6 PARALLEL CIRCUITS

CHAPTER OUTLINE

CHAPTER OBJECTIVES

- Identify a parallel resistive circuit
- Determine the voltage across each parallel branch
- Apply Kirchhoff's current law
- Determine total parallel resistance
- Apply Ohm's law in a parallel circuit
- Determine the total effect of current sources in parallel
- Use a parallel circuit as a current divider
- Determine power in a parallel circuit
- Describe some basic applications of parallel circuits
- Troubleshoot parallel circuits

KEY TERMS

- Branch
- Parallel
- Kirchhoff's current law
- Node
- Current divider

A CIRCUIT APPLICATION PREVIEW

In this application, a panel-mounted power supply will be modified by adding a milliammeter to indicate current to a load. Expansion of the meter for multiple current ranges using parallel (shunt) resistors will be demonstrated. The problem with very low-value resistors when a switch is used to select the current ranges will be introduced and the effect of switch contact resistance will be demonstrated. A way of eliminating the contact resistance problem will be presented. Finally, the ammeter circuit will be installed in the power supply. The knowledge of parallel circuits and of basic ammeters that you will acquire in this chapter plus your understanding of Ohm's law, current dividers, and the resistor color code will be put to good use.

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INTRODUCTION

In Chapter 5, you learned about series circuits and how to apply Ohm's law and Kirchhoff's voltage law. You also saw how a series circuit can be used as a voltage divider to obtain several specified voltages from a single source voltage. The effects of opens and shorts in series circuits were also examined.

In this chapter, you will see how Ohm's law is used in parallel circuits; and you will learn Kirchhoff's current law. Also, several applications of parallel circuits, including automotive lighting, residential wiring, and the internal wiring of analog ammeters are presented. You will learn how to determine total parallel resistance and how to troubleshoot for open resistors.

When resistors are connected in parallel and a voltage is applied across the parallel circuit, each resistor provides a separate path for current. The total resistance of a parallel circuit is reduced as more resistors are connected in parallel. The voltage across each of the parallel resistors is equal to the voltage applied across the entire parallel circuit.

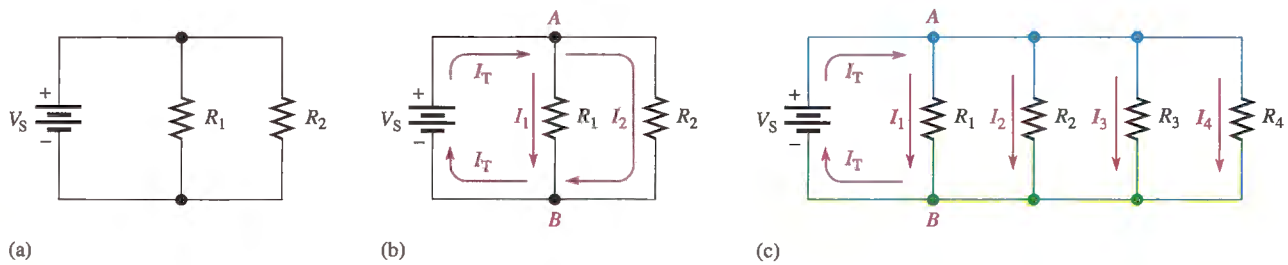
6–1 RESISTORS IN PARALLEL

When two or more resistors are individually connected between two separate points, they are in parallel with each other. A parallel circuit provides more than one path for current.

After completing this section, you should be able to

- **Identify a parallel resistive circuit**
    - Translate a physical arrangement of parallel resistors into a schematic

Each current path is called a **branch**, and a **parallel** circuit is one that has more than one branch. Two resistors connected in parallel are shown in Figure 6–1(a). As shown in part (b), the current out of the source (I_T) divides when it gets to point A. I_1 goes through R_1 and I_2 goes through R_2. If additional resistors are connected in parallel with the first two, more current paths are provided between point A and point B, as shown in Figure 6–1(c). All points along the top shown in blue are electrically the same as point A, and all points along the bottom shown in green are electrically the same as point B.



▲ FIGURE 6–1

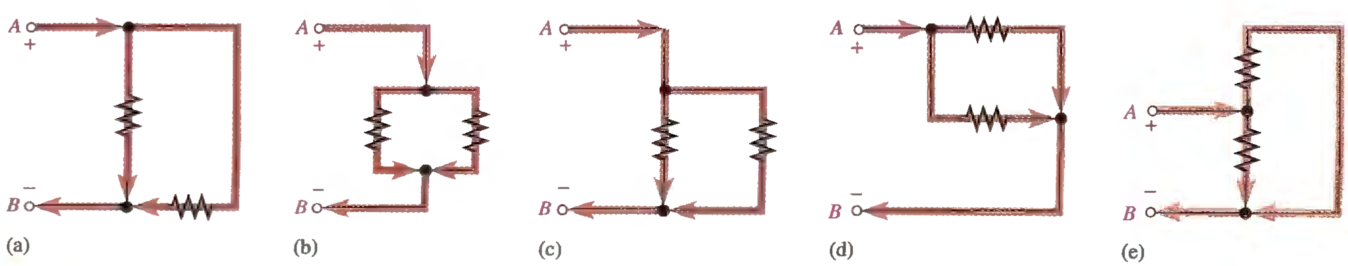
Resistors in parallel.

In Figure 6–1, it is obvious that the resistors are connected in parallel. Often, in actual circuit diagrams, the parallel relationship is not as clear. It is important that you learn to recognize parallel circuits regardless of how they may be drawn.

A rule for identifying parallel circuits is as follows:

If there is more than one current path (branch) between two separate points and if the voltage between those two points also appears across each of the branches, then there is a parallel circuit between those two points.

Figure 6–2 shows parallel resistors drawn in different ways between two separate points labeled A and B. Notice that in each case, the current has two paths going from A to B, and



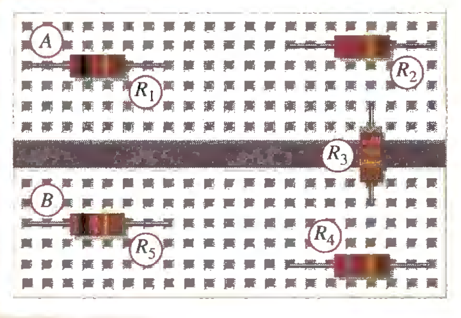
▲ FIGURE 6–2

Examples of circuits with two parallel paths.

the voltage across each branch is the same. Although these examples show only two parallel paths, there can be any number of resistors in parallel.

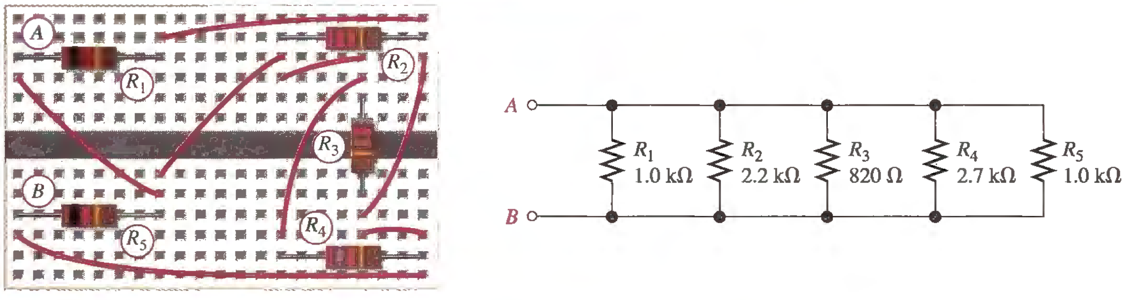
EXAMPLE 6–1

Five resistors are positioned on a protoboard as shown in Figure 6–3. Show the wiring required to connect all of the resistors in parallel between *A* and *B*. Draw a schematic and label each of the resistors with its value.



▲ **FIGURE 6–3**

Solution Wires are connected as shown in the assembly diagram of Figure 6–4(a). The schematic is shown in Figure 6–4(b). Again, note that the schematic does not necessarily have to show the actual physical arrangement of the resistors. The schematic shows how components are connected electrically.



(a) Assembly wiring diagram (b) Schematic

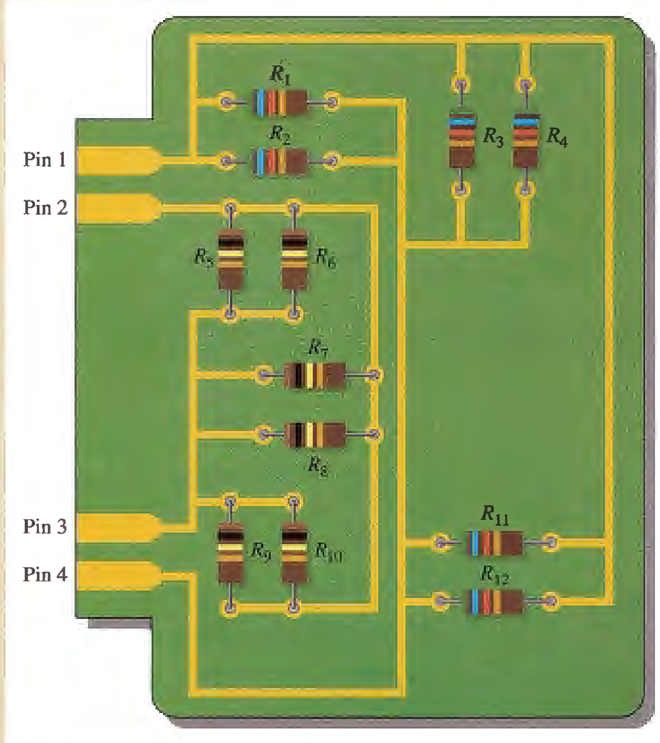
▲ **FIGURE 6–4**

Related Problem* How would the circuit have to be rewired if R_2 is removed?

*Answers are at the end of the chapter.

EXAMPLE 6–2

Determine the parallel groupings in Figure 6–5 and the value of each resistor.



▲ **FIGURE 6–5**

Solution Resistors R_1 through R_4 and R_{11} and R_{12} are all in parallel. This parallel combination is connected to pins 1 and 4. Each resistor in this group is 56 kΩ.

Resistors R_5 through R_{10} are all in parallel. This combination is connected to pins 2 and 3. Each resistor in this group is 100 kΩ.

Related Problem How would you connect all of the resistors in Figure 6–5 in parallel?

SECTION 6–1 REVIEW

Answers are at the end of the chapter.

1. How are the resistors connected in a parallel circuit?
2. How do you identify a parallel circuit?
3. Complete the schematics for the circuits in each part of Figure 6–6 by connecting the resistors in parallel between points *A* and *B*.

4. Connect each group of parallel resistors in Figure 6–6 in parallel with each other.

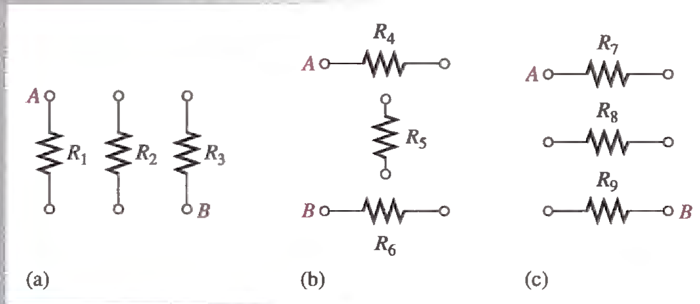


FIGURE 6–6

6–2 VOLTAGE IN A PARALLEL CIRCUIT

The voltage across any given branch of a parallel circuit is equal to the voltage across each of the other branches in parallel. As you know, each current path in a parallel circuit is called a branch.

After completing this section, you should be able to

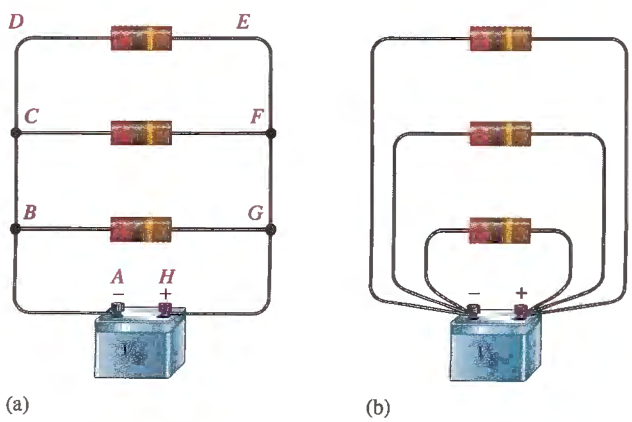
- **Determine the voltage across each parallel branch**
  - Explain why the voltage is the same across all parallel resistors

To illustrate voltage in a parallel circuit, let's examine Figure 6–7(a). Points *A*, *B*, *C*, and *D* along the left side of the parallel circuit are electrically the same point because the voltage is the same along this line. You can think of all of these points as being connected by a single wire to the negative terminal of the battery. The points *E*, *F*, *G*, and *H* along the right side of the circuit are all at a voltage equal to that of the positive terminal of the source. Thus, voltage across each parallel resistor is the same, and each is equal to the source voltage. Note that the parallel circuit in Figure 6–7 resembles a ladder.

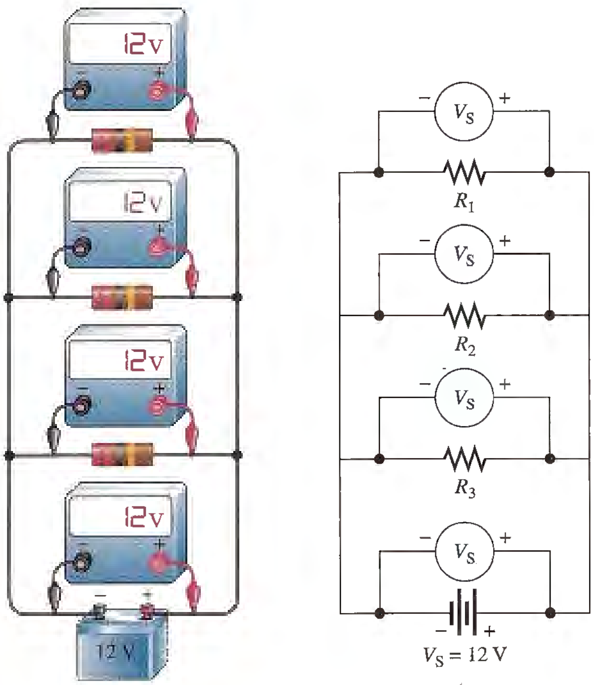
Figure 6–7(b) is the same circuit as in part (a), drawn in a slightly different way. Here the left side of each resistor is connected to a single point, which is the negative battery terminal. The right side of each resistor is connected to a single point, which is the positive battery terminal. The resistors are still all in parallel across the source.

FIGURE 6–7

Voltage across parallel branches is the same.



In Figure 6–8, a 12 V battery is connected across three parallel resistors. When the voltage is measured across the battery and then across each of the resistors, the readings are the same. As you can see, the same voltage appears across each branch in a parallel circuit.



(a) Pictorial

(b) Schematic

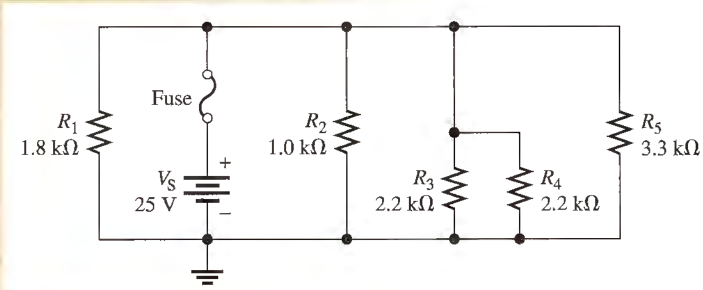
◄ **FIGURE 6–8**

The same voltage appears across each resistor in parallel.

EXAMPLE 6–3

Determine the voltage across each resistor in Figure 6–9.

FIGURE 6–9



Solution The five resistors are in parallel, so the voltage across each one is equal to the applied source voltage. There is no voltage across the fuse. The voltage across the resistors is

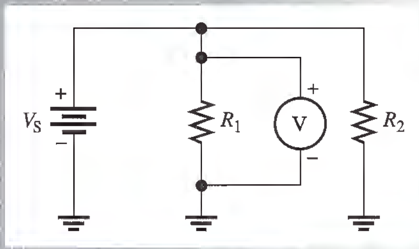
$$V_1 = V_2 = V_3 = V_4 = V_5 = V_S = \mathbf{25\ V}$$

Related Problem If R_4 is removed from the circuit, what is the voltage across R_3?

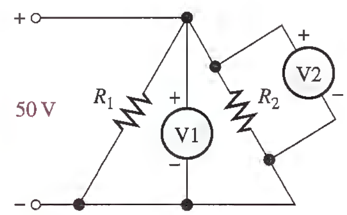
Use Multisim file E06-03 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 6–2 REVIEW

1. A 10 Ω and a 22 Ω resistor are connected in parallel with a 5 V source. What is the voltage across each of the resistors?
2. A voltmeter is connected across R_1 in Figure 6–10. It measures 118 V. If you move the meter and connect it across R_2, how much voltage will it indicate? What is the source voltage?
3. In Figure 6–11, how much voltage does voltmeter 1 indicate? Voltmeter 2?
4. How are voltages across each branch of a parallel circuit related?



▲ FIGURE 6–10



▲ FIGURE 6–11

6–3 KIRCHHOFF'S CURRENT LAW

Kirchhoff's voltage law deals with voltages in a single closed path. Kirchhoff's current law applies to currents in multiple paths.

After completing this section, you should be able to

- **Apply Kirchhoff's current law**
  - State Kirchhoff's current law
  - Define *node*
  - Determine the total current by adding the branch currents
  - Determine an unknown branch current

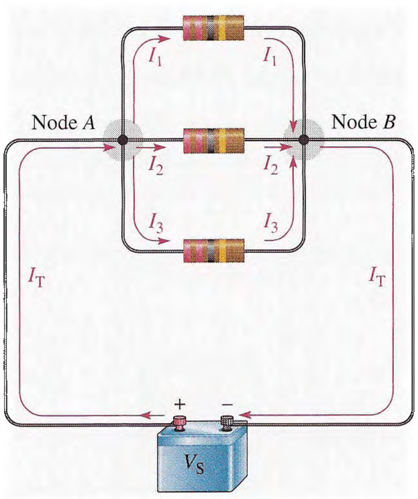
Kirchhoff's current law, often abbreviated KCL, can be stated as follows:

The sum of the currents into a node (total current in) is equal to the sum of the currents out of that node (total current out).

