7 Series-Parallel Circuits

CHAPTER OUTLINE

7–1 Identifying Series-Parallel Relationships
7–2 Analysis of Series-Parallel Resistive Circuits
7–3 Voltage Dividers with Resistive Loads
7–4 Loading Effect of a Voltmeter
7–5 Ladder Networks
7–6 The Wheatstone Bridge
7–7 Troubleshooting
A Circuit Application

CHAPTER OBJECTIVES

- Identify series-parallel relationships
- Analyze series-parallel circuits
- Analyze loaded voltage dividers
- Determine the loading effect of a voltmeter on a circuit
- Analyze ladder networks
- Analyze and apply a Wheatstone bridge
- Troubleshoot series-parallel circuits

KEY TERMS

- Bleeder current
- Wheatstone bridge
- Balanced bridge
- Unbalanced bridge

A CIRCUIT APPLICATION PREVIEW

In the circuit application, you will learn how a Wheatstone bridge in conjunction with a thermistor can be used in a temperature-control application. The circuit in this application is designed to turn a heating element on and off in order to keep the temperature of a liquid in a tank at a desired level.

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INTRODUCTION

In Chapters 5 and 6, series circuits and parallel circuits were studied individually. In this chapter, both series and parallel resistors are combined into series-parallel circuits. In many practical situations, you will have both series and parallel combinations within the same circuit, and the analysis methods you learned for series circuits and for parallel circuits will apply.

Important types of series-parallel circuits are introduced in this chapter. These circuits include the voltage divider with a resistive load, the ladder network, and the Wheatstone bridge.

The analysis of series-parallel circuits requires the use of Ohm's law, Kirchhoff's voltage and current laws, and the methods for finding total resistance and power that you learned in the last two chapters. The topic of loaded voltage dividers is important because this type of circuit is found in many practical situations. One example is the voltage-divider bias circuit for a transistor amplifier, which you will study in a later course. Ladder networks are important in several areas, including a major type of digital-to-analog conversion, which you will study in a digital fundamentals course. The Wheatstone bridge is used in many types of systems for the measurement of unknown parameters, including most electronic scales.

7–1 IDENTIFYING SERIES-PARALLEL RELATIONSHIPS

A series-parallel circuit consists of combinations of both series and parallel current paths. It is important to be able to identify how the components in a circuit are arranged in terms of their series and parallel relationships.

After completing this section, you should be able to

- **Identify series-parallel relationships**
    - Recognize how each resistor in a given circuit is related to the other resistors
    - Determine series and parallel relationships on a PC board

Figure 7–1(a) shows an example of a simple series-parallel combination of resistors. Notice that the resistance from point A to point B is R_1. The resistance from point B to point C is R_2 and R_3 in parallel ($R_2 \parallel R_3$). The total resistance from point A to point C is R_1 in series with the parallel combination of R_2 and R_3, as indicated in Figure 7–1(b).

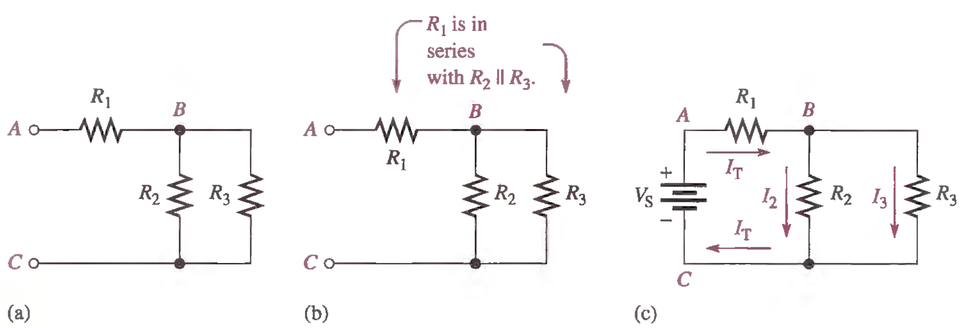


FIGURE 7–1

A simple series-parallel resistive circuit.

When the circuit of Figure 7–1(a) is connected to a voltage source as shown in Figure 7–1(c), the total current is through R_1 and divides at point B into the two parallel paths. These two branch currents then recombine, and the total current is into the negative source terminal as shown.

Now, to illustrate series-parallel relationships, let's increase the complexity of the circuit in Figure 7–1(a) step-by-step. In Figure 7–2(a), another resistor (R_4) is connected in series with R_1. The resistance between points A and B is now $R_1 + R_4$, and this combination is in series with the parallel combination of R_2 and R_3, as illustrated in Figure 7–2(b).

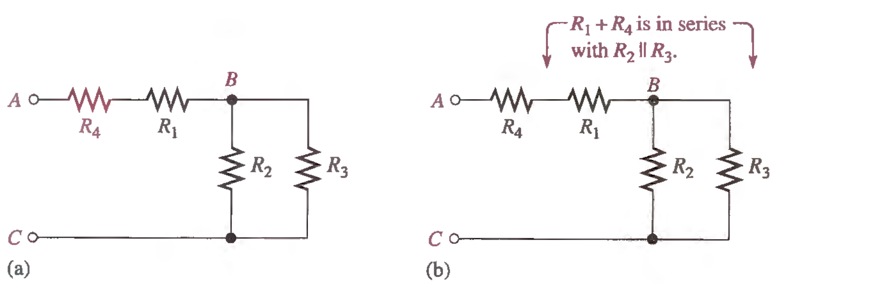


FIGURE 7–2

R_4 is added to the circuit in series with R_1.

In Figure 7–3(a), R_5 is connected in series with R_2. The series combination of R_2 and R_5 is in parallel with R_3. This entire series-parallel combination is in series with the series combination of R_1 and R_4, as illustrated in Figure 7–3(b).

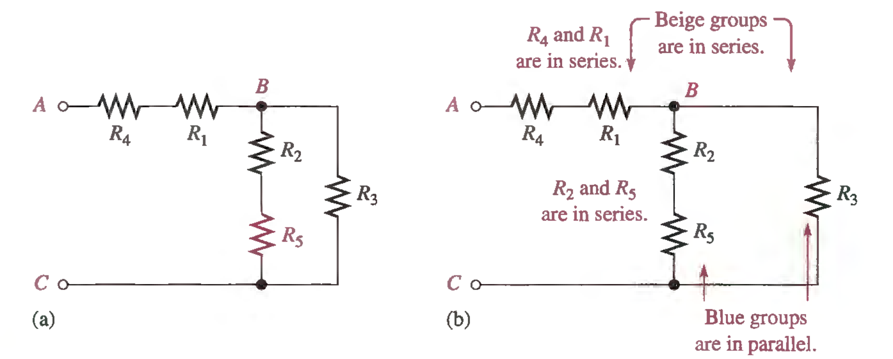


FIGURE 7–3

R_5 is added to the circuit in series with R_2.

In Figure 7–4(a), R_6 is connected in parallel with the series combination of R_1 and R_4. The series-parallel combination of R_1, R_4, and R_6 is in series with the series-parallel combination of R_2, R_3, and R_5, as indicated in Figure 7–4(b).

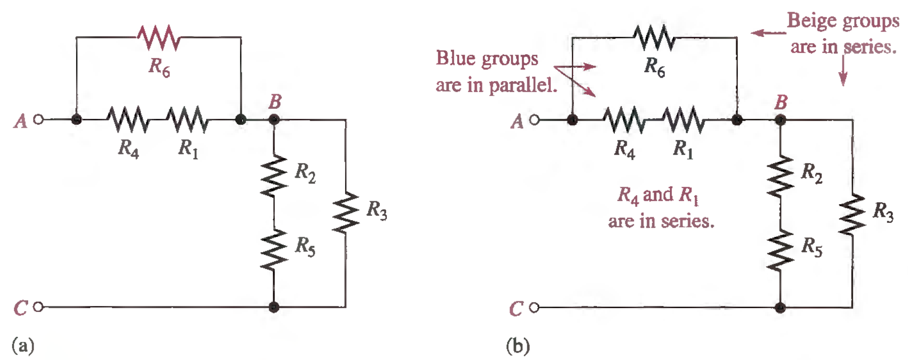


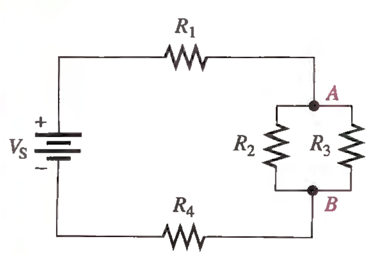
FIGURE 7–4

R_6 is added to the circuit in parallel with the series combination of R_1 and R_4.

EXAMPLE 7–1

Identify the series-parallel relationships in Figure 7–5.

FIGURE 7–5

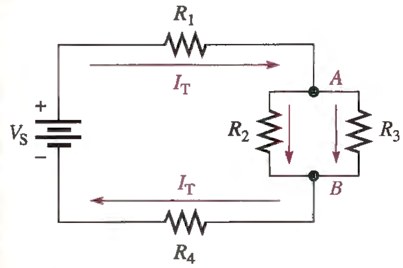


Solution Starting at the positive terminal of the source, follow the current paths. All of the current produced by the source must go through R_1, which is in series with the rest of the circuit.

The total current takes two paths when it gets to node A. Part of it is through R_2, and part of it is through R_3. Resistors R_2 and R_3 are in parallel with each other, and this parallel combination is in series with R_1.

At node B, the currents through R_2 and R_3 come together again. Thus, the total current is through R_4. Resistor R_4 is in series with R_1 and the parallel combination of R_2 and R_3. The currents are shown in Figure 7–6, where I_T is the total current.

► FIGURE 7–6



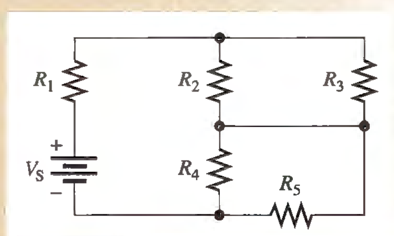
In summary, R_1 and R_4 are in series with the parallel combination of R_2 and R_3 as stated by the following expression:

$$R_1 + R_2 \| R_3 + R_4$$

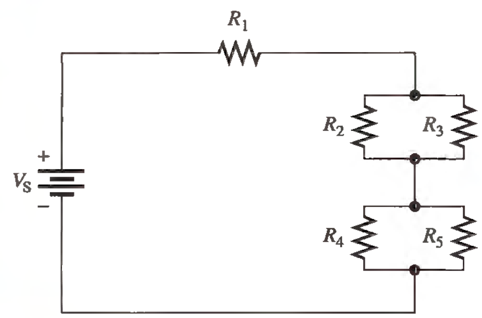
Related Problem* If another resistor, R_5, is connected from node A to the negative side of the source in Figure 7–6, what is its relationship to the other resistors?

*Answers are at the end of the chapter.

EXAMPLE 7–2 Identify the series-parallel relationships in Figure 7–7.



▲ FIGURE 7–7



▲ FIGURE 7–8

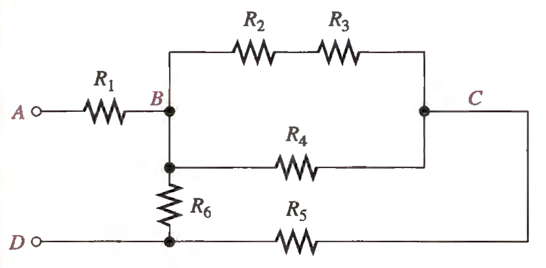
Solution Sometimes it is easier to see a particular circuit arrangement if it is drawn in a different way. In this case, the circuit schematic is redrawn in Figure 7–8, which better illustrates the series-parallel relationships. Now you can see that R_2 and R_3 are in parallel with each other and also that R_4 and R_5 are in parallel with each other. Both parallel combinations are in series with each other and with R_1 as stated by the following expression:

$$R_1 + R_2 \| R_3 + R_4 \| R_5$$

Related Problem If a resistor is connected from the bottom end of R_3 to the top end of R_5 in Figure 7–8, what effect does it have on the circuit?

EXAMPLE 7–3 Describe the series-parallel combination between terminals *A* and *D* in Figure 7–9.

▶ FIGURE 7–9



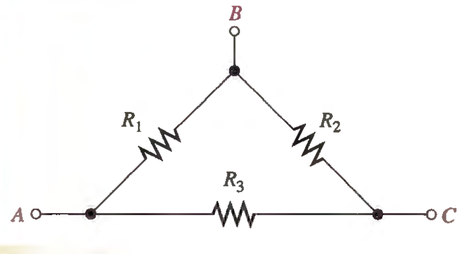
Solution Between nodes *B* and *C*, there are two parallel paths. The lower path consists of R_4, and the upper path consists of a series combination of R_2 and R_3. This parallel combination is in series with R_5. The R_2, R_3, R_4, R_5 combination is in parallel with R_6. Resistor R_1 is in series with this entire combination as stated by the following expression:

$$R_1 + R_6 \| (R_5 + R_4 \| (R_2 + R_3))$$

Related Problem If a resistor is connected from *C* to *D* in Figure 7–9, describe its parallel relationship.

EXAMPLE 7–4 Describe the total resistance between each pair of terminals in Figure 7–10.

▶ FIGURE 7–10



Solution

1. From *A* to *B*: R_1 is in parallel with the series combination of R_2 and R_3.

$$R_1 \| (R_2 + R_3)$$

2. From *A* to *C*: R_3 is in parallel with the series combination of R_1 and R_2.

$$R_3 \| (R_1 + R_2)$$

3. From *B* to *C*: R_2 is in parallel with the series combination of R_1 and R_3.

$$R_2 \| (R_1 + R_3)$$

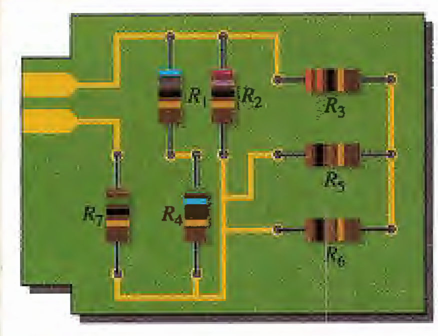
Related Problem In Figure 7–10, describe the total resistance between each terminal and an added ground if a new resistor, R_4, is connected from C to ground. None of the existing resistors connect directly to the ground.

Usually, the physical arrangement of components on a PC or protoboard bears no resemblance to the actual circuit relationships. By tracing out the circuit and rearranging the components on paper into a recognizable form, you can determine the series-parallel relationships.

EXAMPLE 7–5

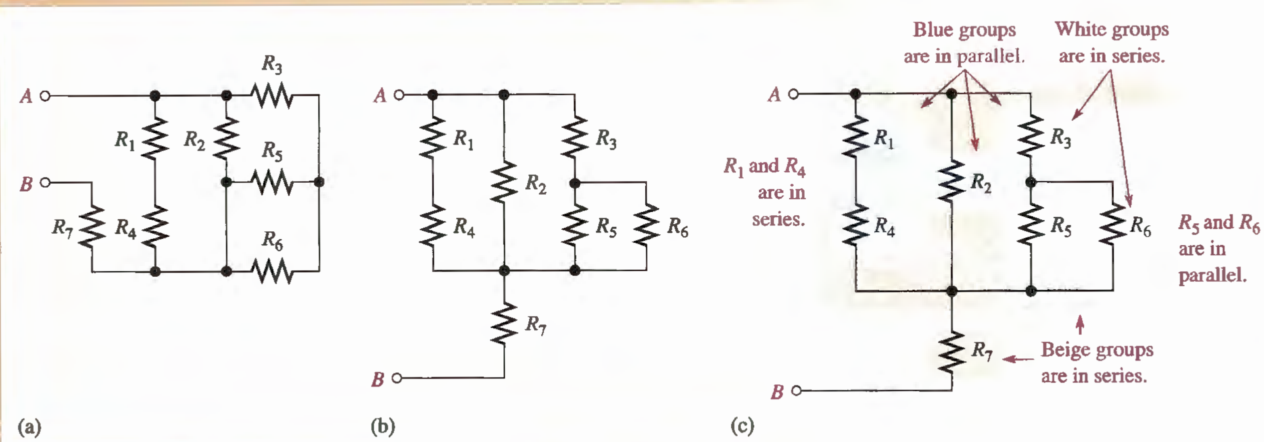
Determine the relationships of the resistors on the PC board in Figure 7–11.

▶ FIGURE 7–11



Solution In Figure 7–12(a), the schematic is drawn in the same arrangement as that of the resistors on the board. In part (b), the resistors are rearranged so that the series-parallel relationships are more obvious. Resistors R_1 and R_4 are in series; $R_1 + R_4$ is in parallel with R_2; R_5 and R_6 are in parallel and this combination is in series with R_3. The R_3, R_5, and R_6 series-parallel combination is in parallel with both R_2 and the $R_1 + R_4$ combination. This entire series-parallel combination is in series with R_7. Figure 7–12(c) illustrates these relationships. Summarizing in equation form,

$$R_{AB} = (R_5 \parallel R_6 + R_3) \parallel R_2 \parallel (R_1 + R_4) + R_7$$



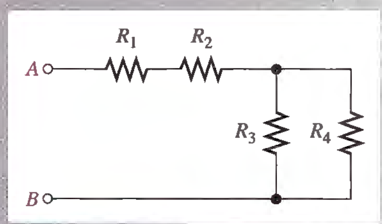
▲ FIGURE 7–12

Related Problem If R_5 were removed from the circuit, what would be the relationship of R_3 and R_6?

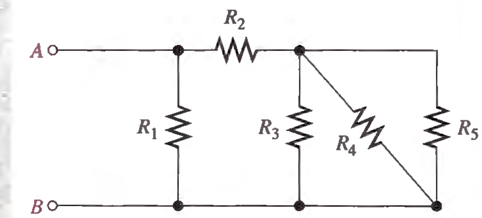
SECTION 7-1 REVIEW

Answers are at the end of the chapter.

1. Define *series-parallel resistive circuit.*
2. A certain series-parallel circuit is described as follows: R_1 and R_2 are in parallel. This parallel combination is in series with another parallel combination of R_3 and R_4. Draw the circuit.
3. In the circuit of Figure 7–13, describe the series-parallel relationships of the resistors.
4. Which resistors are in parallel in Figure 7–14?



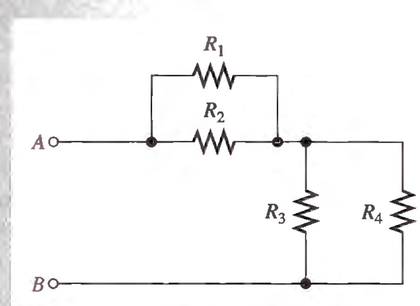
▲ FIGURE 7–13



▲ FIGURE 7–14

5. Describe the parallel arrangements in Figure 7–15.
6. Are the parallel combinations in Figure 7–15 in series?

▶ FIGURE 7–15



7–2 ANALYSIS OF SERIES-PARALLEL RESISTIVE CIRCUITS

The analysis of series-parallel circuits can be approached in many ways, depending on what information you need and what circuit values you know. The examples in this section do not represent an exhaustive coverage, but they give you an idea of how to approach series-parallel circuit analysis.

After completing this section, you should be able to

- **Analyze series-parallel circuits**
  - Determine total resistance
  - Determine all the currents
  - Determine all the voltage drops

If you know Ohm's law, Kirchhoff's laws, the voltage-divider formula, and the current-divider formula, and if you know how to apply these laws, you can solve most resistive circuit analysis problems. The ability to recognize series and parallel combinations is, of course, essential. A few circuits, such as the unbalanced Wheatstone bridge, do not have basic series and parallel combinations. Other methods are needed for these cases, as we will discuss later.

