Bi-nomial Frequency Distribution:=

If the bi-nomial probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bi-nomial frequency distribution.

$$N\binom{n}{x}p^{x}q^{n-x}$$

Example:=

Six dice are thrown 729 times. How many times do you expect at least three dice to show a 5 or a 6.

Sol:=

The probability of getting a 5 or a 6 with one dice is $p = \frac{2}{6}$

Since 6 dice are thrown and there are 729 sets

So

$N = 729$

$p = \frac{2}{6}$

$q = 1 - p$

$= 1 - \frac{2}{6}$

$= 4/6$

$= 2/3$

So Using formula

$$729\left(\frac{2}{3} + \frac{1}{3}\right)^{6}$$

$$729\left[\sum_{x=3}^{6}\binom{6}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{6-x}\right]$$

i.e. the expected number of times at least 3 dice Showing 5 or 6

$$= 729\left[\binom{6}{3}\left(\frac{1}{3}\right)^{3}\left(\frac{2}{3}\right)^{3}+\binom{6}{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)^{2}+\binom{6}{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{1}+\binom{6}{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{0}\right]$$

$\approx$ 233 Ans (by Calculating)

Properties of the Binomial Probability Distribution:=

i) let "x" be a random variable with the bi-nomial distribution $b(x; n, p)$ Then its mean is given by

$$E(x) = \mu = np$$

ii) let "x" be a random variable with the bi-nomial distribution $b(x; n, p)$ Then its variance is given by

$$Var(x) = \delta^2 = npq$$

iii) let "x" be a random variable with the bi-nomial distribution $b(x; n, p)$, Then its Standard Deviation is given by

$$\delta = \sqrt{npq}$$

Example:

In an Examination 24 candidates offered statistics. If the probability of passing the

~~the~~ subject be $\frac{1}{3}$ Find mean and variance, S.D of the distribution.

Sol:-

Here $n = 24$

$p = \frac{1}{3}$

$q = 1 - p$

$1 - \frac{1}{3}$

$= \frac{2}{3}$

Mean

$\mu = np$

$= 24 \times \frac{1}{3}$

$= 8$

Variance

$\delta^2 = npq$

$= 24 \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{16}{3}$

$= 5.33$

S.D

$\delta = \sqrt{npq}$

$= \sqrt{5.33}$

$=$

The Recurrence Formula for the Bi-nomial Distribu.

Begining with the value of $P(X=0)$, probabilities for other values of "x", the number of successes can be computed more easily by the recurrence formula i.e.

$$P(X=x) = \frac{n-x+1}{x} \frac{p}{q} P(X=x-1), \text{ for } x=1,2,3\ldots$$

Example :=

If "X" is binomially distributed with mean 3.20 and variance 1.152. Find the complete binomial probability distribution.

Sol :=

we know that

$$\mu = E(X) = np \quad \text{and}$$
$$\sigma^2 = Var(X) = npq$$

Now

$$E(X) = 3.20 \quad \text{So} \quad np = 3.20$$
$$Var(X) = 1.152 \quad \text{So} \quad npq = 1.152$$

Substituting for np in 2^{nd} equation we get

$$npq = 1.152$$
$$(3.20)q = 1.152$$

$$q = \frac{1.152}{3.20}$$
$$= 0.36$$

Now $p = 1 - q$

$= 1 - 0.36$

$= 0.64$

So $np = 3.20$

~~$n(0.64) = 3.20$~~

$n(0.36) = 3.20$

$$n = \frac{3.20}{0.36}$$

$n = 5$

Therefore $n = 5$ $p = 0.64$ So that "X" is

$$b(x; 5, 0.64)$$

Now

$$P(X = x) = \binom{5}{x}(0.64)^{x}(0.36)^{5-x}$$

and $P(X=0) = \binom{5}{0}(0.64)^{0}(0.36)^{5-0}$

$P(X=0) = 0.006047$

Begining with the value of $P(X=0)$, we compute the probabilities of other values of X, using the recurrence formula

$$P(X=x) = \frac{n-x+1}{x} \frac{p}{q} \cdot P(X=x-1)$$

$\therefore$ $P(X=1) = \frac{5-1+1}{1}\left(\frac{0.64}{0.36}\right)(0.006047)$

$= 5(1.7777)(0.006047) = 0.05374$

$P(X=2) = \frac{4}{2}\left(\frac{0.64}{0.36}\right)(0.053751) = 0.191115$

$P(X=3) = \frac{3}{3}\left(\frac{0.64}{0.36}\right)(0.191115) = 0.339760$

$P(X=4) = \frac{2}{4}\left(\frac{0.64}{0.36}\right)(0.339760) = 0.302009$

$P(X=5) = \frac{1}{5}\left(\frac{0.64}{0.36}\right)(0.302009) = 0.107381$

The sum of probabilities is 1.00063 instead of 1. Error has been introduced by rounding process.

