

8

CIRCUIT THEOREMS AND CONVERSIONS

CHAPTER OUTLINE

- B-1 The DC Voltage Source
- B-2 The Current Source
- B-3 Source Conversions
- B-4 The Superposition Theorem
- B-5 Thevenin's Theorem
- B-6 Norton's Theorem
- B-7 Maximum Power Transfer Theorem
- B-8 Delta-to-Wye (Δ -to-Y) and Wye-to-Delta (Y-to- Δ) Conversions
- A Circuit Application

CHAPTER OBJECTIVES

- ◆ Describe the characteristics of a dc voltage source
- ◆ Describe the characteristics of a current source
- ◆ Perform source conversions
- ◆ Apply the superposition theorem to circuit analysis
- ◆ Apply Thevenin's theorem to simplify a circuit for analysis
- ◆ Apply Norton's theorem to simplify a circuit
- ◆ Apply the maximum power transfer theorem
- ◆ Perform Δ -to-Y and Y-to- Δ conversions

KEY TERMS

- ◆ Terminal equivalency
- ◆ Superposition theorem
- ◆ Thevenin's theorem
- ◆ Norton's theorem
- ◆ Maximum power transfer

A CIRCUIT APPLICATION PREVIEW

In the circuit application, a temperature-measurement and control circuit uses a Wheatstone bridge like the one you studied in Chapter 7. You will utilize Thevenin's theorem as well as other techniques in the evaluation of this circuit.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at <http://www.prenhall.com/floyd>

INTRODUCTION

In previous chapters, you analyzed various types of circuits using Ohm's law and Kirchhoff's laws. Some types of circuits are difficult to analyze using only those basic laws and require additional methods in order to simplify the analysis.

The theorems and conversions in this chapter make analysis easier for certain types of circuits. These methods do not replace Ohm's law and Kirchhoff's laws, but they are normally used in conjunction with the laws in certain situations.

Because all electric circuits are driven by either voltage sources or current sources, it is important to understand how to work with these elements. The superposition theorem will help you to deal with circuits that have multiple sources. Thevenin's and Norton's theorems provide methods for reducing a circuit to a simple equivalent form for ease of analysis. The maximum power transfer theorem is used in applications where it is important for a given circuit to provide maximum power to a load. An example of this is an audio amplifier that provides maximum power to a speaker. Delta-to-wye and wye-to-delta conversions are sometimes useful when you analyze bridge circuits that are commonly found in systems that measure physical parameters such as temperature, pressure, and strain.

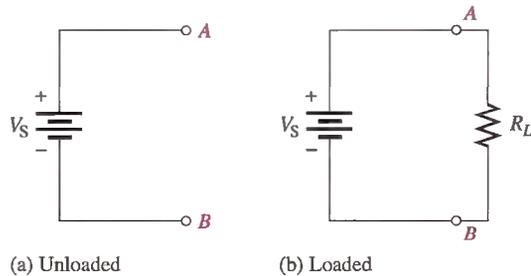
8-1 THE DC VOLTAGE SOURCE

As you learned in Chapter 2, the dc voltage source is one of the principal types of energy source in electronic applications, so it is important to understand its characteristics. The dc voltage source ideally provides constant voltage to a load even when the load resistance varies.

After completing this section, you should be able to

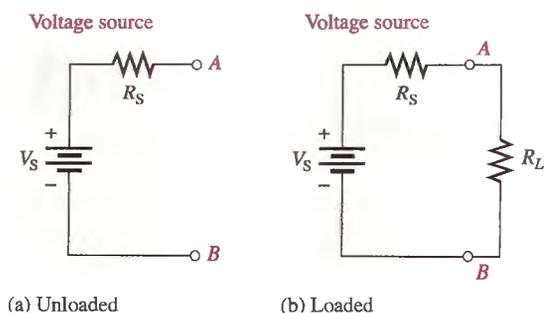
- ♦ Describe the characteristics of a dc voltage source
 - ♦ Compare a practical voltage source to an ideal source
 - ♦ Discuss the effect of loading on a practical voltage source

Figure 8-1(a) is the familiar symbol for an ideal dc voltage source. The voltage across its terminals, A and B , remains fixed regardless of the value of load resistance that may be connected across its output. Figure 8-1(b) shows a load resistor, R_L , connected. All of the source voltage, V_S , is dropped across R_L . Ideally, R_L can be changed to any value except zero, and the voltage will remain fixed. The ideal voltage source has an internal resistance of zero.



◀ FIGURE 8-1
Ideal dc voltage source.

In reality, no voltage source is ideal; however, regulated power supplies can approach ideal when operated within the specified output current. All voltage sources have some inherent internal resistance as a result of their physical and/or chemical makeup, which can be represented by a resistor in series with an ideal source, as shown in Figure 8-2(a). R_S is the internal source resistance and V_S is the source voltage. With no load, the output voltage (voltage from A to B) is V_S . This voltage is sometimes called the *open circuit voltage*.



◀ FIGURE 8-2
Practical voltage source.

Loading of the Voltage Source

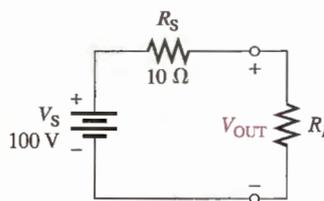
When a load resistor is connected across the output terminals, as shown in Figure 8–2(b), all of the source voltage does not appear across R_L . Some of the voltage is dropped across R_S because R_S and R_L are in series.

If R_S is very small compared to R_L , the source approaches ideal because almost all of the source voltage, V_S , appears across the larger resistance, R_L . Very little voltage is dropped across the internal resistance, R_S . If R_L changes, most of the source voltage remains across the output as long as R_L is much larger than R_S . As a result, very little change occurs in the output voltage. The larger R_L is, compared to R_S , the less change there is in the output voltage. Example 8–1 illustrates the effect of changes in R_L on the output voltage when R_L is much greater than R_S .

EXAMPLE 8–1

Calculate the voltage output of the source in Figure 8–3 for the following values of R_L : 100 Ω , 560 Ω , and 1.0 k Ω .

► FIGURE 8–3



Solution For $R_L = 100 \Omega$, the voltage output is

$$V_{\text{OUT}} = \left(\frac{R_L}{R_S + R_L} \right) V_S = \left(\frac{100 \Omega}{110 \Omega} \right) 100 \text{ V} = \mathbf{90.9 \text{ V}}$$

For $R_L = 560 \Omega$,

$$V_{\text{OUT}} = \left(\frac{560 \Omega}{570 \Omega} \right) 100 \text{ V} = \mathbf{98.2 \text{ V}}$$

For $R_L = 1.0 \text{ k}\Omega$,

$$V_{\text{OUT}} = \left(\frac{1000 \Omega}{1010 \Omega} \right) 100 \text{ V} = \mathbf{99.0 \text{ V}}$$

Notice that the output voltage is within 10% of the source voltage, V_S , for all three values of R_L because R_L is at least ten times R_S .

Related Problem* Determine V_{OUT} in Figure 8–3 if $R_S = 50 \Omega$ and $R_L = 10 \text{ k}\Omega$.



Use Multisim file E08-01 to verify the calculated results in this example and to confirm your calculation for the related problem.

*Answers are at the end of the chapter.

The output voltage decreases significantly as the load resistance is made smaller compared to the internal source resistance. Example 8–2 illustrates the effect of a smaller R_L and confirms the requirement that R_L must be much larger than R_S (at least 10 times) in order to maintain the output voltage near its open circuit value.

EXAMPLE 8–2

Determine V_{OUT} for $R_L = 10\ \Omega$ and for $R_L = 1.0\ \Omega$ in Figure 8–3.

Solution For $R_L = 10\ \Omega$, the voltage output is

$$V_{\text{OUT}} = \left(\frac{R_L}{R_S + R_L} \right) V_S = \left(\frac{10\ \Omega}{20\ \Omega} \right) 100\ \text{V} = 50\ \text{V}$$

For $R_L = 1.0\ \Omega$,

$$V_{\text{OUT}} = \left(\frac{1.0\ \Omega}{11\ \Omega} \right) 100\ \text{V} = 9.09\ \text{V}$$

Related Problem What is V_{OUT} with no load resistor in Figure 8–3?

SECTION 8–1**REVIEW**

Answers are at the end of the chapter.

1. What is the symbol for the ideal voltage source?
2. Draw a practical voltage source.
3. What is the internal resistance of the ideal voltage source?
4. What effect does the load have on the output voltage of the practical voltage source?

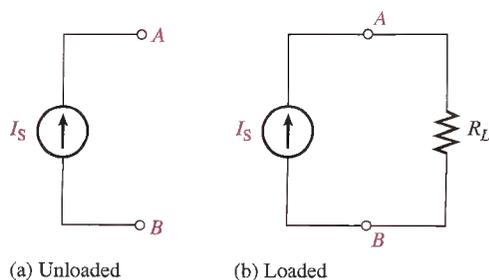
8–2 THE CURRENT SOURCE

As you learned in Chapter 2, the current source is another type of energy source that ideally provides a constant current to a load even when the resistance of the load varies. The concept of the current source is important in certain types of transistor circuits.

After completing this section, you should be able to

- ♦ Describe the characteristics of a current source
 - ♦ Compare a practical current source to an ideal source
 - ♦ Discuss the effect of loading on a practical current source

Figure 8–4(a) shows a symbol for the ideal current source. The arrow indicates the direction of source current, I_S . An ideal current source produces a constant value of current through a load, regardless of the value of the load. This concept is illustrated in Figure 8–4(b), where a load resistor is connected to the current source between terminals A and B . The ideal current source has an infinitely large internal parallel resistance.

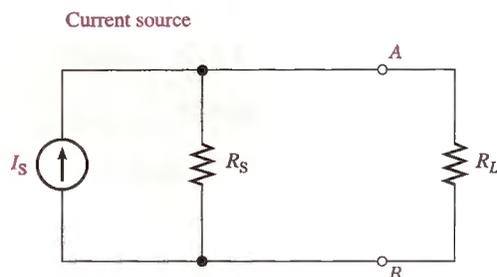


♦ **FIGURE 8–4**
Ideal current source.

Transistors act basically as current sources, and for this reason, knowledge of the current source concept is important. You will find that the equivalent model of a transistor does contain a current source.

Although the ideal current source can be used in most analysis work, no actual device is ideal. A practical current source representation is shown in Figure 8–5. Here the internal resistance appears in parallel with the ideal current source.

► **FIGURE 8–5**
Practical current source with load.



If the internal source resistance, R_S , is much larger than a load resistor, the practical source approaches ideal. The reason is illustrated in the practical current source shown in Figure 8–5. Part of the current, I_S , is through R_S , and part is through R_L . The internal source resistance, R_S , and the load resistor, R_L , act as a current divider. If R_S is much larger than R_L , most of the current is through R_L and very little through R_S . As long as R_L remains much smaller than R_S , the current through R_L will stay almost constant, no matter how much R_L changes.

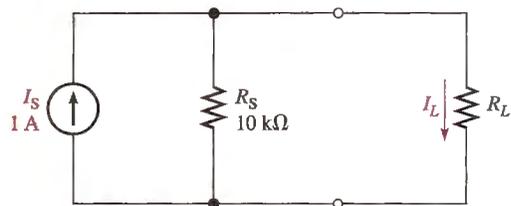
If there is a constant-current source, you can normally assume that R_S is so much larger than the load resistance that R_S can be neglected. This simplifies the source to ideal, making the analysis easier.

Example 8–3 illustrates the effect of changes in R_L on the load current when R_L is much smaller than R_S . Generally, R_L should be at least ten times smaller than R_S ($10R_L \leq R_S$) for a source to act as a reasonable current source.

EXAMPLE 8–3

Calculate the load current (I_L) in Figure 8–6 for the following values of R_L : 100 Ω , 560 Ω , and 1.0 k Ω .

► **FIGURE 8–6**



Solution For $R_L = 100 \Omega$, the load current is

$$I_L = \left(\frac{R_S}{R_S + R_L} \right) I_S = \left(\frac{10 \text{ k}\Omega}{10.1 \text{ k}\Omega} \right) 1 \text{ A} = 990 \text{ mA}$$

For $R_L = 560 \Omega$,

$$I_L = \left(\frac{10 \text{ k}\Omega}{10.56 \text{ k}\Omega} \right) 1 \text{ A} = 947 \text{ mA}$$

For $R_L = 1.0 \text{ k}\Omega$,

$$I_L = \left(\frac{10 \text{ k}\Omega}{11 \text{ k}\Omega} \right) 1 \text{ A} = 909 \text{ mA}$$

Notice that the load current, I_L , is within 10% of the source current for each value of R_L because R_L is at least ten times smaller than R_S in each case.

Related Problem At what value of R_L in Figure 8–6 will the load current equal 750 mA?

SECTION 8–2 REVIEW

1. What is the symbol for an ideal current source?
2. Draw the practical current source.
3. What is the internal resistance of the ideal current source?
4. What effect does the load have on the load current of the practical current source?

8–3 SOURCE CONVERSIONS

In circuit analysis, it is sometimes useful to convert a voltage source to an equivalent current source, or vice versa.

After completing this section, you should be able to

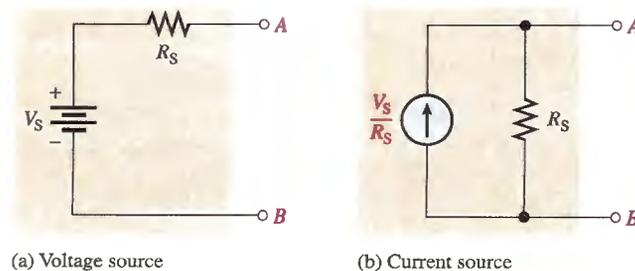
- ♦ Perform source conversions
 - ♦ Convert a voltage source to a current source
 - ♦ Convert a current source to a voltage source
 - ♦ Define *terminal equivalency*

Converting a Voltage Source to a Current Source

The source voltage, V_S , divided by the internal source resistance, R_S , gives the value of the equivalent source current.

$$I_S = \frac{V_S}{R_S}$$

The value of R_S is the same for both the voltage and current sources. As illustrated in Figure 8–7, the directional arrow for the current points from minus to plus. The equivalent current source is in parallel with R_S .



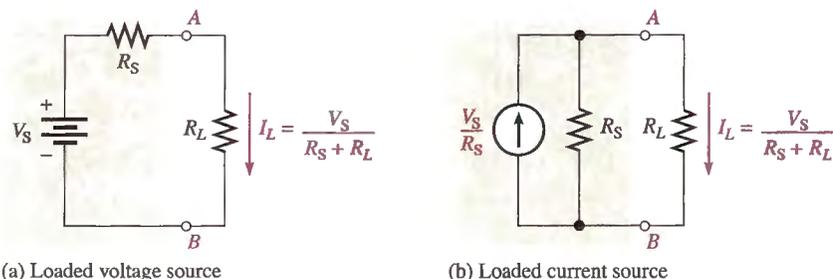
◀ FIGURE 8–7

Conversion of voltage source to equivalent current source.

Equivalency of two sources means that for any given load resistance connected to the two sources, the same load voltage and load current are produced by both sources. This concept is called **terminal equivalency**.

You can show that the voltage source and the current source in Figure 8–7 are equivalent by connecting a load resistor to each, as shown in Figure 8–8, and then calculating the load current. For the voltage source, the load current is

$$I_L = \frac{V_S}{R_S + R_L}$$



▲ FIGURE 8–8

Equivalent sources with loads.

For the current source,

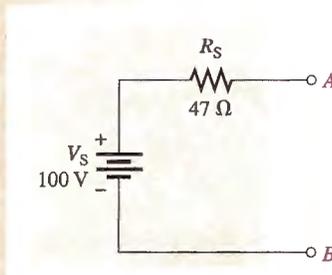
$$I_L = \left(\frac{R_S}{R_S + R_L} \right) \frac{V_S}{R_S} = \frac{V_S}{R_S + R_L}$$

As you see, both expressions for I_L are the same. These equations show that the sources are equivalent as far as the load or terminals A and B are concerned.

EXAMPLE 8–4

Convert the voltage source in Figure 8–9 to an equivalent current source and show the equivalent circuit.

► FIGURE 8–9

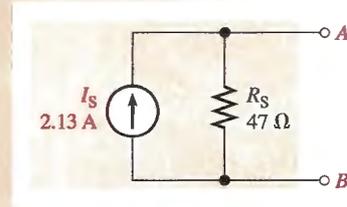


Solution The value of the internal resistance, R_S , of the equivalent current source is the same as the internal resistance of the voltage source. Therefore, the equivalent current source is

$$I_S = \frac{V_S}{R_S} = \frac{100 \text{ V}}{47 \Omega} = 2.13 \text{ A}$$

Figure 8–10 shows the equivalent circuit.

► **FIGURE 8–10**



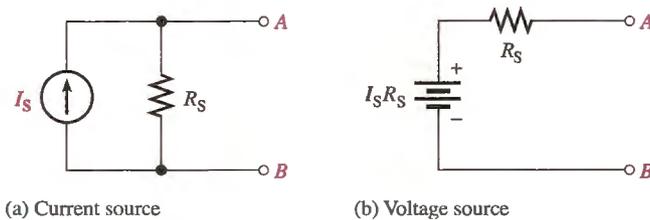
Related Problem Determine I_S and R_S of a current source equivalent to a voltage source with $V_S = 12\text{ V}$ and $R_S = 10\ \Omega$.

Converting a Current Source to a Voltage Source

The source current, I_S , multiplied by the internal source resistance, R_S , gives the value of the equivalent source voltage.

$$V_S = I_S R_S$$

Again, R_S remains the same. The polarity of the voltage source is minus to plus in the direction of the current. The equivalent voltage source is the voltage in series with R_S , as illustrated in Figure 8–11.



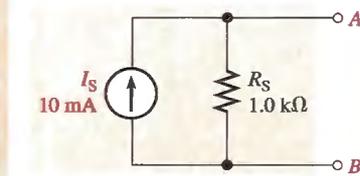
▲ **FIGURE 8–11**

Conversion of current source to equivalent voltage source.

EXAMPLE 8–5

Convert the current source in Figure 8–12 to an equivalent voltage source and show the equivalent circuit.

