University of Engineering & Technology, Peshawar, Pakistan

CE-409: Introduction to Structural Dynamics and Earthquake Engineering

MODULE 6

RESPONSE OF LINEAR ELASTIC S.D.O.F SYSTEMS TO EARTHQUAKE LOADING

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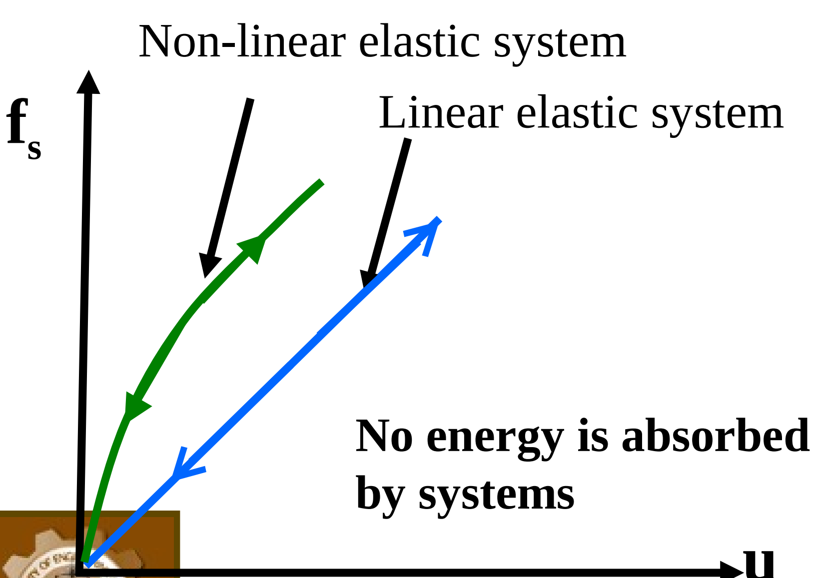
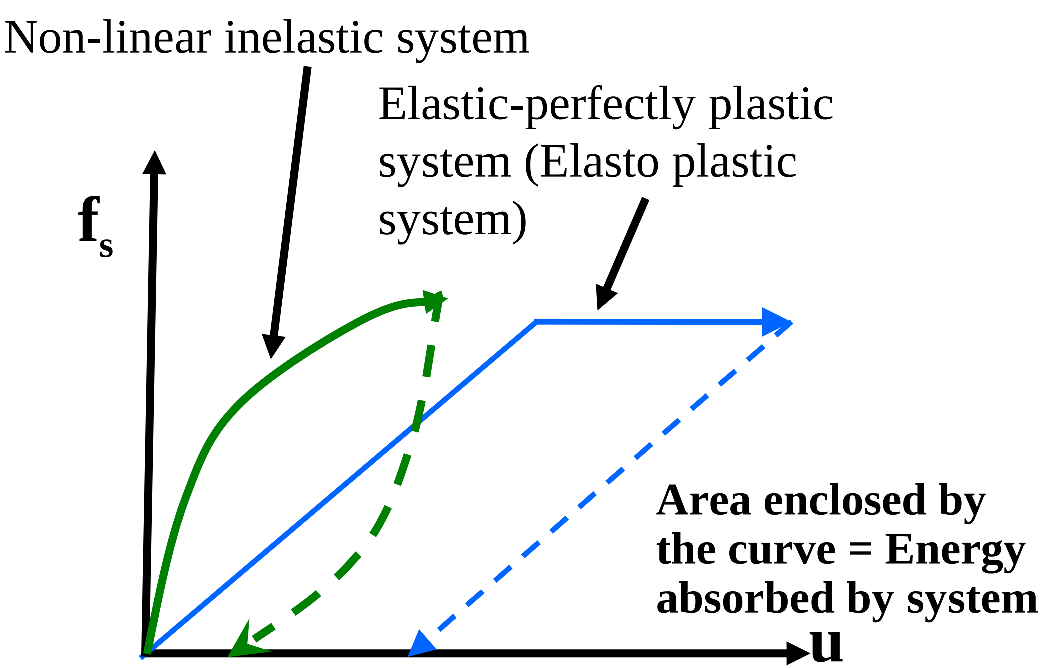
Earthquake Response of Linear System

- In this lecture, we will study the earthquake response of linear SDOF systems subjected to earthquake excitations.
- By definition, *linear systems* are *elastic systems*.
- They are also referred to as *linearly elastic systems* to emphasize both properties.