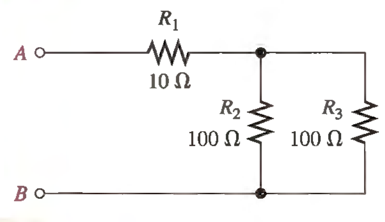
Total Resistance

In Chapter 5, you learned how to determine total series resistance. In Chapter 6, you learned how to determine total parallel resistance. To find the total resistance (R_T) of a series-parallel combination, simply define the series and parallel relationships; then perform the calculations that you have previously learned. The following two examples illustrate this general approach.

EXAMPLE 7–6

Determine R_T of the circuit in Figure 7–16 between terminals A and B.

▶ FIGURE 7–16



Solution First, calculate the equivalent parallel resistance of R_2 and R_3. Since R_2 and R_3 are equal in value, you can use Equation 6–4.

$$R_{2\|3} = \frac{R}{n} = \frac{100\ \Omega}{2} = 50\ \Omega$$

Notice that the term $R_{2\|3}$ is used here to designate the total resistance of a portion of a circuit in order to distinguish it from the total resistance, R_T, of the complete circuit.

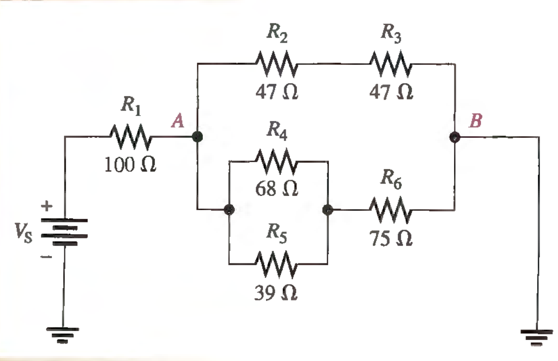
Now, since R_1 is in series with $R_{2\|3}$, add their values as follows:

$$R_T = R_1 + R_{2\|3} = 10\ \Omega + 50\ \Omega = \mathbf{60\ \Omega}$$

Related Problem Determine R_T in Figure 7–16 if R_3 is changed to 82 Ω.

EXAMPLE 7–7

Find the total resistance between the positive and negative terminals of the battery in Figure 7–17.



▲ FIGURE 7–17

Solution In the upper branch, R_2 is in series with R_3. This series combination is designated R_{2+3} and is equal to $R_2 + R_3$.

$$R_{2+3} = R_2 + R_3 = 47\ \Omega + 47\ \Omega = 94\ \Omega$$

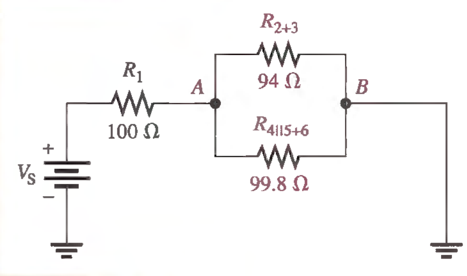
In the lower branch, R_4 and R_5 are in parallel with each other. This parallel combination is designated $R_{4\|5}$.

$$R_{4\|5} = \frac{R_4R_5}{R_4 + R_5} = \frac{(68\ \Omega)(39\ \Omega)}{68\ \Omega + 39\ \Omega} = 24.8\ \Omega$$

Also in the lower branch, the parallel combination of R_4 and R_5 is in series with R_6. This series-parallel combination is designated $R_{4\|5+6}$.

$$R_{4\|5+6} = R_6 + R_{4\|5} = 75\ \Omega + 24.8\ \Omega = 99.8\ \Omega$$

Figure 7–18 shows the original circuit in a simplified equivalent form.



▲ FIGURE 7–18

Now you can find the equivalent resistance between A and B. It is R_{2+3} in parallel with $R_{4\|5+6}$. Calculate the equivalent resistance as follows:

$$R_{AB} = \frac{1}{\frac{1}{R_{2+3}} + \frac{1}{R_{4\|5+6}}} = \frac{1}{\frac{1}{94\ \Omega} + \frac{1}{99.8\ \Omega}} = 48.4\ \Omega$$

Finally, the total resistance is R_1 in series with R_{AB}.

$$R_T = R_1 + R_{AB} = 100\ \Omega + 48.4\ \Omega = \mathbf{148.4\ \Omega}$$

Related Problem Determine R_T if a 68 Ω resistor is added in parallel from A to B in Figure 7–17.

Total Current

Once you know the total resistance and the source voltage, you can apply Ohm's law to find the total current in a circuit. Total current is the source voltage divided by the total resistance.

$$I_T = \frac{V_S}{R_T}$$

For example, assuming that the source voltage is 30 V, the total current in the circuit of Example 7–7 (Figure 7–17) is

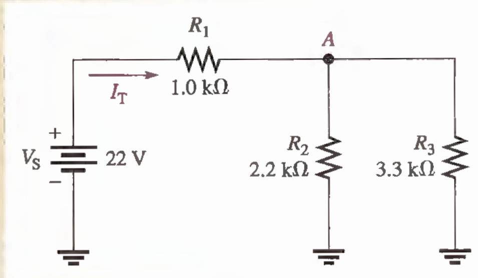
$$I_T = \frac{V_S}{R_T} = \frac{30\ \text{V}}{148.4\ \Omega} = 202\ \text{mA}$$

Branch Currents

Using the current-divider formula, Kirchhoff's current law, Ohm's law, or combinations of these, you can find the current in any branch of a series-parallel circuit. In some cases, it may take repeated application of the formula to find a given current. The following two examples will help you understand the procedure. (Notice that the subscripts for the current variables (I) match the R subscripts; for example, current through R_1 is referred to as I_1.)

EXAMPLE 7–8

Find the current through R_2 and the current through R_3 in Figure 7–19.



▲ FIGURE 7–19

Solution First, identify the series and parallel relationship. Next, determine how much current is into node A. This is the total circuit current. To find I_T, you must know R_T.

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} = 1.0\,\text{k}\Omega + \frac{(2.2\,\text{k}\Omega)(3.3\,\text{k}\Omega)}{2.2\,\text{k}\Omega + 3.3\,\text{k}\Omega} = 1.0\,\text{k}\Omega + 1.32\,\text{k}\Omega = 2.32\,\text{k}\Omega$$

$$I_T = \frac{V_S}{R_T} = \frac{22\,\text{V}}{2.32\,\text{k}\Omega} = 9.48\,\text{mA}$$

Use the current-divider rule for two branches as given in Chapter 6 to find the current through R_2.

$$I_2 = \left(\frac{R_3}{R_2 + R_3}\right)I_T = \left(\frac{3.3\,\text{k}\Omega}{5.5\,\text{k}\Omega}\right)9.48\,\text{mA} = \mathbf{5.69\,mA}$$

Now you can use Kirchhoff's current law to find the current through R_3.

$$I_T = I_2 + I_3$$

$$I_3 = I_T - I_2 = 9.48\,\text{mA} - 5.69\,\text{mA} = \mathbf{3.79\,mA}$$

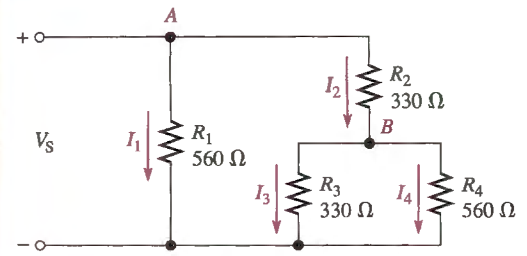
Related Problem A 4.7 kΩ resistor is connected in parallel with R_3 in Figure 7–19. Determine the current through the new resistor.

Use Multisim file E07-08 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 7–9

Determine the current through R_4 in Figure 7–20 if $V_S = 50$ V.

▶ FIGURE 7–20



Solution First, find the current (I_2) into node B. Once you know this current, use the current-divider formula to find I_4, the current through R_4.

Notice that there are two main branches in the circuit. The left-most branch consists of only R_1. The right-most branch has R_2 in series with the parallel combination of R_3 and R_4. The voltage across both of these main branches is the same and equal to 50 V. Calculate the equivalent resistance ($R_{2+3\|4}$) of the right-most main branch and then apply Ohm's law; I_2 is the total current through this main branch. Thus,

$$R_{2+3\|4} = R_2 + \frac{R_3R_4}{R_3 + R_4} = 330\ \Omega + \frac{(330\ \Omega)(560\ \Omega)}{890\ \Omega} = 538\ \Omega$$

$$I_2 = \frac{V_S}{R_{2+3\|4}} = \frac{50\text{ V}}{538\ \Omega} = 93\text{ mA}$$

Use the two-resistor current-divider formula to calculate I_4.

$$I_4 = \left(\frac{R_3}{R_3 + R_4}\right)I_2 = \left(\frac{330\ \Omega}{890\ \Omega}\right)93\text{ mA} = \mathbf{34.5\ mA}$$

Related Problem Determine the current through R_1 and R_3 in Figure 7–20 if $V_S = 20$ V.

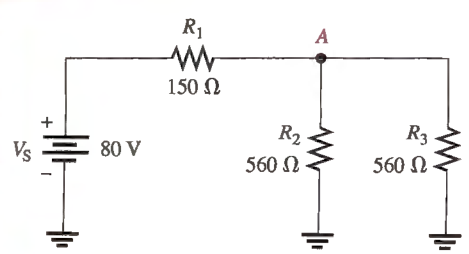
Voltage Drops

To find the voltages across certain parts of a series-parallel circuit, you can use the voltage-divider formula given in Chapter 5, Kirchhoff's voltage law, Ohm's law, or combinations of each. The following three examples illustrate use of the formulas. (The subscripts for V match the subscripts for the corresponding R: V_1 is the voltage across R_1; V_2 is the voltage across R_2, etc.)

EXAMPLE 7–10

Determine the voltage drop from node A to ground in Figure 7–21. Then find the voltage (V_1) across R_1.

▶ FIGURE 7–21



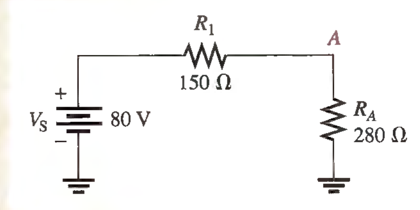
Solution Note that R_2 and R_3 are in parallel in this circuit. Since they are equal in value, their equivalent resistance from node A to ground is

$$R_A = \frac{560\ \Omega}{2} = 280\ \Omega$$

In the equivalent circuit shown in Figure 7–22, R_1 is in series with R_A. The total circuit resistance as seen from the source is

$$R_T = R_1 + R_A = 150\ \Omega + 280\ \Omega = 430\ \Omega$$

▶ FIGURE 7–22



Use the voltage-divider formula to find the voltage across the parallel combination of Figure 7–21 (between node A and ground).

$$V_A = \left(\frac{R_A}{R_T}\right)V_S = \left(\frac{280\ \Omega}{430\ \Omega}\right)80\ \text{V} = \mathbf{52.1\ V}$$

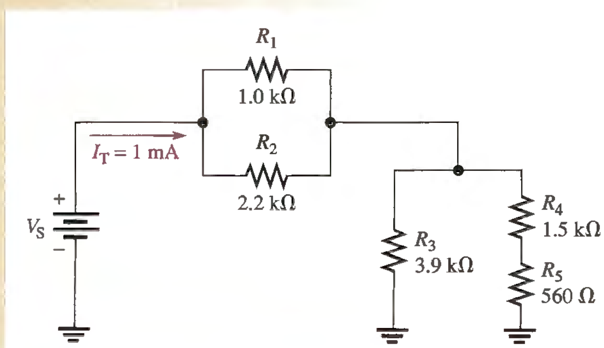
Now use Kirchhoff's voltage law to find V_1.

$$V_S = V_1 + V_A$$
$$V_1 = V_S - V_A = 80\ \text{V} - 52.1\ \text{V} = \mathbf{27.9\ V}$$

Related Problem Determine V_A and V_1 if R_1 is changed to 220 Ω in Figure 7–21.

Use Multisim file E07-10 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 7–11 Determine the voltage drop across each resistor in the circuit of Figure 7–23.



▲ FIGURE 7–23

Solution The source voltage is not given, but you know the total current from the figure. Since R_1 and R_2 are in parallel, they each have the same voltage. The current through R_1 is

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{2.2\,\text{k}\Omega}{3.2\,\text{k}\Omega}\right)1\text{ mA} = 688\,\mu\text{A}$$

The voltages across R_1 and R_2 are

$$V_1 = I_1R_1 = (688\,\mu\text{A})(1.0\,\text{k}\Omega) = \mathbf{688\,mV}$$
$$V_2 = V_1 = \mathbf{688\,mV}$$

The series combination of R_4 and R_5 form the branch resistance, R_{4+5}. Apply the current-divider formula to determine the current through R_3.

$$I_3 = \left(\frac{R_{4+5}}{R_3 + R_{4+5}}\right)I_T = \left(\frac{2.06\,\text{k}\Omega}{5.96\,\text{k}\Omega}\right)1\text{ mA} = 346\,\mu\text{A}$$

The voltage across R_3 is

$$V_3 = I_3R_3 = (346\,\mu\text{A})(3.9\,\text{k}\Omega) = \mathbf{1.35\,V}$$

The currents through R_4 and R_5 are the same because these resistors are in series.

$$I_4 = I_5 = I_T - I_3 = 1\text{ mA} - 346\,\mu\text{A} = 654\,\mu\text{A}$$

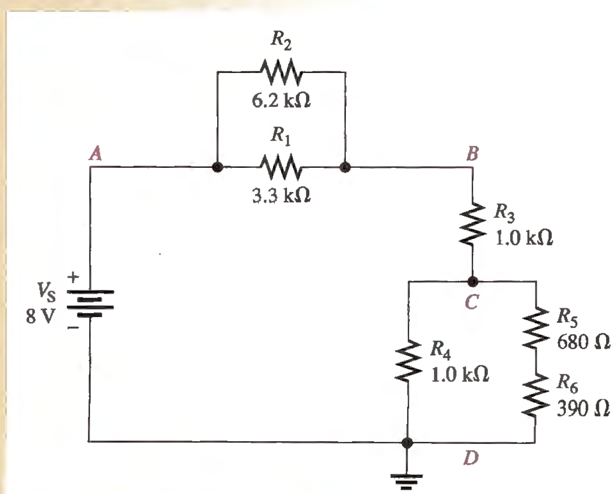
Calculate the voltages across R_4 and R_5 as follows:

$$V_4 = I_4R_4 = (654\,\mu\text{A})(1.5\,\text{k}\Omega) = \mathbf{981\,mV}$$
$$V_5 = I_5R_5 = (654\,\mu\text{A})(560\,\Omega) = \mathbf{366\,mV}$$

Related Problem What is the source voltage, V_S, in the circuit of Figure 7–23?

Use Multisim file E07-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 7–12 Determine the voltage drop across each resistor in Figure 7–24.



▲ FIGURE 7–24

Solution Because the total voltage is given in the figure, you can solve this problem using the voltage-divider formula. First, you need to reduce each parallel combination to an equivalent resistance. Since R_1 and R_2 are in parallel between A and B, combine their values.

$$R_{AB} = \frac{R_1R_2}{R_1 + R_2} = \frac{(3.3\,\text{k}\Omega)(6.2\,\text{k}\Omega)}{9.5\,\text{k}\Omega} = 2.15\,\text{k}\Omega$$

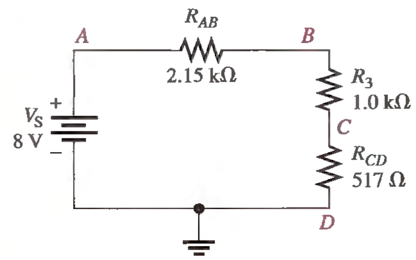
Since R_4 is in parallel with the R_5 and R_6 series combination (R_{5+6}) between C and D, combine these values.

$$R_{CD} = \frac{R_4R_{5+6}}{R_4 + R_{5+6}} = \frac{(1.0\,\text{k}\Omega)(1.07\,\text{k}\Omega)}{2.07\,\text{k}\Omega} = 517\,\Omega$$

The equivalent circuit is drawn in Figure 7–25. The total circuit resistance is

$$R_T = R_{AB} + R_3 + R_{CD} = 2.15\,\text{k}\Omega + 1.0\,\text{k}\Omega + 517\,\Omega = 3.67\,\text{k}\Omega$$

▶ FIGURE 7–25



Next, use the voltage-divider formula to determine the voltages in the equivalent circuit.

$$V_{AB} = \left(\frac{R_{AB}}{R_T}\right)V_S = \left(\frac{2.15\,\text{k}\Omega}{3.67\,\text{k}\Omega}\right)8\text{ V} = 4.69\text{ V}$$

$$V_{CD} = \left(\frac{R_{CD}}{R_T}\right)V_S = \left(\frac{517\,\Omega}{3.67\,\text{k}\Omega}\right)8\text{ V} = 1.13\text{ V}$$

$$V_3 = \left(\frac{R_3}{R_T}\right)V_S = \left(\frac{1.0\,\text{k}\Omega}{3.67\,\text{k}\Omega}\right)8\text{ V} = \mathbf{2.18\text{ V}}$$

Refer to Figure 7–24. V_{AB} equals the voltage across both R_1 and R_2, so

$$V_1 = V_2 = V_{AB} = \mathbf{4.69\text{ V}}$$

V_{CD} is the voltage across R_4 and across the series combination of R_5 and R_6. Therefore,

$$V_4 = V_{CD} = \mathbf{1.13\text{ V}}$$

Now apply the voltage-divider formula to the series combination of R_5 and R_6 to get V_5 and V_6.

$$V_5 = \left(\frac{R_5}{R_5 + R_6}\right)V_{CD} = \left(\frac{680\,\Omega}{1070\,\Omega}\right)1.13\text{ V} = \mathbf{718\text{ mV}}$$

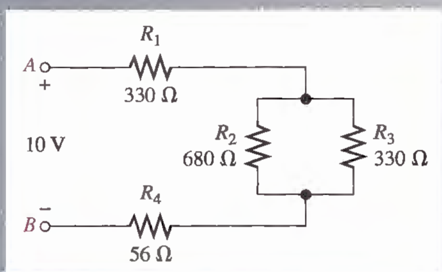
$$V_6 = \left(\frac{R_6}{R_5 + R_6}\right)V_{CD} = \left(\frac{390\,\Omega}{1070\,\Omega}\right)1.13\text{ V} = \mathbf{412\text{ mV}}$$

Related Problem R_2 is removed from the circuit in Figure 7–24. Calculate V_{AB}, V_{BC}, and V_{CD}.

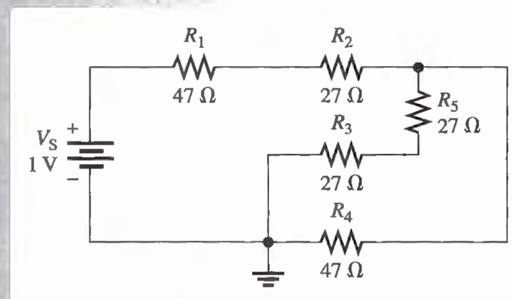
Use Multisim file E07-12 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 7–2 REVIEW

1. List four circuit laws and formulas that may be necessary in the analysis of series-parallel circuits.
2. Find the total resistance between A and B in the circuit of Figure 7–26.
3. Find the current through R_3 in Figure 7–26.
4. Find the voltage drop across R_2 in Figure 7–26.
5. Determine R_T and I_T in Figure 7–27 as "seen" by the source.