► **FIGURE 8–12**

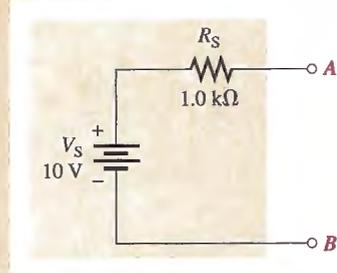


Solution The value of R_S is the same as with a current source. Therefore, the equivalent voltage source is

$$V_S = I_S R_S = (10\text{ mA})(1.0\text{ k}\Omega) = 10\text{ V}$$

Figure 8–13 shows the equivalent circuit.

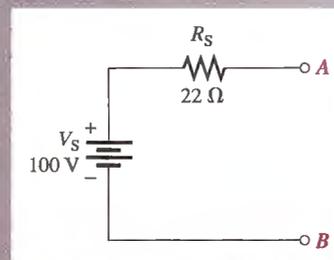
► **FIGURE 8–13**



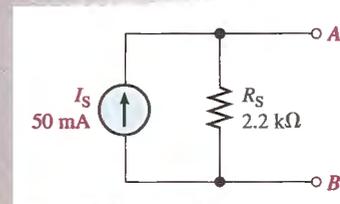
Related Problem Determine V_S and R_S of a voltage source equivalent to a current source with $I_S = 500 \text{ mA}$ and $R_S = 600 \Omega$.

SECTION 8–3 REVIEW

1. Write the formula for converting a voltage source to a current source.
2. Write the formula for converting a current source to a voltage source.
3. Convert the voltage source in Figure 8–14 to an equivalent current source.
4. Convert the current source in Figure 8–15 to an equivalent voltage source.



► **FIGURE 8–14**



► **FIGURE 8–15**

8–4 THE SUPERPOSITION THEOREM

Some circuits require more than one voltage or current source. For example, most amplifiers operate with two voltage sources: an ac and a dc source. Additionally, some amplifiers require both a positive and a negative dc voltage source for proper operation. When multiple sources are used in a circuit, the superposition theorem provides a method for analysis.

After completing this section, you should be able to

- ♦ **Apply the superposition theorem to circuit analysis**
 - ♦ State the superposition theorem
 - ♦ List the steps in applying the theorem

The superposition method is a way to determine currents in a circuit with multiple sources by leaving one source at a time and replacing the other sources by their internal resistances.

Recall that an ideal voltage source has a zero internal resistance and an ideal current source has infinite internal resistance. All sources will be treated as ideal in order to simplify the coverage.

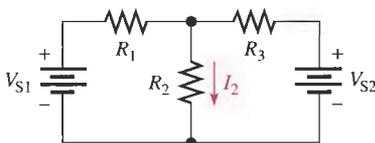
A general statement of the **superposition theorem** is as follows:

The current in any given branch of a multiple-source circuit can be found by determining the currents in that particular branch produced by each source acting alone, with all other sources replaced by their internal resistances. The total current in the branch is the algebraic sum of the individual currents in that branch.

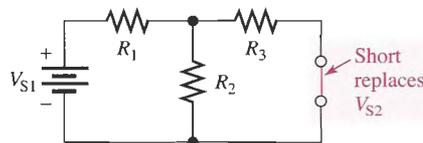
The steps in applying the superposition method are as follows:

- Step 1.** Leave one voltage (or current) source at a time in the circuit and replace each of the other voltage (or current) sources with its internal resistance. For ideal sources a short represents zero internal resistance and an open represents infinite internal resistance.
- Step 2.** Determine the particular current (or voltage) that you want just as if there were only one source in the circuit.
- Step 3.** Take the next source in the circuit and repeat Steps 1 and 2. Do this for each source.
- Step 4.** To find the actual current in a given branch, algebraically sum the currents due to each individual source. (If the currents are in the same direction, they are added. If the currents are in opposite directions, they are subtracted with the direction of the resulting current the same as the larger of the original quantities.) Once you find the current, you can determine the voltage using Ohm's law.

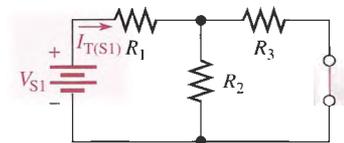
The approach to superposition is demonstrated in Figure 8–16 for a series-parallel circuit with two ideal voltage sources. Study the steps in this figure.



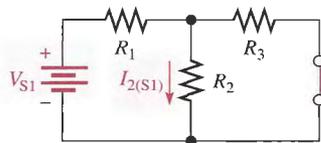
(a) Problem: Find I_2 .



(b) Replace V_{S2} with zero resistance (short).

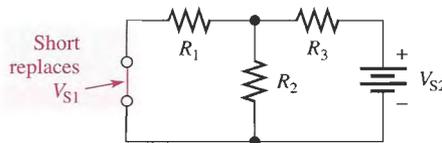


(c) Find R_T and I_T looking from V_{S1} :
 $R_{T(S1)} = R_1 + R_2 \parallel R_3$
 $I_{T(S1)} = V_{S1} / R_{T(S1)}$

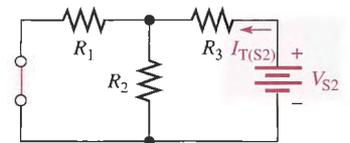


(d) Find I_2 due to V_{S1} (current divider):

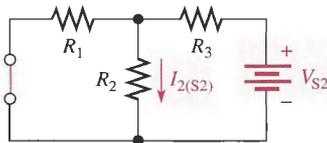
$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3} \right) I_{T(S1)}$$



(e) Replace V_{S1} with zero resistance (short).

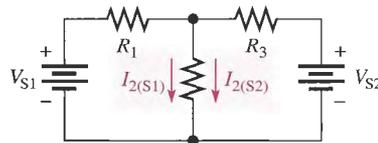


(f) Find R_T and I_T looking from V_{S2} :
 $R_{T(S2)} = R_3 + R_1 \parallel R_2$
 $I_{T(S2)} = V_{S2} / R_{T(S2)}$



(g) Find I_2 due to V_{S2} :

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2} \right) I_{T(S2)}$$



(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):
 $I_2 = I_{2(S1)} + I_{2(S2)}$

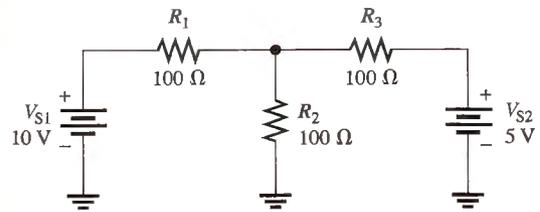
▲ FIGURE 8–16

Demonstration of the superposition method.

EXAMPLE 8-6

Use the superposition theorem to find the current through R_2 of Figure 8-17.

▶ FIGURE 8-17



Solution Step 1: Replace V_{S2} with a short and find the current through R_2 due to voltage source V_{S1} , as shown in Figure 8-18. To find I_2 , use the current-divider formula (Equation 6-6). Looking from V_{S1} ,

$$R_{T(S1)} = R_1 + \frac{R_3}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

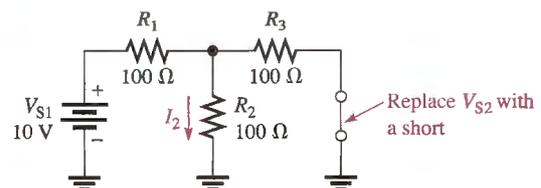
$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{10\ \text{V}}{150\ \Omega} = 66.7\ \text{mA}$$

The current through R_2 due to V_{S1} is

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3} \right) I_{T(S1)} = \left(\frac{100\ \Omega}{200\ \Omega} \right) 66.7\ \text{mA} = 33.3\ \text{mA}$$

Note that this current is downward through R_2 .

▶ FIGURE 8-18



Step 2: Find the current through R_2 due to voltage source V_{S2} by replacing V_{S1} with a short, as shown in Figure 8-19. Looking from V_{S2} ,

$$R_{T(S2)} = R_3 + \frac{R_1}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

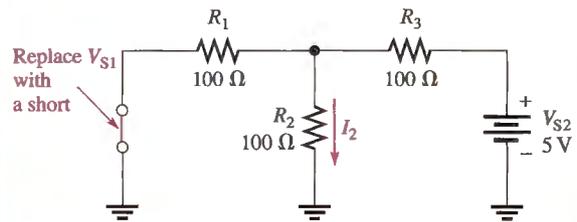
$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{5\ \text{V}}{150\ \Omega} = 33.3\ \text{mA}$$

The current through R_2 due to V_{S2} is

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2} \right) I_{T(S2)} = \left(\frac{100\ \Omega}{200\ \Omega} \right) 33.3\ \text{mA} = 16.7\ \text{mA}$$

Note that this current is downward through R_2 .

► FIGURE 8-19



Step 3: Both component currents are downward through R_2 , so they have the same algebraic sign. Therefore, add the values to get the total current through R_2 .

$$I_{2(\text{tot})} = I_{2(S1)} + I_{2(S2)} = 33.3 \text{ mA} + 16.7 \text{ mA} = 50 \text{ mA}$$

Related Problem

Determine the total current through R_2 if the polarity of V_{S2} in Figure 8-17 is reversed.

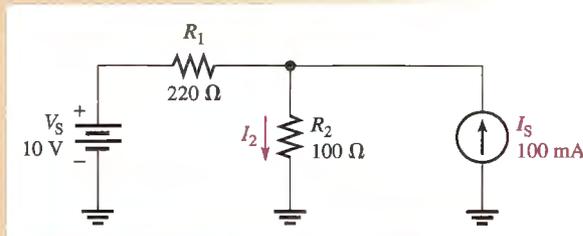


Use Multisim file E08-06 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 8-7

Find the current through R_2 in the circuit of Figure 8-20.

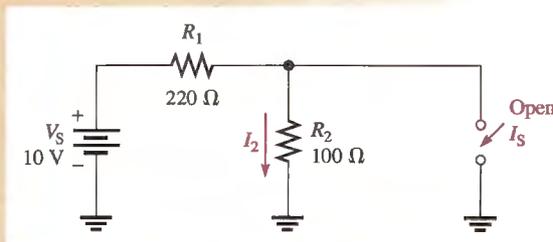
► FIGURE 8-20



Solution

Step 1: Find the current through R_2 due to V_S by replacing I_S with an open, as shown in Figure 8-21.

► FIGURE 8-21



Notice that all of the current produced by V_S is through R_2 . Looking from V_S ,

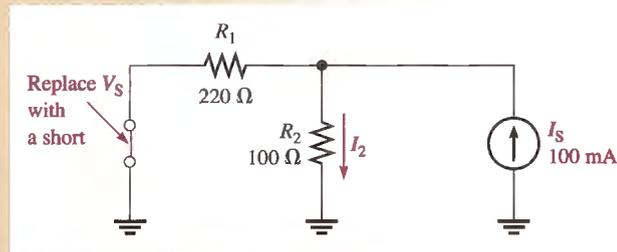
$$R_T = R_1 + R_2 = 320 \Omega$$

The current through R_2 due to V_S is

$$I_{2(V_S)} = \frac{V_S}{R_T} = \frac{10 \text{ V}}{320 \Omega} = 31.2 \text{ mA}$$

Note that this current is downward through R_2 .

Step 2: Find the current through R_2 due to I_S by replacing V_S with a short, as shown in Figure 8–22.



▲ FIGURE 8–22

Use the current-divider formula to determine the current through R_2 due to I_S .

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2} \right) I_S = \left(\frac{220 \Omega}{320 \Omega} \right) 100 \text{ mA} = 68.8 \text{ mA}$$

Note that this current also is downward through R_2 .

Step 3: Both currents are in the same direction through R_2 , so add them to get the total.

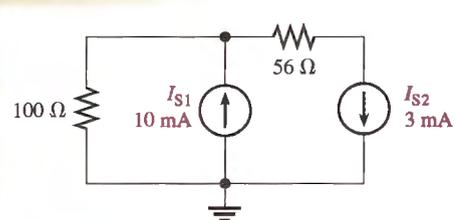
$$I_{2(\text{tot})} = I_{2(V_S)} + I_{2(I_S)} = 31.2 \text{ mA} + 68.8 \text{ mA} = 100 \text{ mA}$$

Related Problem If the polarity of V_S in Figure 8–20 is reversed, how is the value of I_S affected?

EXAMPLE 8–8

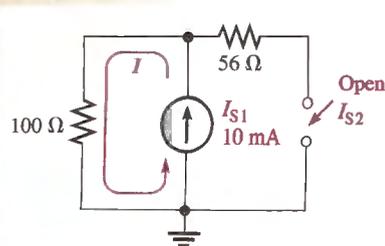
Find the current through the 100 Ω resistor in Figure 8–23.

► FIGURE 8–23

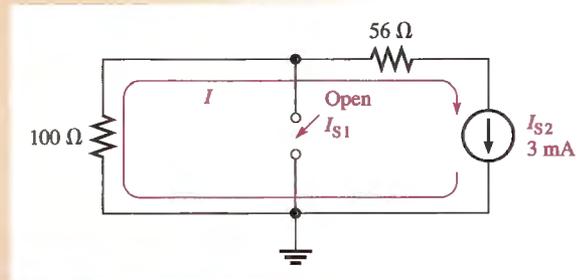


Solution Step 1: Find the current through the 100 Ω resistor due to current source I_{S1} by replacing source I_{S2} with an open, as shown in Figure 8–24. As you can see, the entire 10 mA from the current source I_{S1} is downward through the 100 Ω resistor.

► FIGURE 8–24



Step 2: Find the current through the $100\ \Omega$ resistor due to source I_{S2} by replacing source I_{S1} with an open, as indicated in Figure 8–25. Notice that all of the $3\ \text{mA}$ from source I_{S2} is upward through the $100\ \Omega$ resistor.



▲ FIGURE 8–25

Step 3: To get the total current through the $100\ \Omega$ resistor, subtract the smaller current from the larger because they are in opposite directions. The resulting total current is in the direction of the larger current from source I_{S1} .

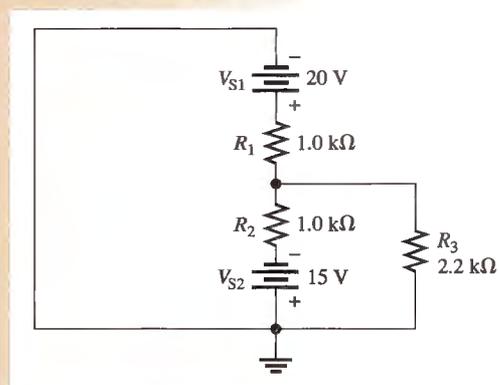
$$\begin{aligned} I_{100\Omega(\text{tot})} &= I_{100\Omega(I_{S1})} - I_{100\Omega(I_{S2})} \\ &= 10\ \text{mA} - 3\ \text{mA} = 7\ \text{mA} \end{aligned}$$

The resulting current is downward through the resistor.

Related Problem If the $100\ \Omega$ resistor in Figure 8–23 is changed to $68\ \Omega$, what will be the current through it?

EXAMPLE 8–9

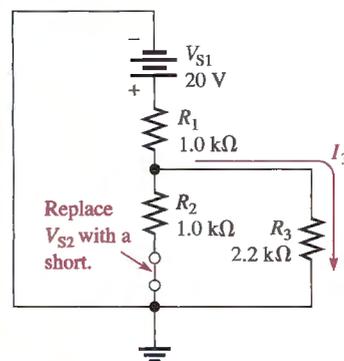
Find the total current through R_3 in Figure 8–26.



▲ FIGURE 8–26

Solution Step 1: Find the current through R_3 due to source V_{S1} by replacing source V_{S2} with a short, as shown in Figure 8–27.

► FIGURE 8-27



Looking from V_{S1} ,

$$R_{T(S1)} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 1.0 \text{ k}\Omega + \frac{(1.0 \text{ k}\Omega)(2.2 \text{ k}\Omega)}{3.2 \text{ k}\Omega} = 1.69 \text{ k}\Omega$$

$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{20 \text{ V}}{1.69 \text{ k}\Omega} = 11.8 \text{ mA}$$

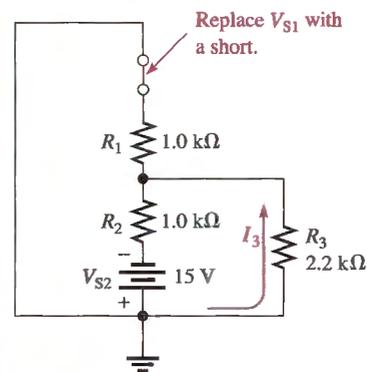
Now apply the current-divider formula to get the current through R_3 due to source V_{S1} .

$$I_{3(S1)} = \left(\frac{R_2}{R_2 + R_3} \right) I_{T(S1)} = \left(\frac{1.0 \text{ k}\Omega}{3.2 \text{ k}\Omega} \right) 11.8 \text{ mA} = 3.69 \text{ mA}$$

Notice that this current is downward through R_3 .

Step 2: Find I_3 due to source V_{S2} by replacing source V_{S1} with a short, as shown in Figure 8-28.

► FIGURE 8-28



Looking from V_{S2} ,

$$R_{T(S2)} = R_2 + \frac{R_1 R_3}{R_1 + R_3} = 1.0 \text{ k}\Omega + \frac{(1.0 \text{ k}\Omega)(2.2 \text{ k}\Omega)}{3.2 \text{ k}\Omega} = 1.69 \text{ k}\Omega$$

$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{15 \text{ V}}{1.69 \text{ k}\Omega} = 8.88 \text{ mA}$$

Now apply the current-divider formula to find the current through R_3 due to source V_{S2} .

$$I_{3(S2)} = \left(\frac{R_1}{R_1 + R_3} \right) I_{T(S2)} = \left(\frac{1.0 \text{ k}\Omega}{3.2 \text{ k}\Omega} \right) 8.88 \text{ mA} = 2.78 \text{ mA}$$

Notice that this current is upward through R_3 .

Step 3: Calculate the total current through R_3 .

$$I_{3(\text{tot})} = I_{3(S1)} - I_{3(S2)} = 3.69 \text{ mA} - 2.78 \text{ mA} = 0.91 \text{ mA} = \mathbf{910 \mu\text{A}}$$

This current is downward through R_3 .