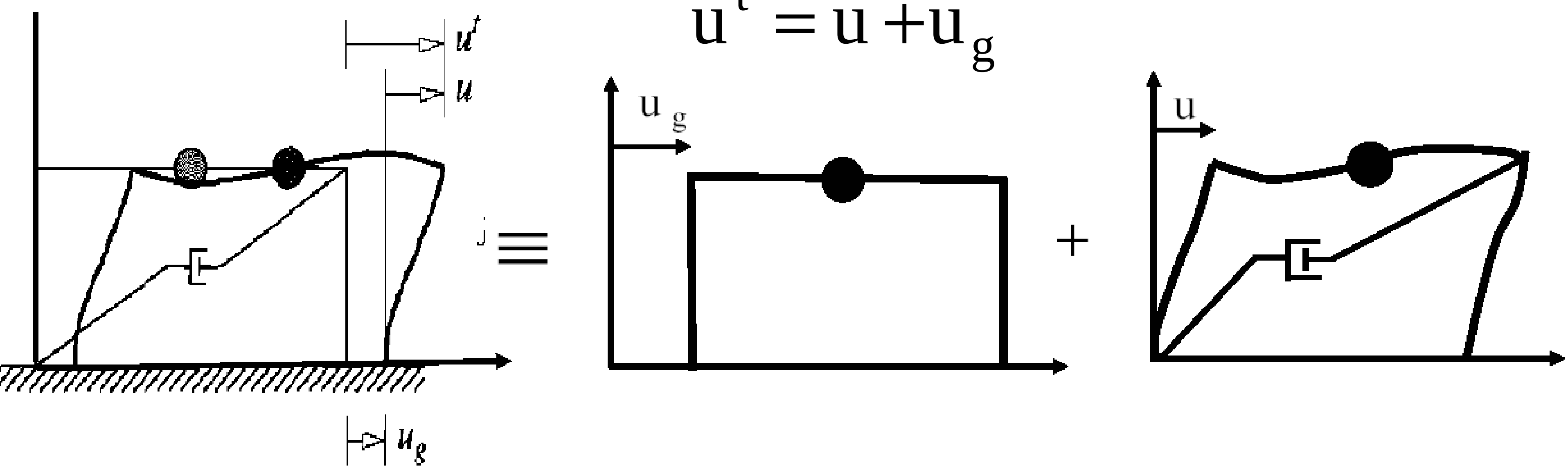




Effective Earthquake Force

Consider a single story frame with lumped mass. Let the frame at the base displaces by an amount u_g due to seismic waves. As a result lumped mass at the top displaces by an amount u^t ,such that:

$$u^t = u + u_g$$

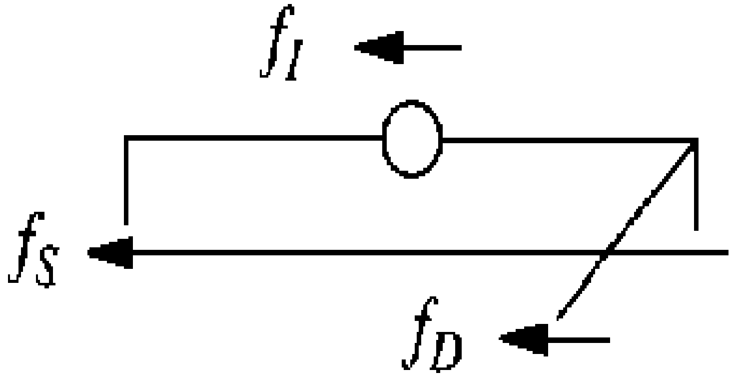


Where u_g= Ground displacement. u^t=Total displacement at the top end and u = Dynamic displacement of lumped mass at the top w.r.t shifted base.

