

Fig. 22-18 A molecule of H_2O , showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment \vec{p} points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

22-9 A Dipole in an Electric Field

We have defined the electric dipole moment \vec{p} of an electric dipole to be a vector that points from the negative to the positive end of the dipole. As you will see, the behavior of a dipole in a uniform external electric field \vec{E} can be described completely in terms of the two vectors \vec{E} and \vec{p} , with no need of any details about the dipole's structure.

A molecule of water (H_2O) is an electric dipole; Fig. 22-18 shows why. There the black dots represent the oxygen nucleus (having eight protons) and the two hydrogen nuclei (having one proton each). The colored enclosed areas represent the regions in which electrons can be located around the nuclei.

In a water molecule, the two hydrogen atoms and the oxygen atom do not lie on a straight line but form an angle of about 105° , as shown in Fig. 22-18. As a result, the molecule has a definite "oxygen side" and "hydrogen side." Moreover, the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei. This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment \vec{p} that points along the symmetry axis of the molecule as shown. If the water molecule is placed in an external electric field, it behaves as would be expected of the more abstract electric dipole of Fig. 22-8.

To examine this behavior, we now consider such an abstract dipole in a uniform external electric field \vec{E} , as shown in Fig. 22-19a. We assume that the dipole is a rigid structure that consists of two centers of opposite charge, each of magnitude q , separated by a distance d . The dipole moment \vec{p} makes an angle θ with field \vec{E} .

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. 22-19a) and with the same magnitude $F = qE$. Thus, *because the field is uniform*, the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance x from one end and thus a distance $d - x$ from the other end. From Eq. 10-39 ($\tau = rF \sin \phi$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \quad (22-32)$$

We can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field E and the dipole moment $p = qd$. To do so, we substitute qE for F and p/q for d in Eq. 22-32, finding that the magnitude of $\vec{\tau}$ is

$$\tau = pE \sin \theta. \quad (22-33)$$

We can generalize this equation to vector form as

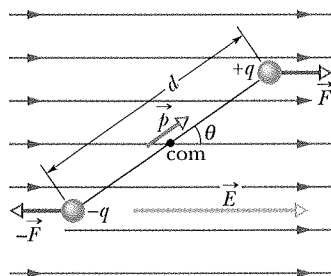
$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{torque on a dipole}). \quad (22-34)$$

Vectors \vec{p} and \vec{E} are shown in Fig. 22-19b. The torque acting on a dipole tends to rotate \vec{p} (hence the dipole) into the direction of field \vec{E} , thereby reducing θ . In Fig. 22-19, such rotation is clockwise. As we discussed in Chapter 10, we can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque. With that notation, the torque of Fig. 22-19 is

$$\tau = -pE \sin \theta. \quad (22-35)$$

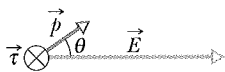
Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment \vec{p} is lined up with the field \vec{E} (then



(a)

The dipole is being torqued into alignment.



(b)

Fig. 22-19 (a) An electric dipole in a uniform external electric field \vec{E} . Two centers of equal but opposite charge are separated by distance d . The line between them represents their rigid connection. (b) Field \vec{E} causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol \otimes .

$\vec{\tau} = \vec{p} \times \vec{E} = 0$). It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has its least gravitational potential energy in its equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

In any situation involving potential energy, we are free to define the zero-potential-energy configuration in a perfectly arbitrary way because only differences in potential energy have physical meaning. It turns out that the expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ in Fig. 22-19 is 90° . We then can find the potential energy U of the dipole at any other value of θ with Eq. 8-1 ($\Delta U = -W$) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90° . With the aid of Eq. 10-53 ($W = \int \tau d\theta$) and Eq. 22-35, we find that the potential energy U at any angle θ is

$$U = -W = -\int_{90^\circ}^{\theta} \tau d\theta = \int_{90^\circ}^{\theta} pE \sin \theta d\theta. \quad (22-36)$$

Evaluating the integral leads to

$$U = -pE \cos \theta. \quad (22-37)$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad (\text{potential energy of a dipole}). \quad (22-38)$$

Equations 22-37 and 22-38 show us that the potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ (\vec{p} and \vec{E} are in the same direction); the potential energy is greatest ($U = pE$) when $\theta = 180^\circ$ (\vec{p} and \vec{E} are in opposite directions).

When a dipole rotates from an initial orientation θ_i to another orientation θ_f , the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i), \quad (22-39)$$

where U_f and U_i are calculated with Eq. 22-38. If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i). \quad (22-40)$$

Microwave Cooking

Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field \vec{E} within the oven and thus also within the food. From Eq. 22-34, we see that any electric field \vec{E} produces a torque on an electric dipole moment \vec{p} to align \vec{p} with \vec{E} . Because the oven's \vec{E} oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with \vec{E} .

Energy is transferred from the electric field to the thermal energy of the water (and thus of the food) where three water molecules happened to have bonded together to form a group. The flip-flop breaks some of the bonds. When the molecules reform the bonds, energy is transferred to the random motion of the group and then to the surrounding molecules. Soon, the thermal energy of the water is enough to cook the food. Sometimes the heating is surprising. If you heat a jelly donut, for example, the jelly (which holds a lot of water) heats far more than the donut material (which holds much less water). Although the exterior of the donut may not be hot, biting into the jelly can burn you. If water molecules were not electric dipoles, we would not have microwave ovens.

CHECKPOINT 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.