Related Problem Find $I_{3(\text{tot})}$ in Figure 8–26 if V_{S1} is changed to 12 V and its polarity reversed.



Use Multisim file E08-09 to verify the calculated results in this example and to confirm your calculation for the related problem.

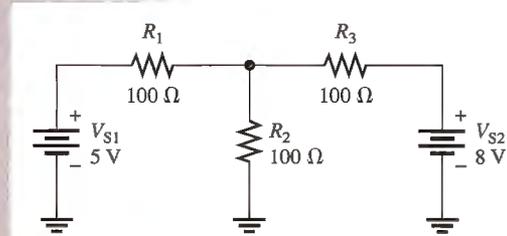
Although regulated dc power supplies are close to ideal voltage sources, many ac sources are not. For example, function generators generally have 50Ω or 600Ω of internal resistance, which appears as a resistance in series with an ideal source. Also, batteries can look ideal when they are fresh; but as they age, the internal resistance increases. When applying the superposition theorem, it is important to recognize when a source is not ideal and replace it with its equivalent internal resistance.

Current sources are not as common as voltage sources and are also not always ideal. If a current source is not ideal, as in the case of many transistors, it should be replaced by its equivalent internal resistance when the superposition theorem is applied.

SECTION 8-4 REVIEW

1. State the superposition theorem.
2. Why is the superposition theorem useful for analysis of multiple-source circuits?
3. Why is an ideal voltage source shorted and an ideal current source opened when the superposition theorem is applied?
4. Using the superposition theorem, find the current through R_1 in Figure 8–29.
5. If, as a result of applying the superposition theorem, two currents are in opposing directions through a branch of a circuit, in which direction is the net current?

FIGURE 8-29



8-5 THEVENIN'S THEOREM

Thevenin's theorem provides a method for simplifying a circuit to a standard equivalent form. This theorem can be used to simplify the analysis of complex circuits.

After completing this section, you should be able to

- ♦ **Apply Thevenin's theorem to simplify a circuit for analysis**
 - ♦ Describe the form of a Thevenin equivalent circuit
 - ♦ Obtain the Thevenin equivalent voltage source
 - ♦ Obtain the Thevenin equivalent resistance

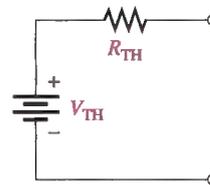
- ♦ Explain terminal equivalency in the context of Thevenin's theorem
- ♦ Thevenize a portion of a circuit
- ♦ Thevenize a bridge circuit

BIOGRAPHY

Leon Charles Thevenin
1857–1926

Leon Charles Thevenin was born in Paris, France. He graduated from the Ecole Polytechnique in 1876 and, in 1878, joined the Corps of Telegraph Engineers where he initially worked on the development of long-distance underground telegraph lines. During his career, Thevenin became increasingly interested in the problems of measurements in electrical circuits and later developed his now famous theorem which made calculations involving complex circuits possible.

The Thevenin equivalent form of any two-terminal resistive circuit consists of an equivalent voltage source (V_{TH}) and an equivalent resistance (R_{TH}), arranged as shown in Figure 8–30. The values of the equivalent voltage and resistance depend on the values in the original circuit. Any resistive circuit can be simplified regardless of its complexity with respect to two output terminals.



◀ **FIGURE 8–30**
The general form of a Thevenin equivalent circuit is a voltage source in series with a resistance.

The equivalent voltage, V_{TH} , is one part of the complete Thevenin equivalent circuit. The other part is R_{TH} .

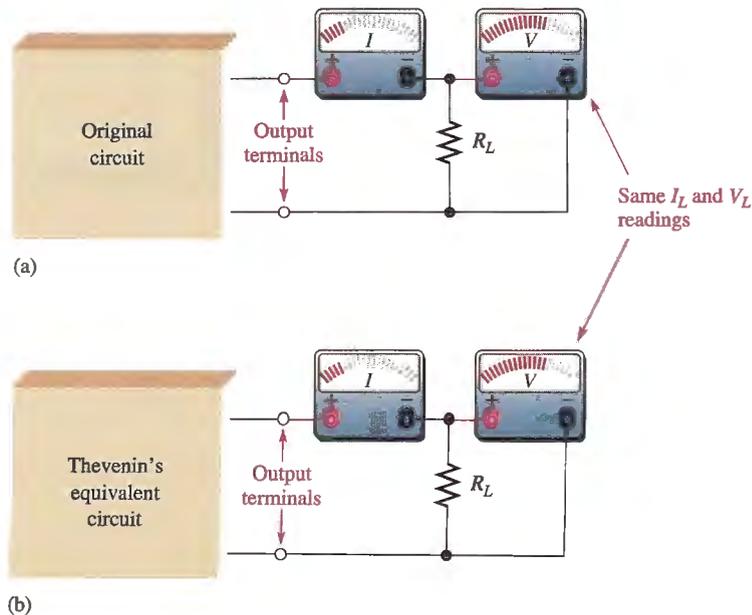
The Thevenin equivalent voltage (V_{TH}) is the open circuit (no-load) voltage between two output terminals in a circuit.

Any component connected between these two terminals effectively “sees” V_{TH} in series with R_{TH} . As defined by **Thevenin's theorem**,

The Thevenin equivalent resistance (R_{TH}) is the total resistance appearing between two terminals in a given circuit with all sources replaced by their internal resistances.

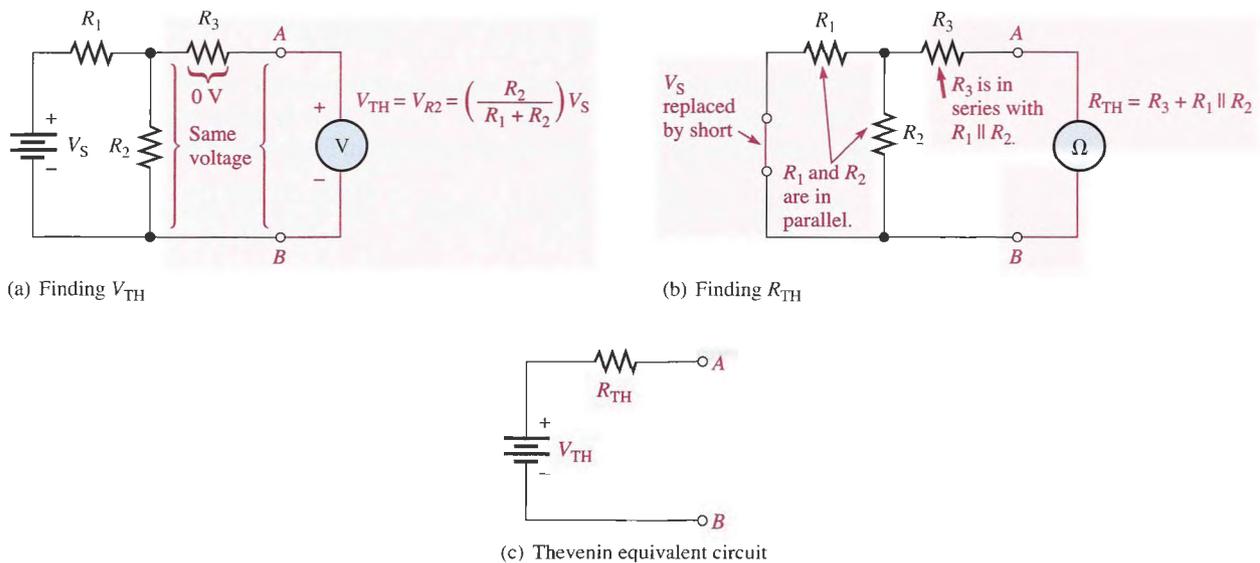
Although a Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of the output voltage and current. Try the following demonstration as illustrated in Figure 8–31. Place a resistive circuit of any complexity in a box with only the

▶ **FIGURE 8–31**
Which box contains the original circuit and which contains the Thevenin equivalent circuit? You cannot tell by observing the meters.



output terminals exposed. Then place the Thevenin equivalent of that circuit in an identical box with, again, only the output terminals exposed. Connect identical load resistors across the output terminals of each box. Next connect a voltmeter and an ammeter to measure the voltage and current for each load as shown in the figure. The measured values will be identical (neglecting tolerance variations), and you will not be able to determine which box contains the original circuit and which contains the Thevenin equivalent. That is, in terms of your observations based on any electrical measurements, both circuits appear to be the same. This condition is sometimes known as *terminal equivalency* because both circuits look the same from the “viewpoint” of the two output terminals.

To find the Thevenin equivalent of any circuit, determine the equivalent voltage, V_{TH} , and the equivalent resistance, R_{TH} , looking from the output terminals. As an example, the Thevenin equivalent for the circuit between terminals A and B is developed in Figure 8–32.



▲ FIGURE 8–32

Example of the simplification of a circuit by Thevenin's theorem.

In Figure 8–32(a), the voltage across the designated terminals A and B is the Thevenin equivalent voltage. In this particular circuit, the voltage from A to B is the same as the voltage across R_2 because there is no current through R_3 and, therefore, no voltage drop across it. The Thevenin voltage is expressed as follows for this particular example:

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) V_S$$

In Figure 8–32(b), the resistance between terminals A and B with the source replaced by a short (zero internal resistance) is the Thevenin equivalent resistance. In this particular circuit, the resistance from A to B is R_3 in series with the parallel combination of R_1 and R_2 . Therefore, R_{TH} is expressed as follows:

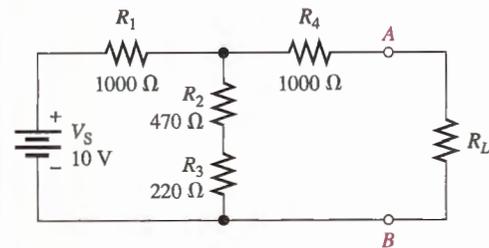
$$R_{TH} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

The Thevenin equivalent circuit is shown in Figure 8–32(c).

EXAMPLE 8-10

Find the Thevenin equivalent circuit between A and B of the circuit in Figure 8-33.

▶ FIGURE 8-33



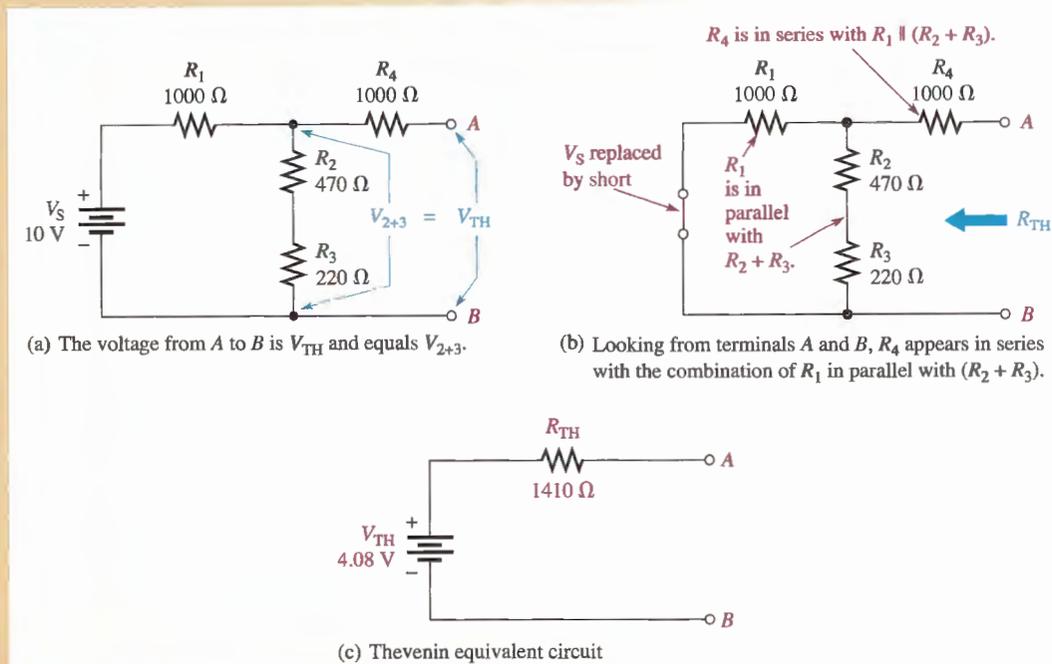
Solution First, remove R_L . Then V_{TH} equals the voltage across $R_2 + R_3$, as shown in Figure 8-34(a), because $V_4 = 0$ V since there is no current through it.

$$V_{TH} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) V_S = \left(\frac{690 \Omega}{1690 \Omega} \right) 10 \text{ V} = 4.08 \text{ V}$$

To find R_{TH} , first replace the source with a short to simulate a zero internal resistance. Then R_1 appears in parallel with $R_2 + R_3$, and R_4 is in series with the series-parallel combination of R_1 , R_2 , and R_3 , as indicated in Figure 8-34(b).

$$R_{TH} = R_4 + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 1000 \Omega + \frac{(1000 \Omega)(690 \Omega)}{1690 \Omega} = 1410 \Omega$$

The resulting Thevenin equivalent circuit is shown in Figure 8-34(c).



▶ FIGURE 8-34

Related Problem

Determine V_{TH} and R_{TH} if a 560Ω resistor is connected in parallel across R_2 and R_3 .



Use Multisim file E08-10 to verify the calculated results in this example and to confirm your calculations for the related problem.

Thevenin Equivalency Depends on the Viewpoint

The Thevenin equivalent for any circuit depends on the location of the two output terminals from which the circuit is “viewed.” In Figure 8–33, you viewed the circuit from between the two terminals labeled *A* and *B*. Any given circuit can have more than one Thevenin equivalent, depending on how the output terminals are designated. For example, if you view the circuit in Figure 8–35 from between terminals *A* and *C*, you obtain a completely different result than if you viewed it from between terminals *A* and *B* or from between terminals *B* and *C*.

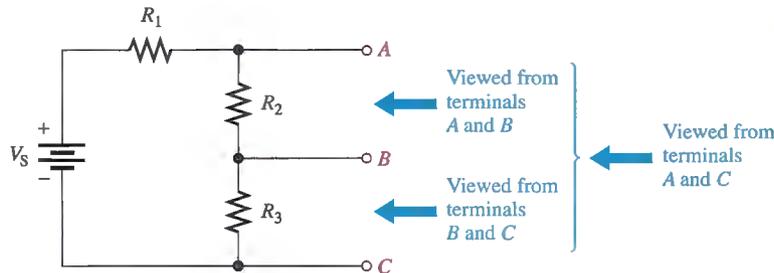


FIGURE 8–35

Thevenin's equivalent depends on the output terminals from which the circuit is viewed.

In Figure 8–36(a), when viewed from between terminals *A* and *C*, V_{TH} is the voltage across $R_2 + R_3$ and can be expressed using the voltage-divider formula as

$$V_{TH(AC)} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) V_S$$

Also, as shown in Figure 8–36(b), the resistance between terminals *A* and *C* is $R_2 + R_3$ in parallel with R_1 (the source is replaced by a short) and can be expressed as

$$R_{TH(AC)} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

The resulting Thevenin equivalent circuit is shown in Figure 8–36(c).

When viewed from between terminals *B* and *C* as indicated in Figure 8–36(d), V_{TH} is the voltage across R_3 and can be expressed as

$$V_{TH(BC)} = \left(\frac{R_3}{R_1 + R_2 + R_3} \right) V_S$$

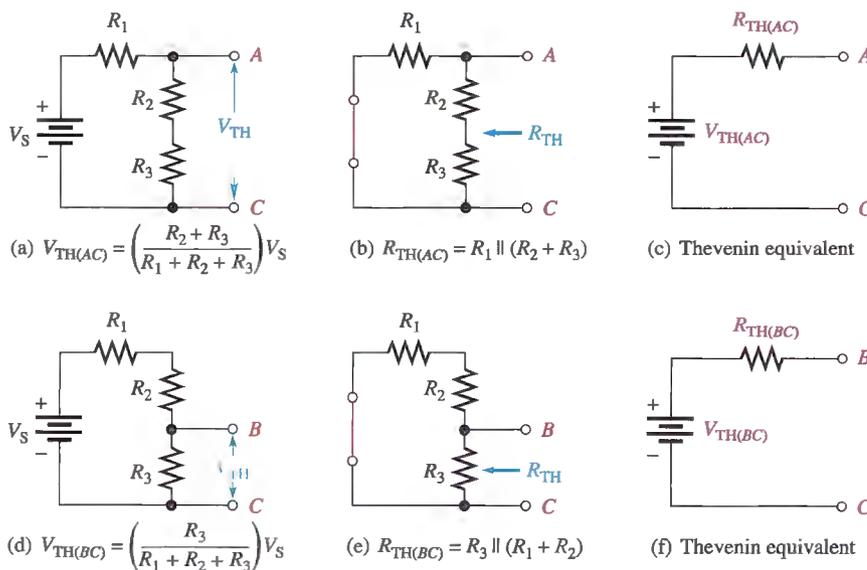


FIGURE 8–36

Example of a circuit thevenized from two different sets of terminals. Parts (a), (b), and (c) illustrate one set of terminals and parts (d), (e), and (f) illustrate another set of terminals. (The V_{TH} and R_{TH} values are different for each case.)

As shown in Figure 8–36(e), the resistance between terminals B and C is R_3 in parallel with the series combination of R_1 and R_2 .

$$R_{TH(BC)} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

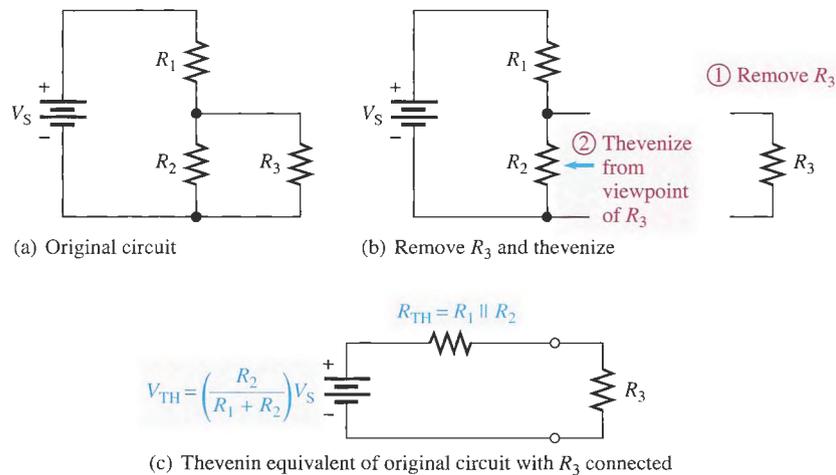
The resulting Thevenin equivalent is shown in Figure 8–36 (f).