Effective Earthquake Force

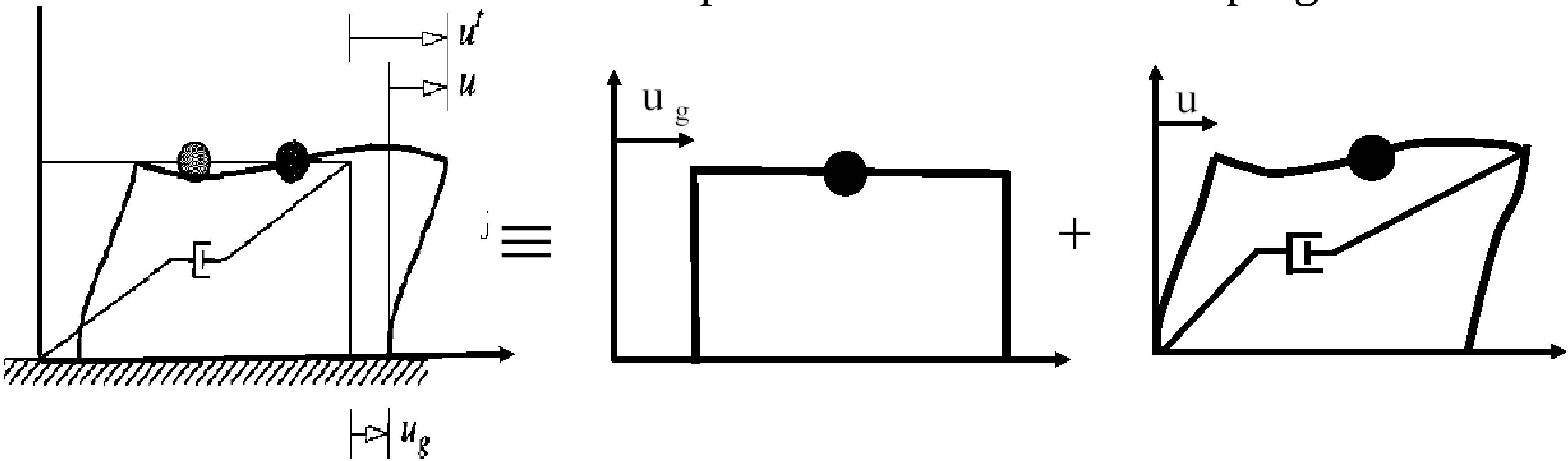
The equation of motion for the frame subjected to the earthquake excitation can be derived by using the using dynamic equilibrium of forces as:

$$f_I + f_D + f_S = 0$$



Effective Earthquake Force

Only the relative motion *u* between the mass and the base cause structural deformation which produces elastic and damping forces.



Thus for a linear system the inertial force f_I is related to the acceleration $\ddot{u}^t$ of the mass by:

$$f_I = m\ddot{u}^t;$$

$$f_D = c\dot{u} \;\text{ and }\; f_s = ku$$

Effective Earthquake Force

By substituting the value of f_I, the equation of motion become:

$$m\ddot{u}^t + c\dot{u} + ku = 0$$

$$or\ m(\ddot{u}_g + \ddot{u}) + c\dot{u} + ku = 0$$

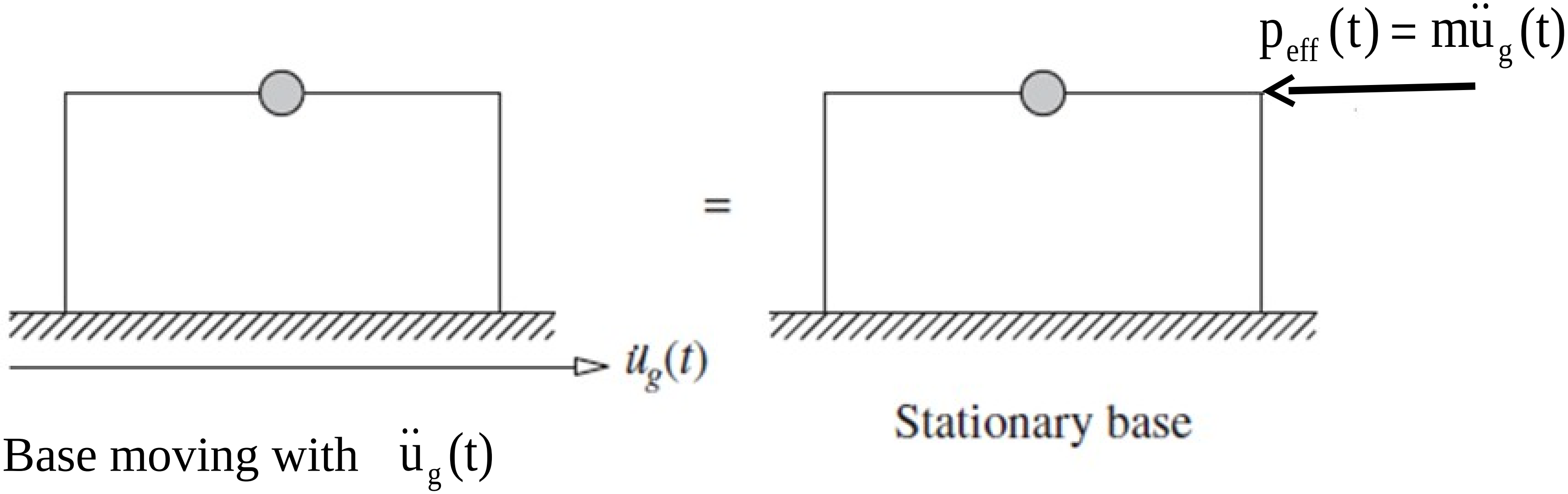
$$or\ m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

Comparing with $m\ddot{u} + c\dot{u} + ku = p(t)$

$$p(t) = p_{eff}(t) = -m\ddot{u}_g(t)$$

The term on the right-hand side of the equation may be regarded as the *Effective earthquake force*.