POISSON DISTRIBUTION :=

A poisson Distribution is a statistical distribution that shows how many times an event is likely to occur within a specified period of time.
It is used for independent events which occur at a constant rate within a given interval of time.
For example
if the average number of people who rent movies on a Friday night at a single video store location is 400, a poisson distribution can answer such question as
what is the probability that more than 600 people will rent the movie?

Formula

$$b(x; n, p) = \frac{\mu^{x} e^{-\mu}}{x!} \qquad x = 0, 1, 2, \dots \infty$$

where $e = 2.71828$

Example:=

If "X" is a Poisson random variable with parameter $\mu = 2$, find the probability for $x = 0, 1, 2, 3$ or more.

Sol:=

The Poisson Distribution would be

$$P(x;2) = \frac{e^{-2}(2)^x}{x!}$$

The desired probabilities for $x = 0, 1, 2, 3$, or more is computed as follows

$$P(X=0) = P(0;2) = \frac{e^{-2}(2)^0}{0!} = e^{-2} = 0.135335$$

$$P(X=1) = P(1;2) = \frac{e^{-2} \cdot 2^1}{1!}$$

$$= e^{-2} \cdot 2$$

$$= 2(0.135335$$

$$= 0.27067$$

$$P(X=2) = P(2;2) = \frac{e^{-2} \cdot (2)^2}{2!} = \frac{4e^{-2}}{2!}$$

$$= 2e^{-2}$$

$$= 2(0.135335) = 0.27067$$

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - [0.135335 + 0.27067 + 0.27067)$$

$= 1 - 0.676675$

$= 0.323325$

Example:= Two hundred (200) passengers have made reservations for an airplane flight. If the probability that a passenger who has a reservation will not show up is 0.01. what is the probability that exactly three will not show up?

Sol:=

Let us assume "no show" as success. then

$n = 200$. $p = 0.01$

Since "p" is very small and "n" is considerably large, we shall apply Poisson distribution using

$$\mu = np = (200)(0.01) = 2$$

$$P(X=3) = P(3;2) = \frac{(2)^3 e^{-2}}{3!}$$

$$= \frac{(8)(0.1353)}{3 \times 2 \times 1} \quad \left[\frac{1}{e^2} = \frac{1}{(2.71828)^2}\right] \quad [= 0.1353]$$

$$= 0.1804$$

NOTE:=

Generally most statisticians use Poisson Distribution when p is 0.05 or less and n is 20 or more

Example:=

The Probability that a man aged 50 years will die within a year is 0.01125. What is the probability that of 12 such man at least 11 will reach their fifty-first birthday?

Sol:= Here

$$P = 0.01125$$
$$n = 12$$

Therefore

$$\mu = np$$
$$= 12 \times (0.01125)$$
$$= 0.135$$

So

$$P(x; 0.135) = \frac{e^{-0.135} (0.135)^x}{x!}$$

Now the probability that no person will die i.e, all will survive (all = 12)

$$P(0; 0.135) = e^{-0.135}$$

$$= 1 - 0.135 + \frac{(0.135)^2}{2!} - \frac{(0.135)^3}{3!} + \cdots$$

$$= 0.8737$$

and the probability that 1 person will die, 11 will survive is

$$P(1; 0.135) = \frac{e^{-0.135} (0.135)^1}{1!}$$

$$= (0.8737)(0.135) = 0.1179$$

Hence the probability that atleast 11 will Survive

$$= P(0; 0.135) + P(1; 0.135)$$

$$= 0.8737 + 0.1179$$

$$= 0.9916$$

Poisson Frequency Distribution:-

When the Poisson distribution is multiplied by "N" the number of sets or experiments, each of "n" trials the resulting distribution is known as Poisson Frequency Distribution.

$$f(x) = N \frac{e^{-\mu} \mu^{x}}{x!}, \quad x = 0, 1, 2, \dots, \infty$$

Example:-

For a machine making parts, there is a small probability of 0.002 for a part to be defective. The parts are supplied in bundle of 10. Calculate approximately the no. of bundles containing no defective, one defective or two defectives in a consignment of 10,000 bundles given that

$$e^{-0.02} = 0.9802$$

Sol:-

Let "P" be the probability of a part being defective. Then

$$P = 0.002$$

$$n = 10$$

$$\mu = nP$$

$$10 \times 0.002 = 0.02$$

Hence the approximate number of bundles containing no defective
one defective or two defectives are terms for
$x = 0, 1$ and 2

$$N\,P(x;\,u) = 10,000\ \frac{(0.02)^x\, e^{-0.02}}{x!}$$

putting $x=0$, we get

$$= 10000\, e^{-0.02}$$

$$= 10000 \times 0.9802$$

$$= 9.802$$

put $x=1$, we get

$$10,000\, e^{-0.02} \cdot (0.02)$$

$$= 10,000 \times 0.9802 \times 0.02$$

$$= 196$$

putting $x=2$, we get

$$\frac{10,000\, e^{-0.02} \cdot (0.02)^2}{2!}$$

$$= \frac{10,000 \times 0.9802 \times (0.02)^2}{2}$$

$$= 2 \text{ approx}$$