Thevenizing a Portion of a Circuit

In many cases, it helps to thevenize only a portion of a circuit. For example, when you need to know the equivalent circuit as viewed by one particular resistor in the circuit, you can remove the resistor and apply Thevenin's theorem to the remaining part of the circuit as viewed from the points between which that resistor was connected. Figure 8–37 illustrates the thevenizing of part of a circuit.

► **FIGURE 8–37**

Example of thevenizing a portion of a circuit. In this case, the circuit is thevenized from the viewpoint of the load resistor, R_3 .



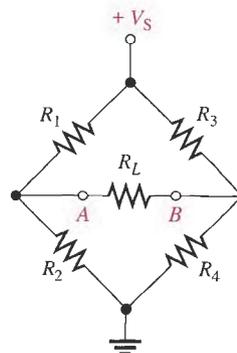
Using this type of approach, you can easily find the voltage and current for a specified resistor for any number of resistor values using only Ohm's law. This method eliminates the necessity of reanalyzing the original circuit for each different resistance value.

Thevenizing a Bridge Circuit

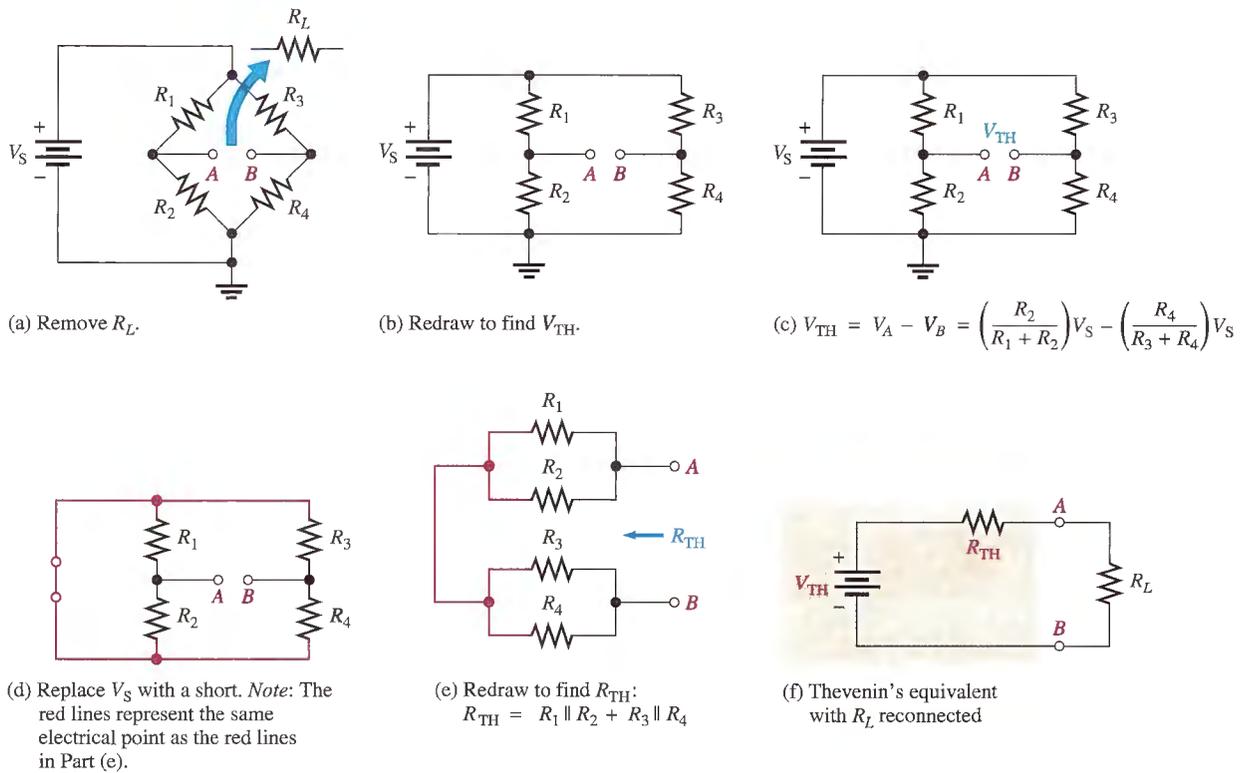
The usefulness of Thevenin's theorem is perhaps best illustrated when it is applied to a Wheatstone bridge circuit. For example, when a load resistor is connected to the output terminals of a Wheatstone bridge, as shown in Figure 8–38, the circuit is difficult to analyze because it is not a straightforward series-parallel arrangement. There are no resistors that are in series or in parallel with another resistor.

► **FIGURE 8–38**

A Wheatstone bridge with a load resistor connected between the output terminals is not a straightforward series-parallel circuit.



Using Thevenin's theorem, you can simplify the bridge circuit to an equivalent circuit viewed from the load resistor as shown step-by-step in Figure 8-39. Study carefully the steps in this figure. Once the equivalent circuit for the bridge is found, the voltage and current for any value of load resistor can easily be determined.

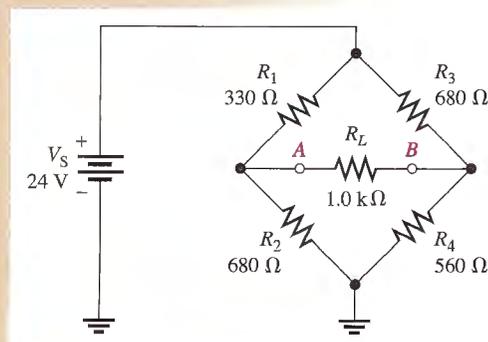


▲ FIGURE 8-39

Simplifying a Wheatstone bridge with Thevenin's theorem.

EXAMPLE 8-11

Determine the voltage and current for the load resistor, R_L , in the bridge circuit of Figure 8-40.



▲ FIGURE 8-40

Solution Step 1: Remove R_L .

Step 2: To thevenize the bridge as viewed from between terminals A and B , as was shown in Figure 8–39, first determine V_{TH} .

$$\begin{aligned} V_{TH} &= V_A - V_B = \left(\frac{R_2}{R_1 + R_2} \right) V_S - \left(\frac{R_4}{R_3 + R_4} \right) V_S \\ &= \left(\frac{680 \Omega}{1010 \Omega} \right) 24 \text{ V} - \left(\frac{560 \Omega}{1240 \Omega} \right) 24 \text{ V} = 16.16 \text{ V} - 10.84 \text{ V} = 5.32 \text{ V} \end{aligned}$$

Step 3: Determine R_{TH} .

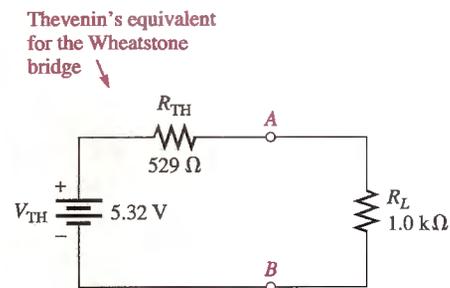
$$\begin{aligned} R_{TH} &= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} \\ &= \frac{(330 \Omega)(680 \Omega)}{1010 \Omega} + \frac{(680 \Omega)(560 \Omega)}{1240 \Omega} = 222 \Omega + 307 \Omega = 529 \Omega \end{aligned}$$

Step 4: Place V_{TH} and R_{TH} in series to form the Thevenin equivalent circuit.

Step 5: Connect the load resistor between terminals A and B of the equivalent circuit, and determine the load voltage and current as illustrated in Figure 8–41.

$$\begin{aligned} V_L &= \left(\frac{R_L}{R_L + R_{TH}} \right) V_{TH} = \left(\frac{1.0 \text{ k}\Omega}{1.529 \text{ k}\Omega} \right) 5.32 \text{ V} = 3.48 \text{ V} \\ I_L &= \frac{V_L}{R_L} = \frac{3.48 \text{ V}}{1.0 \text{ k}\Omega} = 3.48 \text{ mA} \end{aligned}$$

FIGURE 8–41

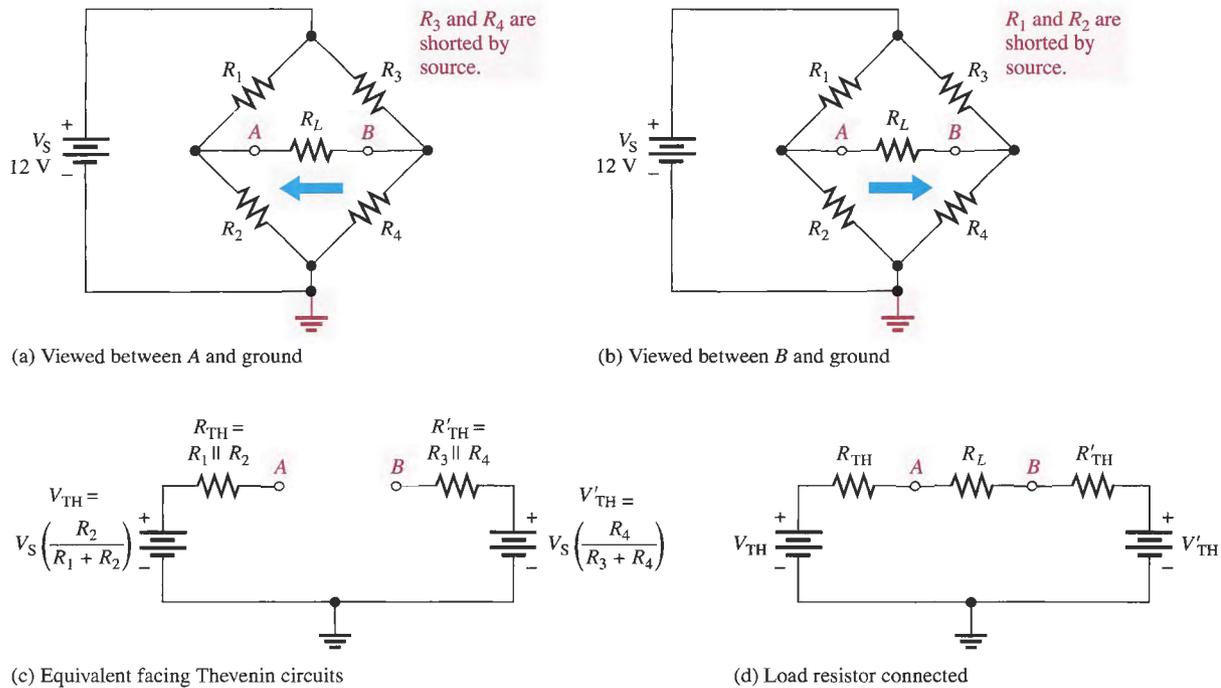


Related Problem Calculate I_L for $R_1 = 2.2 \text{ k}\Omega$, $R_2 = 3.3 \text{ k}\Omega$, $R_3 = 3.9 \text{ k}\Omega$, and $R_4 = 2.7 \text{ k}\Omega$.



Use Multisim file E08-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

An Alternate Approach An alternate way to thevenize the Wheatstone bridge is to consider a different viewpoint. Instead of viewing it from between the A and B terminals, you can view it from terminal A to ground and from terminal B to ground, as illustrated in Figure 8–42(a) and (b). The resulting equivalent circuit is simplified into two facing Thevenin circuits that still include ground, as illustrated in Figure 8–42(c). When calculating the Thevenin resistance, the voltage source is replaced by a short; thus, two of the bridge resistors are shorted out. In Figure 8–42(a), R_3 and R_4 are shorted and in part (b) R_1 and R_2 are shorted. In each case, the remaining two resistors appear in parallel to form the Thevenin resistance. The load resistor can be replaced as in Figure 8–42(d), which is seen to be a simple series circuit with two opposing sources. The advantage of this method is that ground is still shown



▲ FIGURE 8-42

in this equivalent circuit, so it is easy to find the voltage at terminal A or B with respect to ground by applying the superposition theorem to the equivalent circuit.

Summary of Thevenin's Theorem

Remember, the Thevenin equivalent circuit is *always* in the form of an equivalent voltage source in series with an equivalent resistance regardless of the original circuit that it replaces. The significance of Thevenin's theorem is that the equivalent circuit can replace the original circuit as far as any external load is concerned. Any load resistor connected between the terminals of a Thevenin equivalent circuit will have the same current through it and the same voltage across it as if it were connected to the terminals of the original circuit.

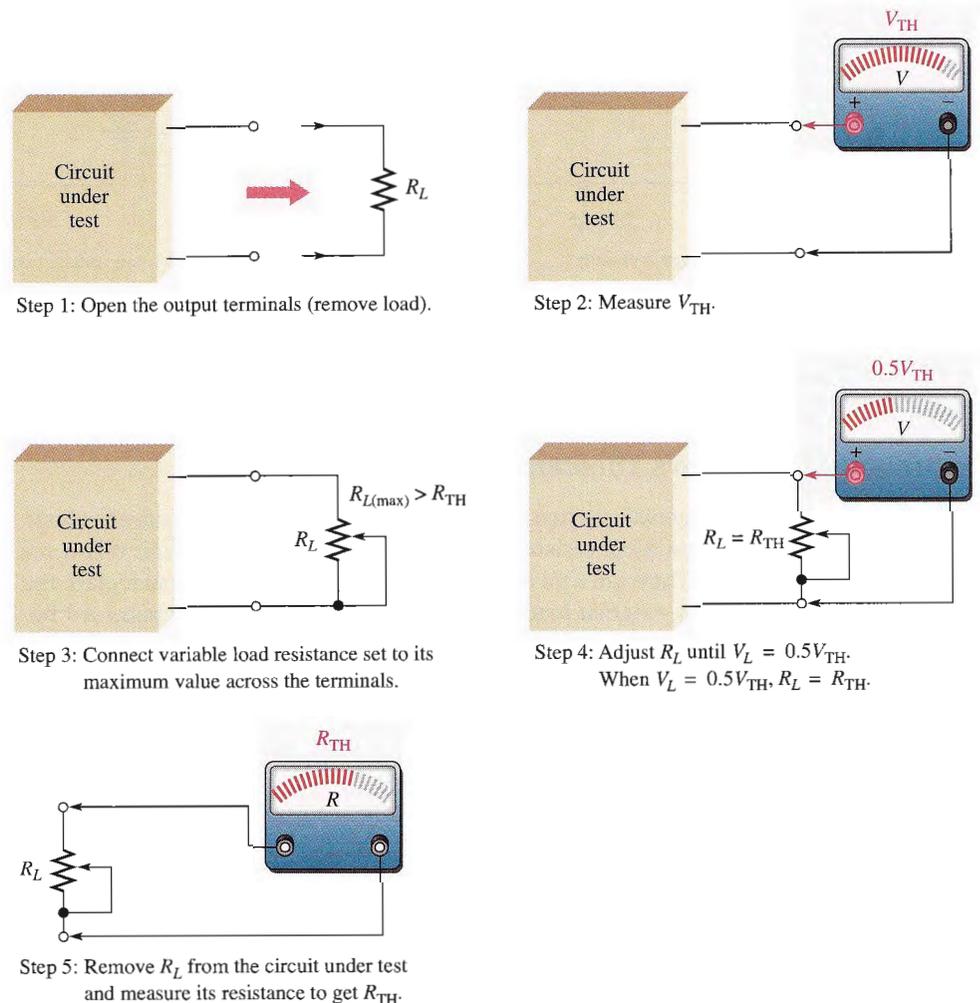
A summary of steps for applying Thevenin's theorem is as follows:

- Step 1.** Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.
- Step 2.** Determine the voltage (V_{TH}) across the two open terminals.
- Step 3.** Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).
- Step 4.** Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.
- Step 5.** Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

Determining V_{TH} and R_{TH} by Measurement

Thevenin's theorem is largely an analytical tool that is applied theoretically in order to simplify circuit analysis. However, you can find Thevenin's equivalent for an actual circuit by the following general measurement methods. These steps are illustrated in Figure 8-43.

- Step 1.** Remove any load from the output terminals of the circuit.
- Step 2.** Measure the open terminal voltage. The voltmeter used must have an internal resistance much greater (at least 10 times greater) than the R_{TH} of the circuit so that it has negligible loading effect. (V_{TH} is the open terminal voltage.)
- Step 3.** Connect a variable resistor (rheostat) across the output terminals. Set it at its maximum value, which must be greater than R_{TH} .
- Step 4.** Adjust the rheostat until the terminal voltage equals $0.5V_{TH}$. At this point, the resistance of the rheostat is equal to R_{TH} .
- Step 5.** Disconnect the rheostat from the terminals and measure its resistance with an ohmmeter. This measured resistance is equal to R_{TH} .



▲ FIGURE 8-43

Determination of Thevenin's equivalent by measurement.

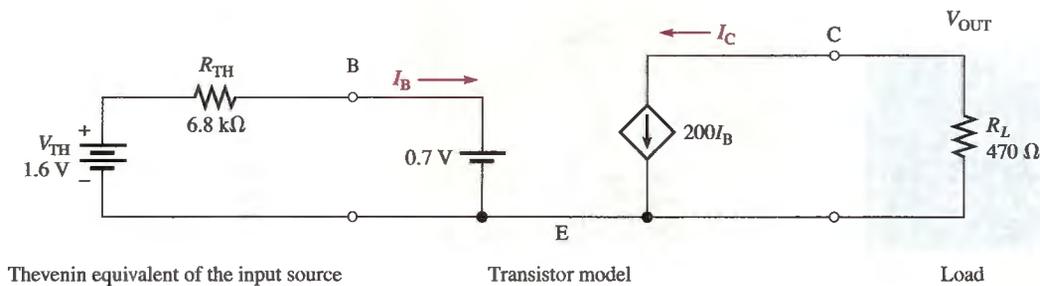
This procedure for determining R_{TH} differs from the theoretical procedure because it is impractical to short voltage sources or open current sources in an actual circuit. Also, when measuring R_{TH} , be certain that the circuit is capable of providing the required current to the variable resistor load and that the variable resistor can handle the required power. These considerations may make the procedure impractical in some cases.

