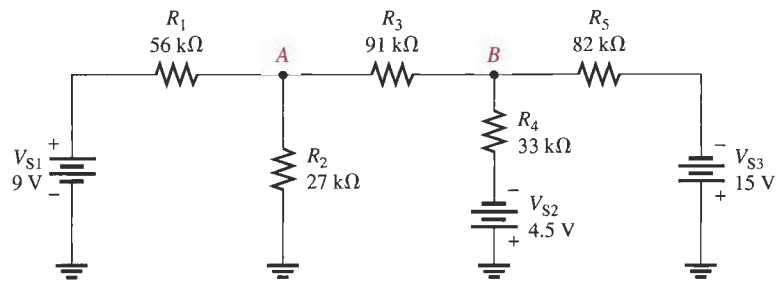
SECTION 9-4 Node Voltage Method

26. In Figure 9-32, use the node voltage method to find the voltage at point *A* with respect to ground.
27. What are the branch current values in Figure 9-32? Show the actual direction of current in each branch.
28. Write the node voltage equations for Figure 9-29. Use your calculator to find the node voltages.



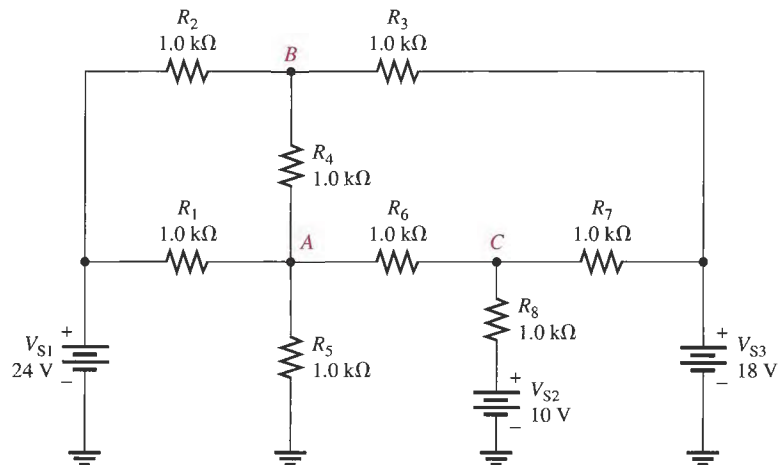
▲ FIGURE 9-32

29. Use node analysis to determine the voltage at points *A* and *B* with respect to ground in Figure 9–33.



▲ FIGURE 9–33

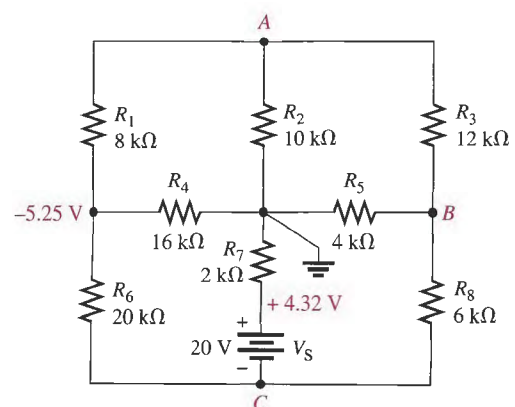
- *30. Find the voltage at points *A*, *B*, and *C* in Figure 9–34.



▲ FIGURE 9–34

- *31. Use node analysis, loop analysis, or any other procedure to find all currents and the voltages at each unknown node in Figure 9–35.

► FIGURE 9–35





Multisim Troubleshooting and Analysis

These problems require your Multisim CD-ROM.

32. Open file P09-32 and measure the current through each resistor.
33. Open file P09-33 and measure the current through each resistor.
34. Open file P09-34 and measure the voltages with respect to ground at nodes *A* and *B*.
35. Open file P09-35. Determine if there is a fault and, if so, specify the fault.
36. Open file P09-36 and measure the voltages with respect to ground at output terminals 1 and 2.
37. Open file P09-37 and determine what the fault is.
38. Open file P09-38 and determine what the fault is.
39. Open file P09-39 and determine what the fault is.

ANSWERS

SECTION REVIEWS

SECTION 9-1 Simultaneous Equations in Circuit Analysis

1. (a) 4 (b) 0.415 (c) -98

$$2. \begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix}$$

$$3. -0.286 \text{ A} = -286 \text{ mA}$$

$$4. I_1 = -.038893513289$$

$$I_2 = .084110232475$$

$$I_3 = .041925798204$$

$$I_4 = -.067156192401$$

$$5. I_1 = -.056363148617$$

$$I_2 = .07218287729$$

$$I_3 = .065684612774$$

$$I_4 = -.041112571034$$

SECTION 9-2 Branch Current Method

1. Kirchhoff's voltage law and Kirchhoff's current law are used in the branch current method.
2. False, but write the equations so that they are consistent with your assigned directions.
3. A loop is a closed path within a circuit.
4. A node is a junction of two or more components.

SECTION 9-3 Loop Current Method

1. No, loop currents are not necessarily the same as branch currents.
2. A negative value means the direction should be reversed.
3. Kirchhoff's voltage law is used in loop analysis.

SECTION 9-4 Node Voltage Method

1. Kirchhoff's current law is the basis for node analysis.
2. A reference node is the junction to which all circuit voltages are referenced.

A Circuit Application

1. The output voltage is unaffected.
2. The output voltage is doubled.

RELATED PROBLEMS FOR EXAMPLES

9-1 $20x_1 - 11x_2 = -15$

$18x_1 + 25x_2 = 10$

9-2 $10V_1 - 21V_2 - 50V_3 = -15$

$18V_1 + 25V_2 - 12V_3 = 10$

$18V_1 - 25V_2 + 12V_3 = 9$

9-3 3.71 A

9-4 -298 mA

9-5 $X_1 = -1.76923076923$; $X_2 = -18.5384615385$; $X_3 = -34.4615384615$

9-6 Same results as answer for 9-5

9-7 $I_1 = 17.2$ mA; $I_2 = 8.74$ mA; $I_3 = -8.44$ mA

9-8 $I_1 = X_1 = .013897190675$ (≈ 13.9 mA); $I_2 = X_2 = -.001867901972$ (≈ -1.87 mA)

9-9 They are correct.

9-10 $V_1 = 7.68$ V, $V_2 = 8.25$ V, $V_3 = 0.6$ V, $V_4 = 3.73$ V, $V_L = 4.32$ V

9-11 1.92 V

9-12 $I_1 = 18.8$ mA, $I_2 = 19.3$ mA, $I_3 = 16.2$ mA, $I_4 = 15.8$ mA, $I_L = 0.39$ mA

9-13 $V_B = 4.04$ V

SELF-TEST

1. (e) **2.** (d) **3.** (b) **4.** (c) **5.** (c)

6. (b) **7.** (d) **8.** (a) **9.** (e) **10.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (a) **4.** (a) **5.** (c) **6.** (b)

7. (a) **8.** (a) **9.** (c) **10.** (b) **11.** (b) **12.** (b)