A **node** is any point or junction in a circuit where two or more components are connected. In a parallel circuit, a node or junction is a point where the parallel branches come together. For example, in the circuit of Figure 6–12, point *A* is one node and point *B* is another. Let's start at the positive terminal of the source and follow the current. The total current I_T from the source is *into* node *A*. At this point, the current splits up among the three branches as indicated. Each of the three branch currents (I_1, I_2, and I_3) is *out of* node *A*. Kirchhoff's current law says that the total current into node *A* is equal to the total current out of node *A*; that is,

$$I_T = I_1 + I_2 + I_3$$

Now, following the currents in Figure 6–12 through the three branches, you see that they come back together at node *B*. Currents I_1, I_2, and I_3 are into node *B*, and



◄ **FIGURE 6–12**

Kirchhoff's current law: The current into a node equals the current out of that node.

I_T is out of node B. Kirchhoff's current law formula at node B is therefore the same as at node A.

$$I_T = I_1 + I_2 + I_3$$

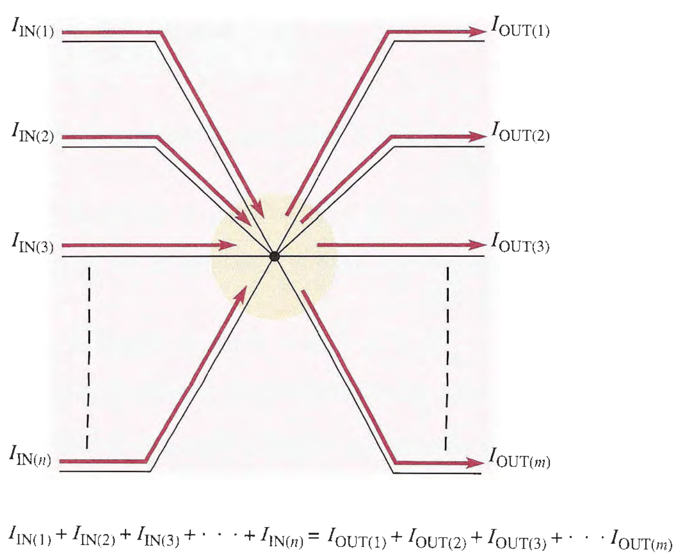
Figure 6–13 shows a generalized circuit node where a number of branches are connected at a point in a circuit. Currents $I_{IN(1)}$ through $I_{IN(n)}$ are into the node (n can be any number). Currents $I_{OUT(1)}$ through $I_{OUT(m)}$ are out of the node (m can be any number, but not necessarily equal to n). By Kirchhoff's current law, the sum of the currents into a node must equal the sum of the currents out of the node. With reference to Figure 6–13, a general formula for Kirchhoff's current law is

$$I_{IN(1)} + I_{IN(2)} + \cdots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + \cdots + I_{OUT(m)}$$

Equation 6–1

If all the terms on the right side of Equation 6–1 are brought over to the left side, their signs change to negative, and a zero is left on the right side as follows:

$$I_{IN(1)} + I_{IN(2)} + \cdots + I_{IN(n)} - I_{OUT(1)} - I_{OUT(2)} - \cdots - I_{OUT(m)} = 0$$



◄ **FIGURE 6–13**

Generalized circuit node illustrating Kirchhoff's current law.

Based on this last equation, Kirchhoff's current law can also be stated in this way:

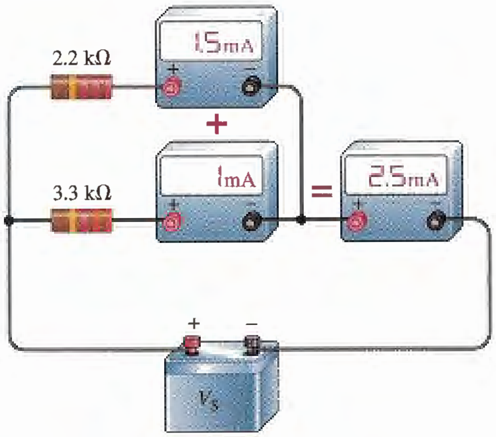
The algebraic sum of all the currents entering and leaving a node is equal to zero.

You can verify Kirchhoff's current law by connecting a circuit and measuring each branch current and the total current from the source, as illustrated in Figure 6–14. When the branch currents are added together, their sum will equal the total current. This rule applies for any number of branches.

The following three examples illustrate use of Kirchhoff's current law.

▶ FIGURE 6–14

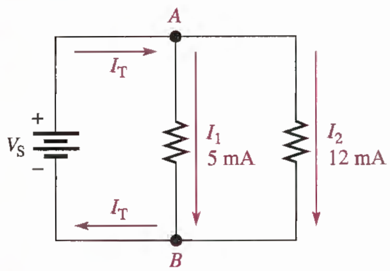
An illustration of Kirchhoff's current law.



EXAMPLE 6–4

The branch currents are shown in the circuit of Figure 6–15. Determine the total current entering node A and the total current leaving node B.

▶ FIGURE 6–15



Solution The total current out of node A is the sum of the two branch currents. So the total current into node A is

$$I_T = I_1 + I_2 = 5\text{ mA} + 12\text{ mA} = \mathbf{17\text{ mA}}$$

The total current entering node B is the sum of the two branch currents. So the total current out of node B is

$$I_T = I_1 + I_2 = 5\text{ mA} + 12\text{ mA} = \mathbf{17\text{ mA}}$$

Note that this equation can be equivalently expressed as $I_T - I_1 - I_2 = 0$.

Related Problem If a third branch is added to the circuit in Figure 6–15 and its current is 3 mA, what is the total current into node A and out of node B?

EXAMPLE 6–5

Determine the current I_2 through R_2 in Figure 6–16.

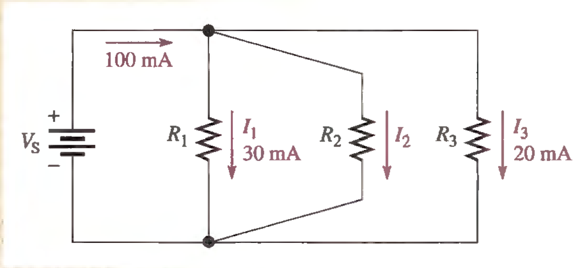


FIGURE 6–16

Solution The total current into the junction of the three branches is $I_T = I_1 + I_2 + I_3$. From Figure 6–16, you know the total current and the branch currents through R_1 and R_3. Solve for I_2 as follows:

$$I_2 = I_T - I_1 - I_3 = 100\,\text{mA} - 30\,\text{mA} - 20\,\text{mA} = \mathbf{50\,mA}$$

Related Problem Determine I_T and I_2 if a fourth branch is added to the circuit in Figure 6–16 and it has 12 mA through it.

EXAMPLE 6–6

Use Kirchhoff's current law to find the current measured by ammeters A3 and A5 in Figure 6–17.

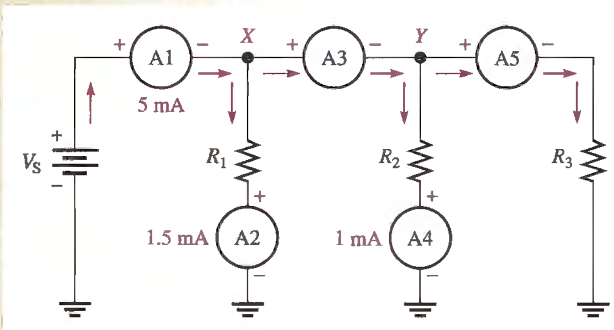


FIGURE 6–17

Solution The total current into node X is 5 mA. Two currents are out of node X: 1.5 mA through resistor R_1 and the current through A3. Kirchhoff's current law applied at node X gives

$$5\,\text{mA} = 1.5\,\text{mA} + I_{A3}$$

Solving for I_{A3} yields

$$I_{A3} = 5\,\text{mA} - 1.5\,\text{mA} = \mathbf{3.5\,mA}$$

The total current into node Y is $I_{A3} = 3.5$ mA. Two currents are out of node Y: 1 mA through resistor R_2 and the current through A5 and R_3. Kirchhoff's current law applied at node Y gives

$$3.5\text{ mA} = 1\text{ mA} + I_{A5}$$

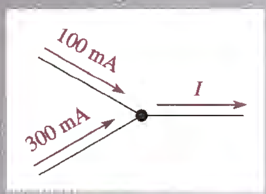
Solving for I_{A5} yields

$$I_{A5} = 3.5\text{ mA} - 1\text{ mA} = \mathbf{2.5\text{ mA}}$$

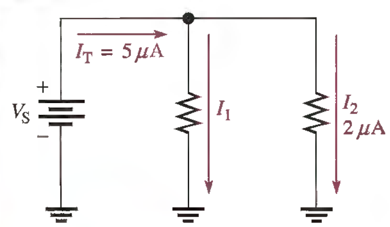
Related Problem How much current will an ammeter measure when it is placed in the circuit right below R_3 in Figure 6–17? Below the negative battery terminal?

SECTION 6–3 REVIEW

1. State Kirchhoff's current law in two ways.
2. There is a total current of 2.5 mA into a node and the current out of the node divides into three parallel branches. What is the sum of all three branch currents?
3. In Figure 6–18, 100 mA and 300 mA are into the node. What is the amount of current out of the node?
4. Determine I_1 in the circuit of Figure 6–19.
5. Two branch currents enter a node, and two branch currents leave the same node. One of the currents entering the node is 1 mA, and one of the currents leaving the node is 3 mA. The total current entering and leaving the node is 8 mA. Determine the value of the unknown current entering the node and the value of the unknown current leaving the node.



▲ FIGURE 6–18



▲ FIGURE 6–19

6–4 TOTAL PARALLEL RESISTANCE

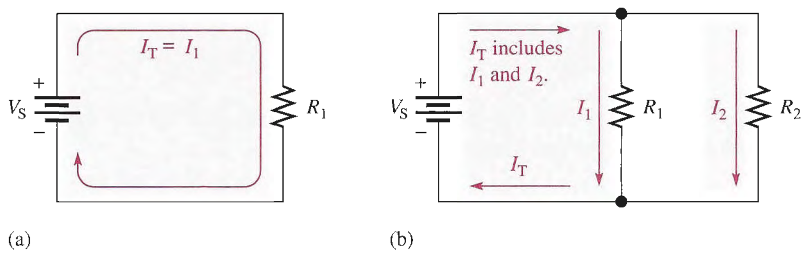
When resistors are connected in parallel, the total resistance of the circuit decreases. The total resistance of a parallel circuit is always less than the value of the smallest resistor. For example, if a 10 Ω resistor and a 100 Ω resistor are connected in parallel, the total resistance is less than 10 Ω.

After completing this section, you should be able to

- **Determine total parallel resistance**
  - Explain why resistance decreases as resistors are connected in parallel
  - Apply the parallel-resistance formula

As you know, when resistors are connected in parallel, the current has more than one path. The number of current paths is equal to the number of parallel branches.

In Figure 6–20(a), there is only one current path because it is a series circuit. There is a certain amount of current, I_1, through R_1. If resistor R_2 is connected in parallel with R_1, as shown in Figure 6–20(b), there is an additional amount of current, I_2, through R_2. The total current from the source has increased with the addition of the parallel resistor. Assuming that the source voltage is constant, an increase in the total current from the source means that the total resistance has decreased, in accordance with Ohm's law. Additional resistors connected in parallel will further reduce the resistance and increase the total current.



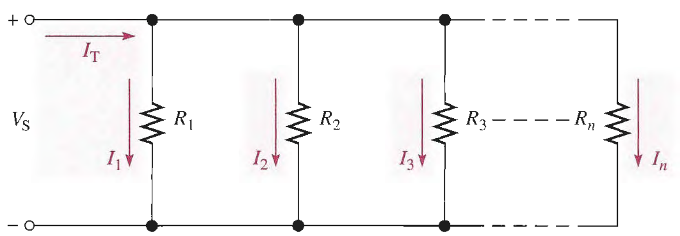
▲ FIGURE 6–20

Addition of resistors in parallel reduces total resistance and increases total current.

Formula for Total Parallel Resistance

The circuit in Figure 6–21 shows a general case of n resistors in parallel (n can be any number). From Kirchhoff's current law, the equation for current is

$$I_T = I_1 + I_2 + I_3 + \cdots + I_n$$



▲ FIGURE 6–21

Circuit with n resistors in parallel.

Since V_S is the voltage across each of the parallel resistors, by Ohm's law, $I_1 = V_S/R_1$, $I_2 = V_S/R_2$, and so on. By substitution into the equation for current,

$$\frac{V_S}{R_T} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} + \cdots + \frac{V_S}{R_n}$$

The term V_S can be factored out of the right side of the equation and canceled with V_S on the left side, leaving only the resistance terms.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Recall that the reciprocal of resistance ($1/R$) is called *conductance,* which is symbolized by G. The unit of conductance is the siemens (S). The equation for $1/R_T$ can be expressed in terms of conductance as

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

Solve for R_T by taking the reciprocal of (that is, by inverting) both sides of the equation for $1/R_T$.

Equation 6–2

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_n}\right)}$$

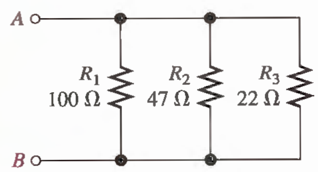
Equation 6–2 shows that to find the total parallel resistance, add all the $1/R$ (or conductance, G) terms and then take the reciprocal of the sum.

$$R_T = \frac{1}{G_T}$$

EXAMPLE 6–7

Calculate the total parallel resistance between points A and B of the circuit in Figure 6–22.

▶ FIGURE 6–22



Solution Use Equation 6–2 to calculate the total parallel resistance when you know the individual resistances. First, find the conductance, which is the reciprocal of the resistance, of each of the three resistors.

$$G_1 = \frac{1}{R_1} = \frac{1}{100\ \Omega} = 10\ \text{mS}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{47\ \Omega} = 21.3\ \text{mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{22\ \Omega} = 45.5\ \text{mS}$$

Next, calculate R_T by adding G_1, G_2, and G_3 and taking the reciprocal of the sum.

$$R_T = \frac{1}{G_T} = \frac{1}{10\ \text{mS} + 21.3\ \text{mS} + 45.5\ \text{mS}} = \frac{1}{76.8\ \text{mS}} = \mathbf{13.0\ \Omega}$$

For a quick accuracy check, notice that the value of R_T (13.0 Ω) is smaller than the smallest value in parallel, which is R_3 (22 Ω), as it should be.

Related Problem If a 33 Ω resistor is connected in parallel in Figure 6–22, what is the new value of R_T?

Calculator Tip

The parallel-resistance formula is easily solved on a calculator using Equation 6–2. The general procedure is to enter the value of R_1 and then take its reciprocal by pressing the x^{-1} key. (The reciprocal is a secondary function on some calculators.) Next press the + key; then enter the value of R_2 and take its reciprocal using the x^{-1} key and press the + key. Repeat this procedure until all of the resistor values have been entered; then press ENTER. The final step is to press the x^{-1} key and the ENTER key to get R_T. The total parallel resistance is now on the display. The display format may vary, depending on the particular calculator. For example, the steps required for a typical calculator solution of Example 6–7 are as follows:

1. Enter 100. Display shows 100.
2. Press x^{-1} (or 2nd then x^{-1}). Display shows 100^{-1}.
3. Press +. Display shows 100^{-1} +.
4. Enter 47. Display shows $100^{-1} + 47$.
5. Press x^{-1} (or 2nd then x^{-1}). Display shows $100^{-1} + 47^{-1}$.
6. Press +. Display shows $100^{-1} + 47^{-1}$ +.
7. Enter 22. Display shows $100^{-1} + 47^{-1} + 22$.
8. Press x^{-1} (or 2nd then x^{-1}). Display shows $100^{-1} + 47^{-1} + 22^{-1}$.
9. Press ENTER. Display shows a result of 76.7311411992E^{-3}.
10. Press x^{-1} (or 2nd then x^{-1}) and then ENTER. Display shows a result of 13.0325182758E0.

The number displayed in Step 10 is the total resistance in ohms. Round it to 13.0 Ω.

The Case of Two Resistors in Parallel

Equation 6–2 is a general formula for finding the total resistance for any number of resistors in parallel. The combination of two resistors in parallel occurs commonly in practice. Also, any number of resistors in parallel can be broken down into pairs as an alternate way to find the R_T. Based on Equation 6–2, the formula for the total resistance of two resistors in parallel is

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)}$$

Combining the terms in the denominator yields

$$R_T = \frac{1}{\left(\frac{R_1 + R_2}{R_1 R_2}\right)}$$

which can be rewritten as follows:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$ Equation 6–3

Equation 6–3 states

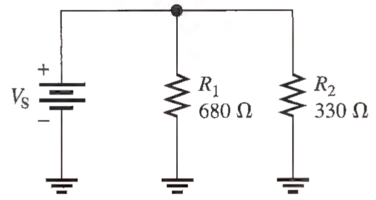
The total resistance for two resistors in parallel is equal to the product of the two resistors divided by the sum of the two resistors.

This equation is sometimes referred to as the "product over the sum" formula.

EXAMPLE 6–8

Calculate the total resistance connected to the voltage source of the circuit in Figure 6–23.

► FIGURE 6–23



Solution Use Equation 6–3.

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(680\ \Omega)(330\ \Omega)}{680\ \Omega + 330\ \Omega} = \frac{224{,}400\ \Omega^2}{1010\ \Omega} = \mathbf{222\ \Omega}$$

Related Problem Determine R_T if a 220 Ω replaces R_1 in Figure 6–23.