▲ FIGURE 7–26



▲ FIGURE 7–27

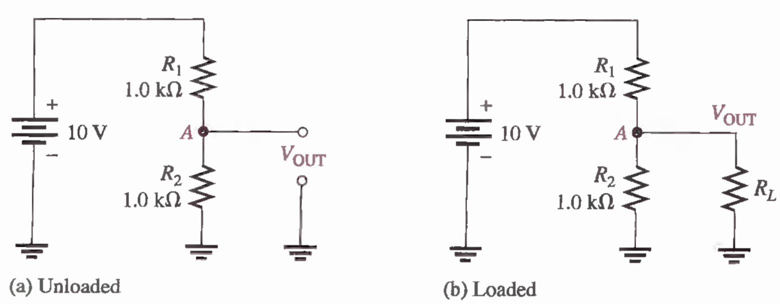
7–3 VOLTAGE DIVIDERS WITH RESISTIVE LOADS

Voltage dividers were introduced in Chapter 5. In this section, you will learn how resistive loads affect the operation of voltage-divider circuits.

After completing this section, you should be able to

- **Analyze loaded voltage dividers**
  - Determine the effect of a resistive load on a voltage-divider circuit
  - Define *bleeder current*

The voltage divider in Figure 7–28(a) produces an output voltage (V_{OUT}) of 5 V because the two resistors are of equal value. This voltage is the *unloaded output voltage*. When a load resistor, R_L, is connected from the output to ground as shown in Figure 7–28(b), the output voltage is reduced by an amount that depends on the value of R_L. The load resistor is in parallel with R_2, reducing the resistance from node A to ground and, as a result, also

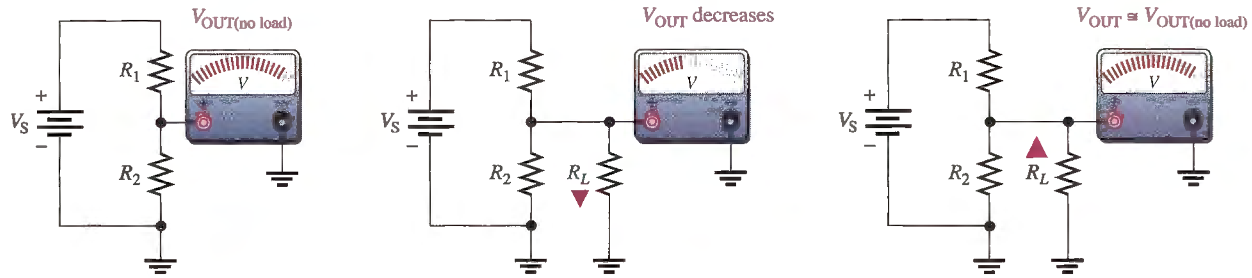


▲ FIGURE 7–28

A voltage divider with both unloaded and loaded outputs.

reducing the voltage across the parallel combination. This is one effect of loading a voltage divider. Another effect of a load is that more current is drawn from the source because the total resistance of the circuit is reduced.

The larger R_L is, compared to R_2, the less the output voltage is reduced from its unloaded value, as illustrated in Figure 7–29. When two resistors are connected in parallel and one of the resistors is much greater than the other, the total resistance is close to the value of the smaller resistance.



(a) No load (b) R_L not significantly greater than R_2 (c) R_L much greater than R_2

▲ FIGURE 7–29

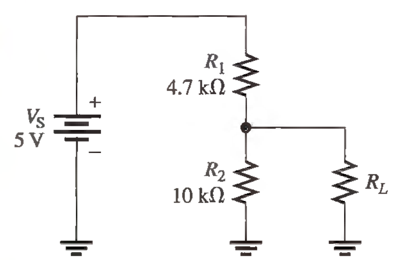
The effect of a load resistor.

EXAMPLE 7–13

(a) Determine the unloaded output voltage of the voltage divider in Figure 7–30.

(b) Find the loaded output voltages of the voltage divider in Figure 7–30 for the following two values of load resistance: $R_L = 10\text{ k}\Omega$ and $R_L = 100\text{ k}\Omega$.

▶ FIGURE 7–30



Solution **(a)** The unloaded output voltage is

$$V_{OUT(unloaded)} = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{10\text{ k}\Omega}{14.7\text{ k}\Omega}\right)5\text{ V} = \mathbf{3.40\text{ V}}$$

(b) With the 10 kΩ load resistor connected, R_L is in parallel with R_2, which gives

$$R_2 \parallel R_L = \frac{R_2 R_L}{R_2 + R_L} = \frac{100\text{ M}\Omega}{20\text{ k}\Omega} = 5\text{ k}\Omega$$

The equivalent circuit is shown in Figure 7–31(a). The loaded output voltage is

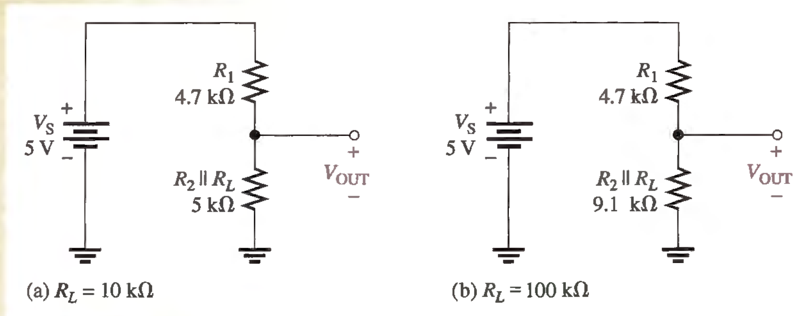
$$V_{OUT(loaded)} = \left(\frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L}\right)V_S = \left(\frac{5\text{ k}\Omega}{9.7\text{ k}\Omega}\right)5\text{ V} = \mathbf{2.58\text{ V}}$$

With the 100 kΩ load, the resistance from output to ground is

$$R_2 \parallel R_L = \frac{R_2 R_L}{R_2 + R_L} = \frac{(10\text{ k}\Omega)(100\text{ k}\Omega)}{110\text{ k}\Omega} = 9.1\text{ k}\Omega$$

The equivalent circuit is shown in Figure 7–31(b). The loaded output voltage is

$$V_{\text{OUT(loaded)}} = \left(\frac{R_2 \parallel R_L}{R_1 + R_2 \parallel R_L}\right)V_S = \left(\frac{9.1\text{ k}\Omega}{13.8\text{ k}\Omega}\right)5\text{ V} = \mathbf{3.30\text{ V}}$$



▲ FIGURE 7–31

For the smaller value of R_L, the reduction in V_{OUT} is

$$3.40\text{ V} - 2.58\text{ V} = 0.82\text{ V}$$

For the larger value of R_L, the reduction in V_{OUT} is

$$3.40\text{ V} - 3.30\text{ V} = 0.10\text{ V}$$

This illustrates the loading effect of R_L on the voltage divider.

Related Problem Determine V_{OUT} in Figure 7–30 for a 1.0 MΩ load resistance.

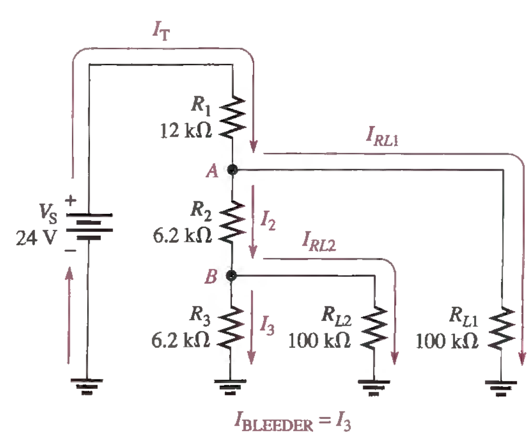
Use Multisim file E07-13 to verify the calculated results in this example and to confirm your calculation for the related problem.

Load Current and Bleeder Current

In a multiple-tap loaded voltage-divider circuit, the total current drawn from the source consists of currents through the load resistors, called *load currents,* and the divider resistors. Figure 7–32 shows a voltage divider with two voltage outputs or two taps. Notice that the total current, I_T, through R_1 enters node A where the current divides into I_{RL1} through R_{L1} and into I_2 through R_2. At node B, the current I_2 divides into I_{RL2} through R_{RL2} and into

▶ FIGURE 7–32

Currents in a two-tap loaded voltage divider.



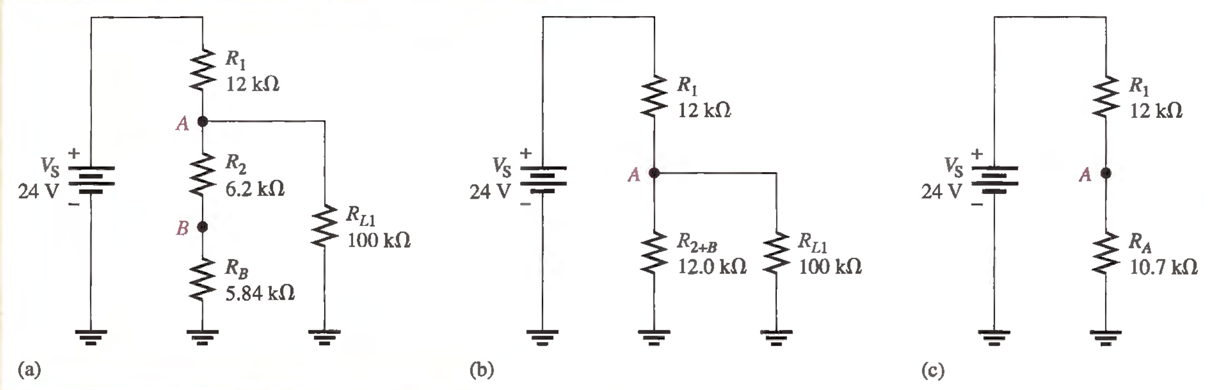
I_3 through R_3. Current I_3 is called the **bleeder current**, which is the current left after the total load current is subtracted from the total current in the circuit.

$$I_{BLEEDER} = I_T - I_{RL1} - I_{RL2}$$ **Equation 7–1**

EXAMPLE 7–14 Determine the load currents I_{RL1} and I_{RL2} and the bleeder current I_3 in the two-tap loaded voltage divider in Figure 7–32.

Solution The equivalent resistance from node A to ground is the 100 kΩ load resistor R_{L1} in parallel with the combination of R_2 in series with the parallel combination of R_3 and R_{L2}. Determine the resistance values first. R_3 in parallel with R_{L2} is designated R_B. The resulting equivalent circuit is shown in Figure 7–33(a).

$$R_B = \frac{R_3 R_{L2}}{R_3 + R_{L2}} = \frac{(6.2\,\text{k}\Omega)(100\,\text{k}\Omega)}{106.2\,\text{k}\Omega} = 5.84\,\text{k}\Omega$$



▲ FIGURE 7–33

R_2 in series with R_B is designated R_{2+B}. The resulting equivalent circuit is shown in Figure 7–33(b).

$$R_{2+B} = R_2 + R_B = 6.2\,\text{k}\Omega + 5.84\,\text{k}\Omega = 12.0\,\text{k}\Omega$$

R_{L1} in parallel with R_{2+B} is designated R_A. The resulting equivalent circuit is shown in Figure 7–33(c).

$$R_A = \frac{R_{L1} R_{2+B}}{R_{L1} + R_{2+B}} = \frac{(100\,\text{k}\Omega)(12.0\,\text{k}\Omega)}{112\,\text{k}\Omega} = 10.7\,\text{k}\Omega$$

R_A is the total resistance from node A to ground. The total resistance for the circuit is

$$R_T = R_A + R_1 = 10.7\,\text{k}\Omega + 12\,\text{k}\Omega = 22.7\,\text{k}\Omega$$

Determine the voltage across R_{L1} as follows, using the equivalent circuit in Figure 7–33(c):

$$V_{RL1} = V_A = \left(\frac{R_A}{R_T}\right)V_S = \left(\frac{10.7\,\text{k}\Omega}{22.7\,\text{k}\Omega}\right)24\text{ V} = 11.3\text{ V}$$

The load current through R_{L1} is

$$I_{RL1} = \frac{V_{RL1}}{R_{L1}} = \left(\frac{11.3\text{ V}}{100\,\text{k}\Omega}\right) = \mathbf{113\ \mu A}$$

Determine the voltage at node B by using the equivalent circuit in Figure 7–33(a) and the voltage at node A.

$$V_B = \left(\frac{R_B}{R_{2+B}}\right)V_A = \left(\frac{5.84\text{ k}\Omega}{12.0\text{ k}\Omega}\right)11.3\text{ V} = 5.50\text{ V}$$

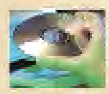
The load current through R_{L2} is

$$I_{RL2} = \frac{V_{RL2}}{R_{L2}} = \frac{V_B}{R_{L2}} = \frac{5.50\text{ V}}{100\text{ k}\Omega} = \mathbf{55\ \mu A}$$

The bleeder current is

$$I_3 = \frac{V_B}{R_3} = \frac{5.50\text{ V}}{6.2\text{ k}\Omega} = \mathbf{887\ \mu A}$$

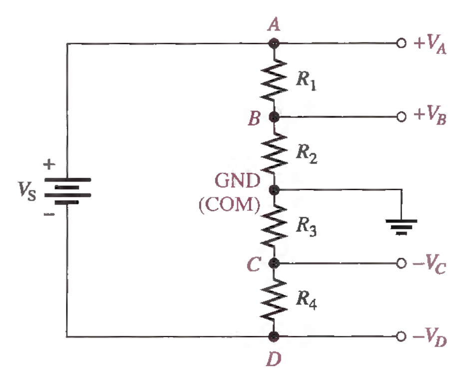
Related Problem How can the bleeder current in Figure 7–32 be reduced without affecting the load currents?



Use Multisim file E07-14 to verify the calculated results in this example.

Bipolar Voltage Dividers

An example of a voltage divider that produces both positive and negative voltages from a single source is shown in Figure 7–34. Notice that neither the positive nor the negative terminal of the source is connected to reference ground or common. The voltages at nodes A and B are positive with respect to reference ground, and the voltages at nodes C and D are negative with respect to reference ground.



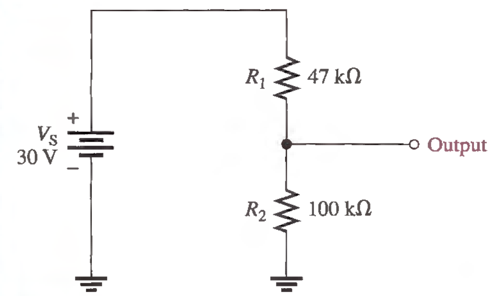
▶ FIGURE 7–34

A bipolar voltage divider. The positive and negative voltages are with respect to reference ground.

SECTION 7–3 REVIEW

1. A load resistor is connected to an output tap on a voltage divider. What effect does the load resistor have on the output voltage at this tap?
2. A larger-value load resistor will cause the output voltage to change less than a smaller-value one will. (T or F)
3. For the voltage divider in Figure 7–35, determine the unloaded output voltage with respect to ground. Also determine the output voltage with a 10 kΩ load resistor connected across the output.

▶ FIGURE 7–35



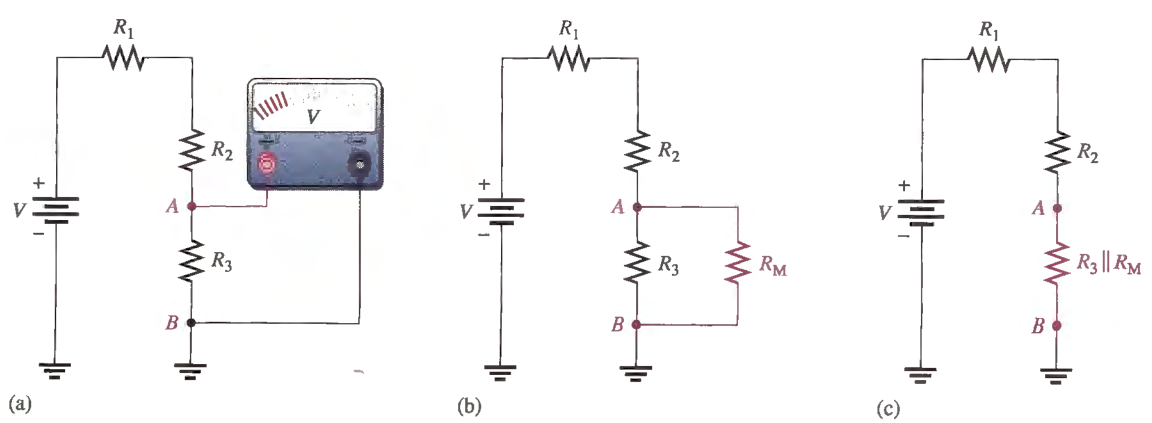
7–4 LOADING EFFECT OF A VOLTMETER

As you have learned, voltmeters must be connected in parallel with a resistor in order to measure the voltage across the resistor. Because of its internal resistance, a voltmeter puts a load on the circuit and will affect, to a certain extent, the voltage that is being measured. Until now, we have ignored the loading effect because the internal resistance of a voltmeter is very high, and normally it has negligible effect on the circuit that is being measured. However, if the internal resistance of the voltmeter is not sufficiently greater than the circuit resistance across which it is connected, the loading effect will cause the measured voltage to be less than its actual value. You should always be aware of this effect.

After completing this section, you should be able to

- **Determine the loading effect of a voltmeter on a circuit**
  - Explain why a voltmeter can load a circuit
  - Discuss the internal resistance of a voltmeter

When a voltmeter is connected to a circuit as shown, for example, in Figure 7–36(a), its internal resistance appears in parallel with R_3, as shown in part (b). The resistance from A



▲ FIGURE 7–36

The loading effect of a voltmeter.

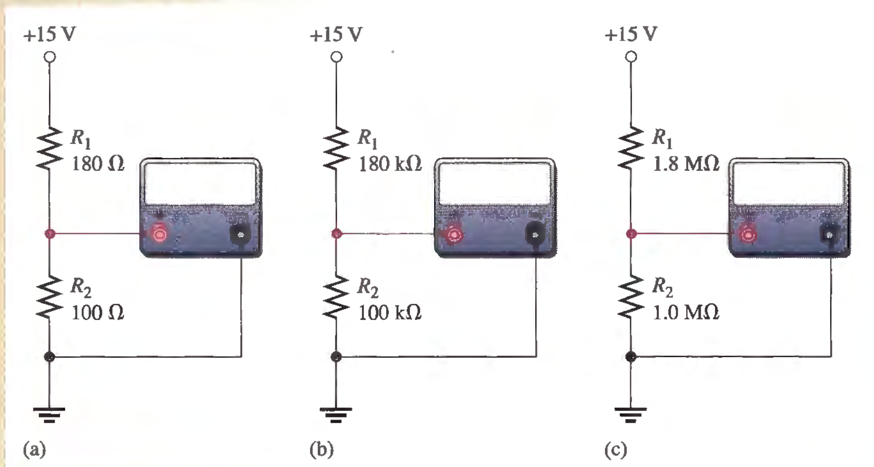
to B is altered by the loading effect of the voltmeter's internal resistance, R_M, and is equal to $R_3 \parallel R_M$, as indicated in part (c).

If R_M is much greater than R_3, the resistance from A to B changes very little, and the meter indicates the actual voltage. If R_M is not sufficiently greater than R_3, the resistance from A to B is reduced significantly, and the voltage across R_3 is altered by the loading effect of the meter. A good rule of thumb is that *if the loading effect is less than 10%, it can usually be neglected, depending on the accuracy required.*

Two categories of voltmeters are the electromagnetic analog voltmeter (commonly called VOM), whose internal resistance is determined by its sensitivity factor, and the digital voltmeter (the most commonly used type and commonly called DMM), whose internal resistance is also typically at least 10 MΩ. The digital voltmeter presents fewer loading problems than the electromagnetic type because the internal resistances of DMMs are much higher.