An Example of a Practical Application

Although you have not studied transistor circuits, a basic amplifier can be used to illustrate the usefulness of Thevenin's equivalent circuit. A transistor circuit can be modeled with basic components including a dependent current source and a Thevenin equivalent circuit. Modeling is generally a mathematical simplification of a complex circuit, retaining only the most important parts of the circuit and eliminating those that have only a minimum effect.

A typical dc model of a transistor is shown in Figure 8–44. This type of transistor (bipolar junction transistor) has three terminals, labeled base (B), collector (C), and emitter (E). In this case, the emitter terminal is both an input and an output, so it is common. The dependent current source (diamond-shaped symbol) is controlled by the base current, I_B . For this example, the current from the dependent source is 200 times larger than the base current as expressed by the term βI_B , where β is a transistor gain parameter and, in this case, $\beta = 200$.

The transistor is part of a dc amplifier circuit, and you can use the basic model to predict the output current. The output current is larger than the input circuit can provide by itself. For example, the source can represent a small transducer, such as a solar cell with an internal resistance of 6.8 k Ω . It is shown as an equivalent Thevenin voltage and Thevenin resistance. The load could be any device that requires higher current than the source can provide directly.



▲ FIGURE 8–44

DC transistor circuit. The diamond-shaped symbol indicates a dependent current source.

EXAMPLE 8–12

- Write KVL around the left part of the circuit in Figure 8–44. Solve for I_B .
- Determine the current from the dependent current source. This current is I_C .
- Calculate the output voltage and the power in the load resistor R_L .
- Compare the power determined in (c) with the power delivered to the load if the load resistor were connected directly to the Thevenin circuit.

Solution

$$(a) \quad V_{TH} - R_{TH}I_B - 0.7 \text{ V} = 0$$

$$I_B = \frac{V_{TH} - 0.7 \text{ V}}{R_{TH}} = \frac{1.6 \text{ V} - 0.7 \text{ V}}{6.8 \text{ k}\Omega} = 132 \mu\text{A}$$

$$(b) \quad I_C = \beta I_B = 200(132 \mu\text{A}) = 26.5 \text{ mA}$$

$$(c) \quad V_{OUT} = I_C R_L = (26.5 \text{ mA})(470 \Omega) = 12.4 \text{ V}$$

$$P_L = \frac{V_{OUT}^2}{R_L} = \frac{(12.4 \text{ V})^2}{470 \Omega} = 327 \text{ mW}$$

$$(d) \quad P_L = I_B^2 R_L = (132 \mu\text{A})^2(470 \Omega) = 8.19 \mu\text{W}$$

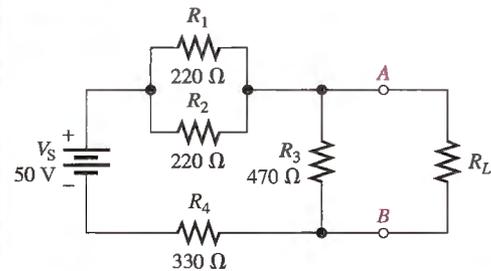
The power in the load resistor is $327 \text{ mW}/8.19 \mu\text{W} = 39,927$ times greater than the power that the Thevenin input circuit could deliver to the same load. This illustrates that the transistor can operate as a power amplifier.

Related Problem Determine the input voltage at the base (B) of the transistor. Compare this value to V_{OUT} . By how much does the amplifier increase the input voltage?

SECTION 8-5 REVIEW

1. What are the two components of a Thevenin equivalent circuit?
2. Draw the general form of a Thevenin equivalent circuit.
3. How is V_{TH} defined?
4. How is R_{TH} defined?
5. For the original circuit in Figure 8-45, draw the Thevenin equivalent circuit as viewed from the output terminals A and B.

▶ FIGURE 8-45

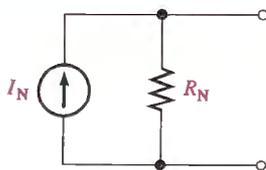


8-6 NORTON'S THEOREM

Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form. The basic difference is that Norton's theorem results in an equivalent current source in parallel with an equivalent resistance.

After completing this section, you should be able to

- ♦ Apply Norton's theorem to simplify a circuit
 - ♦ Describe the form of a Norton equivalent circuit
 - ♦ Obtain the Norton equivalent current source
 - ♦ Obtain the Norton equivalent resistance



▲ FIGURE 8-46
Form of Norton's equivalent circuit.

Norton's theorem is a method for simplifying a two-terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistor. The form of Norton's equivalent circuit is shown in Figure 8-46. Regardless of how complex the original two-terminal circuit is, it can always be reduced to this equivalent form. The equivalent current source is designated I_N , and the equivalent resistance is designated R_N . To apply Norton's theorem, you must know how to find the two quantities I_N and R_N . Once you know them for a given circuit, simply connect them in parallel to get the complete Norton circuit.

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.

Any component connected between these two terminals effectively “sees” a current source I_N in parallel with R_N . To illustrate, suppose that a resistive circuit of some kind has a resistor (R_L) connected between two output terminals in the circuit, as shown in Figure 8–47(a). You want to find the Norton circuit that is equivalent to the one shown as “seen” by R_L . To find I_N , calculate the current between terminals A and B with these two terminals shorted, as shown in Figure 8–47(b). Example 8–13 demonstrates how to find I_N .

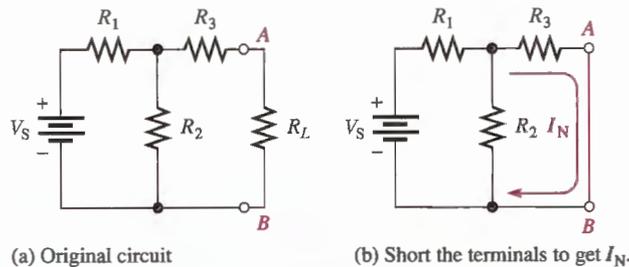


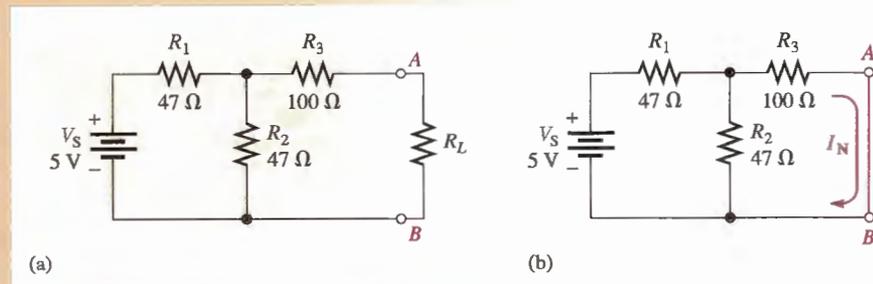
FIGURE 8–47

Determining the Norton equivalent current, I_N .

EXAMPLE 8–13

FIGURE 8–48

Determine I_N for the circuit within the beige area in Figure 8–48(a).



Solution Short terminals A and B as shown in Figure 8–48(b). I_N is the current through the short. First, the total resistance seen by the voltage source is

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 47 \Omega + \frac{(47 \Omega)(100 \Omega)}{147 \Omega} = 79 \Omega$$

The total current from the source is

$$I_T = \frac{V_S}{R_T} = \frac{5 \text{ V}}{79 \Omega} = 63.3 \text{ mA}$$

Now apply the current-divider formula to find I_N (the current through the short).

$$I_N = \left(\frac{R_2}{R_2 + R_3} \right) I_T = \left(\frac{47 \Omega}{147 \Omega} \right) 63.3 \text{ mA} = 20.2 \text{ mA}$$

This is the value for the equivalent Norton current source.

Related Problem Determine I_N in Figure 8–48(a) if the value of R_2 is doubled.



Use Multisim file E08-13 to verify the calculated results in this example and to confirm your calculation for the related problem.

Norton's Equivalent Resistance (R_N)

Norton's equivalent resistance (R_N) is defined in the same way as R_{TH} .

The Norton equivalent resistance, R_N , is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.

Example 8–14 demonstrates how to find R_N .

EXAMPLE 8–14

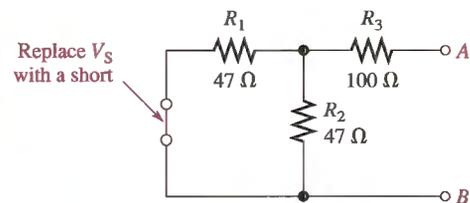
Find R_N for the circuit within the beige area of Figure 8–48(a) (see Example 8–13).

Solution

First reduce V_S to zero by shorting it, as shown in Figure 8–49. Looking in at terminals A and B , you can see that the parallel combination of R_1 and R_2 is in series with R_3 . Thus,

$$R_N = R_3 + \frac{R_1}{2} = 100 \, \Omega + \frac{47 \, \Omega}{2} = 124 \, \Omega$$

► FIGURE 8–49



Related Problem Determine R_N in Figure 8–48(a) if the value of R_2 is doubled.

Examples 8–13 and 8–14 have shown how to find the two equivalent components of a Norton equivalent circuit, I_N and R_N . Keep in mind that these values can be found for any linear circuit. Once these are known, they must be connected in parallel to form the Norton equivalent circuit, as illustrated in Example 8–15.

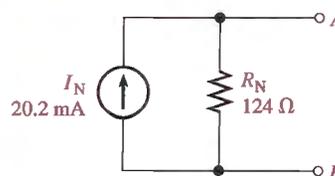
EXAMPLE 8–15

Draw the complete Norton equivalent circuit for the original circuit in Figure 8–48(a) (Example 8–13).

Solution

In Examples 8–13 and 8–14 you found that $I_N = 20.2 \text{ mA}$ and $R_N = 124 \, \Omega$. The Norton equivalent circuit is shown in Figure 8–50.

► FIGURE 8–50



Related Problem Find R_N for the circuit in Figure 8–48(a) if all the resistor values are doubled.

Summary of Norton's Theorem

Any load resistor connected between the output terminals of a Norton equivalent circuit will have the same current through it and the same voltage across it as if it were connected to the output terminals of the original circuit. A summary of steps for theoretically applying Norton's theorem is as follows:

- Step 1.** Short the two terminals between which you want to find the Norton equivalent circuit.
- Step 2.** Determine the current (I_N) through the shorted terminals.
- Step 3.** Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.
- Step 4.** Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

Norton's equivalent circuit can also be derived from Thevenin's equivalent circuit by use of the source conversion method discussed in Section 8-3.

An Example of a Practical Application

A voltage amplifier in a digital light meter is modeled using a Norton equivalent circuit and a dependent voltage source. A block diagram of the light meter is shown in Figure 8-51. The light meter uses a photocell as a sensor. The photocell is a current source that produces a very small current proportional to the incident light. Because it is a current source, a Norton circuit can be used to model the photocell. The very small amount of current from the photocell is converted to a small input voltage across R_N . A dc amplifier is used to increase the voltage to a level sufficient for driving the analog-to-digital converter.



FIGURE 8-51
Light meter block diagram.

In this application, only with the first two blocks in the light meter diagram are of interest. These have been modeled as shown in Figure 8-52. The photocell has been modeled as a Norton circuit on the input. The output of the Norton circuit is fed to the amplifier's input resistance, which converts the current I_N to a small voltage V_{IN} . The amplifier increases this voltage by 33 to drive the analog-to-digital converter which, for simplicity, is modeled simply as a load resistor, R_L . The value of 33 is the gain of this particular amplifier.

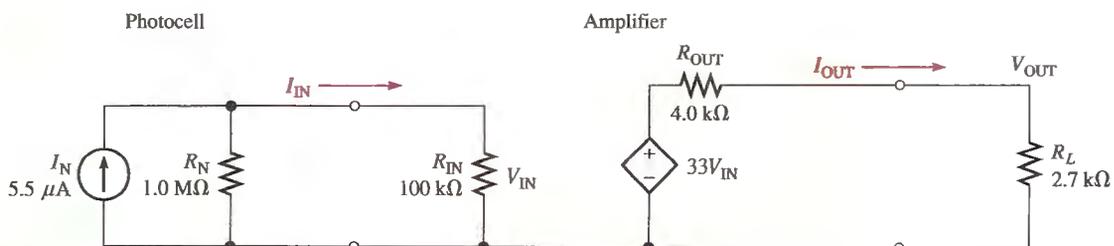


FIGURE 8-52
Photocell and amplifier model. The diamond-shaped symbol indicates a dependent voltage source.

EXAMPLE 8–16

Refer to Figure 8–52.

- Apply the current-divider rule to the input Norton circuit to calculate I_{IN} .
- Use Ohm's law to calculate V_{IN} .
- Determine the voltage from the dependent voltage source. This gain is 33.
- Apply the voltage-divider rule to calculate V_{OUT} .

Solution (a) $I_{IN} = I_N \left(\frac{R_N}{R_N + R_{IN}} \right) = (5.5 \mu\text{A}) \left(\frac{1.0 \text{ M}\Omega}{1.1 \text{ M}\Omega} \right) = 5 \mu\text{A}$

(b) $V_{IN} = I_{IN} R_{IN} = (5 \mu\text{A})(100 \text{ k}\Omega) = 0.5 \text{ V}$

(c) $33V_{IN} = (33)(0.5 \text{ V}) = 16.5 \text{ V}$

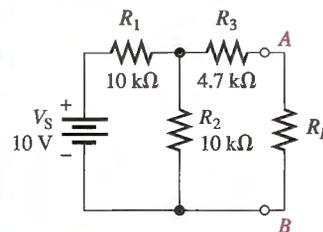
(d) $V_{OUT} = (33 V_{IN}) \left(\frac{R_L}{R_L + R_{OUT}} \right) = (16.5 \text{ V})(0.403) = 6.65 \text{ V}$

Related Problem If the photocell is replaced by one having the same current but a Norton equivalent resistance of $2.0 \text{ M}\Omega$, what is the output voltage?

**SECTION 8–6
REVIEW**

- What are the two components of a Norton equivalent circuit?
- Draw the general form of a Norton equivalent circuit.
- How is I_N defined?
- How is R_N defined?
- Find the Norton circuit as seen by R_L in Figure 8–53.

▶ **FIGURE 8–53**

**8–7 MAXIMUM POWER TRANSFER THEOREM**

The maximum power transfer theorem is important when you need to know the value of the load at which the most power is delivered from the source.

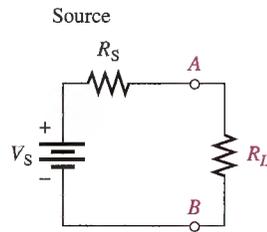
After completing this section, you should be able to

- ♦ **Apply the maximum power transfer theorem**
 - ♦ State the theorem
 - ♦ Determine the value of load resistance for which maximum power is transferred from a given circuit

The **maximum power transfer** theorem is stated as follows:

For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.

The source resistance, R_S , of a circuit is the equivalent resistance as viewed from the output terminals using Thevenin's theorem. A Thevenin equivalent circuit with its output resistance and load is shown in Figure 8–54. When $R_L = R_S$, the maximum power possible is transferred from the voltage source to R_L for a given value of V_S .



◀ **FIGURE 8–54**

Maximum power is transferred to the load when $R_L = R_S$.

Practical applications of the maximum power transfer theorem include audio systems such as stereo, radio, and public address. In these systems the resistance of the speaker is the load. The circuit that drives the speaker is a power amplifier. The systems are typically optimized for maximum power to the speakers. Thus, the resistance of the speaker must equal the internal source resistance of the amplifier.

Example 8–17 shows that maximum power occurs when $R_L = R_S$.

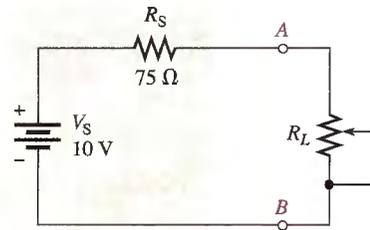
EXAMPLE 8–17

The source in Figure 8–55 has an internal source resistance of $75\ \Omega$. Determine the load power for each of the following values of load resistance:

(a) $0\ \Omega$ (b) $25\ \Omega$ (c) $50\ \Omega$ (d) $75\ \Omega$ (e) $100\ \Omega$ (f) $125\ \Omega$

Draw a graph showing the load power versus the load resistance.

▶ **FIGURE 8–55**



Solution Use Ohm's law ($I = V/R$) and the power formula ($P = I^2R$) to find the load power, P_L , for each value of load resistance.

(a) For $R_L = 0\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{75\ \Omega + 0\ \Omega} = 133\ \text{mA}$$

$$P_L = I^2R_L = (133\ \text{mA})^2(0\ \Omega) = 0\ \text{mW}$$

(b) For $R_L = 25 \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10 \text{ V}}{75 \Omega + 25 \Omega} = 100 \text{ mA}$$

$$P_L = I^2 R_L = (100 \text{ mA})^2 (25 \Omega) = 250 \text{ mW}$$

(c) For $R_L = 50 \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10 \text{ V}}{125 \Omega} = 80 \text{ mA}$$

$$P_L = I^2 R_L = (80 \text{ mA})^2 (50 \Omega) = 320 \text{ mW}$$

(d) For $R_L = 75 \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10 \text{ V}}{150 \Omega} = 66.7 \text{ mA}$$

$$P_L = I^2 R_L = (66.7 \text{ mA})^2 (75 \Omega) = 334 \text{ mW}$$

(e) For $R_L = 100 \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10 \text{ V}}{175 \Omega} = 57.1 \text{ mA}$$

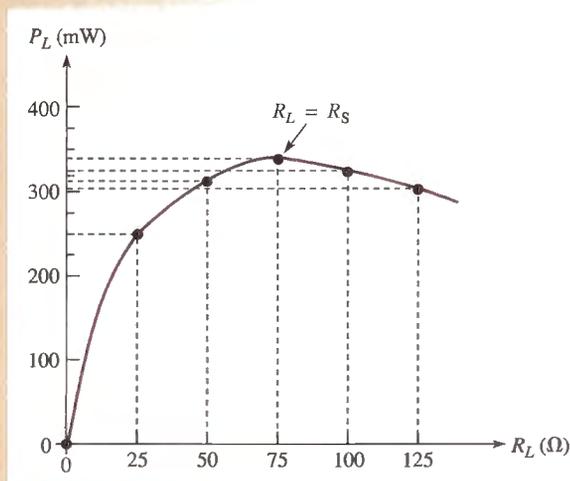
$$P_L = I^2 R_L = (57.1 \text{ mA})^2 (100 \Omega) = 326 \text{ mW}$$

(f) For $R_L = 125 \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10 \text{ V}}{200 \Omega} = 50 \text{ mA}$$

$$P_L = I^2 R_L = (50 \text{ mA})^2 (125 \Omega) = 313 \text{ mW}$$

Notice that the load power is greatest when $R_L = 75 \Omega$, which is the same as the internal source resistance. When the load resistance is less than or greater than this value, the power drops off, as the curve in Figure 8–56 graphically illustrates.