The Case of Equal-Value Resistors in Parallel

Another special case of parallel circuits is the parallel connection of several resistors each having the same resistance value. There is a shortcut method of calculating R_T when this case occurs.

If several resistors in parallel have the same resistance, they can be assigned the same symbol R. For example, $R_1 = R_2 = R_3 = \cdots = R_n = R$. Starting with Equation 6–2, you can develop a special formula for finding R_T.

$$R_T = \frac{1}{\left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \cdots + \left(\frac{1}{R}\right)}$$

Notice that in the denominator, the same term, $1/R$, is added n times (n is the number of equal-value resistors in parallel). Therefore, the formula can be written as

$$R_T = \frac{1}{n/R}$$

or

Equation 6–4

$$R_T = \frac{R}{n}$$

Equation 6–4 says that when any number of resistors (n), all having the same resistance (R), are connected in parallel, R_T is equal to the resistance divided by the number of resistors in parallel.

EXAMPLE 6–9

Four 8 Ω speakers are connected in parallel to the output of an amplifier. What is the total resistance across the output of the amplifier?

Solution There are four 8 Ω resistors in parallel. Use Equation 6–4 as follows:

$$R_T = \frac{R}{n} = \frac{8\ \Omega}{4} = \mathbf{2\ \Omega}$$

Related Problem If two of the speakers are removed, what is the resistance across the output?

Determining an Unknown Parallel Resistor

Sometimes you need to determine the values of resistors that are to be combined to produce a desired total resistance. For example, you use two parallel resistors to obtain a known total resistance. If you know or arbitrarily choose one resistor value, then you can calculate the second resistor value using Equation 6–3 for two parallel resistors. The formula for determining the value of an unknown resistor R_x is developed as follows:

$$\frac{1}{R_T} = \frac{1}{R_A} + \frac{1}{R_x}$$

$$\frac{1}{R_x} = \frac{1}{R_T} - \frac{1}{R_A}$$

$$\frac{1}{R_x} = \frac{R_A - R_T}{R_A R_T}$$

$$R_x = \frac{R_A R_T}{R_A - R_T}$$

Equation 6–5

where R_x is the unknown resistor and R_A is the known or selected value.

EXAMPLE 6–10

Suppose that you wish to obtain a resistance as close to 150 Ω as possible by combining two resistors in parallel. There is a 330 Ω resistor available. What other value do you need?

Solution $R_T = 150\ \Omega$ and $R_A = 330\ \Omega$. Therefore,

$$R_x = \frac{R_A R_T}{R_A - R_T} = \frac{(330\ \Omega)(150\ \Omega)}{330\ \Omega - 150\ \Omega} = 275\ \Omega$$

The closest standard value is **270 Ω**.

Related Problem If you need to obtain a total resistance of 130 Ω, what value can you add in parallel to the parallel combination of 330 Ω and 270 Ω? First find the value of 330 Ω and 270 Ω in parallel and treat that value as a single resistor.

Notation for Parallel Resistors

Sometimes, for convenience, parallel resistors are designated by two parallel vertical marks. For example, R_1 in parallel with R_2 can be written as $R_1 \| R_2$. Also, when several resistors are in parallel with each other, this notation can be used. For example,

$$R_1 \| R_2 \| R_3 \| R_4 \| R_5$$

indicates that R_1 through R_5 are all in parallel.

This notation is also used with resistance values. For example,

$$10\ \text{k}\Omega \| 5\ \text{k}\Omega$$

means that a 10 kΩ resistor is in parallel with a 5 kΩ resistor.

SECTION 6–4 REVIEW

1. Does the total resistance increase or decrease as more resistors are connected in parallel?
2. The total parallel resistance is always less than what value?
3. Write the general formula for R_T with any number of resistors in parallel.

4. Write the special formula for two resistors in parallel.
5. Write the special formula for any number of equal-value resistors in parallel.
6. Calculate R_T for Figure 6–24.
7. Determine R_T for Figure 6–25.
8. Find R_T for Figure 6–26.

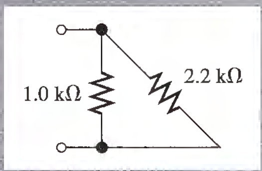


FIGURE 6–24

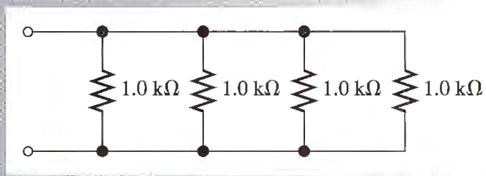


FIGURE 6–25

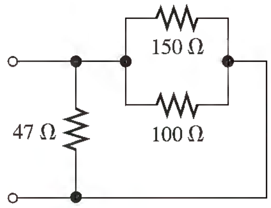


FIGURE 6–26

6–5 APPLICATION OF OHM'S LAW

Ohm's law can be applied to parallel circuit analysis.

After completing this section, you should be able to

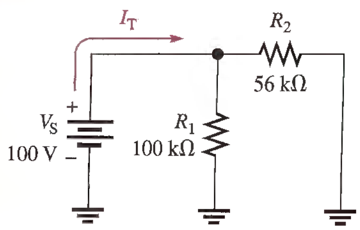
- **Apply Ohm's law in a parallel circuit**
  - Find the total current in a parallel circuit
  - Find each branch current in a parallel circuit
  - Find the voltage across a parallel circuit
  - Find the resistance of a parallel circuit

The following examples illustrate how to apply Ohm's law to determine the total current, branch currents, voltage, and resistance in parallel circuits.

EXAMPLE 6–11

Find the total current produced by the battery in Figure 6–27.

FIGURE 6–27



Solution The battery "sees" a total parallel resistance that determines the amount of current that it generates. First, calculate R_T.

$$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(100\,\text{k}\Omega)(56\,\text{k}\Omega)}{100\,\text{k}\Omega + 56\,\text{k}\Omega} = \frac{5600\,\text{k}\Omega^2}{156\,\text{k}\Omega} = 35.9\,\text{k}\Omega$$

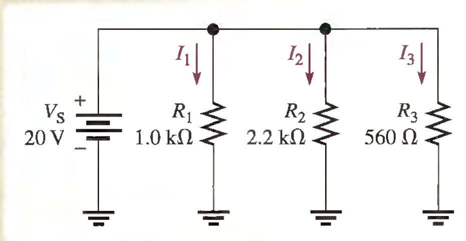
The battery voltage is 100 V. Use Ohm's law to find I_T.

$$I_T = \frac{V_S}{R_T} = \frac{100\text{ V}}{35.9\,\text{k}\Omega} = \mathbf{2.79\ mA}$$

Related Problem What is I_T in Figure 6–27 if R_2 is changed to 120 kΩ? What is the current through R_1?

Use Multisim file E06-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 6–12 Determine the current through each resistor in the parallel circuit of Figure 6–28.



▲ FIGURE 6–28

Solution The voltage across each resistor (branch) is equal to the source voltage. That is, the voltage across R_1 is 20 V, the voltage across R_2 is 20 V, and the voltage across R_3 is 20 V. The current through each resistor is determined as follows:

$$I_1 = \frac{V_S}{R_1} = \frac{20\text{ V}}{1.0\,\text{k}\Omega} = \mathbf{20\ mA}$$

$$I_2 = \frac{V_S}{R_2} = \frac{20\text{ V}}{2.2\,\text{k}\Omega} = \mathbf{9.09\ mA}$$

$$I_3 = \frac{V_S}{R_3} = \frac{20\text{ V}}{560\,\Omega} = \mathbf{35.7\ mA}$$

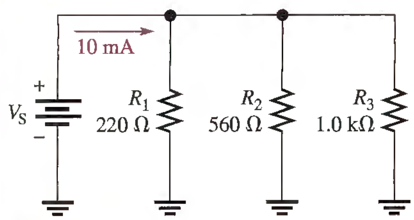
Related Problem If an additional resistor of 910 Ω is connected in parallel to the circuit in Figure 6–28, determine all of the branch currents.

Use Multisim file E06-12 to verify the calculated results in this example and to confirm your calculations for the related problem.

EXAMPLE 6–13

Find the voltage V_S across the parallel circuit in Figure 6–29.

▶ FIGURE 6–29



Solution The total current into the parallel circuit is 10 mA. If you know the total resistance, then you can apply Ohm's law to get the voltage. The total resistance is

$$R_T = \frac{1}{G_1 + G_2 + G_3}$$

$$= \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)}$$

$$= \frac{1}{\left(\frac{1}{220\,\Omega}\right) + \left(\frac{1}{560\,\Omega}\right) + \left(\frac{1}{1.0\,\text{k}\Omega}\right)}$$

$$= \frac{1}{4.55\text{ mS} + 1.79\text{ mS} + 1\text{ mS}} = \frac{1}{7.34\text{ mS}} = 136\,\Omega$$

Therefore, the source voltage is

$$V_S = I_T R_T = (10\text{ mA})(136\,\Omega) = \mathbf{1.36\,V}$$

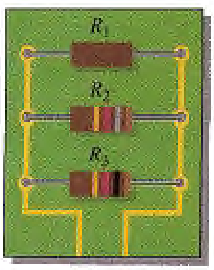
Related Problem Find the voltage if R_3 is decreased to 680 Ω in Figure 6–29 and I_T is 10 mA.

Use Multisim file E06-13 to verify the calculated results in this example and to confirm your calculation for the related problem.

EXAMPLE 6–14

The circuit board in Figure 6–30 has three resistors in parallel. The values of two of the resistors are known from the color bands, but the top resistor is not clearly marked (maybe the bands are worn off from handling). Determine the value of the unknown resistor R_1 using only an ammeter and a dc power supply.

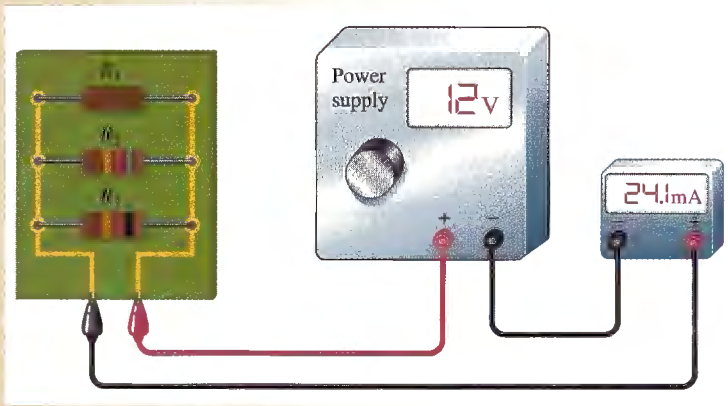
▶ FIGURE 6–30



Solution If you can determine the total resistance of the three resistors in parallel, then you can use the parallel-resistance formula to calculate the unknown resistance. You can use Ohm's law to find the total resistance if voltage and total current are known.

In Figure 6–31, a 12 V source (arbitrary value) is connected across the resistors, and the total current is measured. Using these measured values, find the total resistance.

$$R_T = \frac{V}{I_T} = \frac{12\text{ V}}{24.1\text{ mA}} = 498\ \Omega$$



▲ FIGURE 6–31

Find the unknown resistance as follows:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_1} = \frac{1}{R_T} - \frac{1}{R_2} - \frac{1}{R_3} = \frac{1}{498\ \Omega} - \frac{1}{1.8\text{ k}\Omega} - \frac{1}{1.0\text{ k}\Omega} = 453\ \mu\text{S}$$

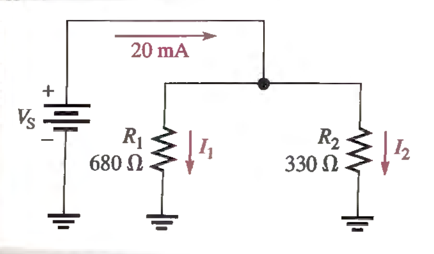
$$R_1 = \frac{1}{453\ \mu\text{S}} = \mathbf{2.21\ k\Omega}$$

Related Problem Explain how to determine the value of R_1 using an ohmmeter and without removing R_1 from the circuit.

SECTION 6–5 REVIEW

1. A 10 V battery is connected across three 680 Ω resistors that are in parallel. What is the total current from the battery?
2. How much voltage is required to produce 20 mA of current through the circuit of Figure 6–32?

► FIGURE 6–32



3. How much current is there through each resistor of Figure 6–32?
4. There are four equal-value resistors in parallel with a 12 V source, and 5.85 mA of current from the source. What is the value of each resistor?
5. A 1.0 kΩ and a 2.2 kΩ resistor are connected in parallel. There is a total of 100 mA through the parallel combination. How much voltage is dropped across the resistors?

6–6 CURRENT SOURCES IN PARALLEL

As you learned in Chapter 2, a current source is a type of energy source that provides a constant current to a load even if the resistance of that load changes. A transistor can be used as a current source; therefore, current sources are important in electronic circuits. Although the study of transistors is beyond the scope of this book, you should understand how current sources act in parallel.

After completing this section, you should be able to

- **Determine the total effect of current sources in parallel**
  - Determine the total current from parallel sources having the same direction
  - Determine the total current from parallel sources having opposite directions

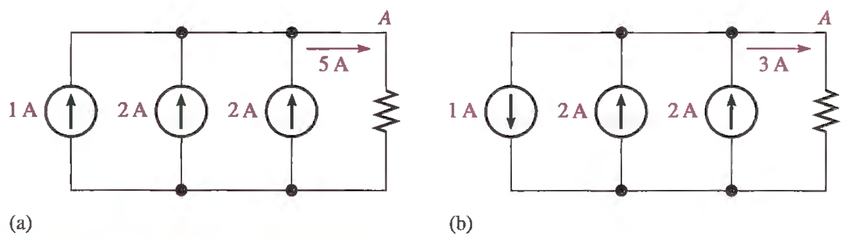
In general, the total current produced by current sources in parallel is equal to the algebraic sum of the individual current sources. The algebraic sum means that you must consider the direction of current when you combine the sources in parallel. For example, in Figure 6–33(a), the three current sources in parallel provide current in the same direction (into node *A*). So the total current into node *A* is

$$I_T = 1\text{ A} + 2\text{ A} + 2\text{ A} = 5\text{ A}$$

In Figure 6–33(b), the 1 A source provides current in a direction opposite to the other two. The total current into node *A* in this case is

$$I_T = 2\text{ A} + 2\text{ A} - 1\text{ A} = 3\text{ A}$$

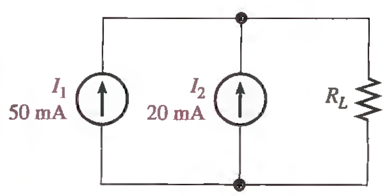
FIGURE 6–33



EXAMPLE 6–15

Determine the current through R_L in Figure 6–34.

FIGURE 6–34



Solution The two current sources are in the same direction; so the current through R_L is

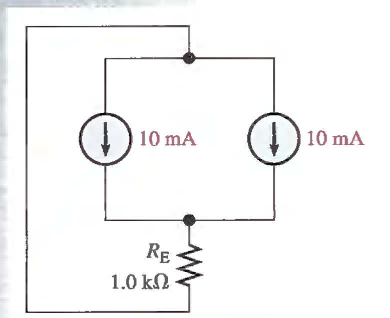
$$I_{R_L} = I_1 + I_2 = 50\,\text{mA} + 20\,\text{mA} = \mathbf{70\,mA}$$

Related Problem Determine the current through R_L if the direction of I_2 is reversed.

SECTION 6–6 REVIEW

1. Four 0.5 A current sources are connected in parallel in the same direction. What current will be produced through a load resistor?
2. How many 100 mA current sources must be connected in parallel to produce a total current output of 300 mA? Draw a schematic showing the sources connected.
3. In a certain transistor amplifier circuit, the transistor can be represented by a 10 mA current source, as shown in Figure 6–35. In a certain transistor amplifier, two transistors act in parallel. How much current is there through the resistor R_E?

FIGURE 6–35



6–7 CURRENT DIVIDERS

A parallel circuit acts as a current divider because the current entering the junction of parallel branches "divides" up into several individual branch currents.

After completing this section, you should be able to

- **Use a parallel circuit as a current divider**
  - Apply the current-divider formula
  - Determine an unknown branch current

In a parallel circuit, the total current into the junction of the parallel branches divides among the branches. Thus, a parallel circuit acts as a **current divider**. This current-divider principle is illustrated in Figure 6–36 for a two-branch parallel circuit in which part of the total current I_T goes through R_1 and part through R_2.

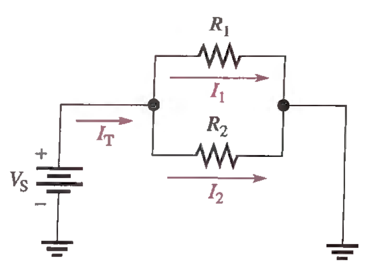


FIGURE 6–36

Total current divides between the two branches.

Since the same voltage is across each of the resistors in parallel, the branch currents are inversely proportional to the values of the resistors. For example, if the value of R_2 is twice that of R_1, then the value of I_2 is one-half that of I_1. In other words,

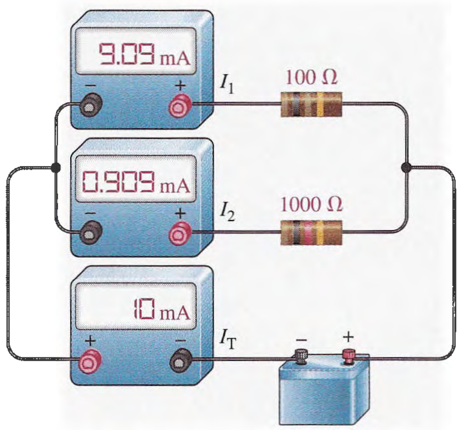
The total current divides among parallel resistors into currents with values inversely proportional to the resistance values.

The branches with higher resistance have less current, and the branches with lower resistance have more current, in accordance with Ohm's law. If all the branches have the same resistance, the branch currents are all equal.

Figure 6–37 shows specific values to demonstrate how the currents divide according to the branch resistances. Notice that in this case the resistance of the upper branch is one-tenth the resistance of the lower branch, but the upper branch current is ten times the lower branch current.