EXAMPLE 7–15

How much does the digital voltmeter affect the voltage being measured for each circuit indicated in Figure 7–37? Assume the meter has an input resistance (R_M) of 10 MΩ.



▲ FIGURE 7–37

Solution To show the small differences more clearly, the results are expressed in more than three significant figures in this example.

(a) Refer to Figure 7–37(a). The unloaded voltage across R_2 in the voltage-divider circuit is

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{100\ \Omega}{280\ \Omega}\right)15\text{ V} = 5.357\text{ V}$$

The meter's resistance in parallel with R_2 is

$$R_2 \parallel R_M = \left(\frac{R_2 R_M}{R_2 + R_M}\right) = \frac{(100\ \Omega)(10\text{ M}\Omega)}{10.0001\text{ M}\Omega} = 99.999\ \Omega$$

The voltage actually measured by the meter is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{99.999\ \Omega}{279.999\ \Omega}\right)15\text{ V} = 5.357\text{ V}$$

The voltmeter has no measurable loading effect.

(b) Refer to Figure 7–37(b).

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{100\ \text{k}\Omega}{280\ \text{k}\Omega}\right)15\ \text{V} = 5.357\ \text{V}$$

$$R_2 \parallel R_M = \frac{R_2 R_M}{R_2 + R_M} = \frac{(100\ \text{k}\Omega)(10\ \text{M}\Omega)}{10.1\ \text{M}\Omega} = 99.01\ \text{k}\Omega$$

The voltage actually measured by the meter is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{99.01\ \text{k}\Omega}{279.01\ \text{k}\Omega}\right)15\ \text{V} = 5.323\ \text{V}$$

The loading effect of the voltmeter reduces the voltage by a very small amount.

(c) Refer to Figure 7–37(c).

$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{1.0\ \text{M}\Omega}{2.8\ \text{M}\Omega}\right)15\ \text{V} = 5.357\ \text{V}$$

$$R_2 \parallel R_M = \frac{R_2 R_M}{R_2 + R_M} = \frac{(1.0\ \text{M}\Omega)(10\ \text{M}\Omega)}{11\ \text{M}\Omega} = 909.09\ \text{k}\Omega$$

The voltage actually measured is

$$V_2 = \left(\frac{R_2 \parallel R_M}{R_1 + R_2 \parallel R_M}\right)V_S = \left(\frac{909.09\ \text{k}\Omega}{2.709\ \text{M}\Omega}\right)15\ \text{V} = 5.034\ \text{V}$$

The loading effect of the voltmeter reduces the voltage by a noticeable amount. As you can see, the higher the resistance across which a voltage is measured, the more the loading effect.

Related Problem Calculate the voltage across R_2 in Figure 7–37(c) if the meter resistance is 20 MΩ.

SECTION 7–4 REVIEW

1. **Explain why a voltmeter can potentially load a circuit.**
2. **If a voltmeter with a 10 MΩ internal resistance is measuring the voltage across a 1.0 kΩ resistor, should you normally be concerned about the loading effect?**
3. **If a voltmeter with a 10 MΩ resistance is measuring the voltage across a 3.3 MΩ resistor, should you be concerned about the loading effect?**

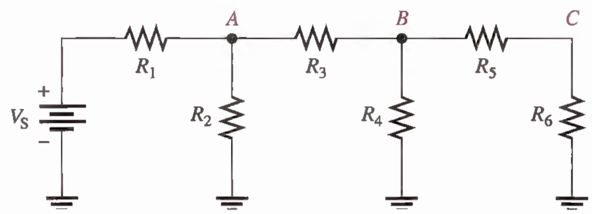
7–5 LADDER NETWORKS

A resistive ladder network is a special type of series-parallel circuit. The $R/2R$ ladder network is commonly used to scale down voltages to certain weighted values for digital-to-analog conversion, which is a process that you will study in another course.

After completing this section, you should be able to

- **Analyze ladder networks**
  - Determine the voltages in a three-step ladder network
  - Analyze an $R/2R$ ladder

One approach to the analysis of a ladder network such as the one shown in Figure 7–38 is to simplify it one step at a time, starting at the side farthest from the source. In this way, you can determine the current in any branch or the voltage at any node, as illustrated in Example 7–16.

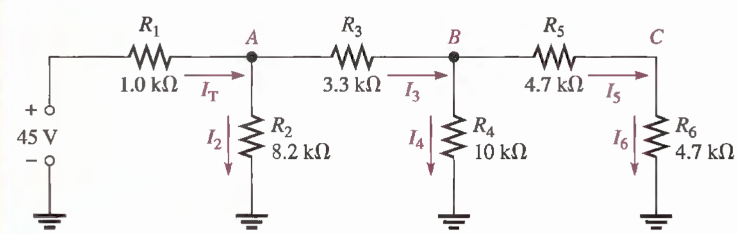


▲ **FIGURE 7–38**

Basic three-step ladder network.

EXAMPLE 7–16

Determine the current through each resistor and the voltage at each labeled node with respect to ground in the ladder network of Figure 7–39.



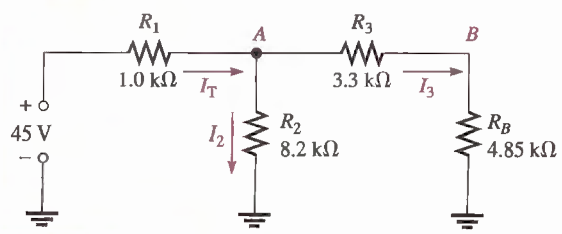
▲ **FIGURE 7–39**

Solution To find the current through each resistor, you must know the total current from the source (I_T). To obtain I_T, you must find the total resistance "seen" by the source.

Determine R_T in a step-by-step process, starting at the right of the circuit diagram. First, notice that R_5 and R_6 are in series across R_4. Neglecting the circuit to the left of node B, the resistance from node B to ground is

$$R_B = \frac{R_4(R_5 + R_6)}{R_4 + (R_5 + R_6)} = \frac{(10\ \text{k}\Omega)(9.4\ \text{k}\Omega)}{19.4\ \text{k}\Omega} = 4.85\ \text{k}\Omega$$

Using R_B, you can draw the equivalent circuit as shown in Figure 7–40.

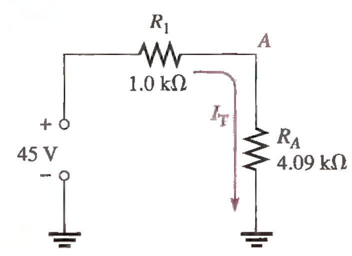


▲ **FIGURE 7–40**

Next, neglecting the circuit to the left of node A, the resistance from node A to ground (R_A) is R_2 in parallel with the series combination of R_3 and R_B. Calculate resistance R_A.

$$R_A = \frac{R_2(R_3 + R_B)}{R_2 + (R_3 + R_B)} = \frac{(8.2\,\text{k}\Omega)(8.15\,\text{k}\Omega)}{16.35\,\text{k}\Omega} = 4.09\,\text{k}\Omega$$

Using R_A, you can further simplify the equivalent circuit of Figure 7–40 as shown in Figure 7–41.



▲ **FIGURE 7–41**

Finally, the total resistance "seen" by the source is R_1 in series with R_A.

$$R_T = R_1 + R_A = 1.0\,\text{k}\Omega + 4.09\,\text{k}\Omega = 5.09\,\text{k}\Omega$$

The total circuit current is

$$I_T = \frac{V_S}{R_T} = \frac{45\text{ V}}{5.09\,\text{k}\Omega} = \mathbf{8.84\,mA}$$

As indicated in Figure 7–40, I_T is into node A and divides between R_2 and the branch containing $R_3 + R_B$. Since the branch resistances are approximately equal in this particular example, half the total current is through R_2 and half into node B. So the currents through R_2 and R_3 are

$$I_2 = \mathbf{4.42\,mA}$$
$$I_3 = \mathbf{4.42\,mA}$$

If the branch resistances are not equal, use the current-divider formula. As indicated in Figure 7–39, I_3 is into node B and is divided between R_4 and the branch containing $R_5 + R_6$. Therefore, the currents through R_4, R_5, and R_6 can be calculated.

$$I_4 = \left(\frac{R_5 + R_6}{R_4 + (R_5 + R_6)}\right)I_3 = \left(\frac{9.4\,\text{k}\Omega}{19.4\,\text{k}\Omega}\right)4.42\text{ mA} = \mathbf{2.14\,mA}$$
$$I_5 = I_6 = I_3 - I_4 = 4.42\text{ mA} - 2.14\text{ mA} = \mathbf{2.28\,mA}$$

To determine V_A, V_B, and V_C, apply Ohm's law.

$$V_A = I_2R_2 = (4.42\text{ mA})(8.2\,\text{k}\Omega) = \mathbf{36.2\,V}$$
$$V_B = I_4R_4 = (2.14\text{ mA})(10\,\text{k}\Omega) = \mathbf{21.4\,V}$$
$$V_C = I_6R_6 = (2.28\text{ mA})(4.7\,\text{k}\Omega) = \mathbf{10.7\,V}$$

Related Problem Recalculate the currents through each resistor and the voltages at each node in Figure 7–39 if R_1 is increased to 2.2 kΩ.

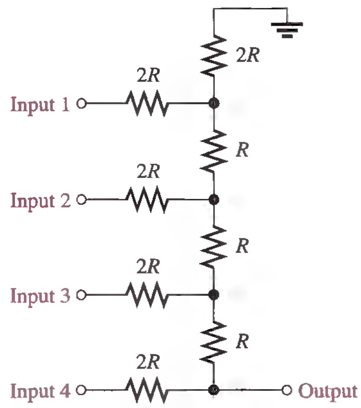
Use Multisim file E07-16 to verify the calculated results in this example and to confirm your calculations for the related problem.

The *R*/2*R* Ladder Network

A basic *R*/2*R* ladder network is shown in Figure 7–42. As you can see, the name comes from the relationship of the resistor values. *R* represents a common value, and one set of resistors has twice the value of the others. This type of ladder network is used in applications where digital codes are converted to speech, music, or other types of analog signals as found, for example, in the area of digital recording and reproduction. This application is called *digital-to-analog* (*D/A*) *conversion.*

FIGURE 7–42

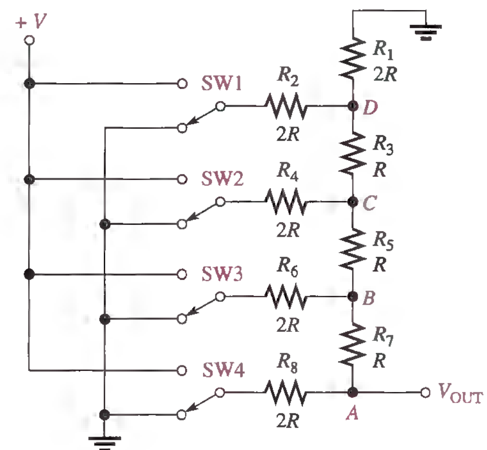
A basic four-step *R*/2*R* ladder network.



Let's examine the general operation of a basic *R*/2*R* ladder using the four-step circuit in Figure 7–43. In a later course in digital fundamentals, you will learn specifically how this type of circuit is used in D/A conversion.

FIGURE 7–43

R/2*R* ladder with switch inputs to simulate a two-level (digital) code.



The switches used in this illustration simulate the digital (two-level) inputs. One switch position is connected to ground (0 V), and the other position is connected to a positive voltage (*V*). The analysis is as follows: Start by assuming that switch SW4 in Figure 7–43 is at the *V* position and the others are at ground so that the inputs are as shown in Figure 7–44(a).

The total resistance from node *A* to ground is found by first combining R_1 and R_2 in parallel from node *D* to ground. The simplified circuit is shown in Figure 7–44(b).

$$R_1 \parallel R_2 = \frac{2R}{2} = R$$

$R_1 \parallel R_2$ is in series with R_3 from node *C* to ground as illustrated in part (c).

$$R_1 \parallel R_2 + R_3 = R + R = 2R$$

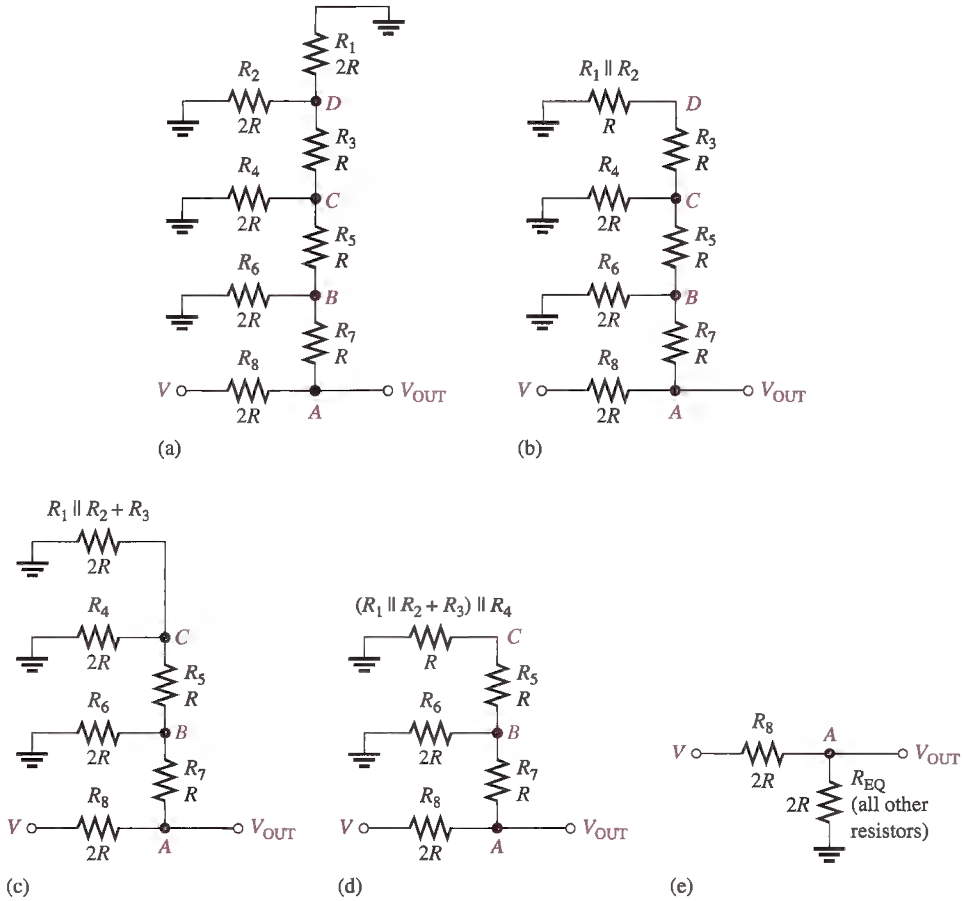


FIGURE 7–44

Simplification of $R/2R$ ladder for analysis.

Next, this combination is in parallel with R_4 from node C to ground as shown in part (d).

$$(R_1 \parallel R_2 + R_3) \parallel R_4 = 2R \parallel 2R = \frac{2R}{2} = R$$

Continuing this simplification process results in the circuit in part (e) in which the output voltage can be expressed using the voltage-divider formula as

$$V_{OUT} = \left(\frac{2R}{4R}\right)V = \frac{V}{2}$$

A similar analysis, except with switch SW3 in Figure 7–43 connected to V and the other switches connected to ground, results in the simplified circuit shown in Figure 7–45.

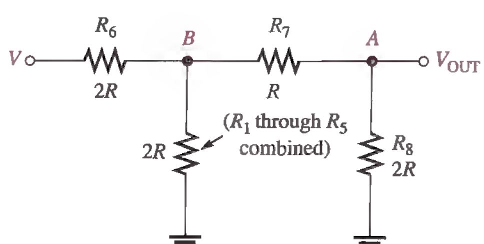


FIGURE 7–45

Simplified ladder with only V input at SW3 in Figure 7–43.

The analysis for this case is as follows: The resistance from node B to ground is

$$R_B = (R_7 + R_8) \parallel 2R = 3R \parallel 2R = \frac{6R}{5}$$

Using the voltage-divider formula, we can express the voltage at node B with respect to ground as

$$V_B = \left(\frac{R_B}{R_6 + R_B}\right)V = \left(\frac{6R/5}{2R + 6R/5}\right)V = \left(\frac{6R/5}{10R/5 + 6R/5}\right)V = \left(\frac{6R/5}{16R/5}\right)V$$
$$= \left(\frac{6R}{16R}\right)V = \frac{3V}{8}$$

The output voltage is, therefore,

$$V_{OUT} = \left(\frac{R_8}{R_7 + R_8}\right)V_B = \left(\frac{2R}{3R}\right)\left(\frac{3V}{8}\right) = \frac{V}{4}$$

Notice that the output voltage in this case ($V/4$) is one half the output voltage ($V/2$) for the case where V is connected at switch SW4.

A similar analysis for each of the remaining switch inputs in Figure 7–43 results in output voltages as follows: For SW2 connected to V and the other switches connected to ground,

$$V_{OUT} = \frac{V}{8}$$

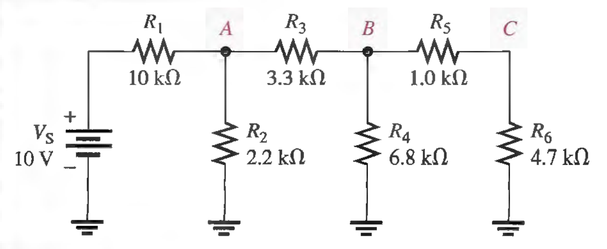
For SW1 connected to V and the other switches connected to ground,

$$V_{OUT} = \frac{V}{16}$$

When more than one input at a time are connected to V, the total output is the sum of the individual outputs, according to the superposition theorem that is covered in Section 8–4. These particular relationships among the output voltages for the various levels of inputs are important in the application of $R/2R$ ladder networks to digital-to-analog conversion.

SECTION 7–5 REVIEW

1. Draw a basic four-step ladder network.
2. Determine the total circuit resistance presented to the source by the ladder network of Figure 7–46.
3. What is the total current in Figure 7–46?
4. What is the current through R_2 in Figure 7–46?
5. What is the voltage at node A with respect to ground in Figure 7–46?



▲ FIGURE 7–46

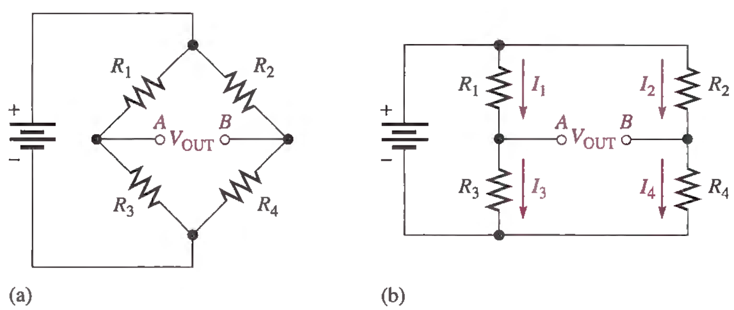
7–6 THE WHEATSTONE BRIDGE

The Wheatstone bridge circuit can be used to precisely measure resistance. However, the bridge is most commonly used in conjunction with transducers to measure physical quantities such as strain, temperature, and pressure. Transducers are devices that sense a change in a physical parameter and convert that change into an electrical quantity such as a change in resistance. For example, a strain gauge exhibits a change in resistance when it is exposed to mechanical factors such as force, pressure, or displacement. A thermistor exhibits a change in its resistance when it is exposed to a change in temperature. The Wheatstone bridge can be operated in a balanced or an unbalanced condition. The condition of operation depends on the type of application.