▲ FIGURE 8–56

Curve showing that the load power is maximum when $R_L = R_S$.

Related Problem If the source resistance in Figure 8–55 is 600Ω , what is the maximum power that can be delivered to a load?

**SECTION 8-7
REVIEW**

1. State the maximum power transfer theorem.
2. When is maximum power delivered from a source to a load?
3. A given circuit has an internal source resistance of $50\ \Omega$. What will be the value of the load to which the maximum power is delivered?

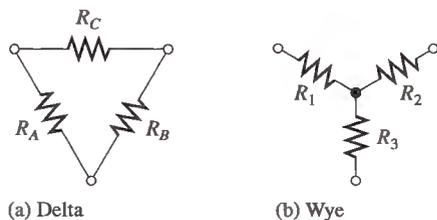
8-8 DELTA-TO-WYE (Δ -TO-Y) AND WYE-TO-DELTA (Y-TO- Δ) CONVERSIONS

Conversions between delta-type and wye-type circuit arrangements are useful in certain specialized three-terminal applications. One example is in the analysis of a loaded Wheatstone bridge circuit.

After completing this section, you should be able to

- ♦ Perform Δ -to-Y and Y-to- Δ conversions
- ♦ Apply Δ -to-Y conversion to a bridge circuit

A resistive delta (Δ) circuit is a three-terminal arrangement as shown in Figure 8-57(a). A wye (Y) circuit is shown in Figure 8-57(b). Notice that letter subscripts are used to designate resistors in the delta circuit and that numerical subscripts are used to designate resistors in the wye circuit.

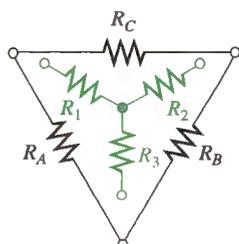


◀ **FIGURE 8-57**
Delta and wye circuits.

Δ -to-Y Conversion

It is convenient to think of the wye positioned within the delta, as shown in Figure 8-58. To convert from delta to wye, you need R_1 , R_2 , and R_3 in terms of R_A , R_B , and R_C . The conversion rule is as follows:

Each resistor in the wye is equal to the product of the resistors in two adjacent delta branches, divided by the sum of all three delta resistors.



◀ **FIGURE 8-58**
“Y within Δ ” aid for conversion formulas.

In Figure 8–58, R_A and R_C are adjacent to R_1 ; therefore,

$$\text{Equation 8-1} \quad R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

Also, R_B and R_C are adjacent to R_2 , so

$$\text{Equation 8-2} \quad R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

and R_A and R_B are adjacent to R_3 , so

$$\text{Equation 8-3} \quad R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Y-to-Δ Conversion

To convert from wye to delta, you need R_A , R_B , and R_C in terms of R_1 , R_2 , and R_3 . The conversion rule is as follows:

Each resistor in the delta is equal to the sum of all possible products of wye resistors taken two at a time, divided by the opposite wye resistor.

In Figure 8–58, R_2 is opposite to R_A ; therefore,

$$\text{Equation 8-4} \quad R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

Also, R_1 is opposite to R_B , so

$$\text{Equation 8-5} \quad R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

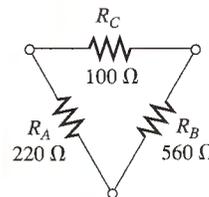
and R_3 is opposite to R_C , so

$$\text{Equation 8-6} \quad R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

EXAMPLE 8–18

Convert the delta circuit in Figure 8–59 to a wye circuit.

► FIGURE 8–59



Solution Use Equations 8–1, 8–2, and 8–3.

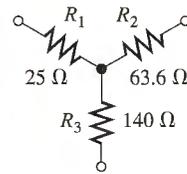
$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(220 \Omega)(100 \Omega)}{220 \Omega + 560 \Omega + 100 \Omega} = 25 \Omega$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(560 \Omega)(100 \Omega)}{880 \Omega} = 63.6 \Omega$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(220 \Omega)(560 \Omega)}{880 \Omega} = 140 \Omega$$

The resulting wye circuit is shown in Figure 8–60.

► **FIGURE 8–60**

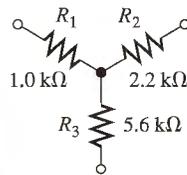


Related Problem Convert the delta circuit to a wye network for $R_A = 2.2 \text{ k}\Omega$, $R_B = 1.0 \text{ k}\Omega$, and $R_C = 1.8 \text{ k}\Omega$.

EXAMPLE 8–19

Convert the wye circuit in Figure 8–61 to a delta circuit.

► **FIGURE 8–61**



Solution Use Equations 8–4, 8–5, and 8–6.

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$= \frac{(1.0 \text{ k}\Omega)(2.2 \text{ k}\Omega) + (1.0 \text{ k}\Omega)(5.6 \text{ k}\Omega) + (2.2 \text{ k}\Omega)(5.6 \text{ k}\Omega)}{2.2 \text{ k}\Omega} = 9.15 \text{ k}\Omega$$

$$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

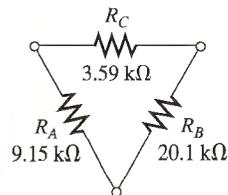
$$= \frac{(1.0 \text{ k}\Omega)(2.2 \text{ k}\Omega) + (1.0 \text{ k}\Omega)(5.6 \text{ k}\Omega) + (2.2 \text{ k}\Omega)(5.6 \text{ k}\Omega)}{1.0 \text{ k}\Omega} = 20.1 \text{ k}\Omega$$

$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

$$= \frac{(1.0 \text{ k}\Omega)(2.2 \text{ k}\Omega) + (1.0 \text{ k}\Omega)(5.6 \text{ k}\Omega) + (2.2 \text{ k}\Omega)(5.6 \text{ k}\Omega)}{5.6 \text{ k}\Omega} = 3.59 \text{ k}\Omega$$

The resulting delta circuit is shown in Figure 8–62.

► **FIGURE 8–62**

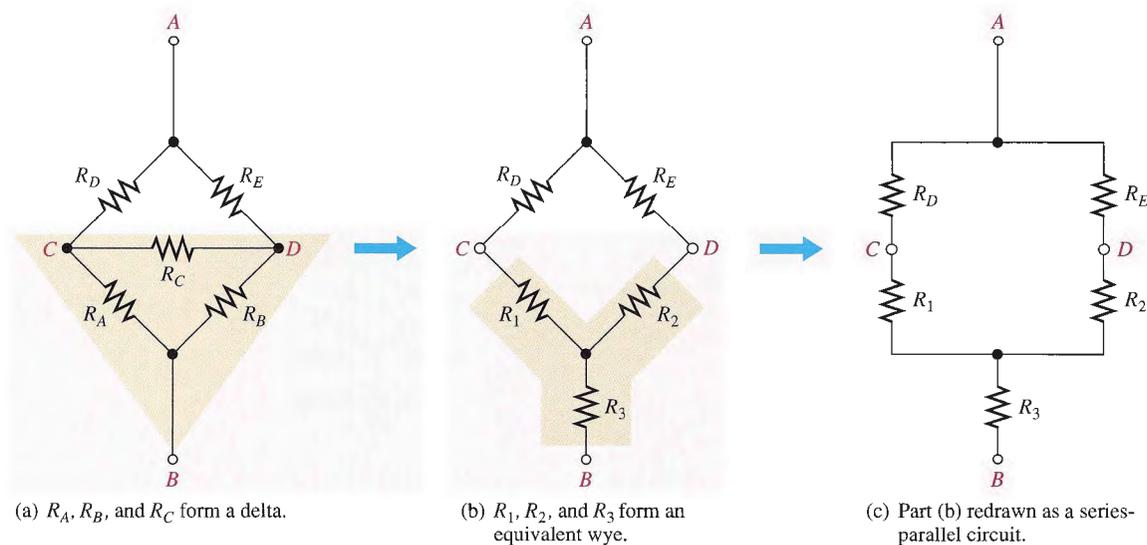


Related Problem Convert the wye circuit to a delta circuit for $R_1 = 100 \Omega$, $R_2 = 330 \Omega$, and $R_3 = 470 \Omega$.

Application of Δ -to-Y Conversion to a Bridge Circuit

In Section 8–5 you learned how Thevenin’s theorem can be used to simplify a bridge circuit. Now you will see how Δ -to-Y conversion can be used for converting a bridge circuit to a series-parallel form for easier analysis.

Figure 8–63 illustrates how the delta (Δ) formed by R_A , R_B , and R_C can be converted to a wye (Y), thus creating an equivalent series-parallel circuit. Equations 8–1, 8–2, and 8–3 are used in this conversion.



▲ FIGURE 8–63

Conversion of a bridge circuit to a series-parallel configuration.

In a bridge circuit, the load is connected between terminals C and D . In Figure 8–63(a), R_C represents the load resistor. When voltage is applied across terminals A and B , the voltage from C to D (V_{CD}) can be determined using the equivalent series-parallel circuit in Figure 8–63(c) as follows. The total resistance from terminal A to terminal B is

$$R_T = \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)} + R_3$$

Then,

$$I_T = \frac{V_{AB}}{R_T}$$

The resistance of the parallel portion of the circuit in Figure 8–63(c) is

$$R_{T(p)} = \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)}$$

The current through the left branch is

$$I_{AC} = \left(\frac{R_{T(p)}}{R_1 + R_D} \right) I_T$$

The current through the right branch is

$$I_{AD} = \left(\frac{R_{T(p)}}{R_2 + R_E} \right) I_T$$

The voltage at terminal C with respect to terminal A is

$$V_{CA} = V_A - I_{AC}R_D$$

The voltage at terminal D with respect to terminal A is

$$V_{DA} = V_A - I_{AD}R_E$$

The voltage from terminal C to terminal D is

$$\begin{aligned} V_{CD} &= V_{CA} - V_{DA} \\ &= (V_A - I_{AC}R_D) - (V_A - I_{AD}R_E) = I_{AD}R_E - I_{AC}R_D \end{aligned}$$

V_{CD} is the voltage across the load (R_C) in the bridge circuit of Figure 8–63(a).

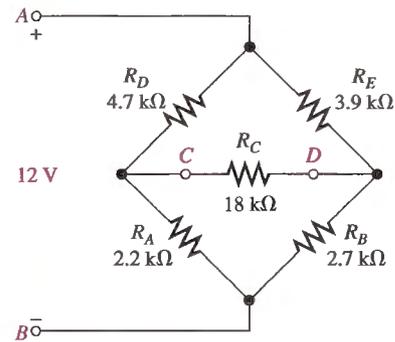
The load current through R_C can be found by Ohm's law.

$$I_{R_C} = \frac{V_{CD}}{R_C}$$

EXAMPLE 8–20

Determine the voltage across the load resistor and the current through the load resistor in the bridge circuit in Figure 8–64. Notice that the resistors are labeled for convenient conversion using Equations 8–1, 8–2, and 8–3. R_C is the load resistor.

► FIGURE 8–64



Solution First, convert the delta formed by R_A , R_B , and R_C to a wye.

$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(2.2 \text{ k}\Omega)(18 \text{ k}\Omega)}{2.2 \text{ k}\Omega + 2.7 \text{ k}\Omega + 18 \text{ k}\Omega} = 1.73 \text{ k}\Omega$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(2.7 \text{ k}\Omega)(18 \text{ k}\Omega)}{22.9 \text{ k}\Omega} = 2.12 \text{ k}\Omega$$

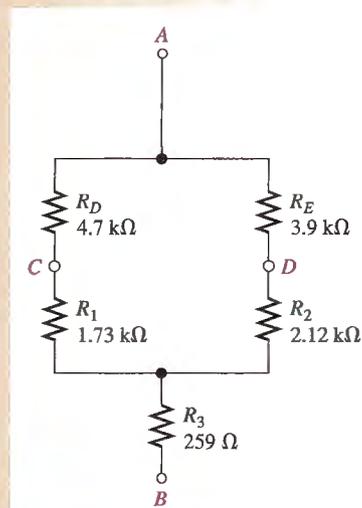
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(2.2 \text{ k}\Omega)(2.7 \text{ k}\Omega)}{22.9 \text{ k}\Omega} = 259 \Omega$$

The resulting equivalent series-parallel circuit is shown in Figure 8–65.

Next, determine R_T and the branch currents in Figure 8–65.

$$\begin{aligned} R_T &= \frac{(R_1 + R_D)(R_2 + R_E)}{(R_1 + R_D) + (R_2 + R_E)} + R_3 \\ &= \frac{(6.43 \text{ k}\Omega)(6.02 \text{ k}\Omega)}{6.43 \text{ k}\Omega + 6.02 \text{ k}\Omega} + 259 \Omega = 3.11 \text{ k}\Omega + 259 \Omega = 3.37 \text{ k}\Omega \end{aligned}$$

$$I_T = \frac{V_{AB}}{R_T} = \frac{12 \text{ V}}{3.37 \text{ k}\Omega} = 3.56 \text{ mA}$$



▲ FIGURE 8-65

The total resistance of the parallel part of the circuit, $R_{T(p)}$, is $3.11 \text{ k}\Omega$.

$$I_{AC} = \left(\frac{R_{T(p)}}{R_1 + R_D} \right) I_T = \left(\frac{3.11 \text{ k}\Omega}{1.73 \text{ k}\Omega + 4.7 \text{ k}\Omega} \right) 3.56 \text{ mA} = 1.72 \text{ mA}$$

$$I_{AD} = \left(\frac{R_{T(p)}}{R_2 + R_E} \right) I_T = \left(\frac{3.11 \text{ k}\Omega}{2.12 \text{ k}\Omega + 3.9 \text{ k}\Omega} \right) 3.56 \text{ mA} = 1.84 \text{ mA}$$

The voltage from terminal C to terminal D is

$$\begin{aligned} V_{CD} &= I_{AD}R_E - I_{AC}R_D = (1.84 \text{ mA})(3.9 \text{ k}\Omega) - (1.72 \text{ mA})(4.7 \text{ k}\Omega) \\ &= 7.18 \text{ V} - 8.08 \text{ V} = -0.9 \text{ V} \end{aligned}$$

V_{CD} is the voltage across the load (R_C) in the bridge circuit shown in Figure 8-64.

The load current through R_C is

$$I_{R_C} = \frac{V_{CD}}{R_C} = \frac{-0.9 \text{ V}}{18 \text{ k}\Omega} = -50 \mu\text{A}$$

Related Problem Determine the load current, I_{R_C} , in Figure 8-64 for the following resistor values: $R_A = 27 \text{ k}\Omega$, $R_B = 33 \text{ k}\Omega$, $R_D = 39 \text{ k}\Omega$, $R_E = 47 \text{ k}\Omega$, and $R_C = 100 \text{ k}\Omega$.



Use Multisim file E08-20 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 8-8 REVIEW

1. Draw a delta circuit.
2. Draw a wye circuit.
3. Write the formulas for delta-to-wye conversion.
4. Write the formulas for wye-to-delta conversion.



A Circuit Application

The Wheatstone bridge circuit was introduced in Chapter 7 and expanded in this chapter to include the use of Thevenin's theorem. In Chapter 7, the circuit application used a thermistor in one arm of the bridge to sense temperature. The bridge was used to compare the resistance of the thermistor with the resistance of a rheostat, which set the temperature at which the output would switch from one polarity to the opposite for turning on a heater in a tank filled with a liquid. In this circuit application, you will work with a similar circuit, but this time it will be used to monitor the temperature in the tank to provide a visual indication that the temperature is within a specified range.

The Temperature Monitor

The basic measuring circuit in the temperature monitor is a Wheatstone bridge with an ammeter and a series resistor acting as the load. The meter is an analog panel meter with a sensitivity of $50 \mu\text{A}$ full scale. The Wheatstone bridge temperature-measuring circuit is shown in Figure 8-66(a), and the meter panel is shown in Figure 8-66(b).

The Thermistor

The thermistor is the same one used in the circuit application of Chapter 7, specifically a Thermometrics RL2006-13.3K-140-D1 thermistor with a specified resistance of $25 \text{ k}\Omega$ at 25°C and a β of 4615K. Recall that β is a constant supplied by the manufacturer that indicates the shape of the temperature-resistance characteristic. As given earlier, the exponential equation for the resistance of a thermistor is approximated by

$$R_T = R_0 e^{\beta \left(\frac{T_0 - T}{T_0 T} \right)}$$

where:

R_T = the resistance at a given temperature

R_0 = the resistance at a reference temperature

T_0 = the reference temperature in K (typically 298 K, which is 25°C)

T = temperature (K)

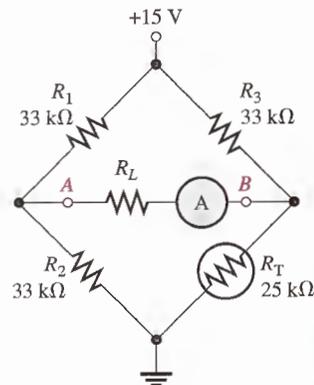
β = a constant (K) provided by the manufacturer

A plot of this equation was given previously in Figure 7-60. You can confirm that your thermistor resistance calculations in this circuit application are in reasonable agreement with this plot.