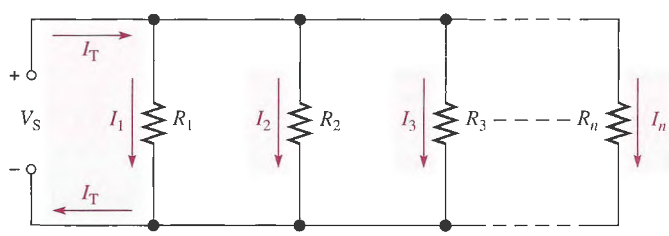
▶ FIGURE 6–37

The branch with the lower resistance has more current, and the branch with the higher resistance has less current.



Current-Divider Formula

You can develop a formula for determining how currents divide among any number of parallel resistors as shown in Figure 6–38, where n is the total number of resistors.



▲ FIGURE 6–38

A parallel circuit with n branches.

The current through any one of the parallel resistors is I_x, where x represents the number of a particular resistor (1, 2, 3, and so on). By Ohm's law, you can express the current through any one of the resistors in Figure 6–38 as follows:

$$I_x = \frac{V_S}{R_x}$$

The source voltage, V_S, appears across each of the parallel resistors, and R_x represents any one of the parallel resistors. The total source voltage, V_S, is equal to the total current times the total parallel resistance.

$$V_S = I_T R_T$$

Substituting $I_T R_T$ for V_S in the expression for I_x results in

$$I_x = \frac{I_T R_T}{R_x}$$

Rearranging terms yields

$$I_x = \left(\frac{R_T}{R_x}\right) I_T$$

Equation 6–6

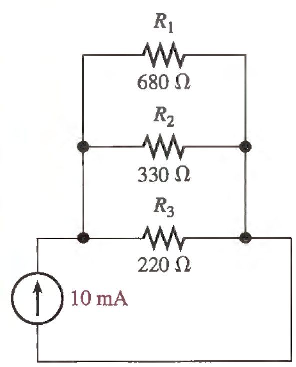
where $x = 1, 2, 3,$ etc.

Equation 6–6 is the general current-divider formula and applies to a parallel circuit with any number of branches.

The current (I_x) through any branch equals the total parallel resistance (R_T) divided by the resistance (R_x) of that branch, and then multiplied by the total current (I_T) into the junction of parallel branches.

EXAMPLE 6–16 Determine the current through each resistor in the circuit of Figure 6–39.

▶ FIGURE 6–39



Solution First calculate the total parallel resistance.

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right)} = \frac{1}{\left(\frac{1}{680\ \Omega}\right) + \left(\frac{1}{330\ \Omega}\right) + \left(\frac{1}{220\ \Omega}\right)} = 111\ \Omega$$

The total current is 10 mA. Use Equation 6–6 to calculate each branch current.

$$I_1 = \left(\frac{R_T}{R_1}\right) I_T = \left(\frac{111\ \Omega}{680\ \Omega}\right) 10\ \text{mA} = \mathbf{1.63\ mA}$$

$$I_2 = \left(\frac{R_T}{R_2}\right) I_T = \left(\frac{111\ \Omega}{330\ \Omega}\right) 10\ \text{mA} = \mathbf{3.36\ mA}$$

$$I_3 = \left(\frac{R_T}{R_3}\right) I_T = \left(\frac{111\ \Omega}{220\ \Omega}\right) 10\ \text{mA} = \mathbf{5.05\ mA}$$

Related Problem Determine the current through each resistor in Figure 6–39 if R_3 is removed.

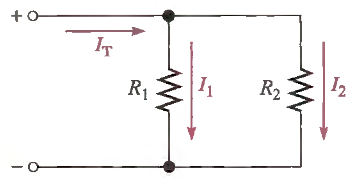


FIGURE 6–40

Current-Divider Formulas for Two Branches Two parallel resistors are common in practical circuits, as shown in Figure 6–40. As you know from Equation 6–3,

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

Using the general current-divider formula in Equation 6–6, the formulas for I_1 and I_2 can be written as follows:

$$I_1 = \left(\frac{R_T}{R_1}\right)I_T \quad \text{and} \quad I_2 = \left(\frac{R_T}{R_2}\right)I_T$$

Substituting $R_1R_2/(R_1 + R_2)$ for R_T and canceling terms result in

$$I_1 = \frac{\left(\dfrac{\cancel{R_1}R_2}{R_1 + R_2}\right)}{\cancel{R_1}}I_T \quad \text{and} \quad I_2 = \frac{\left(\dfrac{R_1\cancel{R_2}}{R_1 + R_2}\right)}{\cancel{R_2}}I_T$$

Therefore, the current-divider formulas for the special case of two branches are

Equation 6–7

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T$$

Equation 6–8

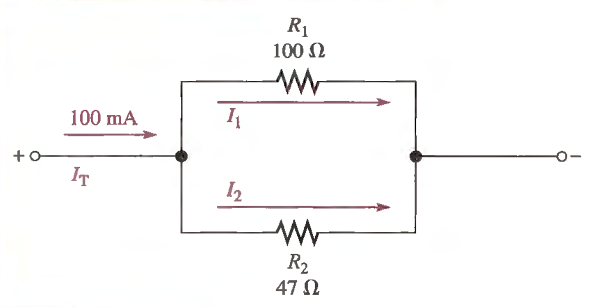
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T$$

Note that in Equations 6–7 and 6–8, the current in one of the branches is equal to the opposite branch resistance divided by the sum of the two resistors, all multiplied by the total current. In all applications of the current-divider equations, you must know the total current into the parallel branches.

EXAMPLE 6–17

Find I_1 and I_2 in Figure 6–41.

FIGURE 6–41



Solution Use Equation 6–7 to determine I_1.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right)100\text{ mA} = \mathbf{32.0\ mA}$$

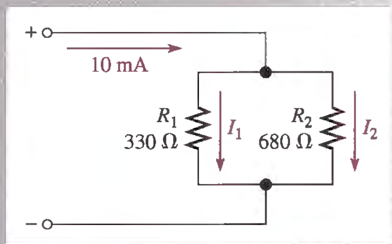
Use Equation 6–8 to determine I_2.

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T = \left(\frac{100\ \Omega}{147\ \Omega}\right)100\text{ mA} = \mathbf{68.0\ mA}$$

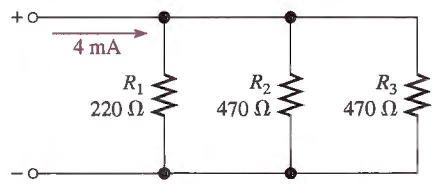
Related Problem If $R_1 = 56\ \Omega$, and $R_2 = 82\ \Omega$ in Figure 6–41 and I_T stays the same, what will each branch current be?

SECTION 6–7 REVIEW

1. Write the general current-divider formula.
2. Write the two special formulas for calculating each branch current for a two-branch circuit.
3. A circuit has the following resistors in parallel with a voltage source: 220 kΩ, 100 kΩ, 82 kΩ, 47 kΩ, and 22 kΩ. Which resistor has the most current through it? The least current?
4. Find I_1 and I_2 in the circuit of Figure 6–42.
5. Determine the current through R_3 in Figure 6–43.



▲ FIGURE 6–42



▲ FIGURE 6–43

6–8 POWER IN PARALLEL CIRCUITS

Total power in a parallel circuit is found by adding up the powers of all the individual resistors, the same as for series circuits.

After completing this section, you should be able to

- **Determine power in a parallel circuit**

Equation 6–9 states the formula for finding total power in a concise way for any number of resistors in parallel.

$$P_T = P_1 + P_2 + P_3 + \cdots + P_n$$

Equation 6–9

where P_T is the total power and P_n is the power in the last resistor in parallel. As you can see, the powers are additive, just as in a series circuit.

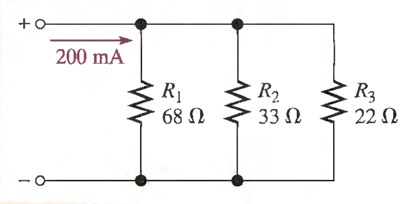
The power formulas in Chapter 4 are directly applicable to parallel circuits. The following formulas are used to calculate the total power P_T:

$$P_T = VI_T$$
$$P_T = I_T^2 R_T$$
$$P_T = \frac{V^2}{R_T}$$

where V is the voltage across the parallel circuit, I_T is the total current into the parallel circuit, and R_T is the total resistance of the parallel circuit. Examples 6–18 and 6–19 show how total power can be calculated in a parallel circuit.

EXAMPLE 6–18

Determine the total amount of power in the parallel circuit in Figure 6–44.



▲ FIGURE 6–44

Solution The total current is 200 mA. The total resistance is

$$R_T = \frac{1}{\left(\frac{1}{68\ \Omega}\right) + \left(\frac{1}{33\ \Omega}\right) + \left(\frac{1}{22\ \Omega}\right)} = 11.1\ \Omega$$

The easiest power formula to use is $P_T = I_T^2 R_T$ because you know both I_T and R_T.

$$P_T = I_T^2 R_T = (200\text{ mA})^2(11.1\ \Omega) = \mathbf{444\ mW}$$

Let's demonstrate that if you determine the power in each resistor and if you add all of these values together, you will get the same result. First, find the voltage across each branch of the circuit.

$$V = I_T R_T = (200\text{ mA})(11.1\ \Omega) = 2.22\text{ V}$$

Remember that the voltage across all branches is the same.

Next, use $P = V^2/R$ to calculate the power for each resistor.

$$P_1 = \frac{(2.22\text{ V})^2}{68\ \Omega} = 72.5\text{ mW}$$

$$P_2 = \frac{(2.22\text{ V})^2}{33\ \Omega} = 149\text{ mW}$$

$$P_3 = \frac{(2.22\text{ V})^2}{22\ \Omega} = 224\text{ mW}$$

Add these powers to get the total power.

$$P_T = 72.5\text{ mW} + 149\text{ mW} + 224\text{ mW} = 446\text{ mW}$$

This calculation shows that the sum of the individual powers is equal (approximately) to the total power as determined by one of the power formulas. Rounding to three significant figures accounts for the difference.

Related Problem Find the total power in Figure 6–44 if the total current is doubled.

EXAMPLE 6–19

The amplifier in one channel of a stereo system as shown in Figure 6–45 drives two speakers. If the maximum voltage* to the speakers is 15 V, how much power must the amplifier be able to deliver to the speakers?

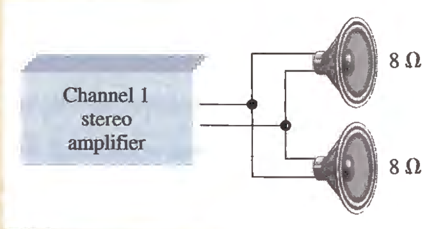


FIGURE 6–45

Solution The speakers are connected in parallel to the amplifier output, so the voltage across each is the same. The maximum power to each speaker is

$$P_{max} = \frac{V_{max}^2}{R} = \frac{(15\text{ V})^2}{8\ \Omega} = 28.1\text{ W}$$

The total power that the amplifier must be capable of delivering to the speaker system is twice the power in an individual speaker because the total power is the sum of the individual powers.

$$P_{T(max)} = P_{(max)} + P_{(max)} = 2P_{(max)} = 2(28.1\text{ W}) = \mathbf{56.2\text{ W}}$$

Related Problem If the amplifier can produce a maximum of 18 V, what is the maximum total power to the speakers?

*Voltage is ac in this case; but power is determined the same for ac voltage as for dc voltage, as you will see later.

SECTION 6–8 REVIEW

1. If you know the power in each resistor in a parallel circuit, how can you find the total power?
2. The resistors in a parallel circuit dissipate the following powers: 238 mW, 512 mW, 109 mW, and 876 mW. What is the total power in the circuit?
3. A circuit has a 1.0 kΩ, a 2.7 kΩ, and a 3.9 kΩ resistor in parallel. There is a total current of 1 A into the parallel circuit. What is the total power?

6–9 PARALLEL CIRCUIT APPLICATIONS

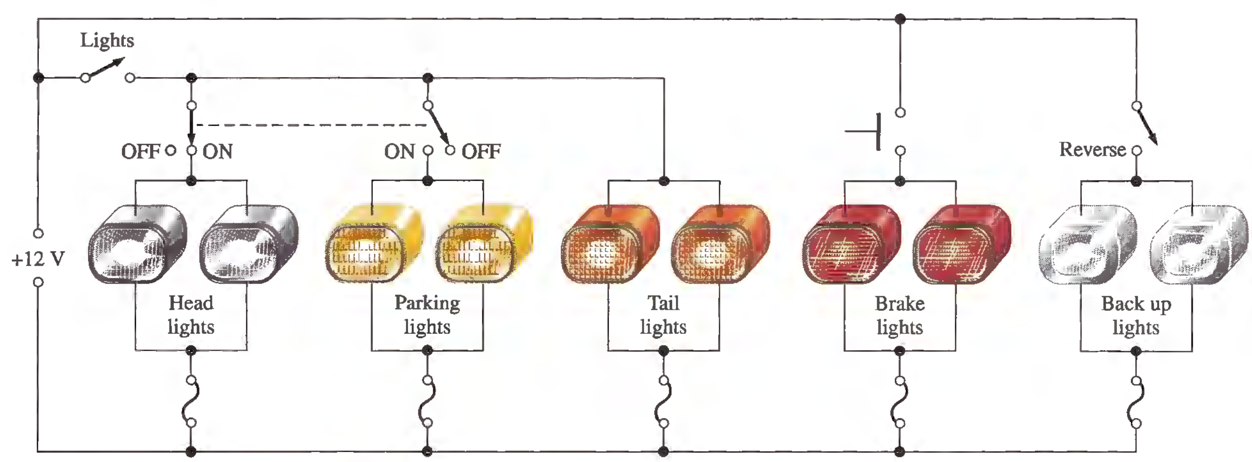
Parallel circuits are found in some form in virtually every electronic system. In many of these applications, the parallel relationship of components may not be obvious until you have covered some advanced topics that you will study later. For now, let's look at some examples of common and familiar applications of parallel circuits.

After completing this section, you should be able to

- **Describe some basic applications of parallel circuits**
  - Discuss the lighting system in automobiles
  - Discuss residential wiring
  - Explain basically how a multiple-range ammeter works

Automotive

One advantage of a parallel circuit over a series circuit is that when one branch opens, the other branches are not affected. For example, Figure 6–46 shows a simplified diagram of an automobile lighting system. When one headlight on a car goes out, it does not cause the other lights to go out because they are all in parallel.



▲ FIGURE 6–46

Simplified diagram of the exterior light system of an automobile.

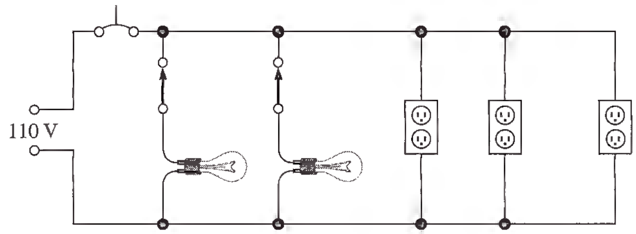
Notice that the brake lights are switched on independently of the headlights and taillights. They come on only when the driver closes the brake light switch by depressing the brake pedal. When the lights switch is closed, both headlights and both taillights are on. When the headlights are on, the parking lights are off and vice versa. If any one of the lights burns out (opens), there is still current in each of the other lights. The back-up lights are switched on when the reverse gear is engaged.

Residential

Another common use of parallel circuits is in residential electrical systems. All the lights and appliances in a home are wired in parallel. Figure 6–47 shows a typical room wiring arrangement with two switch-controlled lights and three wall outlets in parallel.

Analog Ammeters

Parallel circuits are used in the analog (needle-type) ammeter or milliammeter. Although analog meters are not as common as they once were, they are still used as panel meters in certain applications and analog multimeters are still available. Parallel circuits are an

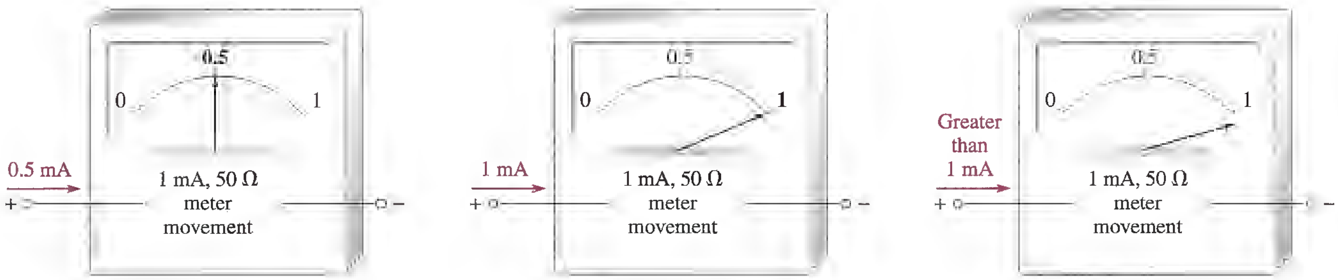


▲ FIGURE 6–47

Example of parallel circuits in residential wiring.

important part of analog ammeter operation because they allow the selection of various ranges in order to measure many different current values.

The mechanism in an ammeter that causes the pointer to move in proportion to the current is called the *meter movement,* which is based on a magnetic principle that you will learn later. Right now, it is sufficient to know that a given meter movement has a certain resistance and a maximum current. This maximum current, called the *full-scale deflection current,* causes the pointer to go all the way to the end of the scale. For example, a certain meter movement has a 50 Ω resistance and a full-scale deflection current of 1 mA. A meter with this particular movement can measure currents of 1 mA or less as indicated in Figure 6–48(a) and (b). Currents greater than 1 mA will cause the pointer to "peg" (or stop) slightly past the full scale mark as indicated in part (c), which can damage the meter.