After completing this section, you should be able to

- **Analyze and apply a Wheatstone bridge**
    - Determine when a bridge is balanced
    - Determine an unknown resistance with a balanced bridge
    - Determine when a bridge is unbalanced
    - Discuss measurements using an unbalanced bridge

A **Wheatstone bridge** circuit is shown in its most common "diamond" configuration in Figure 7–47(a). It consists of four resistors and a dc voltage source connected across the top and bottom points of the "diamond." The output voltage is taken across the left and right points of the "diamond" between A and B. In part (b), the circuit is drawn in a slightly different way to more clearly show its series-parallel configuration.



▲ FIGURE 7–47

Wheatstone bridge.

The Balanced Wheatstone Bridge

The Wheatstone bridge in Figure 7–47 is in the **balanced bridge** condition when the output voltage (V_{OUT}) between terminals A and B is equal to zero.

$$V_{OUT} = 0\text{ V}$$

When the bridge is balanced, the voltages across R_1 and R_2 are equal ($V_1 = V_2$) and the voltages across R_3 and R_4 are equal ($V_3 = V_4$). Therefore, the voltage ratios can be written as

$$\frac{V_1}{V_3} = \frac{V_2}{V_4}$$

Substituting IR for V by Ohm's law gives

$$\frac{I_1R_1}{I_3R_3} = \frac{I_2R_2}{I_4R_4}$$

Since $I_1 = I_3$ and $I_2 = I_4$, all the current terms cancel, leaving the resistor ratios.

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Solving for R_1 results in the following formula:

$$R_1 = R_3\left(\frac{R_2}{R_4}\right)$$

This formula allows you to find the value of resistor R_1 in terms of the other resistor values when the bridge is balanced. You can also find the value of any other resistor in a similar way.

Using the Balanced Wheatstone Bridge to Find an Unknown Resistance Assume that R_1 in Figure 7–47 has an unknown value, which we call R_X. Resistors R_2 and R_4 have fixed values so that their ratio, R_2/R_4, also has a fixed value. Since R_X can be any value, R_3 must be adjusted to make $R_1/R_3 = R_2/R_4$ in order to create a balanced condition. Therefore, R_3 is a variable resistor, which we will call R_V. When R_X is placed in the bridge, R_V is adjusted until the bridge is balanced as indicated by a zero output voltage. Then, the unknown resistance is found as

Equation 7–2

$$R_X = R_V\left(\frac{R_2}{R_4}\right)$$

The ratio R_2/R_4 is the scale factor.

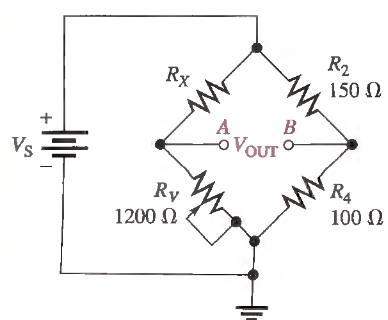
An older type of measuring instrument called a galvanometer can be connected between the output terminals A and B to detect a balanced condition. The galvanometer is essentially a very sensitive ammeter that senses current in either direction. It differs from a regular ammeter in that the midscale point is zero. In modern instruments, an amplifier connected across the bridge output indicates a balanced condition when its output is 0 V.

From Equation 7–2, the value of R_V at balance multiplied by the scale factor R_2/R_4 is the actual resistance value of R_X. If $R_2/R_4 = 1$, then $R_X = R_V$, if $R_2/R_4 = 0.5$, then $R_X = 0.5R_V$, and so on. In a practical bridge circuit, the position of the R_V adjustment can be calibrated to indicate the actual value of R_X on a scale or with some other method of display.

EXAMPLE 7–17

Determine the value of R_X in the balanced bridge shown in Figure 7–48.

FIGURE 7–48



Solution The scale factor is

$$\frac{R_2}{R_4} = \frac{150\ \Omega}{100\ \Omega} = 1.5$$

The bridge is balanced ($V_{OUT} = 0$ V) when R_V is set at 1200 Ω, so the unknown resistance is

$$R_X = R_V\left(\frac{R_2}{R_4}\right) = (1200\ \Omega)(1.5) = \mathbf{1800\ \Omega}$$

Related Problem If R_V must be adjusted to 2.2 kΩ to balance the bridge in Figure 7–48, what is R_X?

Use Multisim file E07-17 to verify the calculated results in this example and to confirm your calculation for the related problem.

The Unbalanced Wheatstone Bridge

An **unbalanced bridge** condition occurs when V_{OUT} is not equal to zero. The unbalanced bridge is used to measure several types of physical quantities such as mechanical strain, temperature, or pressure. This can be done by connecting a transducer in one leg of the bridge, as shown in Figure 7–49. The resistance of the transducer changes proportionally to the changes in the parameter that it is measuring. If the bridge is balanced at a known point, then the amount of deviation from the balanced condition, as indicated by the output voltage, indicates the amount of change in the parameter being measured. Therefore, the value of the parameter being measured can be determined by the amount that the bridge is unbalanced.

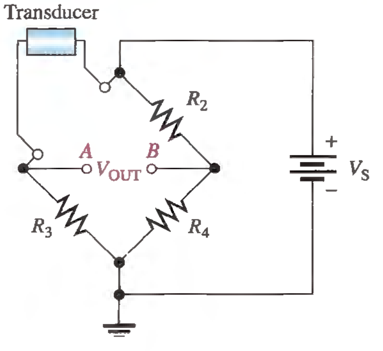


FIGURE 7–49

A bridge circuit for measuring a physical parameter using a transducer.

A Bridge Circuit for Measuring Temperature If temperature is to be measured, the transducer can be a thermistor, which is a temperature-sensitive resistor. The thermistor resistance changes in a predictable way as the temperature changes. A change in temperature causes a change in thermistor resistance, which causes a corresponding change in the output voltage of the bridge as it becomes unbalanced. The output voltage is proportional to the temperature; therefore, either a voltmeter connected across the output can be calibrated to show the temperature or the output voltage can be amplified and converted to digital form to drive a readout display of the temperature.

A bridge circuit used to measure temperature is designed so that it is balanced at a reference temperature and becomes unbalanced at a measured temperature. For example, let's say the bridge is to be balanced at 25°C. A thermistor will have a known value of resistance at 25°C. For simplicity, let's assume the other three bridge resistors are equal to the thermistor resistance at 25°C, so $R_{therm} = R_2 = R_3 = R_4$. For this particular case, the change in output voltage (ΔV_{OUT}) can be shown to be related to the change in R_{therm} by the following formula:

Equation 7–3

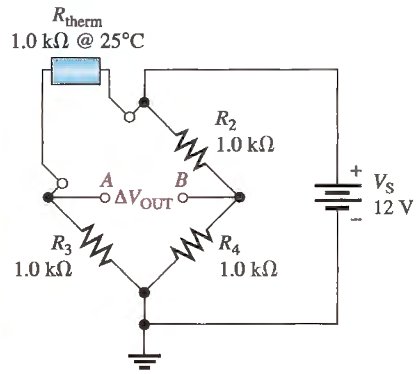
$$\Delta V_{OUT} \cong \Delta R_{therm}\left(\frac{V_S}{4R}\right)$$

The Δ (Greek letter delta) in front of a variable means a change in the variable. This formula applies only to the case where all resistances in the bridge are equal when the bridge is balanced. A derivation is provided in Appendix B. Keep in mind that the bridge can be initially balanced without having all the resistors equal as long as $R_1 = R_2$ and $R_3 = R_4$ (see Figure 7–47), but the formula for ΔV_{OUT} would be more complicated.

EXAMPLE 7–18

Determine the output voltage of the temperature-measuring bridge circuit in Figure 7–50 if the thermistor is exposed to a temperature of 50°C and its resistance at 25°C is 1.0 kΩ. Assume the resistance of the thermistor decreases to 900 Ω at 50°C.

FIGURE 7–50



Solution

$$\Delta R_{therm} = 1.0\,\text{k}\Omega - 900\,\Omega = 100\,\Omega$$

$$\Delta V_{OUT} \cong \Delta R_{therm}\left(\frac{V_S}{4R}\right) = 100\,\Omega\left(\frac{12\text{ V}}{4\text{ k}\Omega}\right) = 0.3\text{ V}$$

Since $V_{OUT} = 0$ V when the bridge is balanced at 25°C and it changes 0.3 V, then

$$V_{OUT} = \mathbf{0.3\ V}$$

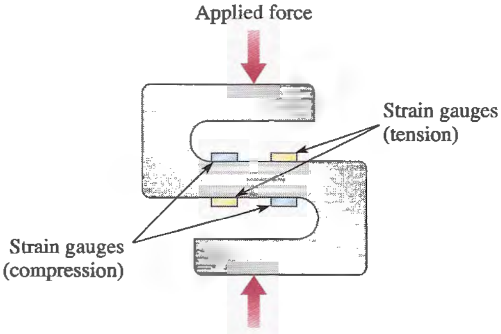
when the temperature is 50°C.

Related Problem If the temperature is increased to 60°C, causing the thermistor resistance in Figure 7–50 to decrease to 850 Ω, what is V_{OUT}?

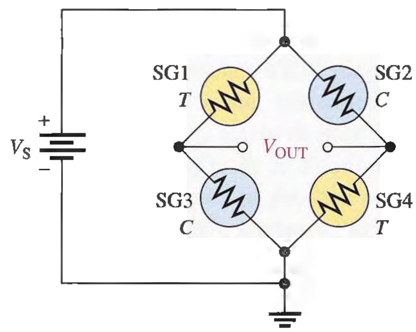
Other Unbalanced Wheatstone Bridge Applications A Wheatstone bridge with a strain gauge can be used to measure certain forces. A strain gauge is a device that exhibits a change in resistance when it is compressed or stretched by the application of an external force. As the resistance of the strain gauge changes, the previously balanced bridge becomes unbalanced. This unbalance causes the output voltage to change from zero, and this change can be measured to determine the amount of strain. In strain gauges, the resistance change is extremely small. This tiny change unbalances a Wheatstone bridge because of its high sensitivity. For example, Wheatstone bridges with strain gauges are commonly used in weight scales.

Some resistive transducers have extremely small resistance changes, and these changes are difficult to measure accurately with a direct measurement. In particular, strain gauges are one of the most useful resistive transducers that convert the stretching or compression of a fine wire into a change in resistance. When strain causes the wire in the gauge to stretch, the resistance increases a small amount; and when it compresses, the resistance of the wire decreases.

Strain gauges are used in many types of scales, from those that are used for weighing small parts to those for weighing huge trucks. Typically, the gauges are mounted on a special block of aluminum that deforms when a weight is on the scale. The strain gauges are extremely delicate and must be mounted properly, so the entire assembly is generally prepared as a single unit called a load cell. A wide variety of load cells with different shapes and sizes are available from manufacturers depending on the application. A typical S-shaped load cell for a weighing application that has four strain gauges is illustrated in Figure 7–51(a). The gauges are mounted so that two of the gauges stretch (tension) when a load is placed on the scale and two of the gauges compress.



(a) A typical load cell with four active strain gauges



(b) Wheatstone bridge

▲ FIGURE 7–51

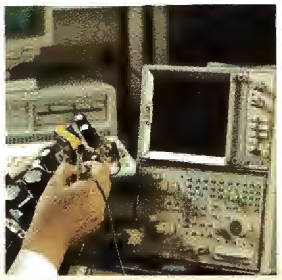
Load cells are usually connected to a Wheatstone bridge as shown in Figure 7–51(b) with strain gauges (SG) in tension (T) and compression (C) in opposite diagonal legs as shown. The output of the bridge is normally digitized and converted to a reading for a display or sent to a computer for processing. The major advantage of the Wheatstone bridge circuit is that it is capable of accurately measuring very small differences in resistance. The use of four active transducers increases the sensitivity of the measurement and makes the bridge the ideal circuit for instrumentation. The Wheatstone bridge circuit has the added benefit of compensating for temperature variations and wire resistance of connecting wires that would otherwise contribute to inaccuracies.

In addition to scales, strain gauges are used with Wheatstone bridges in other types of measurements including pressure measurements, displacement and acceleration measurements to name a few. In pressure measurements, the strain gauges are bonded to a flexible diaphragm that stretches when pressure is applied to the transducer. The amount of flexing is related to the pressure, which again converts to a very small resistance change.

SECTION 7–6 REVIEW

1. Draw a basic Wheatstone bridge circuit.
2. Under what condition is a bridge balanced?
3. What is the unknown resistance in Figure 7–48 when $R_V = 3.3\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, and $R_4 = 2.2\ \text{k}\Omega$?
4. How is a Wheatstone bridge used in the unbalanced condition?

7–7 TROUBLESHOOTING



As you know, troubleshooting is the process of identifying and locating a failure or problem in a circuit. Some troubleshooting techniques and the application of logical thought have already been discussed in relation to both series circuits and parallel circuits. A basic premise of troubleshooting is that you must know what to look for before you can successfully troubleshoot a circuit.

After completing this section, you should be able to

- **Troubleshoot series-parallel circuits**
  - Determine the effects of an open in a circuit
  - Determine the effects of a short in a circuit
  - Locate opens and shorts

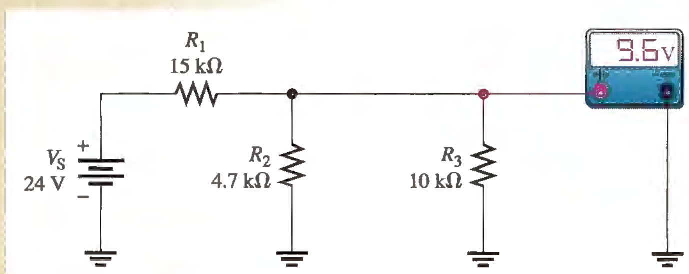
Opens and shorts are typical problems that occur in electric circuits. As mentioned in Chapter 5, if a resistor burns out, it will normally produce an open. Bad solder connections, broken wires, and poor contacts can also be causes of open paths. Pieces of foreign material, such as solder splashes, broken insulation on wires, and so on, can often lead to shorts in a circuit. A short is considered to be a zero resistance path between two points.

In addition to complete opens or shorts, partial opens or partial shorts can develop in a circuit. A partial open would be a much higher than normal resistance, but not infinitely large. A partial short would be a much lower than normal resistance, but not zero.

The following three examples illustrate troubleshooting series-parallel circuits.

EXAMPLE 7–19

From the indicated voltmeter reading in Figure 7–52, determine if there is a fault by applying the APM approach. If there is a fault, identify it as either a short or an open.



▲ FIGURE 7–52

Solution **Step 1:** Analysis

Determine what the voltmeter should be indicating as follows. Since R_2 and R_3 are in parallel, their combined resistance is

$$R_{2\|3} = \frac{R_2R_3}{R_2 + R_3} = \frac{(4.7\,\text{k}\Omega)(10\,\text{k}\Omega)}{14.7\,\text{k}\Omega} = 3.20\,\text{k}\Omega$$

Determine the voltage across the parallel combination by the voltage-divider formula.

$$V_{2\|3} = \left(\frac{R_{2\|3}}{R_1 + R_{2\|3}}\right)V_S = \left(\frac{3.2\,\text{k}\Omega}{18.2\,\text{k}\Omega}\right)24\text{ V} = 4.22\text{ V}$$

This calculation shows that 4.22 V is the voltage reading that you should get on the meter. However, the meter reads 9.6 V across $R_{2\|3}$. This value is incorrect, and, because it is higher than it should be, either R_2 or R_3 is probably open. Why? Because if either of these two resistors is open, the resistance across which the meter is connected is larger than expected. A higher resistance will drop a higher voltage in this circuit.

Step 2: Planning

Start trying to find the open resistor by assuming that R_2 is open. If it is, the voltage across R_3 is

$$V_3 = \left(\frac{R_3}{R_1 + R_3}\right)V_S = \left(\frac{10\text{ k}\Omega}{25\text{ k}\Omega}\right)24\text{ V} = 9.6\text{ V}$$

Since the measured voltage is also 9.6 V, this calculation shows that R_2 is probably open.

Step 3: Measurement

Disconnect power and remove R_2. Measure its resistance to verify it is open. If it is not, inspect the wiring, solder, or connections around R_2, looking for the open.

Related Problem What would be the voltmeter reading if R_3 were open in Figure 7–52? If R_1 were open?

EXAMPLE 7–20 Suppose that you measure 24 V with the voltmeter in Figure 7–53. Determine if there is a fault, and, if there is, identify it.

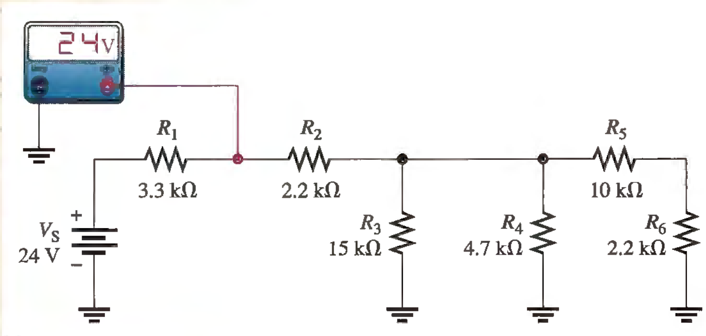


FIGURE 7–53

Solution **Step 1:** Analysis

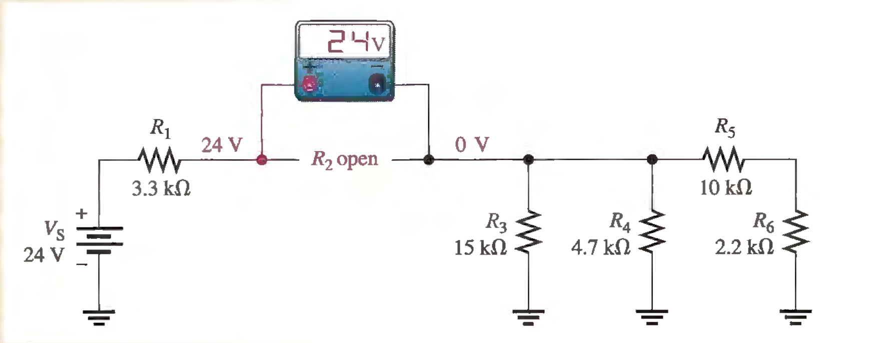
There is no voltage drop across R_1 because both sides of the resistor are at +24 V. Either there is no current through R_1 from the source, which tells you that R_2 is open in the circuit, or R_1 is shorted.

Step 2: Planning

The most probable failure is an open R_2. If it is open, then there will be no current from the source. To verify this, measure across R_2 with the voltmeter. If R_2 is open, the meter will indicate 24 V. The right side of R_2 will be at zero volts because there is no current through any of the other resistors to cause a voltage drop across them.

Step 3: Measurement

The measurement to verify that R_2 is open is shown in Figure 7–54.

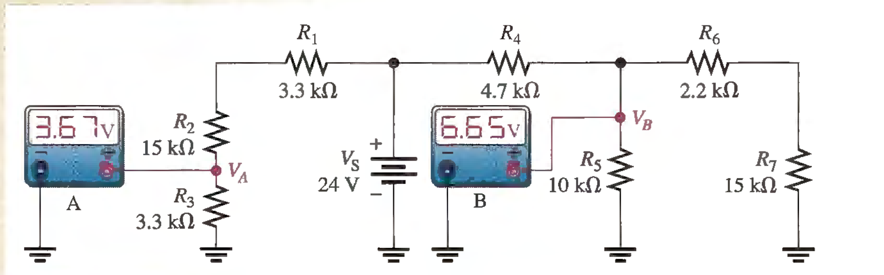


▲ FIGURE 7–54

Related Problem What would be the voltage across an open R_5 in Figure 7–53 assuming no other faults?