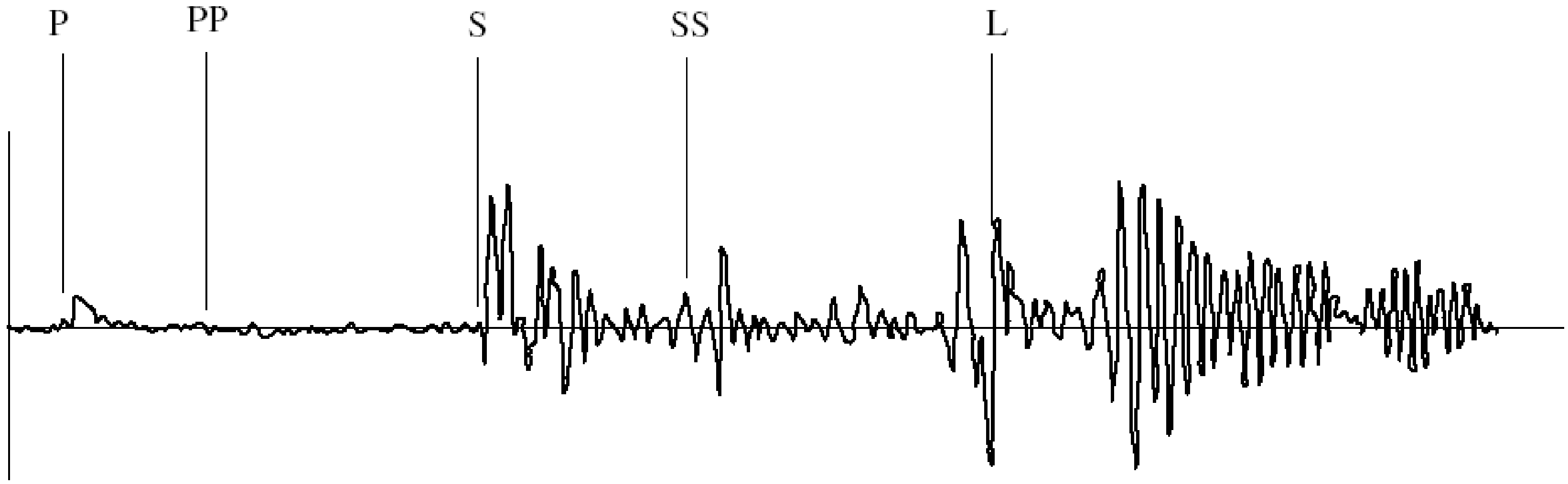
Effective Earthquake Force



Effective earthquake force: horizontal ground motion

Thus the ground motion can be replaced by the *effective earthquake force* (indicated by the subscript "***eff***". Since this force is proportional to the mass, thus, by increasing the mass the structural designer increases the effective earthquake force