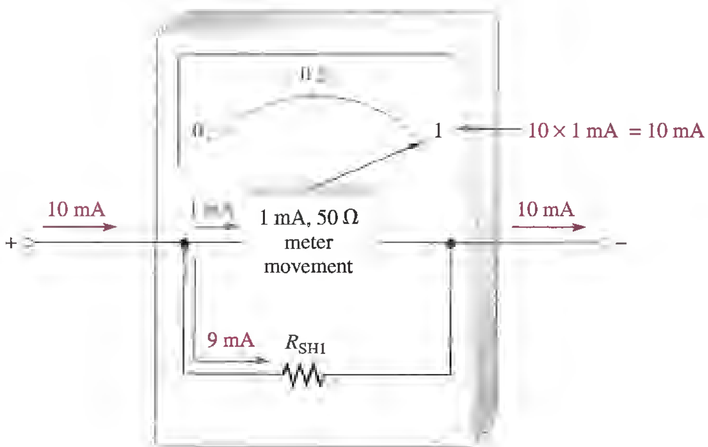


(a) Half-scale deflection (b) Full-scale deflection (c) "Pegged"

▲ FIGURE 6–48

A 1 mA analog ammeter.

Figure 6–49 shows a simple ammeter with a resistor in parallel with the 1 mA meter movement; this resistor is called a *shunt resistor.* Its purpose is to bypass a portion of current around the meter movement to extend the range of current that can be measured. The



◀ FIGURE 6–49

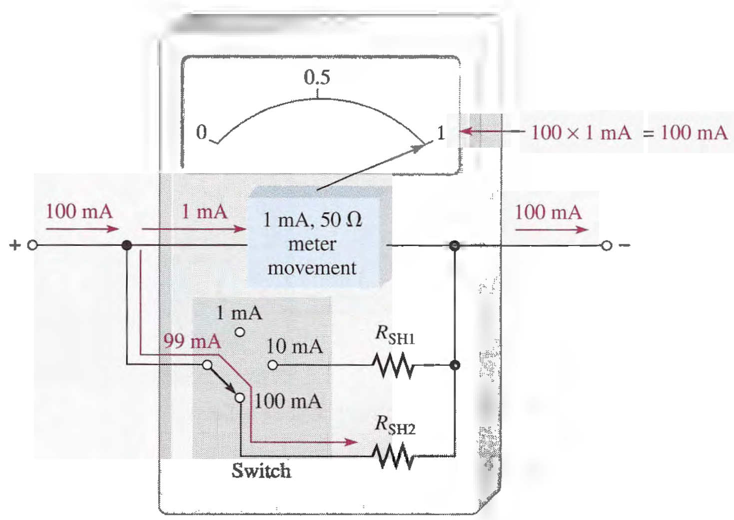
A 10 mA analog ammeter.

figure specifically shows 9 mA through the shunt resistor and 1 mA through the meter movement. Thus, up to 10 mA can be measured. To find the actual current value, simply multiply the reading on the scale by 10.

A multiple-range ammeter has a range switch that permits the selection of several full-scale current settings. In each switch position, a certain amount of current is bypassed through a parallel resistor as determined by the resistance value. In our example, the current though the movement is never greater than 1 mA.

Figure 6–50 illustrates a meter with three ranges: 1 mA, 10 mA, and 100 mA. When the range switch is in the 1 mA position, all of the current into the meter goes through the meter movement. In the 10 mA setting, up to 9 mA goes through R_{SH1} and up to 1 mA through the movement. In the 100 mA setting, up to 99 mA goes through R_{SH2}, and the movement can still have only 1 mA for full-scale.

The scale reading is interpreted based on the range setting. For example, in Figure 6–50, if 50 mA of current are being measured, the needle points at the 0.5 mark on the scale; you must multiply 0.5 by 100 to find the current value. In this situation, 0.5 mA is through the movement (half-scale deflection) and 49.5 mA are through R_{SH2}.



▲ FIGURE 6–50

An analog ammeter with three ranges.

Effect of the Ammeter on a Circuit As you know, an ammeter is connected in series to measure the current in a circuit. Ideally, the meter should not alter the current that it is intended to measure. In practice, however, the meter unavoidably has some effect on the circuit because its internal resistance is connected in series with the circuit resistance. However, in most cases, the meter's internal resistance is so small compared to the circuit resistance that it can be neglected.

For example, if a meter has a 50 Ω movement (R_M) and a 0.1 mA full-scale current (I_M), the maximum voltage dropped across the movement is

$$V_M = I_M R_M = (0.1\text{ mA})(50\ \Omega) = 5\text{ mV}$$

The shunt resistance (R_{SH}) for the 10 mA range, for example, is

$$R_{SH} = \frac{V_M}{I_{SH}} = \frac{5\text{ mV}}{9.9\text{ mA}} = 0.505\ \Omega$$

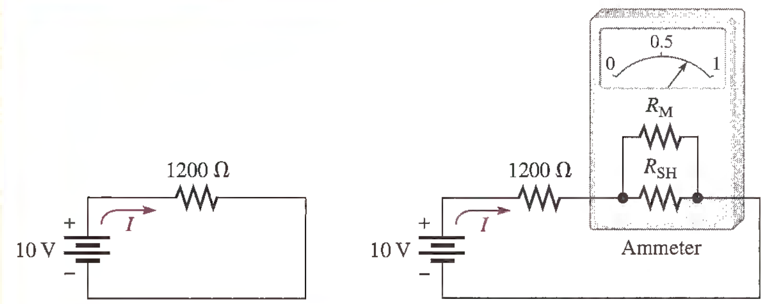
As you can see, the total resistance of the ammeter on the 10 mA range is the resistance of the meter movement in parallel with the shunt resistance.

$$R_{M(tot)} = R_M \| R_{SH} = 50\ \Omega \| 0.505\ \Omega = 0.5\ \Omega$$

EXAMPLE 6–20

How much does a 10 mA ammeter with a 0.1 mA, 50 Ω movement affect the current in the circuit of Figure 6–51?

FIGURE 6–51



(a) Circuit (b) Circuit with ammeter connected

Solution The original current in the circuit (with no meter) is

$$I_{orig} = \frac{10\text{ V}}{1200\ \Omega} = 8.3333\text{ mA}$$

The meter is set on the 10 mA range in order to measure this particular amount of current. The meter's resistance on the 10 mA range is 0.5 Ω. When the meter is connected in the circuit, its resistance is in series with the 1200 Ω resistor. Thus, there is a total of 1200.5 Ω.

The current in the circuit is reduced slightly by inserting the meter.

$$I_{meas} = \frac{10\text{ V}}{1200.5\ \Omega} = 8.3299\text{ mA}$$

The current with the presence of the meter differs from the original circuit current by only **3.4 μA** or **0.04%**.

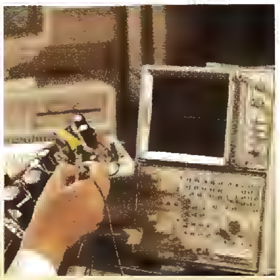
Therefore, the meter does not significantly alter the current value, a situation which, of course, is necessary because the measuring instrument should not change the quantity that is to be measured accurately.

Related Problem How much will the measured current differ from the original current if the circuit resistance in Figure 6–51 is 12 kΩ rather than 1200 Ω?

SECTION 6–9 REVIEW

1. For the ammeter in Figure 6–51, what is the maximum resistance that the meter will have when connected in a circuit? What is the maximum current that can be measured at the setting?
2. Do the shunt resistors have resistance values considerably less than or more than that of the meter movement? Why?

6–10 TROUBLESHOOTING



Recall that an open circuit is one in which the current path is interrupted and there is no current. In this section we examine what happens when a branch of a parallel circuit opens.

After completing this section, you should be able to

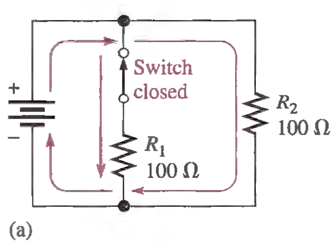
- **Troubleshoot parallel circuits**
  - Check for an open in a circuit

Open Branches

If a switch is connected in a branch of a parallel circuit, as shown in Figure 6–52, an open or a closed path can be made by the switch. When the switch is closed, as in Figure 6–52(a), R_1 and R_2 are in parallel. The total resistance is 50 Ω (two 100 Ω resistors in parallel). Current is through both resistors. If the switch is opened, as in Figure 6–52(b), R_1 is effectively removed from the circuit, and the total resistance is 100 Ω. Current is now only through R_2. In general,

> **When an open occurs in a parallel branch, the total resistance increases, the total current decreases, and the same current continues through each of the remaining parallel paths.**

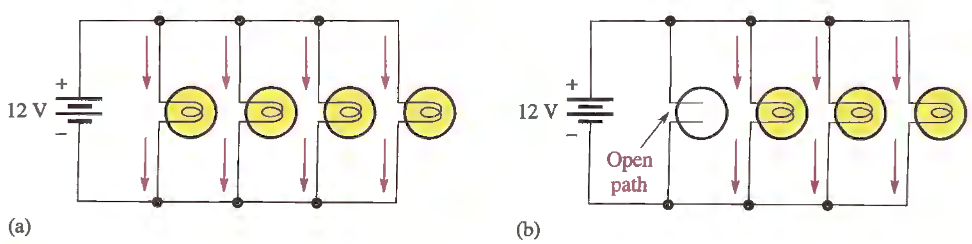
The decrease in total current equals the amount of current that was previously in the open branch. The other branch currents remain the same.



▶ FIGURE 6–52

When switch opens, total current decreases and current through R_2 remains unchanged.

Consider the lamp circuit in Figure 6–53. There are four bulbs in parallel with a 12 V source. In part (a), there is current through each bulb. Now suppose that one of the bulbs burns out, creating an open path as shown in Figure 6–53(b). This light will go out because there is no current through the open path. Notice, however, that current continues through all the other parallel bulbs, and they continue to glow. The open branch does not change the



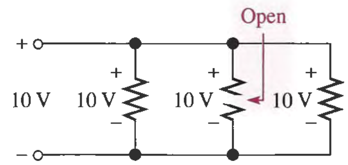
▲ FIGURE 6–53

When one lamp opens, total current decreases and other branch currents remain unchanged.

voltage across the parallel branches; it remains at 12 V, and the current through each branch remains the same.

You can see that a parallel circuit has an advantage over a series circuit in lighting systems because if one or more of the parallel bulbs burn out, the others will stay on. In a series circuit, when one bulb goes out, all of the others go out also because the current path is completely interrupted.

When a resistor in a parallel circuit opens, the open resistor cannot be located by measurement of the voltage across the branches because the same voltage exists across all the branches. Thus, there is no way to tell which resistor is open by simply measuring voltage. The good resistors will always have the same voltage as the open one, as illustrated in Figure 6–54 (note that the middle resistor is open).



▲ FIGURE 6–54

Parallel branches (open or not) have the same voltage.

If a visual inspection does not reveal the open resistor, it must be located by current measurements. In practice, measuring current is more difficult than measuring voltage because you must insert the ammeter in series to measure the current. Thus, a wire or a PC board connection must be cut or disconnected, or one end of a component must be lifted off the circuit board, in order to connect the ammeter in series. This procedure, of course, is not required when voltage measurements are made because the meter leads are simply connected across a component.

Finding an Open Branch by Current Measurement

In a parallel circuit with a suspected open branch, the total current can be measured to find the open. *When a parallel resistor opens, the total current, I_T, is always less than its normal value.* Once you know I_T and the voltage across the branches, a few calculations will determine the open resistor when all the resistors are of different resistance values.

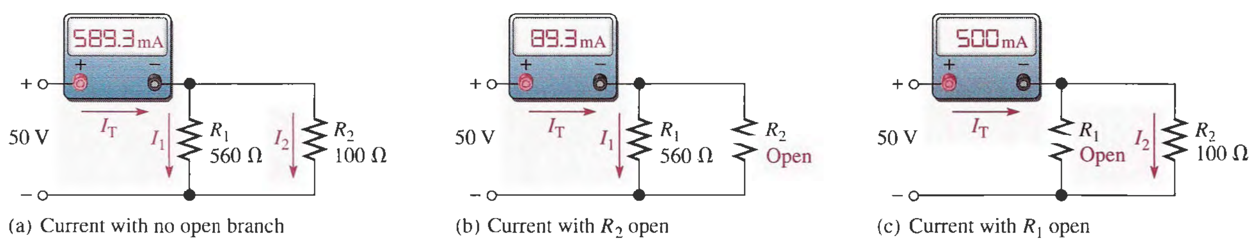
Consider the two-branch circuit in Figure 6–55(a). If one of the resistors opens, the total current will equal the current in the good resistor. Ohm's law quickly tells you what the current in each resistor should be.

$$I_1 = \frac{50\text{ V}}{560\ \Omega} = 89.3\text{ mA}$$

$$I_2 = \frac{50\text{ V}}{100\ \Omega} = 500\text{ mA}$$

$$I_T = I_1 + I_2 = 589.3\text{ mA}$$

If R_2 is open, the total current is 89.3 mA, as indicated in Figure 6–55(b). If R_1 is open, the total current is 500 mA, as indicated in Figure 6–55(c).



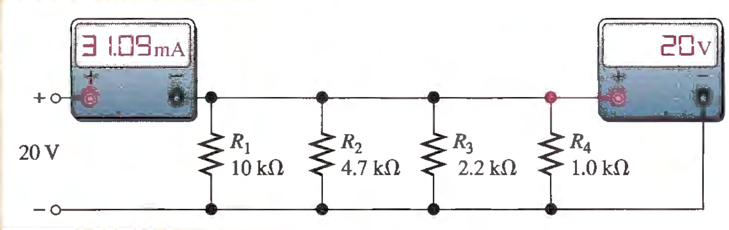
▲ FIGURE 6–55

Finding an open path by current measurement.

This procedure can be extended to any number of branches having unequal resistances. If the parallel resistances are all equal, the current in each branch must be checked until a branch is found with no current. This is the open resistor.

EXAMPLE 6–21

In Figure 6–56, there is a total current of 31.09 mA, and the voltage across the parallel branches is 20 V. Is there an open resistor, and, if so, which one is it?



▲ FIGURE 6–56

Solution Calculate the current in each branch.

$$I_1 = \frac{V}{R_1} = \frac{20\text{ V}}{10\text{ k}\Omega} = 2\text{ mA}$$

$$I_2 = \frac{V}{R_2} = \frac{20\text{ V}}{4.7\text{ k}\Omega} = 4.26\text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{20\text{ V}}{2.2\text{ k}\Omega} = 9.09\text{ mA}$$

$$I_4 = \frac{V}{R_4} = \frac{20\text{ V}}{1.0\text{ k}\Omega} = 20\text{ mA}$$

The total current should be

$$I_T = I_1 + I_2 + I_3 + I_4 = 2\text{ mA} + 4.26\text{ mA} + 9.09\text{ mA} + 20\text{ mA} = 35.35\text{ mA}$$

The actual measured current is 31.09 mA, as stated, which is 4.26 mA less than normal, indicating that the branch carrying 4.26 mA is open. Thus, **R_2 must be open.**

Related Problem What is the total current measured in Figure 6–56 if R_4 and not R_2 is open?

Finding an Open Branch by Resistance Measurement

If the parallel circuit to be checked can be disconnected from its voltage source and from any other circuit to which it may be connected, a measurement of the total resistance can be used to locate an open branch.

Recall that conductance, G, is the reciprocal of resistance ($1/R$) and its unit is the siemens (S). The total conductance of a parallel circuit is the sum of the conductances of all the resistors.

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

To locate an open branch, do the following steps:

1. Calculate what the total conductance should be using the individual resistor values.

$$G_{T(calc)} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

2. Measure the total resistance with an ohmmeter and calculate the total measured conductance.

$$G_{T(meas)} = \frac{1}{R_{T(meas)}}$$

3. Subtract the measured total conductance (Step 2) from the calculated total conductance (Step 1). The result is the conductance of the open branch and the resistance is obtained by taking its reciprocal ($R = 1/G$).

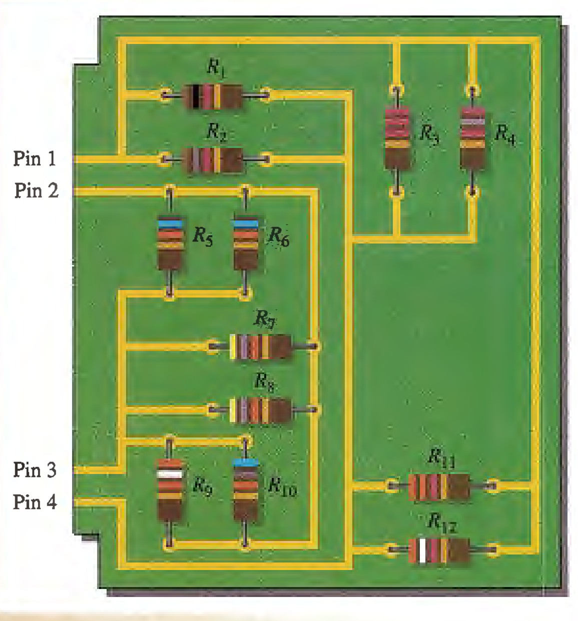
$$R_{open} = \frac{1}{G_{T(calc)} - G_{T(meas)}}$$

Equation 6–10

EXAMPLE 6–22

Check the PC board in Figure 6–57 for open branches.

FIGURE 6–57



Solution There are two separate parallel circuits on the board. The circuit between pin 1 and pin 4 is checked as follows (we will assume one of the resistors is open):

1. Calculate what the total conductance should be using the individual resistor values.

$$G_{T(calc)} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{11}} + \frac{1}{R_{12}}$$

$$= \frac{1}{1.0\,\text{k}\Omega} + \frac{1}{1.8\,\text{k}\Omega} + \frac{1}{2.2\,\text{k}\Omega} + \frac{1}{2.7\,\text{k}\Omega} + \frac{1}{3.3\,\text{k}\Omega} + \frac{1}{3.9\,\text{k}\Omega} = 2.94\text{ mS}$$

2. Measure the total resistance with an ohmmeter and calculate the total measured conductance. Assume that your ohmmeter measures 402 Ω.

$$G_{T(meas)} = \frac{1}{402\ \Omega} = 2.49\text{ mS}$$

3. Subtract the measured total conductance (Step 2) from the calculated total conductance (Step 1). The result is the conductance of the open branch and the resistance is obtained by taking its reciprocal.

$$G_{open} = G_{T(calc)} - G_{T(meas)} = 2.94\text{ mS} - 2.49\text{ mS} = 0.45\text{ mS}$$

$$R_{open} = \frac{1}{G_{open}} = \frac{1}{0.45\text{ mS}} = 2.2\,\text{k}\Omega$$

Resistor R_3 is open and must be replaced.