EXAMPLE 7–21

The two voltmeters in Figure 7–55 indicate the voltages shown. Apply logical thought and your knowledge of circuit operation to determine if there are any opens or shorts in the circuit and, if so, where they are located.



▲ FIGURE 7–55

Solution **Step 1:** Determine if the voltmeter readings are correct. R_1, R_2, and R_3 act as a voltage divider. Calculate the voltage (V_A) across R_3 as follows:

$$V_A = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)V_S = \left(\frac{3.3\,\text{k}\Omega}{21.6\,\text{k}\Omega}\right)24\text{ V} = 3.67\text{ V}$$

The voltmeter A reading is correct. This indicates that R_1, R_2, and R_3 are connected and are not faulty.

Step 2: See if the voltmeter B reading is correct. $R_6 + R_7$ is in parallel with R_5. The series-parallel combination of R_5, R_6, and R_7 is in series with R_4. Calculate the resistance of the R_5, R_6, and R_7 combination as follows:

$$R_{5\|(6+7)} = \frac{R_5(R_6 + R_7)}{R_5 + R_6 + R_7} = \frac{(10\,\text{k}\Omega)(17.2\,\text{k}\Omega)}{27.2\,\text{k}\Omega} = 6.32\,\text{k}\Omega$$

$R_{5\|(6+7)}$ and R_4 form a voltage divider, and voltmeter B measures the voltage across $R_{5\|(6+7)}$. Is it correct? Check as follows:

$$V_B = \left(\frac{R_{5\|(6+7)}}{R_4 + R_{5\|(6+7)}}\right)V_S = \left(\frac{6.32\,\text{k}\Omega}{11\,\text{k}\Omega}\right)24\text{ V} = 13.8\text{ V}$$

Thus, the actual measured voltage (6.65 V) at this point is incorrect. Some logical thinking will help to isolate the problem.

Step 3: R_4 is not open, because if it were, the meter would read 0 V. If there were a short across it, the meter would read 24 V. Since the actual voltage is much less than it should be, $R_{5\|(6+7)}$ must be less than the calculated value of 6.32 kΩ. The most likely problem is a short across R_7. If there is a short from the top of R_7 to ground, R_6 is effectively in parallel with R_5. In this case,

$$R_5 \parallel R_6 = \frac{R_5R_6}{R_5 + R_6} = \frac{(10\,\text{k}\Omega)(2.2\,\text{k}\Omega)}{12.2\,\text{k}\Omega} = 1.80\,\text{k}\Omega$$

Then V_B is

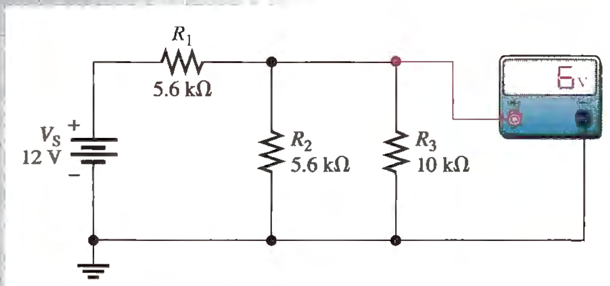
$$V_B = \left(\frac{1.80\,\text{k}\Omega}{6.5\,\text{k}\Omega}\right)24\text{ V} = 6.65\text{ V}$$

This value for V_B agrees with the voltmeter B reading. So there is a short across R_7. If this were an actual circuit, you would try to find the physical cause of the short.

Related Problem If the only fault in Figure 7–55 is that R_2 is shorted, what will voltmeter A read? What will voltmeter B read?

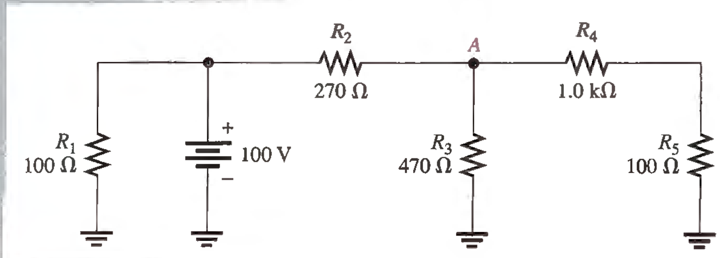
SECTION 7–7 REVIEW

1. Name two types of common circuit faults.
2. In Figure 7–56, one of the resistors in the circuit is open. Based on the meter reading, determine which is the open resistor.

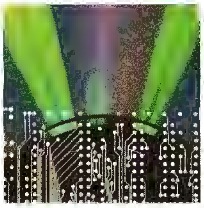


▲ FIGURE 7–56

3. For the following faults in Figure 7–57, what voltage would be measured at node A with respect to ground?
   (a) No faults (b) R_1 open (c) Short across R_5 (d) R_3 and R_4 open
   (e) R_2 open



▲ FIGURE 7–57



A Circuit Application

The Wheatstone bridge is widely used in measurement applications that use sensors to convert a physical parameter to a change in resistance. Modern Wheatstone bridges are automated; with intelligent interface modules, the output can be conditioned and converted to any desired unit for display or processing (for example, the output might be displayed in pounds for a scale application).

The Wheatstone bridge provides a null measurement, which enables it to have great sensitivity. It can also be designed to compensate for changes in temperature, a great advantage for many resistive measurements, particularly when the resistance change of the sensor is very small. Usually, the output voltage of the bridge is increased by an amplifier that has a minimum loading effect on the bridge.

Temperature Controller

In this application, a Wheatstone bridge circuit is used in a temperature controller. The sensor is a thermistor ("thermal resistor"), which is a resistive sensor that changes resistance as temperature changes. Thermistors are available with positive or negative resistance characteristics as a function of temperature. The thermistor in this circuit is one of the resistors in a Wheatstone bridge but is located a short distance from the circuit board for sensing the temperature at a point off the board.

The threshold voltage for the output to change is controlled by the 10 kΩ potentiometer, R_3. The amplifier in this case is constructed using an operational amplifier ("op-amp"), which is an integrated circuit. The term *amplifier* is used in electronics to describe a device that produces a larger replica of the input voltage or current at its output. The term *gain* refers to the amount of amplification. The op-amp in this circuit is configured as a *comparator,* which is be used to compare the voltage on one side of the bridge with the voltage on the other.

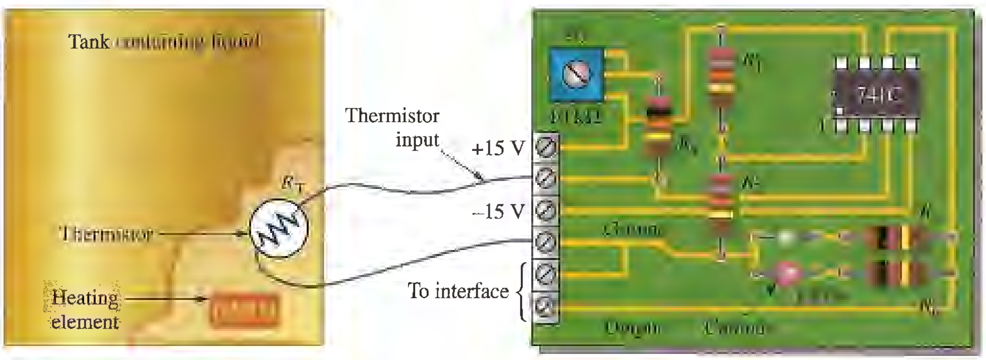
The advantage of a comparator is that it is extremely sensitive to an unbalanced bridge and produces a large output when the bridge is unbalanced. In fact, it is so sensitive, it is virtually impossible to adjust the bridge for perfect balance. Even the tiniest imbalance will cause the output to go to a voltage near either the maximum or minimum possible (the power supply voltages). This is handy for turning on a heater or other device based on the temperature.

The Control Circuit

This application has a tank containing a liquid that needs to be held at a warm temperature, as illustrated in Figure 7–58(a). The circuit board for the temperature controller is shown in Figure 7–58(b). The circuit board controls a heating unit (through an interface, which is not shown) when the temperature is too cold. The thermistor, which is located in the tank, is connected between one of the amplifier inputs and ground as illustrated.

The amplifier is a 741C op-amp, an inexpensive and popular device that will work fine for this application. The op-amp has two inputs, an output, and connections for positive and negative supply voltages. The schematic symbol for the op-amp with light-emitting diodes (LEDs) connected to the output is shown in Figure 7–59. The red LED is on when the op-amp output is a positive voltage, indicating the heater is on. The green LED is on when the output is a negative voltage, indicating the heater is off.

◆ Using the circuit board as a guide, complete the schematic in Figure 7–59. The op-amp inputs are connected to a Wheatstone bridge. Show the values for all resistors.



(a) Tank with heating element and thermistor

(b) Control board

FIGURE 7–58

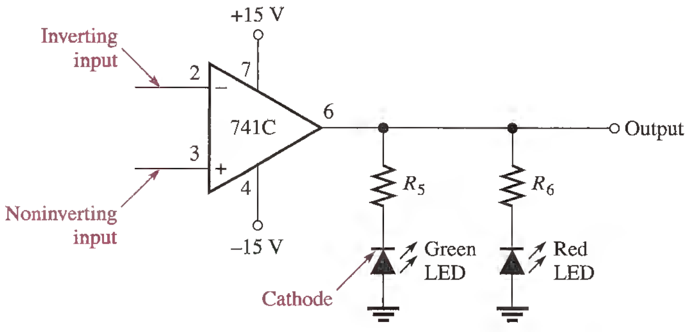


FIGURE 7–59

The op-amp and output LED indicators.

The Thermistor

The thermistor is a mixture of two metal-oxides, which exhibit a large resistance change as a function of temperature. The thermistor in the temperature controller circuit is located off the board near the point where temperature is to be sensed in the tank and is connected between the thermistor input and ground.

Thermistors have a nonlinear resistance-temperature characteristic described by the exponential equation:

$$R_T = R_0 e^{\beta\left(\frac{T_0 - T}{T_0 T}\right)}$$

where:

R_T = the resistance at a given temperature

R_0 = the resistance at a reference temperature

T_0 = the reference temperature in Kelvin (K), typically 298 K, which is 25°C

T = temperature in K

β = a constant (K) provided by the manufacturer

This exponential equation where e is the base of natural logarithms can be solved easily on a scientific calculator. Exponential equations are studied in later chapters.

The thermistor in this application is a Thermometrics RL2006-13.3K-140-D1 thermistor with a specified resistance of 25 kΩ at 25°C and a β of 4615 K. For convenience, the resistance of this thermistor is plotted as a function of temperature in Figure 7–60. Notice that the negative slope indicates this thermistor has a negative temperature coefficient (NTC); that is, its resistance decreases as the temperature increases.

As an example, the calculation for finding the resistance at T = 50°C is shown. First, convert 50°C to K.

$$T = °C + 273 = 50°C + 273 = 323\text{ K}$$

Also,

$$T_0 = °C + 273 = 25°C + 273 = 298\text{ K}$$

$$R_0 = 25\text{ k}\Omega$$

$$R_T = R_0 e^{\beta\left(\frac{T_0 - T}{T_0 T}\right)}$$

$$= (25\text{ k}\Omega)e^{4615\left(\frac{298-323}{298\times323}\right)}$$

$$= (25\text{ k}\Omega)e^{-1.198}$$

$$= (25\text{ k}\Omega)(0.302)$$

$$= 7.54\text{ k}\Omega$$

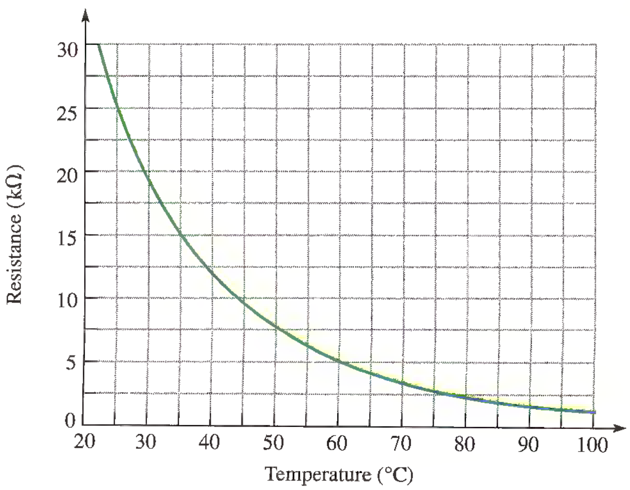


FIGURE 7–60

Using your calculator, first determine the value of the exponent $\beta(T_0 - T)/(T_0T)$. Next determine the value of the term $e^{\beta\left(\frac{T_0-T}{T_0T}\right)}$. Finally, multiply by R_0. On many calculators, e^x is a secondary function.

- Calculate the resistance of the thermistor at a temperature of 40°C using the exponential equation and confirm that your calculation is correct by comparing your result with Figure 7–60. Remember that temperatures in the equation are in kelvin. (K = °C + 273).
- Calculate the resistance setting of R_3 to balance the bridge at 25°C.
- Calculate the output voltage of the bridge (input to the op-amp) when the temperature of the thermistor is 40°C. Assume the bridge was balanced at 25°C and that the only change is the resistance of the thermistor.
- If you needed to set the reference temperature to 0°C, what simple change would you make to the circuit? Show with a calculation that your change will work and draw the revised schematic.

Review

1. At 25°C, the thermistor will have about 7.5 V across it. Calculate the power it dissipates. Is there any loading effect on a temperature measurement due to this?
2. As the temperature increases, does the loading effect go up, down, or remain the same? Explain your answer.
3. Can ⅛ W resistors be used in this application? Explain your answer.
4. Why is only one LED on at a time at the output?

SUMMARY

- A series-parallel circuit is a combination of both series and parallel current paths.
- To determine total resistance in a series-parallel circuit, identify the series and parallel relationships, and then apply the formulas for series resistance and parallel resistance from Chapters 5 and 6.
- To find the total current, apply Ohm's law and divide the total voltage by the total resistance.
- To determine branch currents, apply the current-divider formula, Kirchhoff's current law, or Ohm's law. Consider each circuit problem individually to determine the most appropriate method.
- To determine voltage drops across any portion of a series-parallel circuit, use the voltage-divider formula, Kirchhoff's voltage law, or Ohm's law. Consider each circuit problem individually to determine the most appropriate method.
- When a load resistor is connected across a voltage-divider output, the output voltage decreases.
- The load resistor should be large compared to the resistance across which it is connected, in order that the loading effect may be minimized.
- To find total resistance of a ladder network, start at the point farthest from the source and reduce the resistance in steps.
- A balanced Wheatstone bridge can be used to measure an unknown resistance.
- A bridge is balanced when the output voltage is zero. The balanced condition produces zero current through a load connected across the output terminals of the bridge.
- An unbalanced Wheatstone bridge can be used to measure physical quantities using transducers.
- Opens and shorts are typical circuit faults.
- Resistors normally open when they burn out.

KEY TERMS

These key terms are also in the end-of-book glossary.

Balanced bridge A bridge circuit that is in the balanced state is indicated by 0 V across the output.

Bleeder current The current left after the total load current is subtracted from the total current into the circuit.

Unbalanced bridge A bridge circuit that is in the unbalanced state as indicated by a voltage across the output that is proportional to the amount of deviation from the balanced state.

Wheatstone bridge A 4-legged type of bridge circuit with which an unknown resistance can be accurately measured using the balanced state. Deviations in resistance can be measured using the unbalanced state.

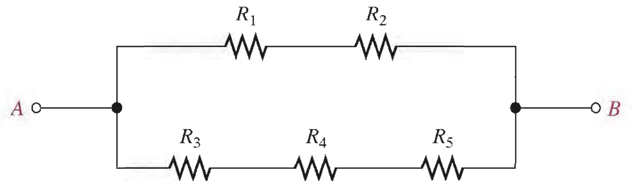
FORMULAS

7–1	$I_{BLEEDER} = I_T - I_{RL1} - I_{RL2}$	Bleeder current
7–2	$R_X = R_V\left(\frac{R_2}{R_4}\right)$	Unknown resistance in a Wheatstone bridge
7–3	$\Delta V_{OUT} = \Delta R_{therm}\left(\frac{V_S}{4R}\right)$	Thermistor bridge output

SELF–TEST

Answers are at the end of the chapter.

1. Which of the following statements are true concerning Figure 7–61?
   (a) R_1 and R_2 are in series with R_3, R_4, and R_5.
   (b) R_1 and R_2 are in series.
   (c) R_3, R_4, and R_5 are in parallel.
   (d) The series combination of R_1 and R_2 is in parallel with the series combination of R_3, R_4, and R_5.
   (e) answers (b) and (d)



▲ FIGURE 7–61

2. The total resistance of Figure 7–61 can be found with which of the following formulas?
   (a) $R_1 + R_2 + R_3 \| R_4 \| R_5$ (b) $R_1 \| R_2 + R_3 \| R_4 \| R_5$
   (c) $(R_1 + R_2) \| (R_3 + R_4 + R_5)$ (d) none of these answers
3. If all of the resistors in Figure 7–61 have the same value, when voltage is applied across terminals A and B, the current is
   (a) greatest in R_5 (b) greatest in R_3, R_4, and R_5
   (c) greatest in R_1 and R_2 (d) the same in all the resistors
4. Two 1.0 kΩ resistors are in series and this series combination is in parallel with a 2.2 kΩ resistor. The voltage across one of the 1.0 kΩ resistors is 6 V. The voltage across the 2.2 kΩ resistor is
   (a) 6 V (b) 3 V (c) 12 V (d) 13.2 V
5. The parallel combination of a 330 Ω resistor and a 470 Ω resistor is in series with the parallel combination of four 1.0 kΩ resistors. A 100 V source is connected across the circuit. The resistor with the most current has a value of
   (a) 1.0 kΩ (b) 330 Ω (c) 470 Ω
6. In the circuit described in Question 5, the resistor(s) with the most voltage has (have) a value of
   (a) 1.0 kΩ (b) 470 Ω (c) 330 Ω

7. In the circuit of Question 5, the percentage of the total current through any single 1.0 kΩ resistor is
   (a) 100% (b) 25% (c) 50% (d) 31.3%
8. The output of a certain voltage divider is 9 V with no load. When a load is connected, the output voltage
   (a) increases (b) decreases (c) remains the same (d) becomes zero
9. A certain voltage divider consists of two 10 kΩ resistors in series. Which of the following load resistors will have the most effect on the output voltage?
   (a) 1.0 MΩ (b) 20 kΩ (c) 100 kΩ (d) 10 kΩ
10. When a load resistance is connected to the output of a voltage-divider circuit, the current drawn from the source
   (a) decreases (b) increases (c) remains the same (d) is cut off
11. In a ladder network, simplification should begin at
   (a) the source (b) the resistor farthest from the source
   (c) the center (d) the resistor closest to the source
12. In a certain four-step $R/2R$ ladder network, the smallest resistor value is 10 kΩ. The largest value is
   (a) indeterminable (b) 20 kΩ (c) 50 kΩ (d) 100 kΩ
13. The output voltage of a balanced Wheatstone bridge is
   (a) equal to the source voltage
   (b) equal to zero
   (c) dependent on all of the resistor values in the bridge
   (d) dependent on the value of the unknown resistor
14. A certain Wheatstone bridge has the following resistor values: $R_V = 8\text{ k}\Omega$, $R_2 = 680\ \Omega$, and $R_4 = 2.2\text{ k}\Omega$. The unknown resistance is
   (a) 2473 Ω (b) 25.9 kΩ (c) 187 Ω (d) 2890 Ω
15. You are measuring the voltage at a given point in a circuit that has very high resistance values and the measured voltage is a little lower than it should be. This is possibly because of
   (a) one or more of the resistance values being off
   (b) the loading effect of the voltmeter
   (c) the source voltage is too low
   (d) all of these answers

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 7–62(b).

