

Q9:- Evaluate  $\int_1^e x \ln x \, dx$

Integrating by parts.

$$\int \ln x \cdot x \, dx = \ln x \cdot \frac{x^2}{2} - \int \left(\frac{1}{x}\right) \left(\frac{x^2}{2}\right) dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$$

$$\int_1^e x \ln x \, dx = \left[ \frac{1}{2} x^2 \ln x - \frac{x^2}{4} \right]_1^e$$

$$= \left( \frac{1}{2} e^2 \ln e - \frac{e^2}{4} \right) - \left( \frac{1}{2} (1)^2 \ln 1 - \frac{(1)^2}{4} \right)$$

$$= \left( \frac{e^2}{2} - \frac{1}{4} - \frac{e^2}{4} \right) - \left( \frac{1}{2} (0) - \frac{1}{4} \right) \quad \left. \begin{array}{l} \ln e = 1 \\ \ln 1 = 0 \end{array} \right\}$$

$$= \frac{e^2}{4} + \frac{1}{4}$$