Temperature Measuring Circuit

The Wheatstone bridge is designed to be balanced at 20°C . The resistance of the thermistor is approximately $33 \text{ k}\Omega$ at this temperature. You can confirm this value by substituting the temperature (in Kelvin) into the equation for R_T . Remember that the temperature in K is $^\circ\text{C} + 273$.

- ◆ Substituting into the equation for R_T , calculate the resistance of the thermistor at a temperature of 50°C (full-scale deflection of the meter).
- ◆ Thevenize the bridge between terminals *A* and *B* by keeping the ground reference and forming two facing Thevenin circuits as was illustrated in Figure 8-42. Assume the thermistor temperature is 50°C and its resistance is the value calculated previously. Draw the Thevenin circuit for this temperature but do not show a load.
- ◆ Show the load resistor for the Thevenin circuit you drew. The load is a resistor in series with the ammeter, which will have a current of $50 \mu\text{A}$ at full scale (50°C). You can find the value of the required load resistor by applying the



(a) Temperature-measuring circuit with thermistor



(b) Meter panel

FIGURE 8-66

superposition theorem to the two sources and calculating the total resistance from Ohm's law (using the full-scale deflection as the current). Subtract the Thevenin resistance of each arm from the total resistance to obtain the required load resistance. Neglect the meter resistance. Show the value calculated on the Thevenin circuit.

- ◆ Calculate the thermistor resistance for the lower and upper limits of temperature (30°C and 40°C). Draw Thevenin circuits for each temperature and calculate the current through the load resistor.

The Meter Scale

A requirement for the temperature monitor is to mark three color bands on the meter to indicate that the temperature is within the

desired range. The desired range is between a low of 30°C and 40°C. The meter should indicate a too-cold range from 20°C to 30°C, a proper operating range from 30°C to 40°C, and a too hot range from 40°C and 50°C. Full-scale deflection of the meter should be set for 50°C.

- ◆ Indicate how you would mark the meter to have a quick visual indication of the temperature in the tank.

Review

1. At 35°C, what is the current in the meter?
2. What change is needed if a 100 μA meter is used instead of a 50 μA meter?

SUMMARY

- ◆ An ideal voltage source has zero internal resistance. It provides a constant voltage across its terminals regardless of the load resistance.
- ◆ A practical voltage source has a nonzero internal resistance.
- ◆ An ideal current source has infinite internal resistance. It provides a constant current regardless of the load resistance.
- ◆ A practical current source has a finite internal resistance.
- ◆ The superposition theorem is useful for multiple-source circuits.
- ◆ Thevenin's theorem provides for the reduction of any two-terminal linear resistive circuit to an equivalent form consisting of an equivalent voltage source in series with an equivalent resistance.
- ◆ The term *equivalency*, as used in Thevenin's and Norton's theorems, means that when a given load resistance is connected to the equivalent circuit, it will have the same voltage across it and the same current through it as when it was connected to the original circuit.
- ◆ Norton's theorem provides for the reduction of any two-terminal linear resistive circuit to an equivalent form consisting of an equivalent current source in parallel with an equivalent resistance.
- ◆ Maximum power is transferred to a load from a source when the load resistance equals the internal source resistance.

KEY TERMS

These key terms are also defined in the end-of-book glossary.

Maximum power transfer For a given source voltage, a transfer of maximum power from a source to a load occurs when the load resistance equals the internal source resistance.

Norton's theorem A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a current source in parallel with a resistance.

Superposition theorem A method for the analysis of circuits with more than one source.

Terminal equivalency The concept that when any given load resistance is connected to two sources, the same load voltage and load current are produced by both sources.

Thevenin's theorem A method for simplifying a two-terminal linear circuit to an equivalent circuit with only a voltage source in series with a resistance.

FORMULAS

 Δ -to-Y Conversions

$$8-1 \quad R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$8-2 \quad R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$8-3 \quad R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Y-to- Δ Conversions

$$8-4 \quad R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$8-5 \quad R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$8-6 \quad R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

SELF-TEST

Answers are at the end of the chapter.

- A 100 Ω load is connected across an ideal voltage source with $V_S = 10$ V. The voltage across the load is
(a) 0 V (b) 10 V (c) 100 V
- A 100 Ω load is connected across a voltage source with $V_S = 10$ V and $R_S = 10$ Ω . The voltage across the load is
(a) 10 V (b) 0 V (c) 9.09 V (d) 0.909 V
- A certain voltage source has the values $V_S = 25$ V and $R_S = 5$ Ω . The values for an equivalent current source are
(a) 5 A, 5 Ω (b) 25 A, 5 Ω (c) 5 A, 125 Ω
- A certain current source has the values $I_S = 3$ μ A and $R_S = 1.0$ M Ω . The values for an equivalent voltage source are
(a) 3 μ V, 1.0 M Ω (b) 3 V, 1.0 M Ω (c) 1 V, 3.0 M Ω
- In a two-source circuit, one source acting alone produces 10 mA through a given branch. The other source acting alone produces 8 mA in the opposite direction through the same branch. The actual current through the branch is
(a) 10 mA (b) 18 mA (c) 8 mA (d) 2 mA
- Thevenin's theorem converts a circuit to an equivalent form consisting of
(a) a current source and a series resistance
(b) a voltage source and a parallel resistance
(c) a voltage source and a series resistance
(d) a current source and a parallel resistance
- The Thevenin equivalent voltage for a given circuit is found by
(a) shorting the output terminals
(b) opening the output terminals
(c) shorting the voltage source
(d) removing the voltage source and replacing it with a short
- A certain circuit produces 15 V across its open output terminals, and when a 10 k Ω load is connected across its output terminals, it produces 12 V. The Thevenin equivalent for this circuit is
(a) 15 V in series with 10 k Ω (b) 12 V in series with 10 k Ω
(c) 12 V in series with 2.5 k Ω (d) 15 V in series with 2.5 k Ω

9. Maximum power is transferred from a source to a load when
- (a) the load resistance is very large
 - (b) the load resistance is very small
 - (c) the load resistance is twice the source resistance
 - (d) the load resistance equals the source resistance
10. For the circuit described in Question 8, maximum power is transferred to a
- (a) 10 k Ω load
 - (b) 2.5 k Ω load
 - (c) an infinitely large resistance load

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 8–69.

1. If a short develops across R_4 , the voltage across R_5
 - (a) increases
 - (b) decreases
 - (c) stays the same
2. If the 2 V source opens, the voltage across R_1
 - (a) increases
 - (b) decreases
 - (c) stays the same
3. If R_2 opens, the current through R_1
 - (a) increases
 - (b) decreases
 - (c) stays the same

Refer to Figure 8–77.

4. If R_L opens, the voltage at the output terminal with respect to ground
 - (a) increases
 - (b) decreases
 - (c) stays the same
5. If either of the 5.6 k Ω resistors are shorted, the current through the load resistor
 - (a) increases
 - (b) decreases
 - (c) stays the same
6. If either of the 5.6 k Ω resistors are shorted, current from the source
 - (a) increases
 - (b) decreases
 - (c) stays the same

Refer to Figure 8–79.

7. If the input to the amplifier becomes shorted to ground, the current drawn from both voltage sources
 - (a) increases
 - (b) decreases
 - (c) stays the same

Refer to Figure 8–82.

8. If R_1 is actually 1.0 k Ω instead of 10 k Ω , the expected voltage between A and B
 - (a) increases
 - (b) decreases
 - (c) stays the same
9. If a 10 M Ω load resistor is connected from A to B , the voltage between A and B
 - (a) increases
 - (b) decreases
 - (c) stays the same
10. If a short develops across R_4 , the magnitude of the voltage between A and B
 - (a) increases
 - (b) decreases
 - (c) stays the same

Refer to Figure 8–84.

11. If the 220 Ω resistor opens, V_{AB}
 - (a) increases
 - (b) decreases
 - (c) stays the same
12. If a short develops across the 330 Ω resistor, V_{AB}
 - (a) increases
 - (b) decreases
 - (c) stays the same

Refer to Figure 8–85(d).

13. If the 680 Ω resistor opens, the current through R_L
 - (a) increases
 - (b) decreases
 - (c) stays the same
14. If the 47 Ω resistor becomes shorted, the voltage across R_L
 - (a) increases
 - (b) decreases
 - (c) stays the same

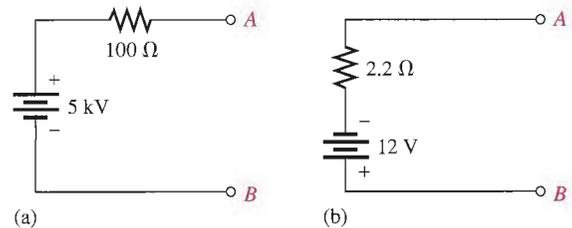
PROBLEMS

More difficult problems are indicated by an asterisk (*).
 Answers to odd-numbered problems are at the end of the book.

SECTION 8-3 Source Conversions

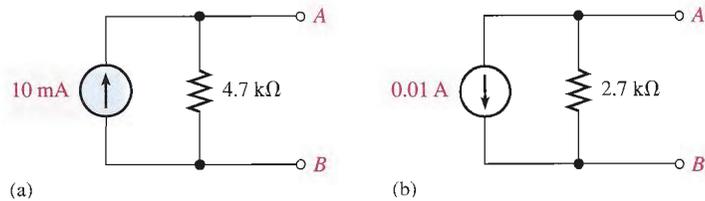
1. A voltage source has the values $V_S = 300\text{ V}$ and $R_S = 50\ \Omega$. Convert it to an equivalent current source.
2. Convert the practical voltage sources in Figure 8-67 to equivalent current sources.

► **FIGURE 8-67**



3. A fresh D cell battery has a terminal voltage of 1.6 V and can supply up to 8.0 A into a short for a very short time. What is the internal resistance of the battery?
4. Draw the voltage and current source equivalent circuits for the D cell in Problem 3.
5. A current source has an I_S of 600 mA and an R_S of 1.2 k Ω . Convert it to an equivalent voltage source.
6. Convert the practical current sources in Figure 8-68 to equivalent voltage sources.

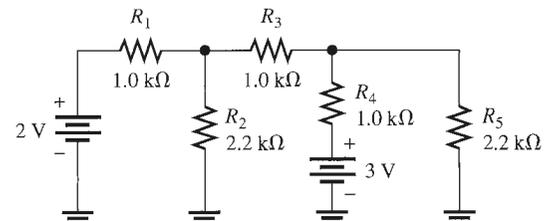
► **FIGURE 8-68**



SECTION 8-4 The Superposition Theorem

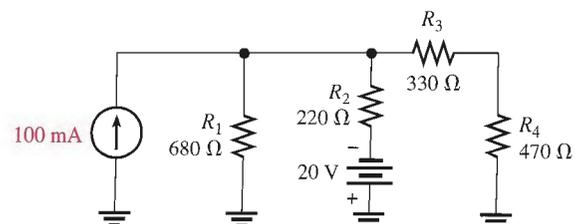
7. Using the superposition method, calculate the current through R_5 in Figure 8-69.
8. Use the superposition theorem to find the current in and the voltage across the R_2 branch of Figure 8-69.

► **FIGURE 8-69**

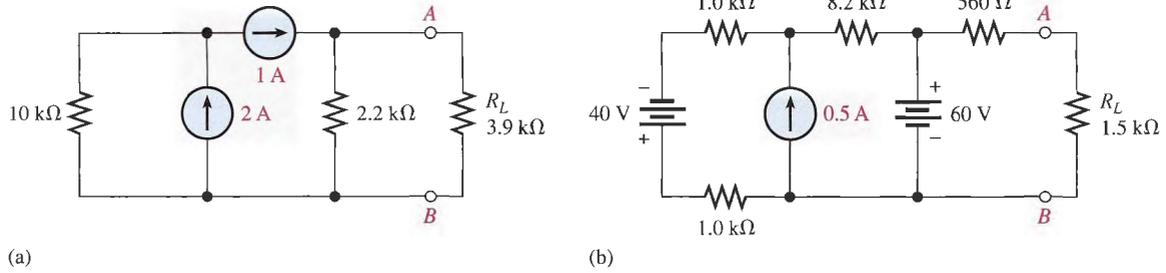


9. Using the superposition theorem, solve for the current through R_3 in Figure 8-70.

► **FIGURE 8-70**



10. Using the superposition theorem, find the load current in each circuit of Figure 8–71.

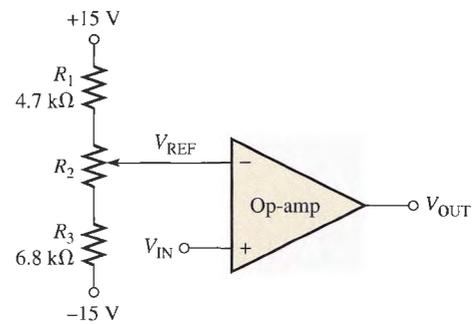


▲ FIGURE 8–71

11. A comparator circuit is shown in Figure 8–72. The input voltage, V_{IN} , is compared to the reference voltage, V_{REF} , and an output is generated that is negative if $V_{REF} > V_{IN}$; otherwise it is positive. The comparator does not load either input. If R_2 is $1.0\text{ k}\Omega$, what is the range of the reference voltage?

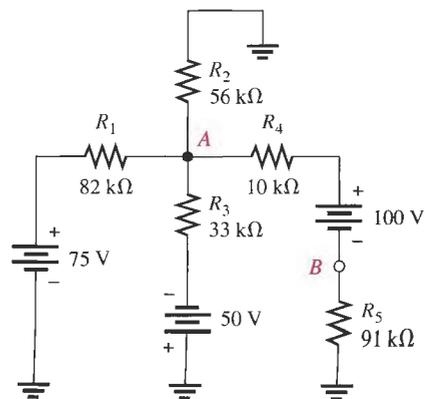
12. Repeat Problem 11 if R_2 is $10\text{ k}\Omega$.

► FIGURE 8–72



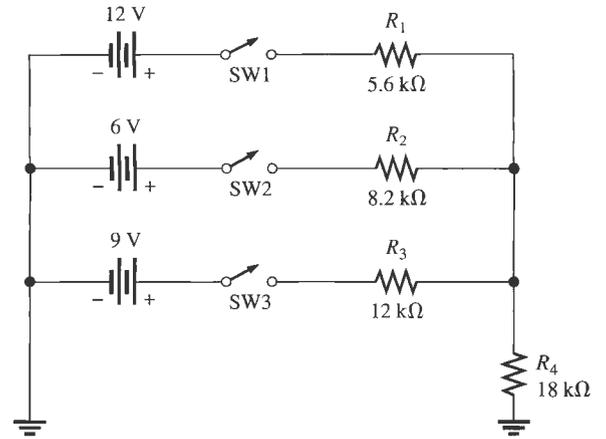
*13. Determine the voltage from point A to point B in Figure 8–73.

► FIGURE 8–73

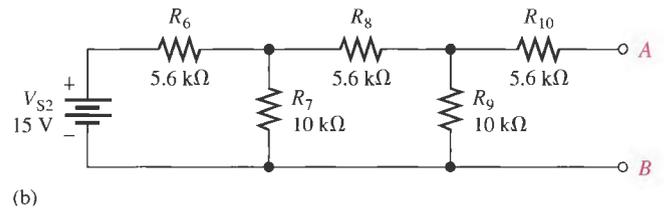
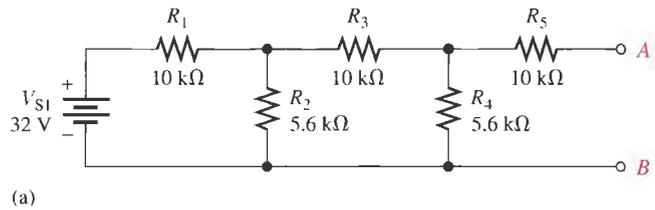


14. The switches in Figure 8–74 are closed in sequence, SW1 first. Find the current through R_4 after each switch closure.

▶ FIGURE 8-74



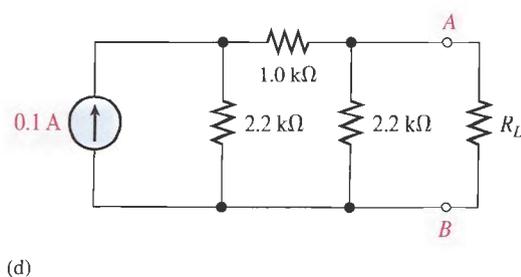
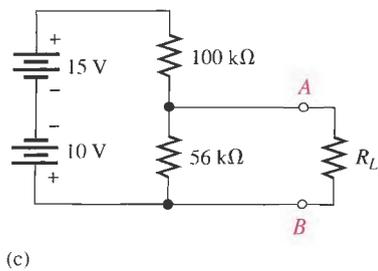
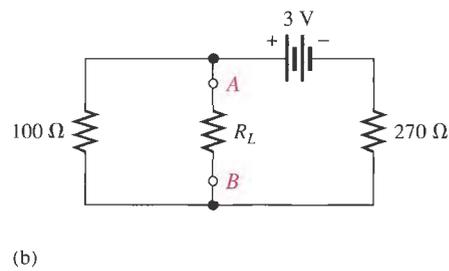
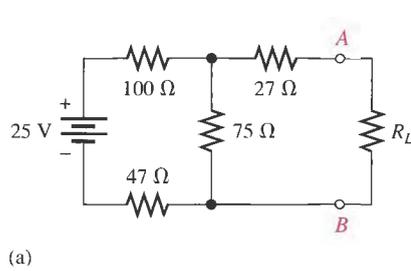
* 15. Figure 8-75 shows two ladder networks. Determine the current provided by each of the batteries when terminals *A* are connected (*A* to *A*) and terminals *B* are connected (*B* to *B*).



▶ FIGURE 8-75

SECTION 8-5 Thevenin's Theorem

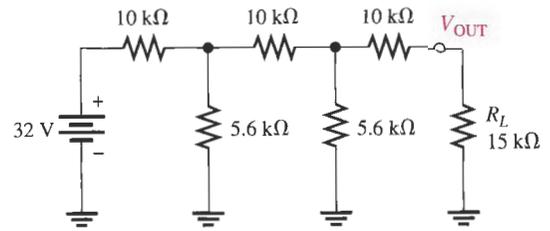
16. For each circuit in Figure 8-76, determine the Thevenin equivalent as seen from terminals *A* and *B*.