1. If R_2 opens, the total current
   (a) increases (b) decreases (c) stays the same
2. If R_3 opens, the current in R_2
   (a) increases (b) decreases (c) stays the same
3. If R_4 opens, the voltage across it
   (a) increases (b) decreases (c) stays the same
4. If R_4 is shorted, the total current
   (a) increases (b) decreases (c) stays the same

Refer to Figure 7–64.

5. If R_{10} opens, with 10 V applied between terminals A and B, the total current
   (a) increases (b) decreases (c) stays the same

6. If R_1 opens with 10 V applied between terminals A and B, the voltage across R_1
   (a) increases (b) decreases (c) stays the same
7. If there is a short between the left contact of R_3 and the bottom contact of R_5, the total resistance between A and B
   (a) increases (b) decreases (c) stays the same

Refer to Figure 7–68.

8. If R_4 opens, the voltage at point C
   (a) increases (b) decreases (c) stays the same
9. If there is a short from point D to ground, the voltage from A to B
   (a) increases (b) decreases (c) stays the same
10. If R_5 opens, the current through R_1
    (a) increases (b) decreases (c) stays the same

Refer to Figure 7–74.

11. If a 10 kΩ load resistor is connected across the output terminals A and B, the output voltage
    (a) increases (b) decreases (c) stays the same
12. If the 10 kΩ load resistor mentioned in Question 11 is replaced by a 100 kΩ load resistor, V_{OUT}
    (a) increases (b) decreases (c) stays the same

Refer to Figure 7–75.

13. If there is a short between the V_2 and V_3 terminals of the switch, the voltage V_1 with respect to ground
    (a) increases (b) decreases (c) stays the same
14. If the switch is in the position shown and if the V_3 terminal of the switch is shorted to ground, the voltage across R_L
    (a) increases (b) decreases (c) stays the same
15. If R_4 opens with the switch in the position shown, the voltage across R_L
    (a) increases (b) decreases (c) stays the same

Refer to Figure 7–80.

16. If R_4 opens, V_{OUT}
    (a) increases (b) decreases (c) stays the same
17. If R_7 is shorted to ground, V_{OUT}
    (a) increases (b) decreases (c) stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

SECTION 7–1 Identifying Series-Parallel Relationships

1. Visualize and draw the following series-parallel combinations:
   (a) R_1 in series with the parallel combination of R_2 and R_3
   (b) R_1 in parallel with the series combination of R_2 and R_3
   (c) R_1 in parallel with a branch containing R_2 in series with a parallel combination of four other resistors
2. Visualize and draw the following series-parallel circuits:
   (a) A parallel combination of three branches, each containing two series resistors
   (b) A series combination of three parallel circuits, each containing two resistors

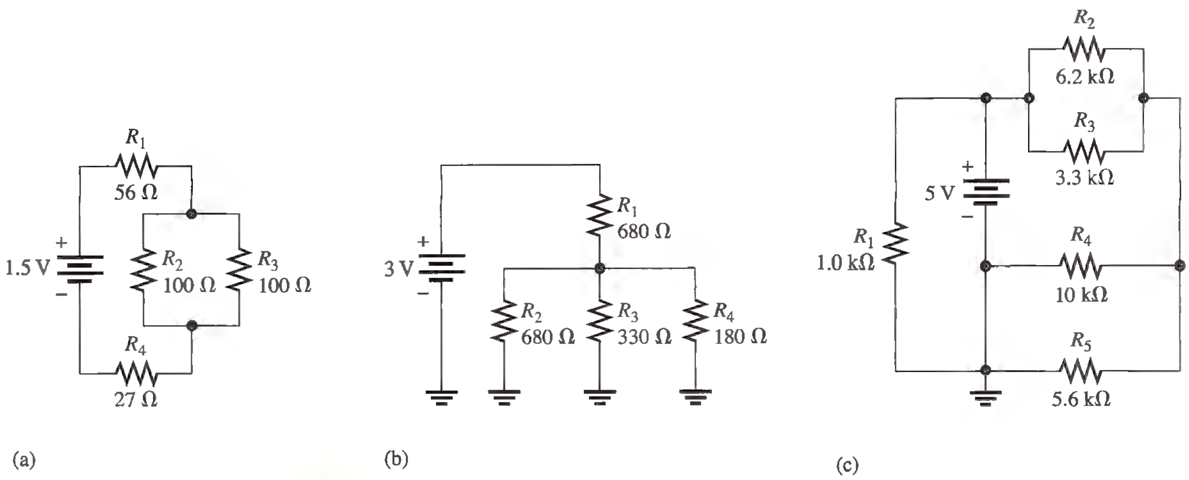


FIGURE 7–62

3. In each circuit of Figure 7–62, identify the series and parallel relationships of the resistors viewed from the source.
4. For each circuit in Figure 7–63, identify the series and parallel relationships of the resistors viewed from the source.

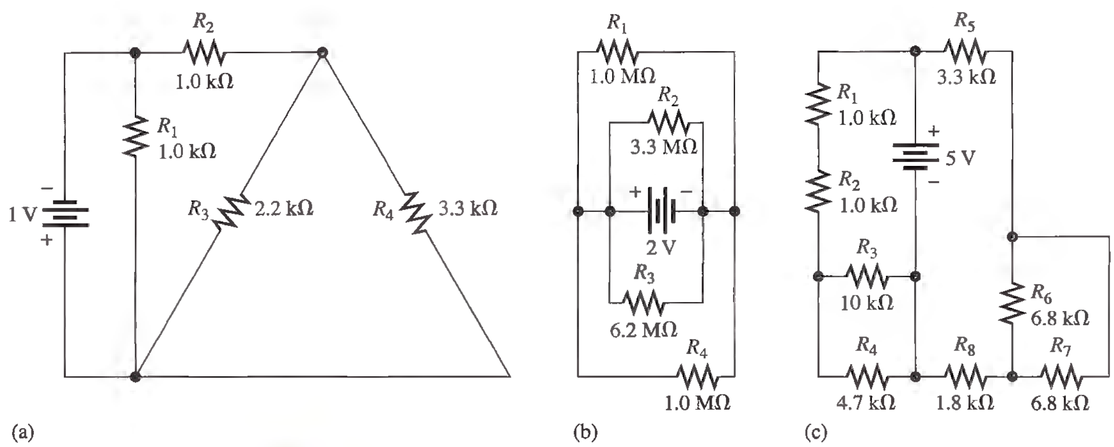
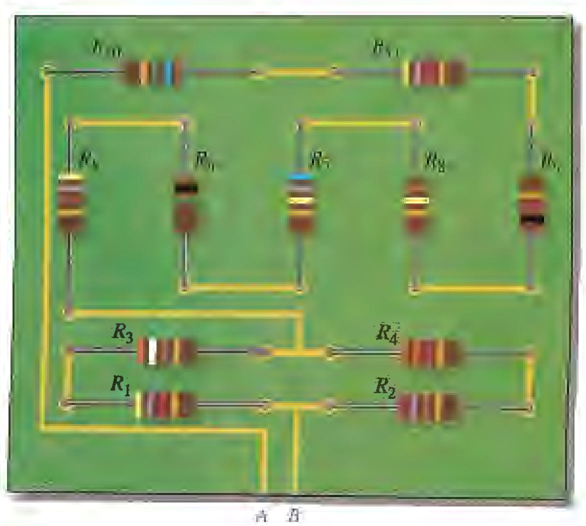


FIGURE 7–63

5. Draw the schematic of the PC board layout in Figure 7–64 showing resistor values and identify the series-parallel relationships.

FIGURE 7–64



*6. Develop a schematic for the double-sided PC board in Figure 7–65 and label the resistor values.

*7. Lay out a PC board for the circuit in Figure 7–63(c). The battery is to be connected external to the board.

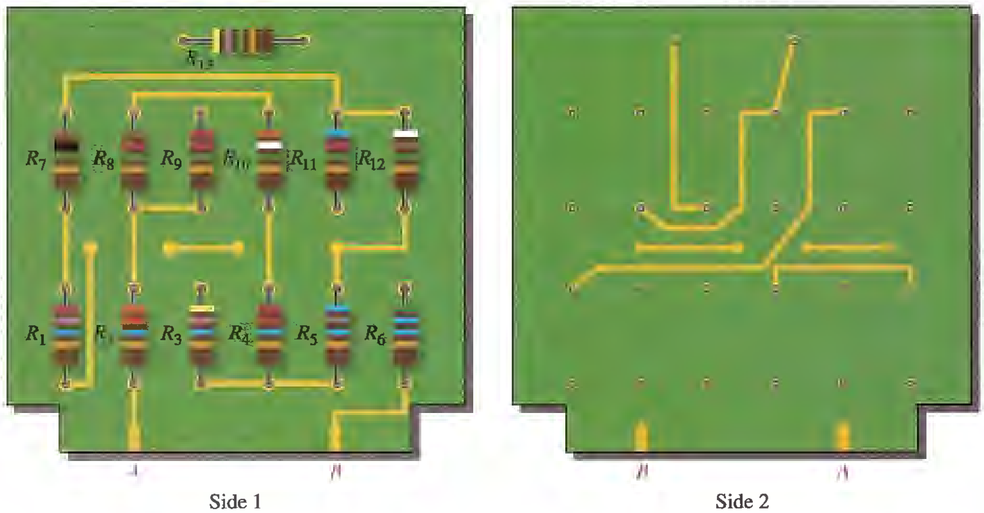


FIGURE 7–65

SECTION 7–2 Analysis of Series-Parallel Resistive Circuits

8. A certain circuit is composed of two parallel resistors. The total resistance is 667 Ω. One of the resistors is 1.0 kΩ. What is the other resistor?

9. For each circuit in Figure 7–62, determine the total resistance presented to the source.

10. Repeat Problem 9 for each circuit in Figure 7–63.

11. Determine the current through each resistor in each circuit in Figure 7–62; then calculate each voltage drop.

12. Determine the current through each resistor in each circuit in Figure 7–63; then calculate each voltage drop.

13. Find R_T for all combinations of the switches in Figure 7–66.

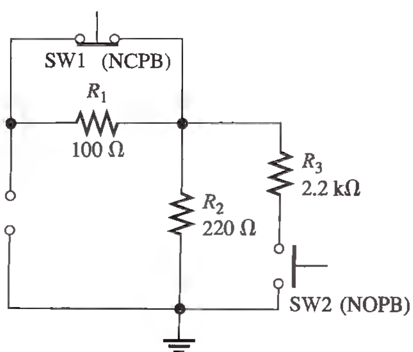
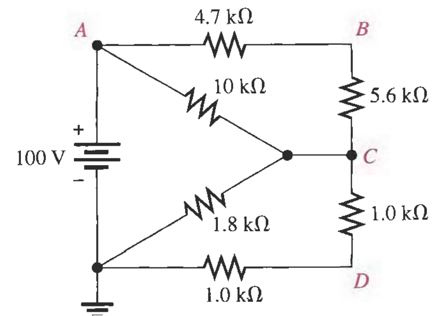


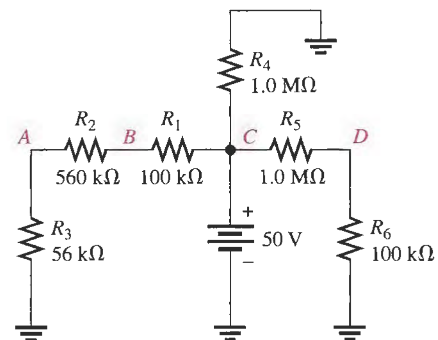
FIGURE 7–66

▶ FIGURE 7–67



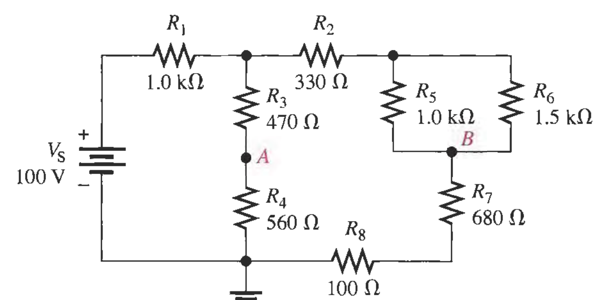
14. Determine the resistance between A and B in Figure 7–67 with the source removed.
15. Determine the voltage at each node with respect to ground in Figure 7–67.
16. Determine the voltage at each node with respect to ground in Figure 7–68.
17. In Figure 7–68, how would you determine the voltage across R_2 by measuring without connecting a meter directly across the resistor?

▶ FIGURE 7–68



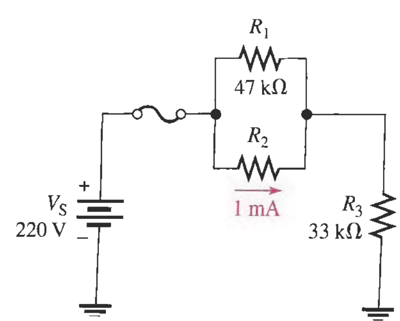
18. Determine the resistance of the circuit in Figure 7–67 as seen from the voltage source.
19. Determine the resistance of the circuit in Figure 7–68 as seen from the voltage source.
20. Determine the voltage, V_{AB}, in Figure 7–69.

▶ FIGURE 7–69

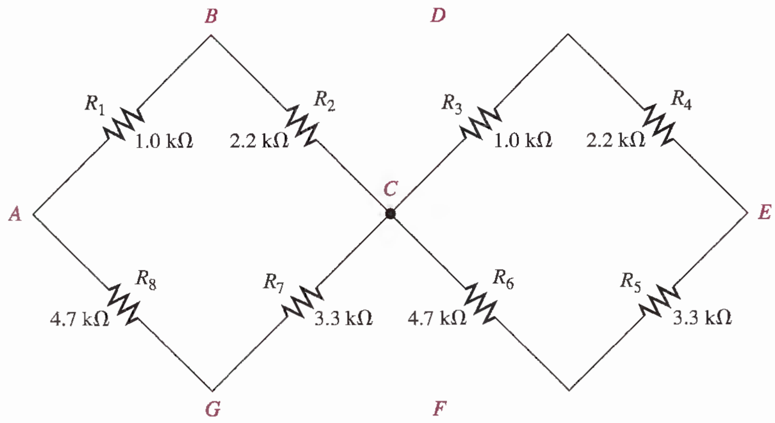


*21. (a) Find the value of R_2 in Figure 7–70. (b) Determine the power in R_2.

▶ FIGURE 7–70



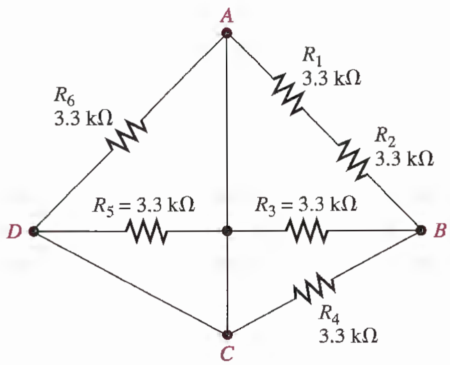
*22. Find the resistance between node A and each of the other nodes (R_{AB}, R_{AC}, R_{AD}, R_{AE}, R_{AF}, and R_{AG}) in Figure 7–71.



▲ FIGURE 7–71

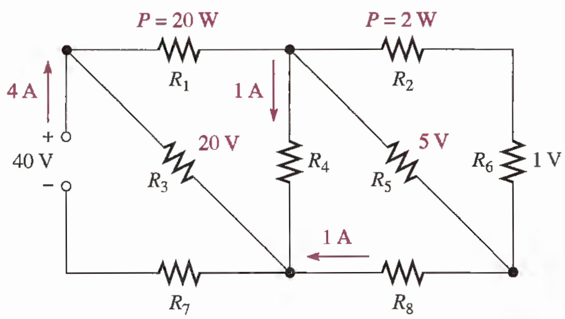
*23. Find the resistance between each of the following sets of nodes in Figure 7–72: AB, BC, and CD.

▶ FIGURE 7–72



*24. Determine the value of each resistor in Figure 7–73.

▶ FIGURE 7–73



SECTION 7–3 Voltage Dividers with Resistive Loads

25. A voltage divider consists of two 56 kΩ resistors and a 15 V source. Calculate the unloaded output voltage. What will the output voltage be if a load resistor of 1.0 MΩ is connected to the output?

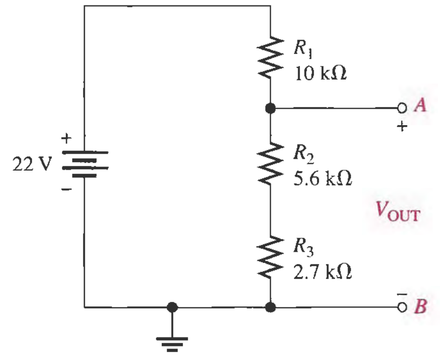
26. A 12 V battery output is divided down to obtain two output voltages. Three 3.3 kΩ resistors are used to provide the two taps. Determine the output voltages. If a 10 kΩ load is connected to the higher of the two outputs, what will its loaded value be?

27. Which will cause a smaller decrease in output voltage for a given voltage divider, a 10 kΩ load or a 47 kΩ load?

28. In Figure 7–74, determine the output voltage with no load across the output terminals. With a 100 kΩ load connected from *A* to *B*, what is the output voltage?

29. In Figure 7–74, determine the output voltage with a 33 kΩ load connected between *A* and *B*.

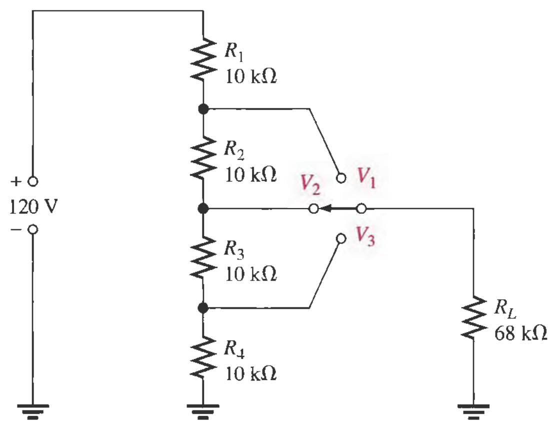
30. In Figure 7–74, determine the continuous current drawn from the source with no load across the output terminals. With a 33 kΩ load, what is the current drain?



▲ FIGURE 7–74

***31.** Determine the resistance values for a voltage divider that must meet the following specifications: The current drawn from the source under unloaded condition is not to exceed 5 mA. The source voltage is to be 10 V, and the required outputs are to be 5 V and 2.5 V. Sketch the circuit. Determine the effect on the output voltages if a 1.0 kΩ load is connected to each tap one at a time.

32. The voltage divider in Figure 7–75 has a switched load. Determine the voltage at each tap (V_1, V_2, and V_3) for each position of the switch.



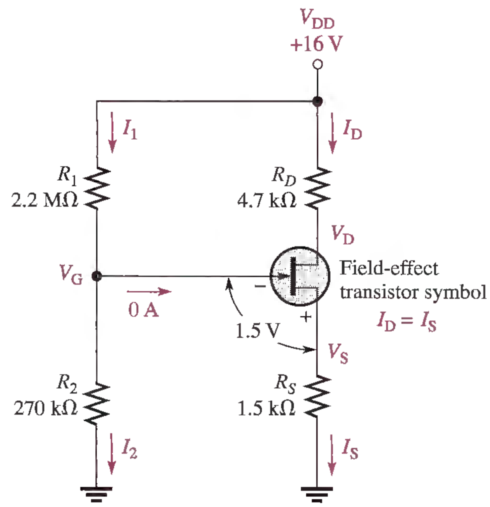
▲ FIGURE 7–75

***33.** Figure 7–76 shows a dc biasing arrangement for a field-effect transistor amplifier. Biasing is a common method for setting up certain dc voltage levels required for proper amplifier operation. Although you are not expected to be familiar with transistor amplifiers at this point, the dc voltages and currents in the circuit can be determined using methods that you already know.