Related Problem Your ohmmeter indicates 9.6 kΩ between pin 2 and pin 3 on the PC board in Figure 6–57. Determine if this is correct and, if not, which resistor is open.

Shorted Branches

When a branch in a parallel circuit shorts, the current increases to an excessive value, causing a fuse or circuit breaker to blow. This results in a difficult troubleshooting problem because it is hard to isolate the shorted branch.

A pulser and a current tracer are tools often used to find shorts in a circuit. They are not restricted to use in digital circuits but can be effective in any type of circuit. The pulser is a pen-shaped tool that applies pulses to a selected point in a circuit, causing pulses of current to flow through the shorted path. The current tracer is also a pen-shaped tool that senses pulses of current. By following the current with the tracer, the current path can be identified.

SECTION 6–10 REVIEW

1. If a parallel branch opens, what changes can be detected in the circuit's voltage and the currents, assuming that the parallel circuit is across a constant-voltage source?
2. What happens to the total resistance if one branch opens?
3. If several light bulbs are connected in parallel and one of the bulbs opens (burns out), will the others continue to glow?
4. There is 100 mA of current in each branch of a parallel circuit. If one branch opens, what is the current in each of the remaining branches?
5. A three-branch circuit normally has the following branch currents: 100 mA, 250 mA, and 120 mA. If the total current measures 350 mA, which branch is open?

A Circuit Application

In this application, a dc power supply is modified by adding a 3-range ammeter to indicate current to the load. As you have learned, parallel resistances can be used to extend the range of an ammeter. These parallel resistors, called *shunts,* bypass current around the meter movement, allowing for the meter to effectively measure higher currents than the maximum current for which the meter movement is designed.

The Power Supply

A rack-mounted power supply is shown in Figure 6–58. The voltmeter indicates the output voltage, which can be adjusted from 0 V

FIGURE 6–58

Front panel view of a rack-mounted power supply.

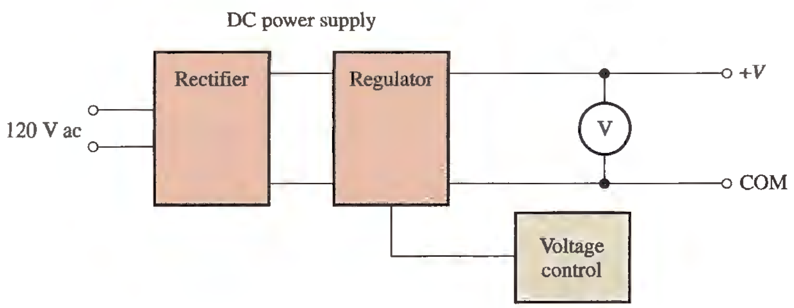


FIGURE 6–59

Basic block diagram of the dc power supply.

to 10 V using the voltage control. The power supply is capable of providing up to 2 A to a load. A basic block diagram of the power supply is shown in Figure 6–59. It consists of a rectifier circuit that converts ac voltage from the wall outlet to dc voltage and a regulator circuit that keeps the output voltage at a constant value.

It is required that the power supply be modified by adding an ammeter with three switch-selected current ranges of 25 mA, 250 mA, and 2.5 A. To accomplish this, two shunt resistances are used that can each be switched into a parallel connection with the meter movement. This approach works fine as long as the required values of the shunt resistors are not too small. However, there are problems at very low values of shunt resistance and you will see why next.

The Shunt Circuit

An ammeter is selected that has a full-scale deflection of 25 mA and a resistance of 6 Ω*. Two shunt resistors must be added—one for 250 mA and one for 2.5 A full-scale deflections. The internal meter movement provides the 25 mA range. This is shown in Figure 6–60. The range selection is provided by a 1-pole, 3-position rotary switch with a contact resistance of 50 mΩ. Contact resistance of switches can be from less than 20 mΩ to about 100 mΩ. The contact resistance of a given switch can vary with temperature, current, and usage and, therefore, cannot be relied upon to remain within a reasonable tolerance of the specified value. Also, the switch is a make-before-break type, which means that contact with the previous position is not broken until contact with the new position is made.

The shunt resistance value for the 2.5 A range is determined as follows where the voltage across the meter movement is

$$V_M = I_M R_M = (25\text{ mA})(6\ \Omega) = 150\text{ mV}$$

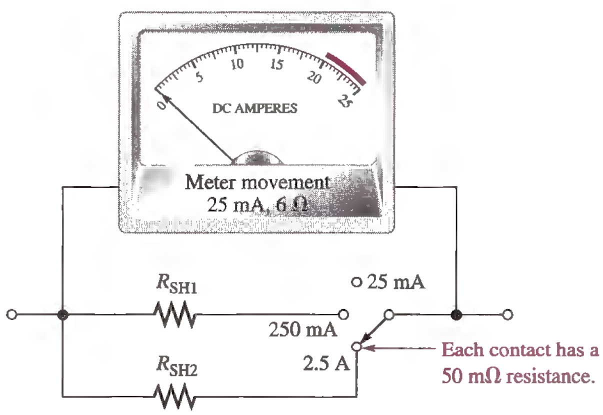


FIGURE 6–60

Ammeter modified to provide three current ranges.

*See the Simpson model 1227 milliammeter at www.simpsonelectric.com.

The current through the shunt resistor for full-scale deflection is

$$I_{SH2} = I_{FULL\ SCALE} - I_M = 2.5\text{ A} - 25\text{ mA} = 2.475\text{ A}$$

The total shunt resistance is

$$R_{SH2(tot)} = \frac{V_M}{I_{SH2}} = \frac{150\text{ mV}}{2.475\text{ A}} = 60.6\text{ m}\Omega$$

Low-ohm precision resistors are generally available in values from 1 mΩ to 10 Ω or greater from various manufacturers.

Notice in Figure 6–60 that the contact resistance, R_{CONT}, of the switch appears in series with R_{SH2}. Since the total shunt resistance must be 60.6 mΩ, the value of the shunt resistor R_{SH2} is

$$R_{SH2} = R_{SH2(tot)} - R_{CONT} = 60.6\text{ m}\Omega - 50\text{ m}\Omega = 10.6\text{ m}\Omega$$

Although this value, or one close to it, may be available, the problem in this case is that the switch contact resistance is almost twice that of R_{SH2} and any variation in it would create a significant inaccuracy in the meter. As you can see, this approach is not acceptable for these particular requirements.

Another Approach

A variation of the standard shunt resistance circuit is shown in Figure 6–61. The shunt resistor, R_{SH}, is connected in parallel for the two higher current range settings and disconnected for the 25 mA setting using a 2-pole, 3-position switch. This circuit avoids dependency on the switch contact resistance by using resistor values that are large enough to make it insignificant. The disadvantages of this meter circuit are that it requires a more complex switch and the voltage drops from input to output are greater than in the previous shunt circuit.

For the 250 mA range, the current through the meter movement for full-scale deflection is 25 mA. The voltage across the meter movement is 150 mV.

$$I_{SH} = 250\text{ mA} - 25\text{ mA} = 225\text{ mA}$$

$$R_{SH} = \frac{150\text{ mV}}{225\text{ mA}} = 0.67\ \Omega = 670\text{ m}\Omega$$

This value of R_{SH} is more than thirty times the expected switch contact resistance of 20 mΩ, thus minimizing the effect of the contact resistance.

For the 2.5 A range, the current through the meter movement for full-scale deflection is still 25 mA. This is also the current through R_1.

$$I_{SH} = 2.5\text{ A} - 25\text{ mA} = 2.475\text{ A}$$

The voltage across the meter circuit from A to B is

$$V_{AB} = I_{SH}R_{SH} = (2.475\text{ A})(670\text{ m}\Omega) = 1.66\text{ V}$$

Applying Kirchhoff's voltage law and Ohm's law to find R_1,

$$V_{R1} + V_M = V_{AB}$$

$$V_{R1} = V_{AB} - V_M = 1.66\text{ V} - 150\text{ mV} = 1.51\text{ V}$$

$$R_1 = \frac{V_{R1}}{I_M} = \frac{1.51\text{ V}}{25\text{ mA}} = 60.4\ \Omega$$

This value is much greater than the contact resistance of the switch.

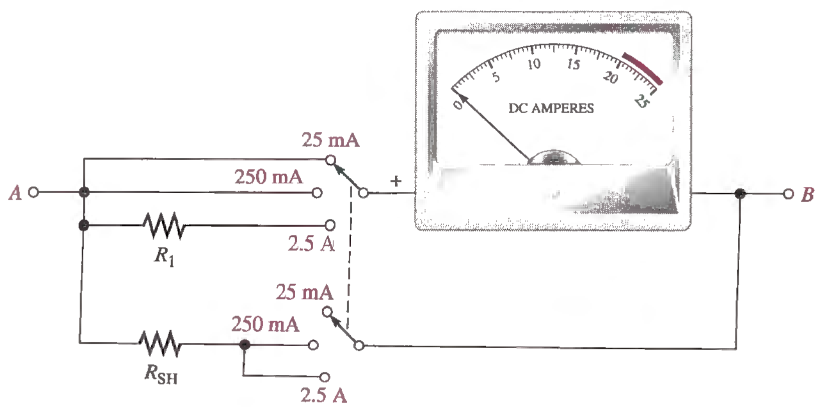
- Determine the maximum power dissipated by R_{SH} in Figure 6–61 for each range setting.
- How much voltage is there from A to B in Figure 6–61 when the switch is set to the 2.5 A range and the current is 1 A?
- The meter indicates 250 mA. How much does the voltage across the meter circuit from A to B change when the switch is moved from the 250 mA position to the 2.5 A position?
- Assume the meter movement has a resistance of 4 Ω instead of 6 Ω. Specify any changes necessary in the circuit of Figure 6–61.

Implementing the Power Supply Modification

Once the proper values are obtained, the resistors are placed on a board which is then mounted in the power supply. The resistors and the range switch are connected to the power supply as shown in Figure 6–62. The ammeter circuit is connected between the rectifier circuit in the power supply and the regulator circuit in order to reduce the impact of the voltage drop across the meter circuit on the output voltage. The regulator maintains, within certain limits, a constant dc output voltage even though its input voltage coming through the meter circuit may change.

FIGURE 6–61

Meter circuit redesigned to eliminate or minimize the effect of switch contact resistance. The switch is a 2-pole, 3-position make-before-break rotary type.



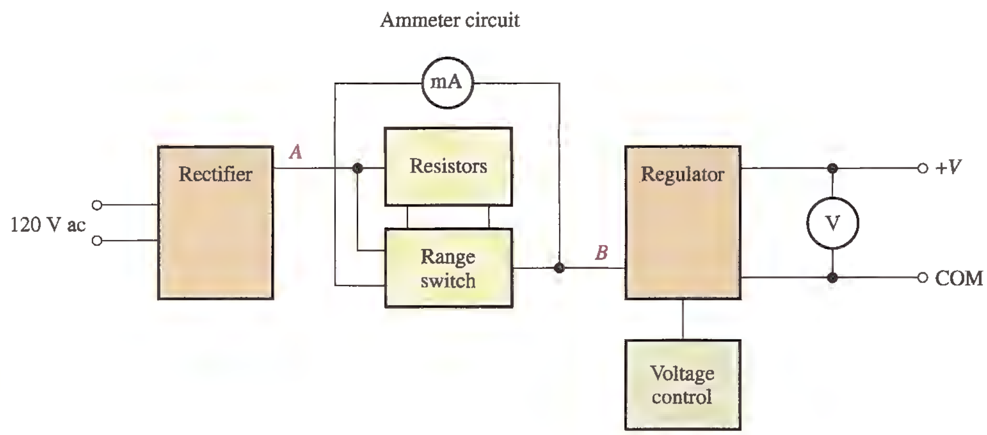


FIGURE 6–62

Block diagram of dc power supply with 3-range milliammeter.

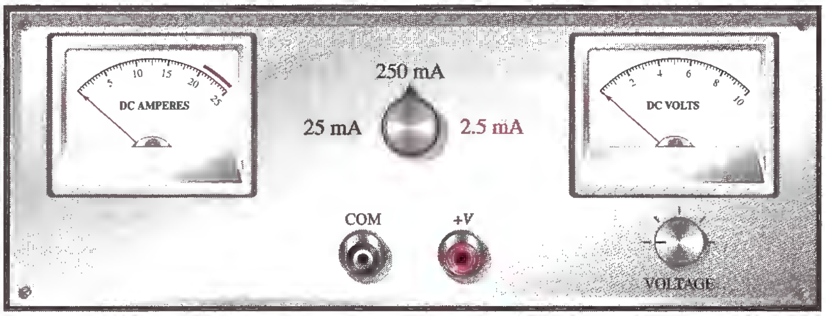


FIGURE 6–63

The power supply with the addition of the milliammeter and the current range selection switch.

Figure 6–63 shows the modified power supply front panel with the rotary range switch and milliammeter installed. The red portion of the scale indicates excess current for the 2.5 A range since the power supply has a maximum current of 2 A for safe operation.

Review

1. When the meter is set to the 250 mA range, which resistance has the most current through it?
2. Determine the total resistance from A to B of the meter circuit in Figure 6–61 for each of the three current ranges.
3. Explain why the circuit in Figure 6–61 was used instead of the one in Figure 6–60.
4. If the pointer is at the 15 and the range switch is set to 250 mA, what is the current?
5. How much current is indicated by the ammeter in Figure 6–64 for each of the three range switch settings in Figure 6–61?

FIGURE 6–64

SUMMARY

- Resistors in parallel are connected between two points (nodes).
- A parallel combination has more than one path for current.
- The total parallel resistance is less than the lowest-value resistor.
- The voltages across all branches of a parallel circuit are the same.
- Current sources in parallel add algebraically.
- Kirchhoff's current law: The sum of the currents into a junction (total current in) equals the sum of the currents out of the junction (total current out).
- The algebraic sum of all the currents entering and leaving a junction is equal to zero.
- A parallel circuit is a current divider, so called because the total current entering the junction of parallel branches divides up into each of the branches.
- If all of the branches of a parallel circuit have equal resistance, the currents through all of the branches are equal.
- The total power in a parallel-resistive circuit is the sum of all of the individual powers of the resistors making up the parallel circuit.
- The total power for a parallel circuit can be calculated with the power formulas using values of total current, total resistance, or total voltage.
- If one of the branches of a parallel circuit opens, the total resistance increases, and therefore the total current decreases.
- If a branch of a parallel circuit opens, there is no change in current through the remaining branches.

KEY TERMS

These key terms are also in the end-of-book glossary.

Branch One current path in a parallel circuit.

Current divider A parallel circuit in which the currents divide inversely proportional to the parallel branch resistances.

Kirchhoff's current law A circuit law stating that the total current into a node equals the total current out of the node. Equivalently, the algebraic sum of all the currents entering and leaving a node is zero.

Node A point in a circuit at which two or more components are connected; also known as a *junction.*

Parallel The relationship in electric circuits in which two or more current paths are connected between two separate nodes.

FORMULAS

6–1 $$I_{IN(1)} + I_{IN(2)} + \cdots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + \cdots + I_{OUT(m)}$$ Kirchhoff's current law

6–2 $$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_n}\right)}$$ Total parallel resistance

6–3 $$R_T = \frac{R_1 R_2}{R_1 + R_2}$$ Special case for two resistors in parallel

6–4 $$R_T = \frac{R}{n}$$ Special case for n equal-value resistors in parallel

6–5 $$R_x = \frac{R_A R_T}{R_A - R_T}$$ Unknown parallel resistor

6–6	$I_x = \left(\frac{R_T}{R_x}\right)I_T$	General current-divider formula
6–7	$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T$	Two-branch current-divider formula
6–8	$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T$	Two-branch current-divider formula
6–9	$P_T = P_1 + P_2 + P_3 + \cdots + P_n$	Total power
6–10	$R_{open} = \frac{1}{G_{T(calc)} - G_{T(meas)}}$	Open branch resistance

SELF-TEST

Answers are at the end of the chapter.

1. In a parallel circuit, each resistor has
(a) the same current (b) the same voltage
(c) the same power (d) all of the above

2. When a 1.2 kΩ resistor and a 100 Ω resistor are connected in parallel, the total resistance is
(a) greater than 1.2 kΩ
(b) greater than 100 Ω but less than 1.2 kΩ
(c) less than 100 Ω but greater than 90 Ω
(d) less than 90 Ω

3. A 330 Ω resistor, a 270 Ω resistor, and a 68 Ω resistor are all in parallel. The total resistance is approximately
(a) 668 Ω (b) 47 Ω (c) 68 Ω (d) 22 Ω

4. Eight resistors are in parallel. The two lowest-value resistors are both 1.0 kΩ. The total resistance
(a) is less than 8 kΩ (b) is greater than 1.0 kΩ
(c) is less than 1.0 kΩ (d) is less than 500 Ω

5. When an additional resistor is connected across an existing parallel circuit, the total resistance
(a) decreases (b) increases
(c) remains the same (d) increases by the value of the added resistor

6. If one of the resistors in a parallel circuit is removed, the total resistance
(a) decreases by the value of the removed resistor (b) remains the same
(c) increases (d) doubles

7. One current into a junction is 500 mA and the other current into the same junction is 300 mA. The total current out of the junction is
(a) 200 mA (b) unknown (c) 800 mA (d) the larger of the two

8. The following resistors are in parallel across a voltage source: 390 Ω, 560 Ω, and 820 Ω. The resistor with the least current is
(a) 390 Ω (b) 560 Ω
(c) 820 Ω (d) impossible to determine without knowing the voltage

9. A sudden decrease in the total current into a parallel circuit may indicate
(a) a short (b) an open resistor
(c) a drop in source voltage (d) either (b) or (c)

10. In a four-branch parallel circuit, there are 10 mA of current in each branch. If one of the branches opens, the current in each of the other three branches is
(a) 13.3 mA (b) 10 mA (c) 0 A (d) 30 mA

11. In a certain three-branch parallel circuit, R_1 has 10 mA through it, R_2 has 15 mA through it, and R_3 has 20 mA through it. After measuring a total current of 35 mA, you can say that
(a) R_1 is open (b) R_2 is open
(c) R_3 is open (d) the circuit is operating properly

12. If there are a total of 100 mA into a parallel circuit consisting of three branches and two of the branch currents are 40 mA and 20 mA, the third branch current is

(a) 60 mA (b) 20 mA (c) 160 mA (d) 40 mA

13. A complete short develops across one of five parallel resistors on a PC board. The most likely result is

(a) the shorted resistor will burn out

(b) one or more of the other resistors will burn out

(c) the fuse in the power supply will blow

(d) the resistance values will be altered

14. The power dissipation in each of four parallel branches is 1 W. The total power dissipation is

(a) 1 W (b) 4 W (c) 0.25 W (d) 16 W

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 6–68.