▶ FIGURE 8-76

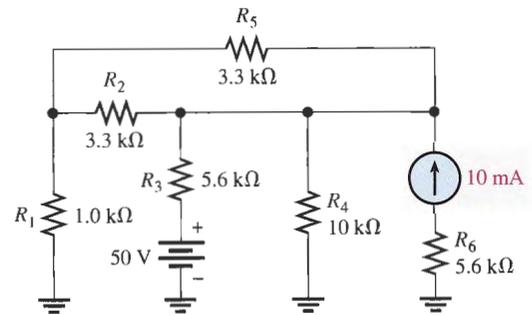
17. Using Thevenin's theorem, determine the current through the load R_L in Figure 8-77.

▶ FIGURE 8-77



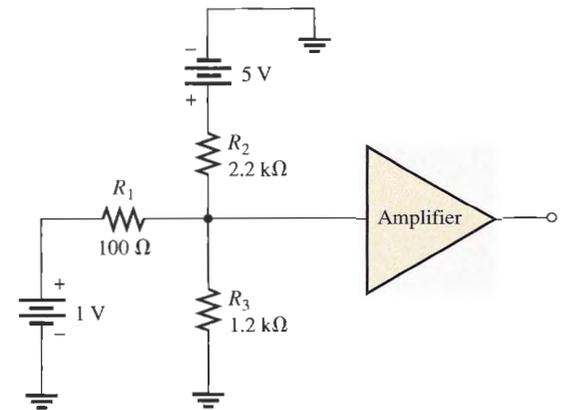
*18. Using Thevenin's theorem, find the voltage across R_4 in Figure 8-78.

▶ FIGURE 8-78

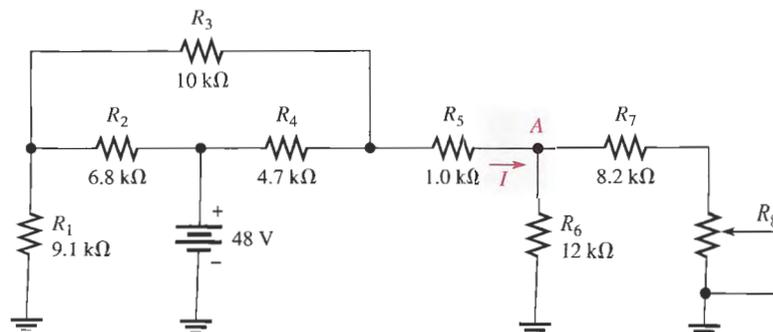


19. Find the Thevenin equivalent for the circuit external to the amplifier in Figure 8-79.

▶ FIGURE 8-79

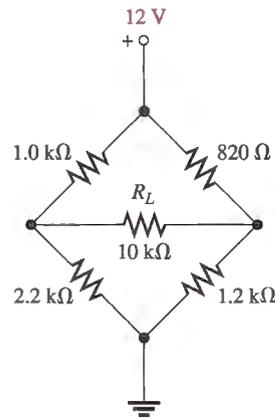


20. Determine the current into point A when R_8 is 1.0 kΩ, 5 kΩ, and 10 kΩ in Figure 8-80.



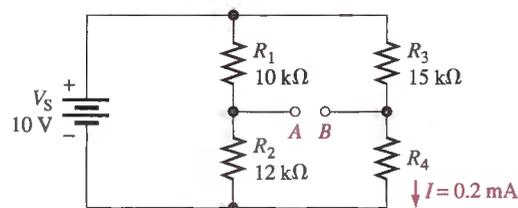
▲ FIGURE 8-80

*21. Find the current through the load resistor in the bridge circuit of Figure 8–81.



▲ FIGURE 8–81

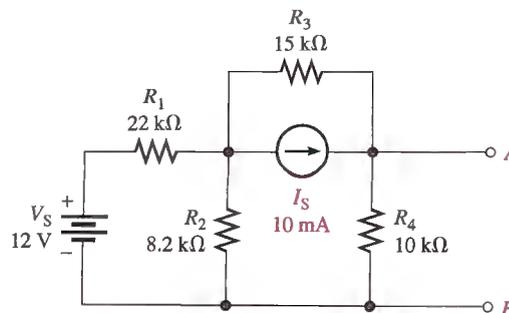
22. Determine the Thevenin equivalent looking from terminals A and B for the circuit in Figure 8–82.



▲ FIGURE 8–82

SECTION 8–6 Norton’s Theorem

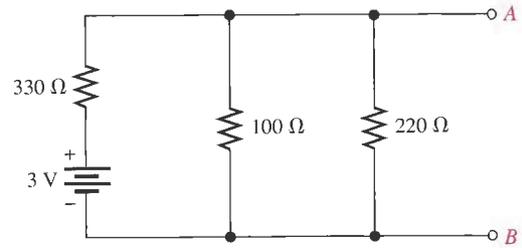
- 23. For each circuit in Figure 8–76, determine the Norton equivalent as seen by R_L .
- 24. Using Norton’s theorem, find the current through the load resistor R_L in Figure 8–77.
- *25. Using Norton’s theorem, find the voltage across R_5 in Figure 8–78.
- 26. Using Norton’s theorem, find the current through R_1 in Figure 8–80 when $R_8 = 8 \text{ k}\Omega$.
- 27. Determine the Norton equivalent circuit for the bridge in Figure 8–81 with R_L removed.
- 28. Reduce the circuit between terminals A and B in Figure 8–83 to its Norton equivalent.



▲ FIGURE 8–83

29. Apply Norton's theorem to the circuit of Figure 8–84.

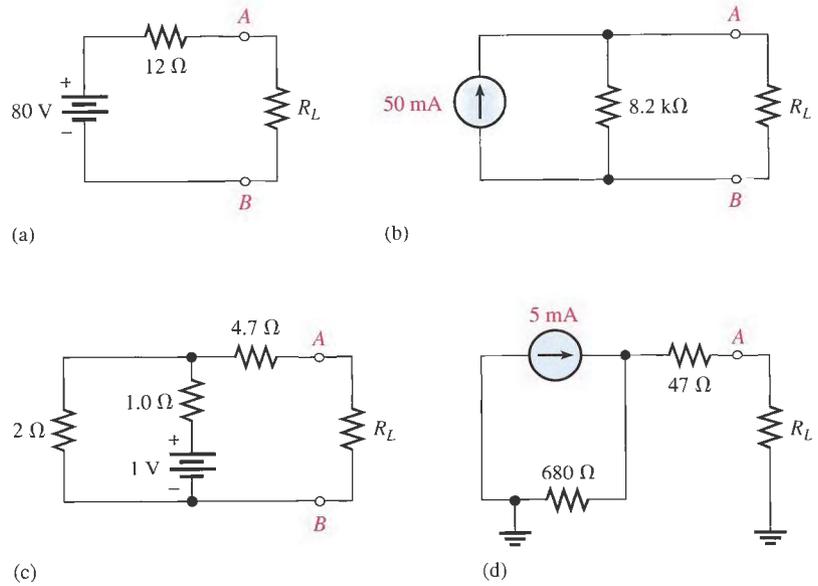
► FIGURE 8–84



SECTION 8–7 Maximum Power Transfer Theorem

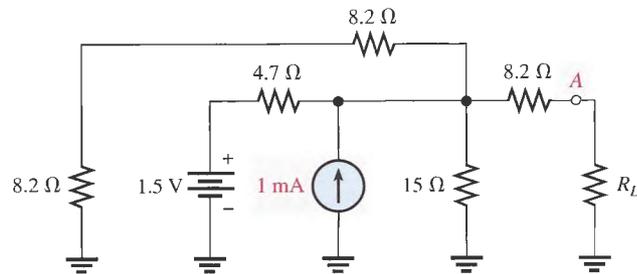
30. For each circuit in Figure 8–85, maximum power is to be transferred to the load R_L . Determine the appropriate value for R_L in each case.

► FIGURE 8–85

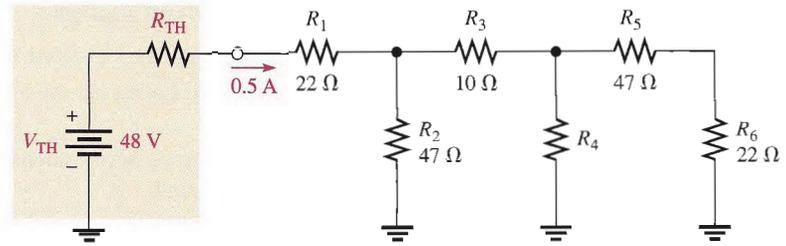


31. Determine the value of R_L for maximum power in Figure 8–86.

► FIGURE 8–86



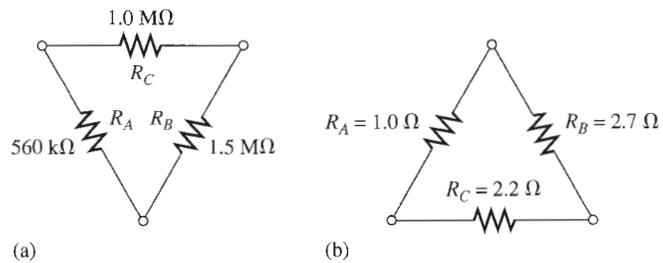
- *32. How much power is delivered to the load when R_L is 10% higher than its value for maximum power in Figure 8–86?
- *33. What are the values of R_4 and R_{TH} when the maximum power is transferred from the thevenized source to the ladder network in Figure 8–87?



▲ FIGURE 8-87

SECTION 8-8 Delta-to-Wye (Δ -to-Y) and Wye-to-Delta (Y-to- Δ) Conversions

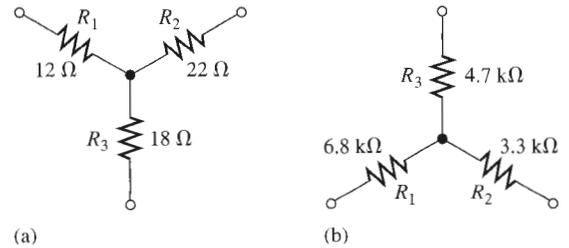
34. In Figure 8-88, convert each delta network to a wye network.



▲ FIGURE 8-88

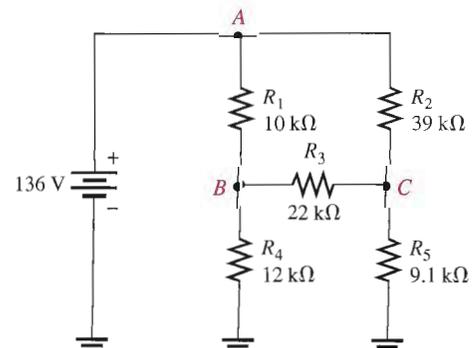
35. In Figure 8-89, convert each wye network to a delta network.

► FIGURE 8-89



*36. Find all currents in the circuit of Figure 8-90.

► FIGURE 8-90





Multisim Troubleshooting and Analysis

These problems require your Multisim CD-ROM.

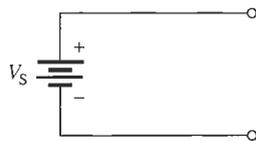
37. Open file P08-37 and verify that the current through each resistor is correct and, if not, determine the fault.
38. Open file P08-38 and determine by measurement the Thevenin equivalent for the circuit between terminal *A* and ground.
39. Open file P08-39 and determine by measurement the Norton equivalent for the circuit between terminal *A* and ground.
40. Open file P08-40 and determine the fault, if any.
41. Open file P08-41 and determine the value of a load resistor to be connected between terminals *A* and *B* to achieve maximum power transfer.

ANSWERS

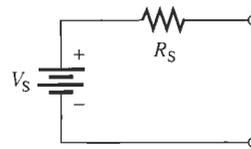
SECTION REVIEWS

SECTION 8-1 The DC Voltage Source

1. For ideal voltage source, see Figure 8-91.
2. For practical voltage source, see Figure 8-92.
3. The internal resistance of an ideal voltage source is zero ohms.
4. Output voltage of a voltage source varies directly with load resistance.



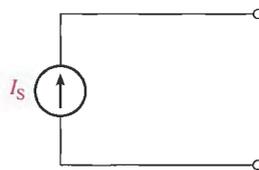
▲ FIGURE 8-91



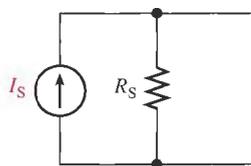
▲ FIGURE 8-92

SECTION 8-2 The Current Source

1. For ideal current source, see Figure 8-93.
2. For practical current source, see Figure 8-94.
3. An ideal current source has infinite internal resistance.
4. Load current from a current source varies inversely with load resistance.



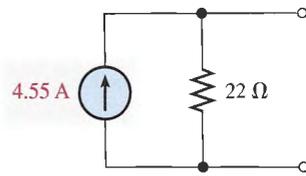
▲ FIGURE 8-93



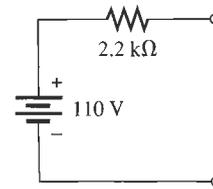
▲ FIGURE 8-94

SECTION 8-3 Source Conversions

1. $I_S = V_S/R_S$
2. $V_S = I_S R_S$
3. See Figure 8-95.
4. See Figure 8-96.



▲ FIGURE 8-95



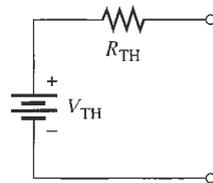
▲ FIGURE 8-96

SECTION 8-4 The Superposition Theorem

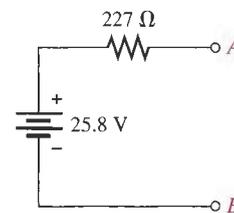
1. The superposition theorem states that the total current in any branch of a multiple-source linear circuit is equal to the algebraic sum of the currents due to the individual sources acting alone, with the other sources replaced by their internal resistances.
2. The superposition theorem allows each source to be treated independently.
3. A short simulates the internal resistance of an ideal voltage source; an open simulates the internal resistance of an ideal current source.
4. $I_{R1} = 6.67 \text{ mA}$
5. The net current is in the direction of the larger current.

SECTION 8-5 Thevenin's Theorem

1. A Thevenin equivalent circuit consists of V_{TH} and R_{TH} .
2. See Figure 8-97 for the general form of a Thevenin equivalent circuit.
3. V_{TH} is the open circuit voltage between two terminals in a circuit.
4. R_{TH} is the resistance as viewed from two terminals in a circuit, with all sources replaced by their internal resistances.
5. See Figure 8-98.



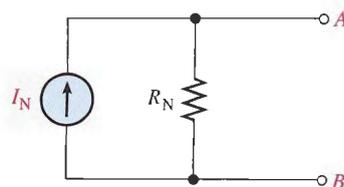
▲ FIGURE 8-97



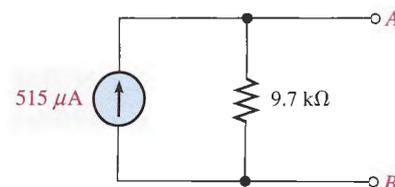
▲ FIGURE 8-98

SECTION 8-6 Norton's Theorem

1. A Norton equivalent circuit consists of I_N and R_N .
2. See Figure 8-99 for the general form of a Norton equivalent circuit.
3. I_N is the short circuit current between two terminals in a circuit.
4. R_N is the resistance as viewed from the two open terminals in a circuit.
5. See Figure 8-100.



▲ FIGURE 8-99



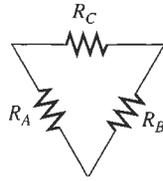
▲ FIGURE 8-100

SECTION 8-7 Maximum Power Transfer Theorem

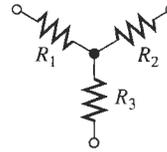
1. For a given source voltage, the maximum power transfer theorem states that maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.
2. Maximum power is delivered to a load when $R_L = R_S$.
3. $R_L = R_S = 50 \Omega$

SECTION 8-8 Delta-to-Wye (Δ -to-Y) and Wye-to-Delta (Y-to- Δ) Conversions

1. For a delta circuit see Figure 8-101.
2. For a wye circuit, see Figure 8-102.



▲ FIGURE 8-101



▲ FIGURE 8-102

3. The delta-to-wye conversion equations are

$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

4. The wye-to-delta conversion equations are

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

A Circuit Application

1. $27.8 \mu\text{A}$
2. The total series resistance at 50°C needs to be $47.1 \text{ k}\Omega$ (bridge arms, thermistor, and limiting resistor). The series resistor needs to be reduced to $26.2 \text{ k}\Omega$. ($47.1 \text{ k}\Omega - (16.5 \text{ k}\Omega + 4.38 \text{ k}\Omega)$)

RELATED PROBLEMS FOR EXAMPLES

- 8-1 99.5 V
 8-2 100 V
 8-3 3.33 k Ω
 8-4 1.2 A; 10 Ω
 8-5 300 V; 600 Ω
 8-6 16.6 mA
 8-7 I_S is not affected.

- 8-8 7 mA
8-9 5 mA
8-10 2.36 V; 1240 Ω
8-11 1.17 mA
8-12 0.7 V; V_{OUT} is 17.7 greater than V_{IN} at the base (B).
8-13 25.4 mA
8-14 131 Ω
8-15 $R_N = 248 \Omega$
8-16 6.93 V
8-17 41.7 mW
8-18 $R_1 = 792 \Omega$, $R_2 = 360 \Omega$, $R_3 = 440 \Omega$
8-19 $R_A = 712 \Omega$, $R_B = 2.35 \text{ k}\Omega$, $R_C = 500 \Omega$
8-20 0.3 μA

SELF-TEST

1. (b) 2. (c) 3. (a) 4. (b) 5. (d) 6. (c) 7. (b) 8. (d)
9. (d) 10. (b)

CIRCUIT DYNAMICS QUIZ

1. (a) 2. (b) 3. (b) 4. (a) 5. (b) 6. (a) 7. (a) 8. (a)
9. (b) 10. (a) 11. (a) 12. (a) 13. (a) 14. (a)