(a) Find V_G and V_S **(b)** Determine I_1, I_2, I_D, and I_S **(c)** Find V_{DS} and V_{DG}

***34.** Design a voltage divider to provide a 6 V output with no load and a minimum of 5.5 V across a 1.0 kΩ load. The source voltage is 24 V, and the unloaded current drain is not to exceed 100 mA.

▶ FIGURE 7–76



SECTION 7–4 Loading Effect of a Voltmeter

35. On which one of the following voltage range settings will a voltmeter present the minimum load on a circuit?

(a) 1 V **(b)** 10 V **(c)** 100 V **(d)** 1000 V

36. Determine the internal resistance of a 20,000 Ω/V voltmeter on each of the following range settings.

(a) 0.5 V **(b)** 1 V **(c)** 5 V **(d)** 50 V **(e)** 100 V **(f)** 1000 V

37. The voltmeter described in Problem 36 is used to measure the voltage across R_4 in Figure 7–62(a).

(a) What range should be used?

(b) How much less is the voltage measured by the meter than the actual voltage?

38. Repeat Problem 37 if the voltmeter is used to measure the voltage across R_4 in the circuit of Figure 7–62(b).

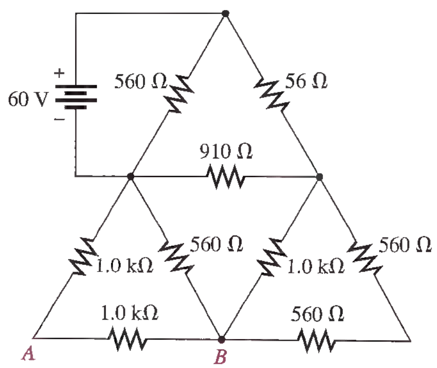
SECTION 7–5 Ladder Networks

39. For the circuit shown in Figure 7–77, calculate the following:

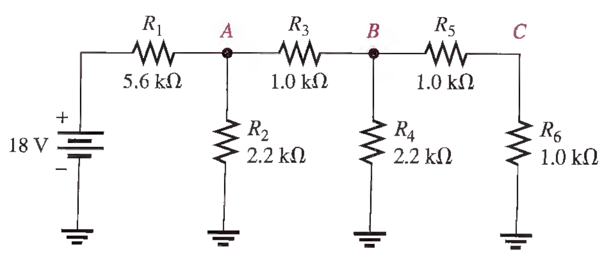
(a) Total resistance across the source **(b)** Total current from the source

(c) Current through the 910 Ω resistor **(d)** Voltage from A to point B

40. Determine the total resistance and the voltage at nodes A, B, and C in the ladder network of Figure 7–78.



▲ FIGURE 7–77



▲ FIGURE 7–78

*41. Determine the total resistance between terminals A and B of the ladder network in Figure 7–79. Also calculate the current in each branch with 10 V between A and B.

42. What is the voltage across each resistor in Figure 7–79 with 10 V between A and B?

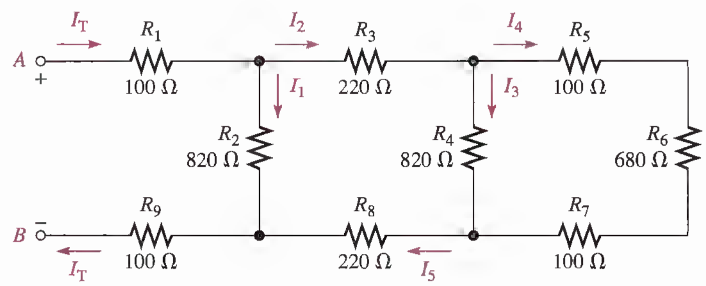
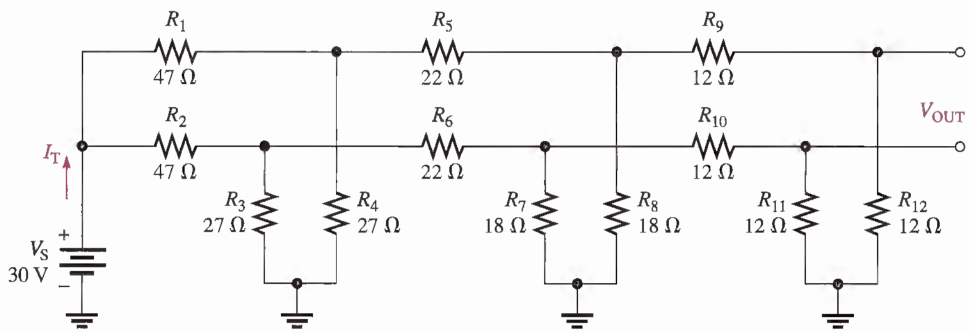


FIGURE 7–79

*43. Find I_T and V_{OUT} in Figure 7–80.

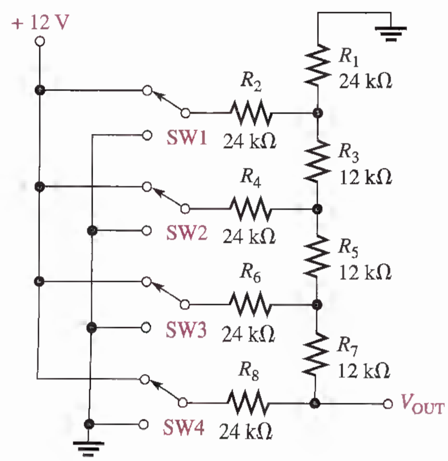


▲ FIGURE 7–80

44. Determine V_{OUT} for the $R/2R$ ladder network in Figure 7–81 for the following conditions:

(a) Switch SW2 connected to +12 V and the others connected to ground

(b) Switch SW1 connected to +12 V and the others connected to ground



▲ FIGURE 7–81

45. Repeat Problem 44 for the following conditions:

(a) SW3 and SW4 to +12 V, SW1 and SW2 to ground

(b) SW3 and SW1 to +12 V, SW2 and SW4 to ground

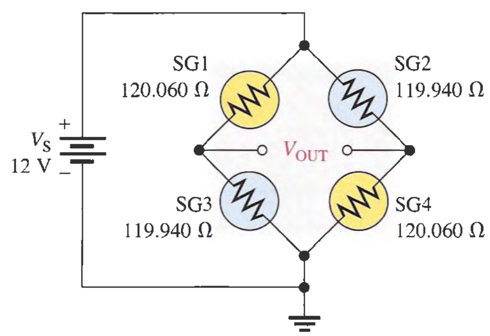
(c) All switches to +12 V

SECTION 7–6 The Wheatstone Bridge

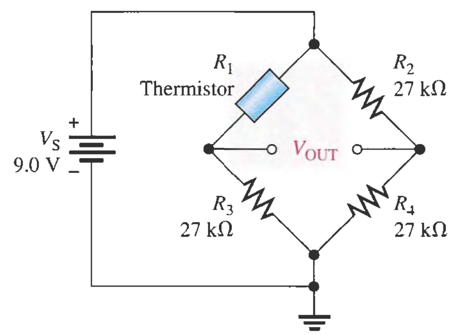
46. A resistor of unknown value is connected to a Wheatstone bridge circuit. The bridge parameters for a balanced condition are set as follows: R_V = 18 kΩ and $R_2/R_4 = 0.02$. What is R_X?

47. A load cell has four identical strain gauges with an unstrained resistance of 120.000 Ω for each gauge (a standard value). When a load is added, the gauges in tension increase their resistance by 60 mΩ to 120.060 Ω and the gauges in compression decrease their resistance by 60 mΩ to 119.940 Ω as shown in Figure 7–82. What is the output voltage under load?

48. Determine the output voltage for the unbalanced bridge in Figure 7–83 for a temperature of 60°C. The temperature resistance characteristic for the thermistor is shown in Figure 7–60.



▲ FIGURE 7–82

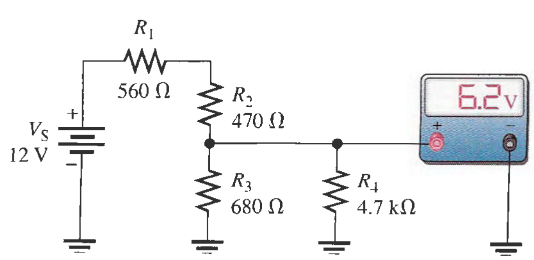


▲ FIGURE 7–83

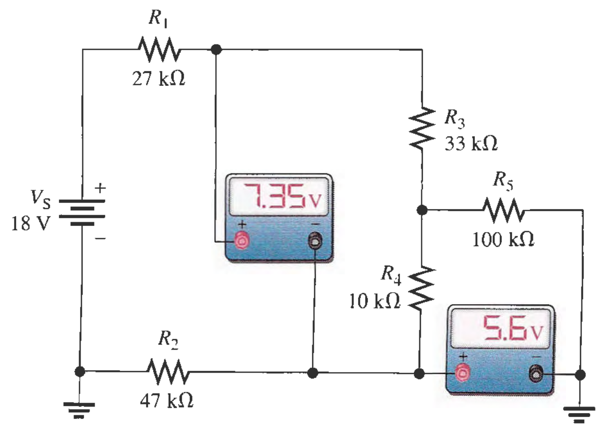
SECTION 7–7 Troubleshooting

49. Is the voltmeter reading in Figure 7–84 correct?

50. Are the meter readings in Figure 7–85 correct?

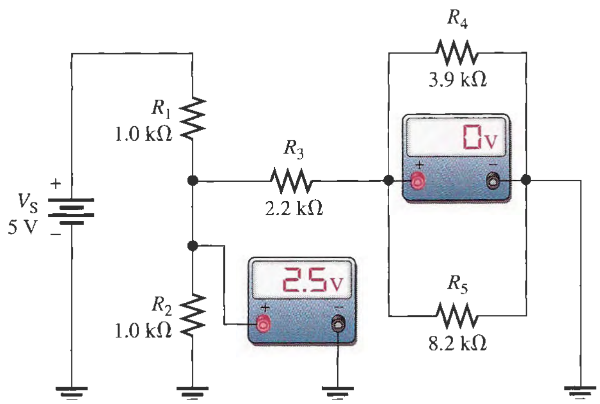


▲ FIGURE 7–84



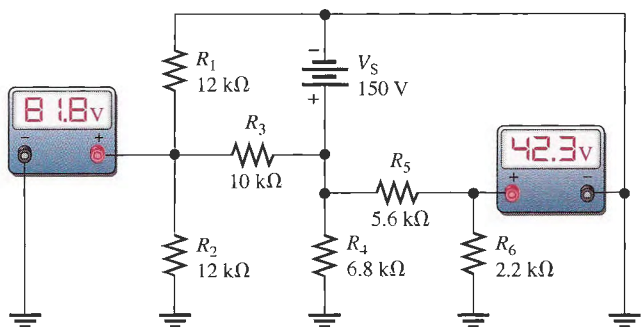
▲ FIGURE 7–85

51. There is one fault in Figure 7–86. Based on the meter indications, determine what the fault is.



▲ FIGURE 7–86

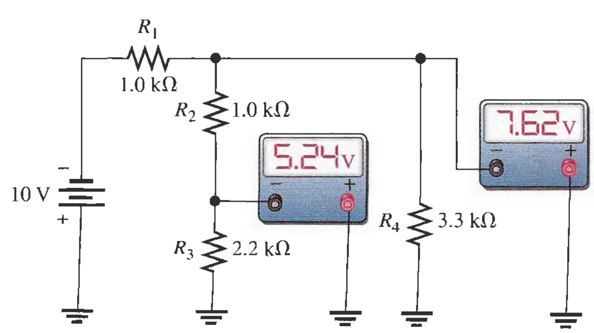
52. Look at the meters in Figure 7–87 and determine if there is a fault in the circuit. If there is a fault, identify it.



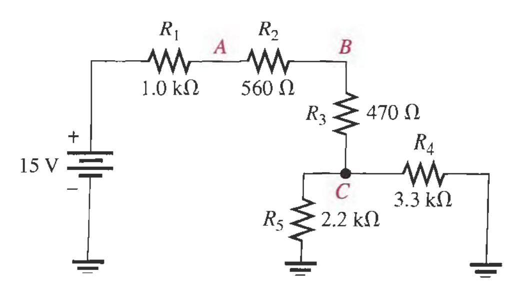
▲ FIGURE 7–87

53. Check the meter readings in Figure 7–88 and locate any fault that may exist.

54. If R_2 in Figure 7–89 opens, what voltages will be read at points A, B, and C?



▲ FIGURE 7–88



▲ FIGURE 7–89

Multisim Troubleshooting and Analysis

These problems require your Multisim CD-ROM.

55. Open file P07-55 and measure the total resistance.

56. Open file P07-56. Determine by measurement if there is an open resistor and, if so, which one.

57. Open file P07-57 and determine the unspecified resistance value.

58. Open file P07-58 and determine how much the load resistance affects each of the resistor voltages.

59. Open file P07-59 and find the shorted resistor, if there is one.

60. Open file P07-60 and adjust the value of R_X until the bridge is approximately balanced.

ANSWERS

SECTION REVIEWS

SECTION 7–1 Identifying Series-Parallel Relationships

1. A series-parallel resistive circuit is a circuit consisting of both series and parallel connections.

2. See Figure 7–90.

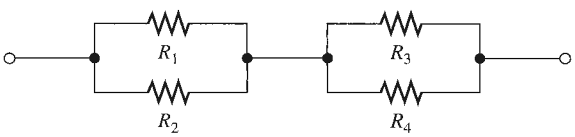
3. Resistors R_1 and R_2 are in series with the parallel combination of R_3 and R_4.

4. R_3, R_4, and R_5 are in parallel. Also the series-parallel combination $R_2 + (R_3 \| R_4 \| R_5)$ is in parallel with R_1.

5. Resistors R_1 and R_2 are in parallel; R_3 and R_4 are in parallel.

6. Yes, the parallel combinations are in series.

▶ FIGURE 7–90



SECTION 7–2 Analysis of Series-Parallel Resistive Circuits

1. Voltage-divider and current-divider formulas, Kirchhoff's laws, and Ohm's law can be used in series-parallel analysis.

2. $R_T = R_1 + R_2 \| R_3 + R_4 = 608\ \Omega$

3. $I_3 = [R_2/(R_2 + R_3)]I_T = 11.1$ mA

4. $V_2 = I_2R_2 = 3.65$ V

5. $R_T = 47\ \Omega + 27\ \Omega + (27\ \Omega + 27\ \Omega) \| 47\ \Omega = 99.1\ \Omega$; $I_T = 1\text{ V}/99.1\ \Omega = 10.1$ mA

SECTION 7–3 Voltage Dividers with Resistive Loads

1. The load resistor decreases the output voltage.

2. True

3. $V_{OUT(unloaded)} = (100\text{ k}\Omega/147\text{ k}\Omega)30\text{ V} = 20.4$ V; $V_{OUT(loaded)} = (9.1\text{ k}\Omega/56.1\text{ k}\Omega)30\text{ V} = 4.87$ V

SECTION 7–4 Loading Effect of a Voltmeter

1. A voltmeter loads a circuit because the internal resistance of the meter appears in parallel with the circuit resistance across which it is connected, reducing the resistance between those two points of the circuit and drawing current from the circuit.

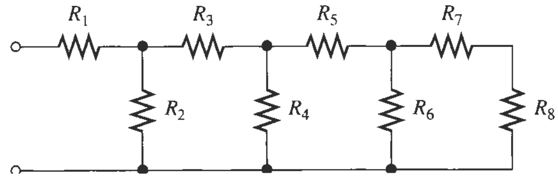
2. No, because the meter resistance is much larger than 1.0 kΩ.

3. Yes.

SECTION 7–5 Ladder Networks

1. See Figure 7–91.
2. $R_T = 11.6\text{ k}\Omega$
3. $I_T = 10\text{ V}/11.6\text{ k}\Omega = 859\ \mu\text{A}$
4. $I_2 = 640\ \mu\text{A}$
5. $V_A = 1.41\text{ V}$

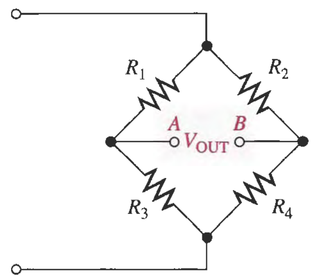
▶ FIGURE 7–91



SECTION 7–6 The Wheatstone Bridge

1. See Figure 7–92.
2. The bridge is balanced when $V_A = V_B$; that is, when $V_{OUT} = 0$
3. $R_X = 15\text{ k}\Omega$
4. An unbalanced bridge is used to measure transducer-sensed quantities.

▶ FIGURE 7–92



SECTION 7–7 Troubleshooting

1. Common circuit faults are opens and shorts.
2. The $10\text{ k}\Omega$ resistor (R_3) is open.
3. **(a)** $V_A = 55\text{ V}$ **(b)** $V_A = 55\text{ V}$ **(c)** $V_A = 54.2\text{ V}$ **(d)** $V_A = 100\text{ V}$ **(e)** $V_A = 0\text{ V}$

A Circuit Application

1. $P = 2.25\text{ W}$; yes, a very tiny effect
2. The loading decreases.
3. Yes, worst case is $R_{THERM} = 0$, so 15 V is across $R_3 + R_4$, making power less than ⅛ W.
4. The output voltage of the op-amp can only be at a maximum or a minimum level, causing one LED or the other to be on at a time, but not both.

RELATED PROBLEMS FOR EXAMPLES

7–1 The new resistor is in parallel with $R_4 + R_2 \| R_3$.

7–2 The resistor has no effect because it is shorted.

7–3 The new resistor is in parallel with R_5.

7–4 A to gnd: $R_T = R_4 + R_3 \| (R_1 + R_2)$
B to gnd: $R_T = R_4 + R_2 \| (R_1 + R_3)$
C to gnd: $R_T = R_4$

7–5 R_3 and R_6 are in series.

7–6 55.1 Ω

7–7 128.3 Ω

7–8 2.38 mA

7–9 $I_1 = 35.7$ mA; $I_3 = 23.4$ mA

7–10 $V_A = 44.8$ V; $V_1 = 35.2$ V

7–11 2.04 V

7–12 $V_{AB} = 5.48$ V; $V_{BC} = 1.66$ V; $V_{CD} = 0.86$ V

7–13 3.39 V

7–14 Increase R_1, R_2, and R_3 proportionally.

7–15 5.19 V

7–16 $I_1 = 7.16$ mA; $I_2 = 3.57$ mA; $I_3 = 3.57$ mA; $I_4 = 1.74$ mA; $I_5 = 1.85$ mA; $I_6 = 1.85$ mA; $V_A = 29.3$ V; $V_B = 17.4$ V; $V_C = 8.70$ V

7–17 3.3 kΩ

7–18 0.45 V

7–19 5.73 V; 0 V

7–20 9.46 V

7–21 $V_A = 12$ V; $V_B = 13.8$ V

SELF-TEST

1. (e) **2.** (c) **3.** (c) **4.** (c) **5.** (b) **6.** (a) **7.** (b) **8.** (b)
9. (d) **10.** (b) **11.** (b) **12.** (b) **13.** (b) **14.** (a) **15.** (d)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (a) **3.** (a) **4.** (a) **5.** (b) **6.** (a)
7. (b) **8.** (c) **9.** (c) **10.** (c) **11.** (b) **12.** (a)
13. (b) **14.** (b) **15.** (a) **16.** (a) **17.** (a)