1. If R_1 opens with the switch in the position shown, the voltage at terminal A with respect to ground

(a) increases (b) decreases (c) stays the same

2. If the switch is thrown from position A to position B, the total current

(a) increases (b) decreases (c) stays the same

3. If R_4 opens with the switch in position C, the total current

(a) increases (b) decreases (c) stays the same

4. If a short develops between B and C while the switch is in position B, the total current

(a) increases (b) decreases (c) stays the same

Refer to Figure 6–74(b).

5. If R_2 opens, the current through R_1

(a) increases (b) decreases (c) stays the same

6. If R_3 opens, the voltage across it

(a) increases (b) decreases (c) stays the same

7. If R_1 opens, the voltage across it

(a) increases (b) decreases (c) stays the same

Refer to Figure 6–75.

8. If the resistance of the rheostat R_2 is increased, the current through R_1

(a) increases (b) decreases (c) stays the same

9. If the fuse opens, the voltage across the rheostat R_2

(a) increases (b) decreases (c) stays the same

10. If the rheostat R_2 develops a short between the wiper and ground, the current through it

(a) increases (b) decreases (c) stays the same

Refer to Figure 6–79.

11. If the 2.25 mA source opens while the switch is in position C, the current through R

(a) increases (b) decreases (c) stays the same

12. If the 2.25 mA source opens while the switch is in position B, the current through R

(a) increases (b) decreases (c) stays the same

Refer to Figure 6–87.

13. If pins 4 and 5 are shorted together, the resistance between pins 3 and 6

(a) increases (b) decreases (c) stays the same

14. If the bottom connection of R_1 is shorted to the top connection of R_5, the resistance between pins 1 and 2

(a) increases **(b)** decreases **(c)** stays the same

15. If R_7 opens, the resistance between pins 5 and 6

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

SECTION 6–1 Resistors in Parallel

1. Show how to connect the resistors in Figure 6–65(a) in parallel across the battery.

2. Determine whether or not all the resistors in Figure 6–65(b) are connected in parallel on the printed circuit (PC) board.

***3.** Identify which groups of resistors are in parallel on the double-sided PC board in Figure 6–66.

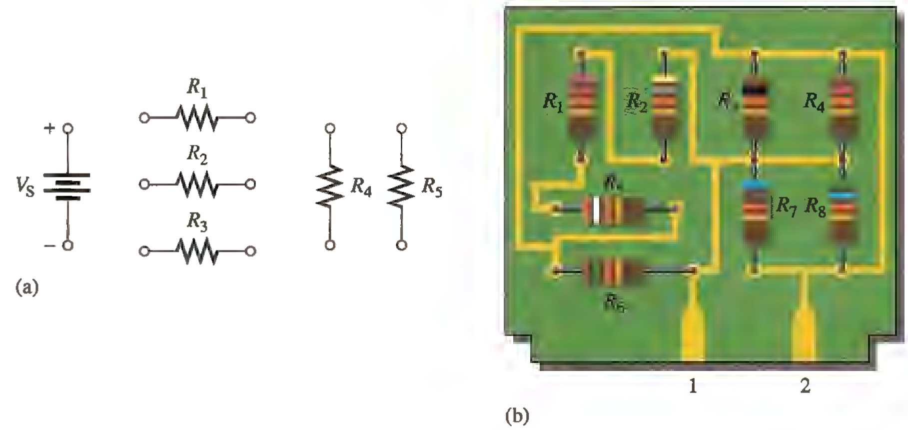


FIGURE 6–65

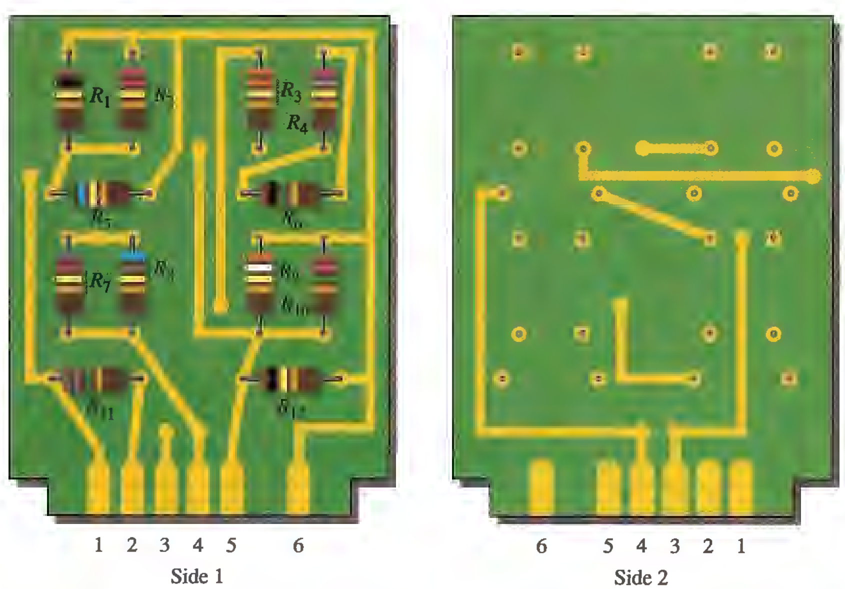
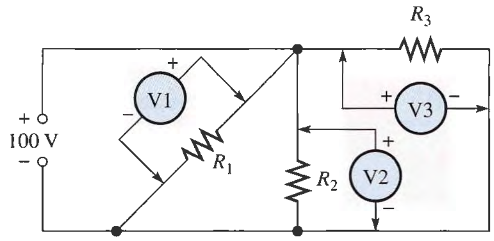


FIGURE 6–66

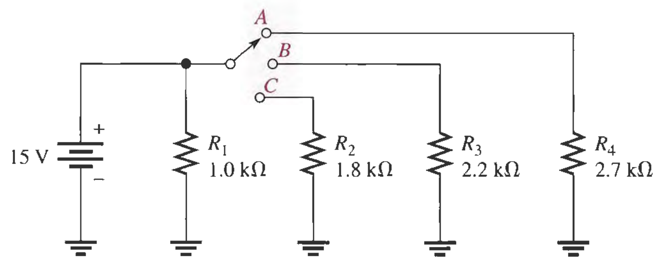
SECTION 6–2 Voltage in a Parallel Circuit

4. What is the voltage across and the current through each parallel resistor if the total voltage is 12 V and the total resistance is 550 Ω? There are four resistors, all of equal value.
5. The source voltage in Figure 6–67 is 100 V. How much voltage does each of the meters read?

▶ FIGURE 6–67



6. What is the total resistance of the circuit as seen from the voltage source for each position of the switch in Figure 6–68?
7. What is the voltage across each resistor in Figure 6–68 for each switch position?
8. What is the total current from the voltage source in Figure 6–68 for each switch position?

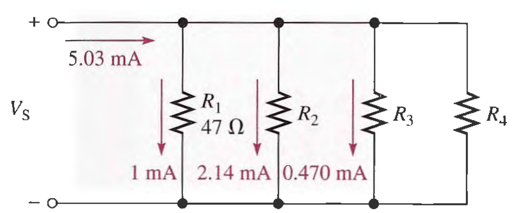


▲ FIGURE 6–68

SECTION 6–3 Kirchhoff's Current Law

9. The following currents are measured in the same direction in a three-branch parallel circuit: 250 mA, 300 mA, and 800 mA. What is the value of the current into the junction of these three branches?
10. There is a total of 500 mA of current into five parallel resistors. The currents through four of the resistors are 50 mA, 150 mA, 25 mA, and 100 mA. What is the current through the fifth resistor?
11. In the circuit of Figure 6–69, determine the resistance R_2, R_3, and R_4.

▶ FIGURE 6–69



*12. The electrical circuit in a room has a ceiling lamp that draws 1.25 A and four wall outlets. Two table lamps that each draw 0.833 A are plugged into two outlets, and an electric heater that draws 10 A is connected to the third outlet. When all of these items are in use, how much current is in the main line serving the room? If the main line is protected by a 15 A circuit breaker, how much current can be drawn from the fourth outlet? Draw a schematic of this wiring.

*13. The total resistance of a parallel circuit is 25 Ω. What is the current through a 220 Ω resistor that makes up part of the parallel circuit if the total current is 100 mA?

SECTION 6–4 Total Parallel Resistance

14. The following resistors are connected in parallel: 1.0 MΩ, 2.2 MΩ, 5.6 MΩ, 12 MΩ, and 22 MΩ. Determine the total resistance.

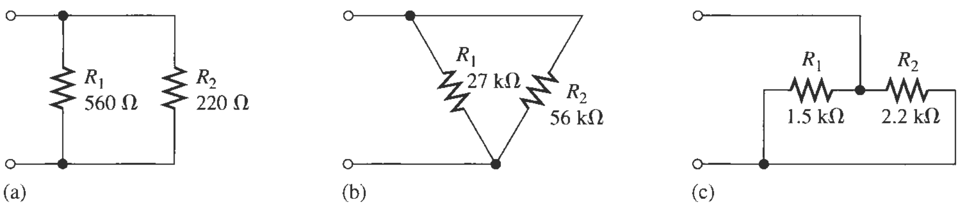
15. Find the total resistance for each of the following groups of parallel resistors:

(a) 560 Ω and 1000 Ω **(b)** 47 Ω and 56 Ω

(c) 1.5 kΩ, 2.2 kΩ, 10 kΩ **(d)** 1.0 MΩ, 470 kΩ, 1.0 kΩ, 2.7 MΩ

16. Calculate R_T for each circuit in Figure 6–70.

▶ FIGURE 6–70



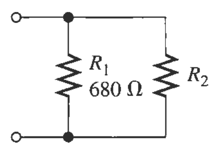
17. What is the total resistance of twelve 6.8 kΩ resistors in parallel?

18. Five 470 Ω, ten 1000 Ω, and two 100 Ω resistors are all connected in parallel. What is the total resistance for each of the three groupings?

19. Find the total resistance for the entire parallel circuit in Problem 18.

20. If the total resistance in Figure 6–71 is 389.2 Ω, what is the value of R_2?

▶ FIGURE 6–71

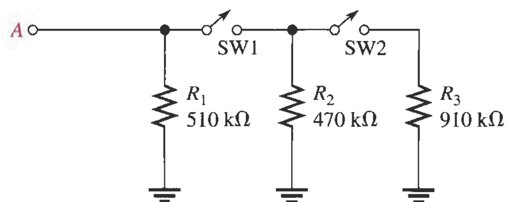


21. What is the total resistance between point *A* and ground in Figure 6–72 for the following conditions?

(a) SW1 and SW2 open **(b)** SW1 closed, SW2 open

(c) SW1 open, SW2 closed **(d)** SW1 and SW2 closed

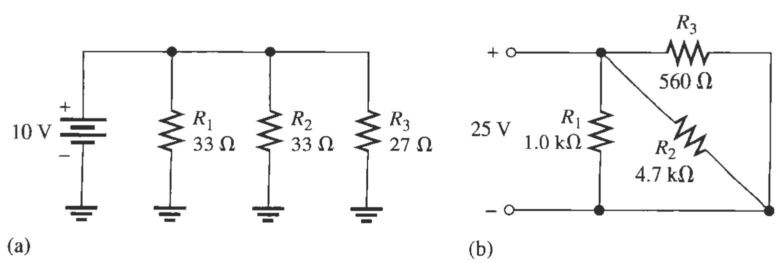
▶ FIGURE 6–72



SECTION 6–5 Application of Ohm's Law

22. What is the total current in each circuit of Figure 6–73?

▶ FIGURE 6–73

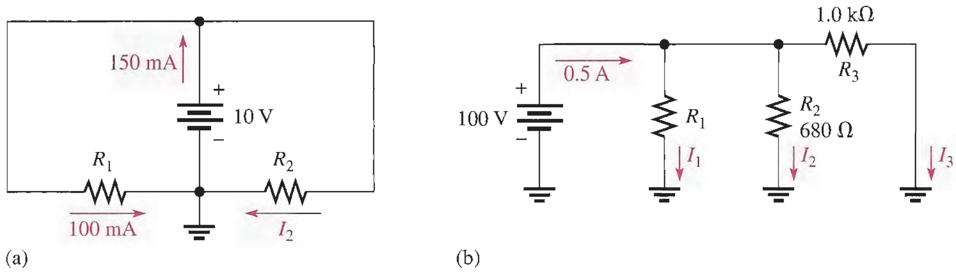


23. Three 33 Ω resistors are connected in parallel with a 110 V source. What is the current from the source?

24. Four equal-value resistors are connected in parallel. Five volts are applied across the parallel circuit, and 1.11 mA are measured from the source. What is the value of each resistor?

25. Many types of decorative lights are connected in parallel. If a set of lights is connected to a 110 V source and the filament of each bulb has a hot resistance of 2.2 kΩ, what is the current through each bulb? Why is it better to have these bulbs in parallel rather than in series?

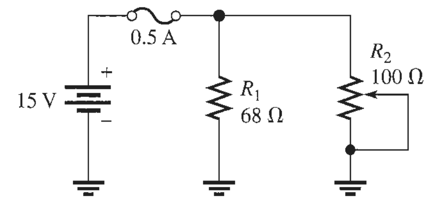
26. Find the values of the unspecified labeled quantities in each circuit of Figure 6–74.



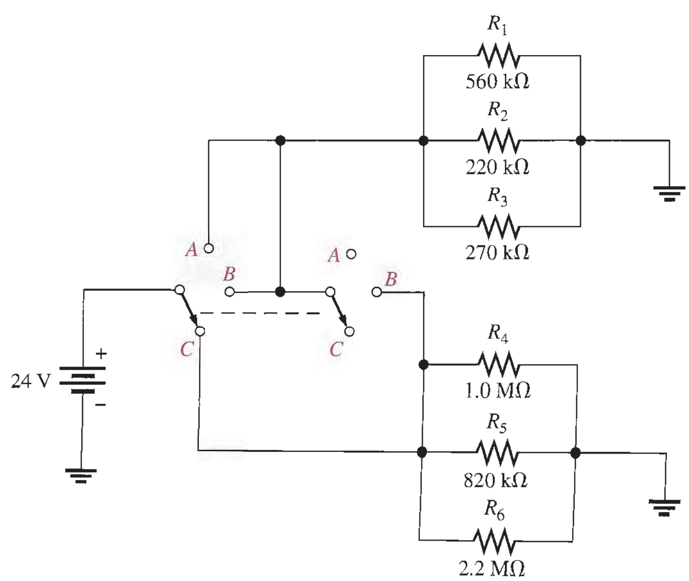
▲ FIGURE 6–74

27. To what minimum value can the 100 Ω rheostat in Figure 6–75 be adjusted before the 0.5 A fuse blows?

▶ FIGURE 6–75



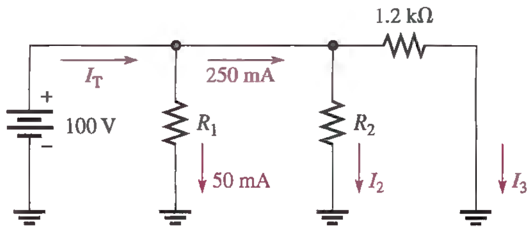
28. Determine the total current from the source and the current through each resistor for each switch position in Figure 6–76.



▲ FIGURE 6–76

29. Find the values of the unspecified quantities in Figure 6–77.

FIGURE 6–77



SECTION 6–6 Current Sources in Parallel

30. Determine the current through R_L in each circuit in Figure 6–78.

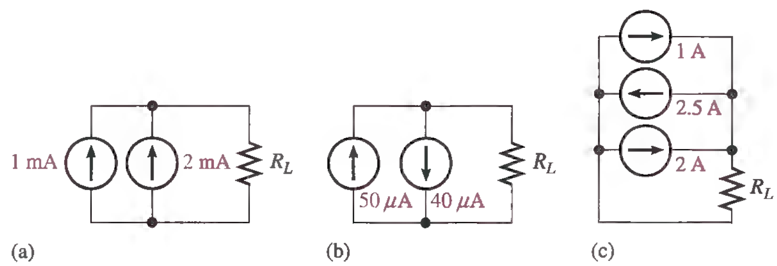
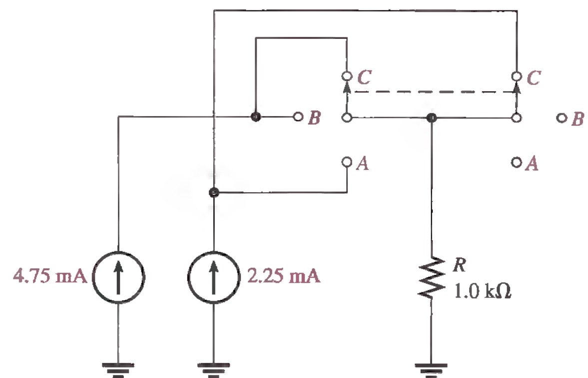


FIGURE 6–78

31. Find the current through the resistor for each position of the ganged switch in Figure 6–79.

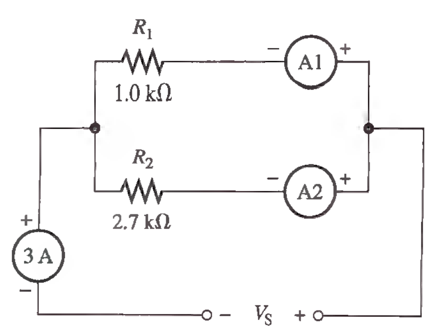
FIGURE 6–79



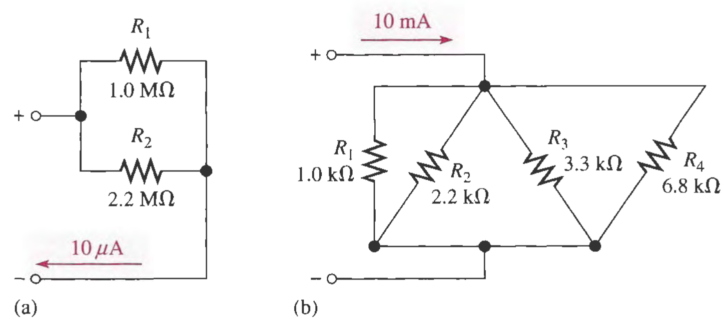
SECTION 6–7 Current Dividers

32. How much branch current should each meter in Figure 6–80 indicate?

FIGURE 6–80



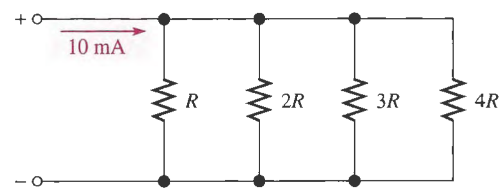
33. Determine the current in each branch of the current dividers of Figure 6–81.