Strong motion record

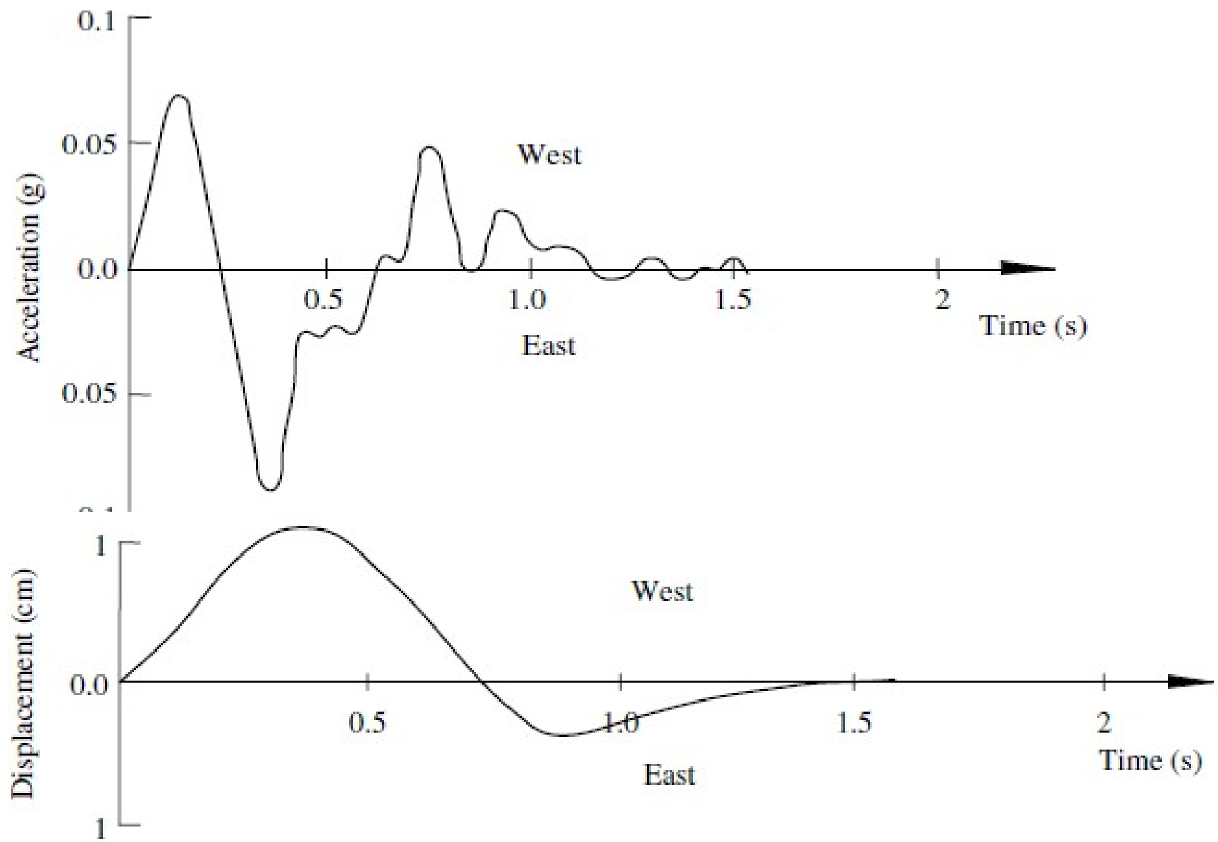


Typical strong motion record

Strong motion record

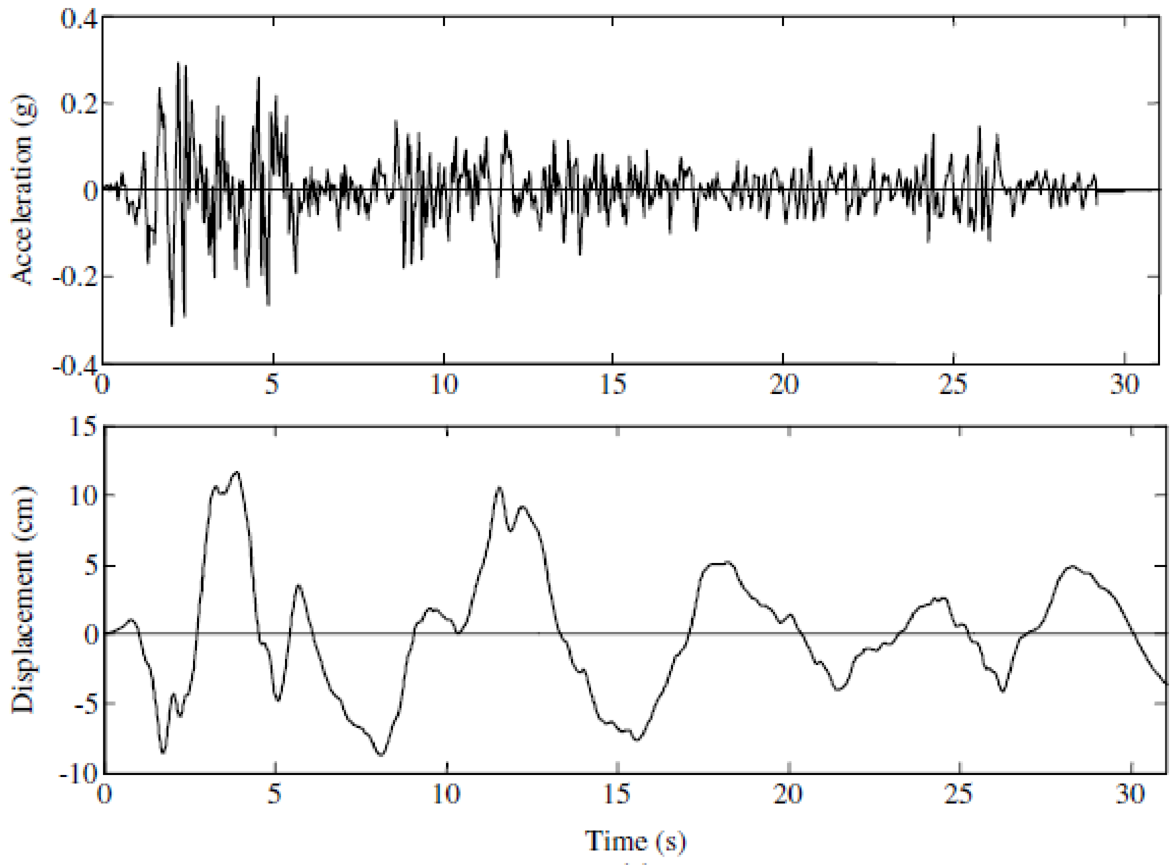
Strong earthquakes can generally be classified into three groups:

1. Practically a single shock: Acceleration, velocity, and displacement records for one such motion are shown in figure. A motion of this type occurs only at short distances from the epicenter, only on firm ground, and only for shallow earthquakes.