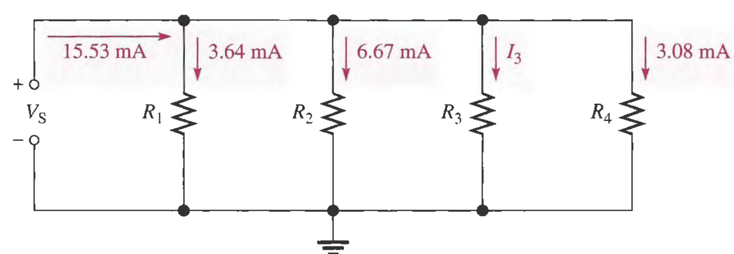
▲ FIGURE 6–81

34. What is the current through each resistor in Figure 6–82? R is the lowest-value resistor, and all others are multiples of that value as indicated.

▶ FIGURE 6–82



35. Determine all of the resistor values in Figure 6–83. $R_T = 773\ \Omega$.



▲ FIGURE 6–83

*36. (a) Determine the required value of the shunt resistor R_{SH1} in the ammeter of Figure 6–49 if the resistance of the meter movement is 50 Ω.

(b) Find the required value for R_{SH2} in the meter circuit of Figure 6–50 ($R_M = 50\ \Omega$).

*37. Special shunt resistors designed to drop 50 mV in high current-measuring applications are available from manufacturers. A 50 mV, 10 kΩ full-scale voltmeter is connected across the shunt to make the measurement.

(a) What value of shunt resistance is required to use a 50 mV meter in a 50 A measurement application?

(b) How much current is through the meter?

SECTION 6–8 Power in Parallel Circuits

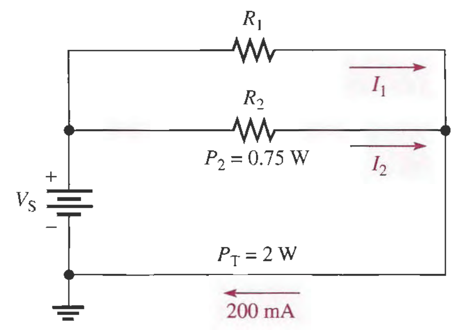
38. Five parallel resistors each handle 250 mW. What is the total power?

39. Determine the total power in each circuit of Figure 6–81.

40. Six light bulbs are connected in parallel across 110 V. Each bulb is rated at 75 W. What is the current through each bulb, and what is the total current?

*41. Find the values of the unspecified quantities in Figure 6–84.

► FIGURE 6–84



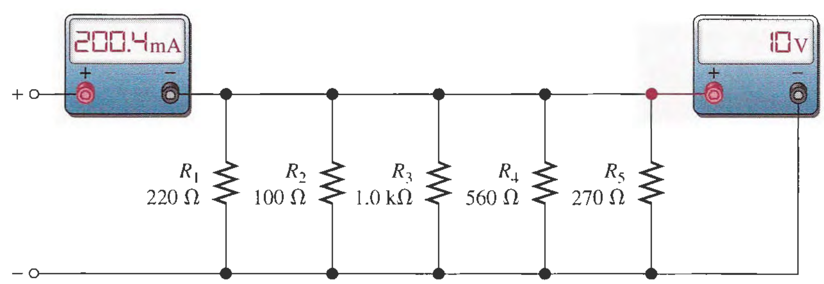
*42. A certain parallel circuit consists of only ½ W resistors. The total resistance is 1.0 kΩ, and the total current is 50 mA. If each resistor is operating at one-half its maximum power level, determine the following:

(a) The number of resistors
(b) The value of each resistor
(c) The current in each branch
(d) The applied voltage

SECTION 6–10 Troubleshooting

43. If one of the bulbs burns out in Problem 40, how much current will be through each of the remaining bulbs? What will the total current be?

44. In Figure 6–85, the current and voltage measurements are indicated. Has a resistor opened, and, if so, which one?

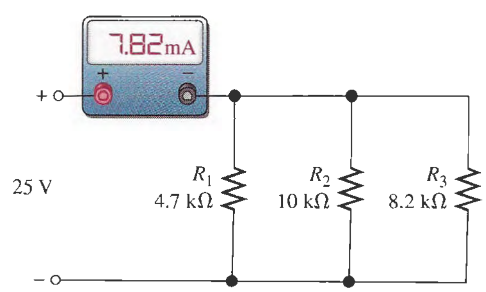


▲ FIGURE 6–85

45. What is wrong with the circuit in Figure 6–86?

46. What is wrong with the circuit in Figure 6–86 if the meter reads 5.55 mA?

► FIGURE 6–86



***47.** Develop a test procedure to check the circuit board in Figure 6–87 to make sure that there are no open components. You must do this test without removing a component from the board. List the procedure in a detailed step-by-step format.

***48.** For the circuit board shown in Figure 6–88, determine the resistance between the following pins if there is a short between pins 2 and 4:

(a) 1 and 2 **(b)** 2 and 3 **(c)** 3 and 4 **(d)** 1 and 4

***49.** For the circuit board shown in Figure 6–88, determine the resistance between the following pins if there is a short between pins 3 and 4:

(a) 1 and 2 **(b)** 2 and 3 **(c)** 2 and 4 **(d)** 1 and 4

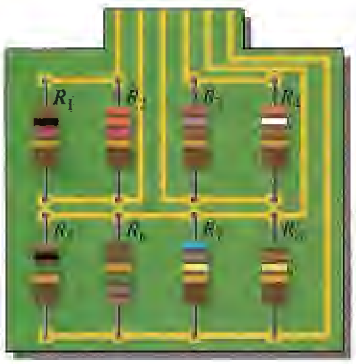


FIGURE 6–87

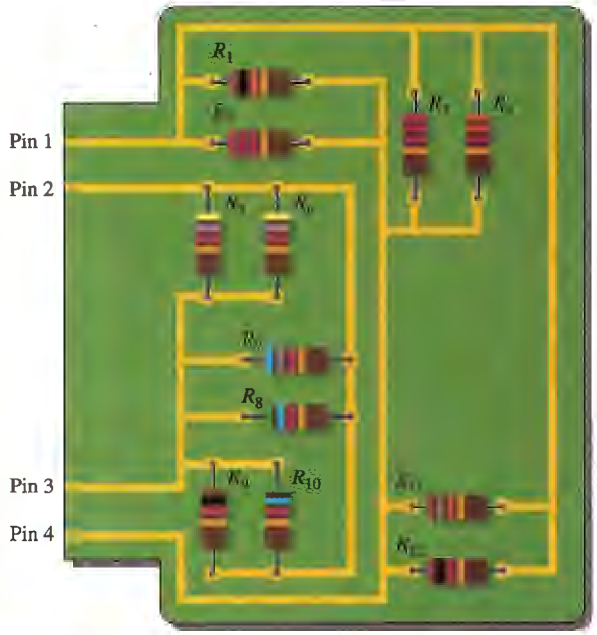
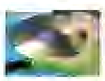


FIGURE 6–88



Multisim Troubleshooting and Analysis

These problems require your Multisim CD-ROM.

50. Open file P06-50 and measure the total parallel resistance.

51. Open file P06-51. Determine by measurement if there is an open resistor and, if so, which one.

52. Open file P06-52 and determine the unspecified resistance value.

53. Open file P06-53 and determine the unspecified source voltage.

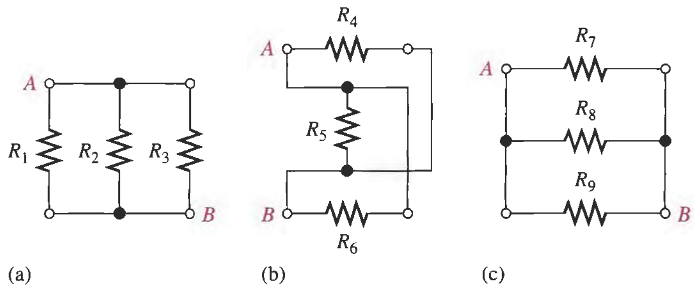
54. Open file P06-54 and find the fault if there is one.

ANSWERS

SECTION REVIEWS

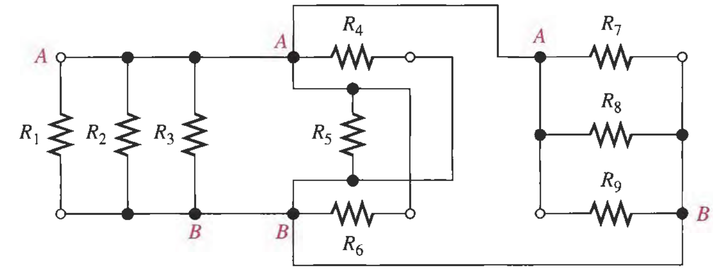
SECTION 6–1 Resistors in Parallel

1. Parallel resistors are connected between the same two separate points.
2. A parallel circuit has more than one current path between two given points.
3. See Figure 6–89.



▲ FIGURE 6–89

4. See Figure 6–90.



▲ FIGURE 6–90

SECTION 6–2 Voltage in a Parallel Circuit

1. $V_{10\Omega} = V_{22\Omega} = 5\text{ V}$
2. $V_{R2} = 118\text{ V}; V_S = 118\text{ V}$
3. $V_{R1} = 50\text{ V}$ and $V_{R2} = 50\text{ V}$
4. Voltage is the same across all parallel branches.

SECTION 6–3 Kirchhoff's Current Law

1. Kirchhoff's law: The algebraic sum of all the currents at a junction is zero; The sum of the currents entering a junction equals the sum of the currents leaving that junction.
2. $I_1 = I_2 = I_3 = I_T = 2.5\text{ mA}$
3. $I_{OUT} = 100\text{ mA} + 300\text{ mA} = 400\text{ mA}$
4. $I_1 = I_T - I_2 = 3\ \mu\text{A}$
5. $I_{IN} = 8\text{ mA} - 1\text{ mA} = 7\text{ mA}; I_{OUT} = 8\text{ mA} - 3\text{ mA} = 5\text{ mA}$

SECTION 6–4 Total Parallel Resistance

1. R_T decreases with more resistors in parallel.
2. The total parallel resistance is less than the smallest branch resistance.
3. $R_T = \dfrac{1}{(1/R_1) + (1/R_2) + \cdots + (1/R_n)}$
4. $R_T = R_1R_2/(R_1 + R_2)$
5. $R_T = R/n$
6. $R_T = (1.0\text{ k}\Omega)(2.2\text{ k}\Omega)/3.2\text{ k}\Omega = 688\ \Omega$
7. $R_T = 1.0\text{ k}\Omega/4 = 250\ \Omega$
8. $R_T = \dfrac{1}{1/47\ \Omega + 1/150\ \Omega + 1/100\ \Omega} = 26.4\ \Omega$

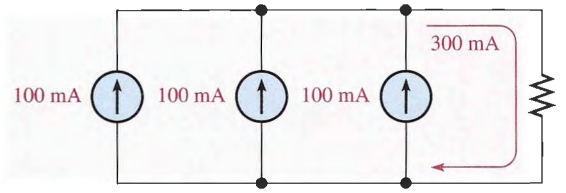
SECTION 6–5 Application of Ohm's Law

1. $I_T = 10\text{ V}/22.7\ \Omega = 44.1\text{ mA}$
2. $V_S = (20\text{ mA})(222\ \Omega) = 4.44\text{ V}$
3. $I_1 = 4.44\text{ V}/680\ \Omega = 6.53\text{ mA}; I_2 = 4.44\text{ V}/330\ \Omega = 13.5\text{ mA}$
4. $R_T = 12\text{ V}/5.85\text{ mA} = 2.05\text{ k}\Omega; R = (2.05\text{ k}\Omega)(4) = 8.2\text{ k}\Omega$
5. $V = (100\text{ mA})(688\ \Omega) = 68.8\text{ V}$

SECTION 6–6 Current Sources in Parallel

1. $I_T = 4(0.5\text{ A}) = 2\text{ A}$
2. Three sources; See Figure 6–91.
3. $I_{R_E} = 10\text{ mA} + 10\text{ mA} = 20\text{ mA}$

▶ FIGURE 6–91



SECTION 6–7 Current Dividers

1. $I_x = (R_T/R_x)I_T$
2. $I_1 = \left(\dfrac{R_2}{R_1 + R_2}\right)I_T \qquad I_2 = \left(\dfrac{R_1}{R_1 + R_2}\right)I_T$
3. The 22 kΩ has the most current; the 220 kΩ has the least current.
4. $I_1 = (680\ \Omega/1010\ \Omega)10\text{ mA} = 6.73\text{ mA}; I_2 = (330\ \Omega/1010\ \Omega)10\text{ mA} = 3.27\text{ mA}$
5. $I_3 = (114\ \Omega/470\ \Omega)4\text{ mA} = 970\ \mu\text{A}$

SECTION 6–8 Power in Parallel Circuits

1. Add the power of each resistor to get total power.
2. $P_T = 238\text{ mW} + 512\text{ mW} + 109\text{ mW} + 876\text{ mW} = 1.74\text{ W}$
3. $P_T = (1\text{ A})^2(615\ \Omega) = 615\text{ W}$

SECTION 6–9 Parallel Circuit Applications

1. $R_{max} = 50\ \Omega; I_{max} = 1\text{ mA}$
2. R_{SH} is less than R_M because the shunt resistors must allow currents much greater than the current through the meter movement.

SECTION 6–10 Troubleshooting

1. When a branch opens, there is no change in voltage; the total current decreases.
2. If a branch opens, total parallel resistance increases.
3. The remaining bulbs continue to glow.
4. All remaining branch currents are 100 mA.
5. The branch with 120 mA is open.

A Circuit Application

1. R_{SH} has the most current.
2. 25 mA range: $R_{AB} = R_M = 6\ \Omega$

   250 mA range: $R_{AB} = R_M \| R_{SH} = 6\ \Omega \| 670\text{ m}\Omega = 603\text{ m}\Omega$

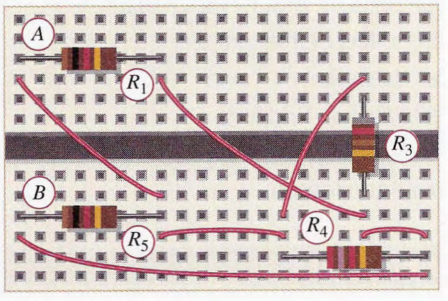
   2.5 A range: $R_{AB} = (R_1 + R_M) \| R_{SH} = (60.4\ \Omega + 6\ \Omega) \| 670\text{ m}\Omega =$
   $66.4\ \Omega \| 670\text{ m}\Omega = 663\text{ m}\Omega$

3. The meter circuit in Figure 6–61 negates the effect of the switch contact resistance.
4. 150 mA
5. 25 mA range: 7.5 mA
   250 mA range: 75 mA
   2.5 A range: 750 mA

RELATED PROBLEMS FOR EXAMPLES

6–1 See Figure 6–92.

▶ **FIGURE 6–92**



6–2 Connect pin 1 to pin 2 and pin 3 to pin 4.

6–3 25 V

6–4 20 mA into node A and out of node B

6–5 $I_T = 112\text{ mA}, I_2 = 50\text{ mA}$

6–6 2.5 mA; 5 mA

6–7 9.33 Ω

6–8 132 Ω

6–9 4 Ω

6–10 1044 Ω

6–11 1.83 mA; 1 mA

6–12 $I_1 = 20\text{ mA}; I_2 = 9.09\text{ mA}; I_3 = 35.7\text{ mA}, I_4 = 22.0\text{ mA}$

6–13 1.28 V

6–14 Measure R_T with an ohmmeter and calculate R_1 using $R_1 = 1/[(1/R_T) - (1/R_2) - (1/R_3)]$

6–15 30 mA

6–16 $I_1 = 3.27\text{ mA}; I_2 = 6.73\text{ mA}$

6–17 $I_1 = 59.4\text{ mA}; I_2 = 40.6\text{ mA}$

6–18 1.78 W

6–19 81 W

6–20 0.0347 μA

6–21 15.4 mA

6–22 Not correct, R_{10} (68 kΩ) must be open.

SELF-TEST

1. (b) **2.** (c) **3.** (b) **4.** (d) **5.** (a) **6.** (c) **7.** (c) **8.** (c)
9. (d) **10.** (b) **11.** (a) **12.** (d) **13.** (c) **14.** (b)

CIRCUIT DYNAMICS QUIZ

1. (c) **2.** (a) **3.** (c) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (c)
9. (b) **10.** (c) **11.** (b) **12.** (c) **13.** (a) **14.** (c) **15.** (a)