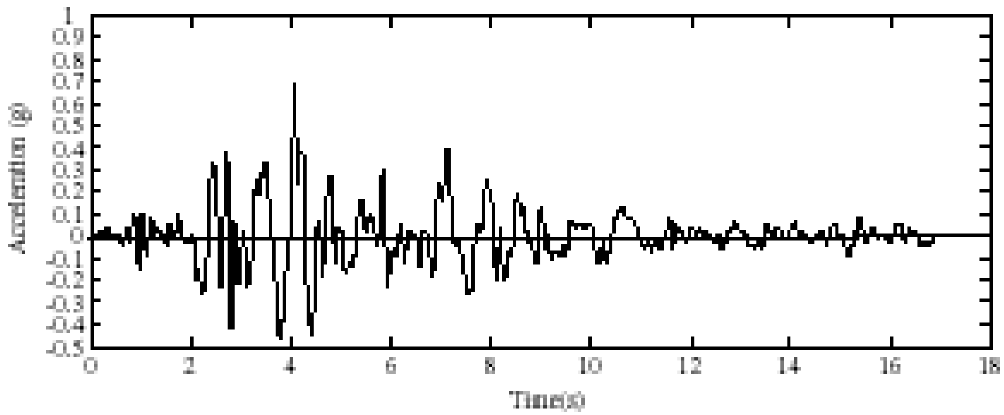
Strong motion record

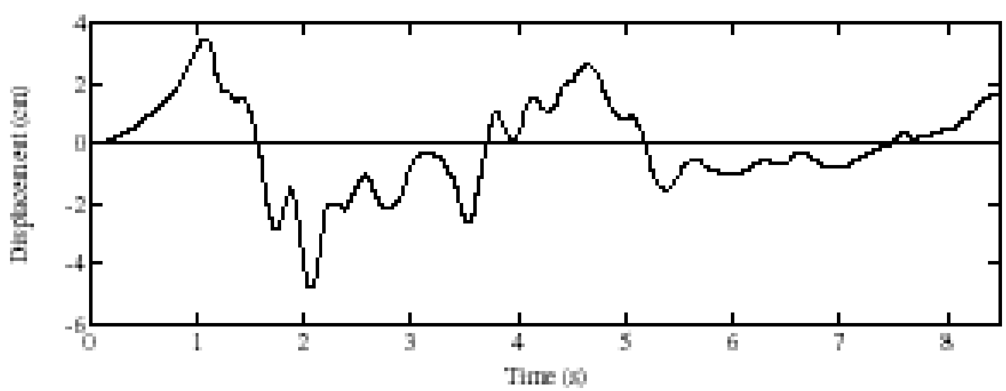
A moderately long, extremely irregular motion : The record of the earthquake of El Centro, California in 1940, NS component exemplifies this type of motion. It is associated with moderate distances from the focus and occurs only on firm ground. On such ground, almost all the major earthquakes originating along the Circumpacific Belt are of this type.



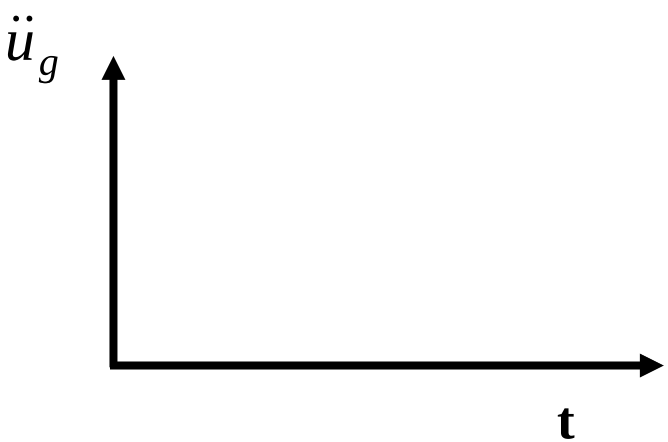
Strong motion record

A long ground motion exhibiting pronounced prevailing periods of vibration: A portion of the accelerogram obtained during the earthquake of 1989 in Loma Prieta is shown in figure to illustrate this type. Such motions result from the filtering of earthquakes of the preceding types through layers of soft soil within the range of linear or almost linear soil behavior and from the successive wave reflections at the interfaces of these layers.





Strong ground motions recorded in various earthquakes



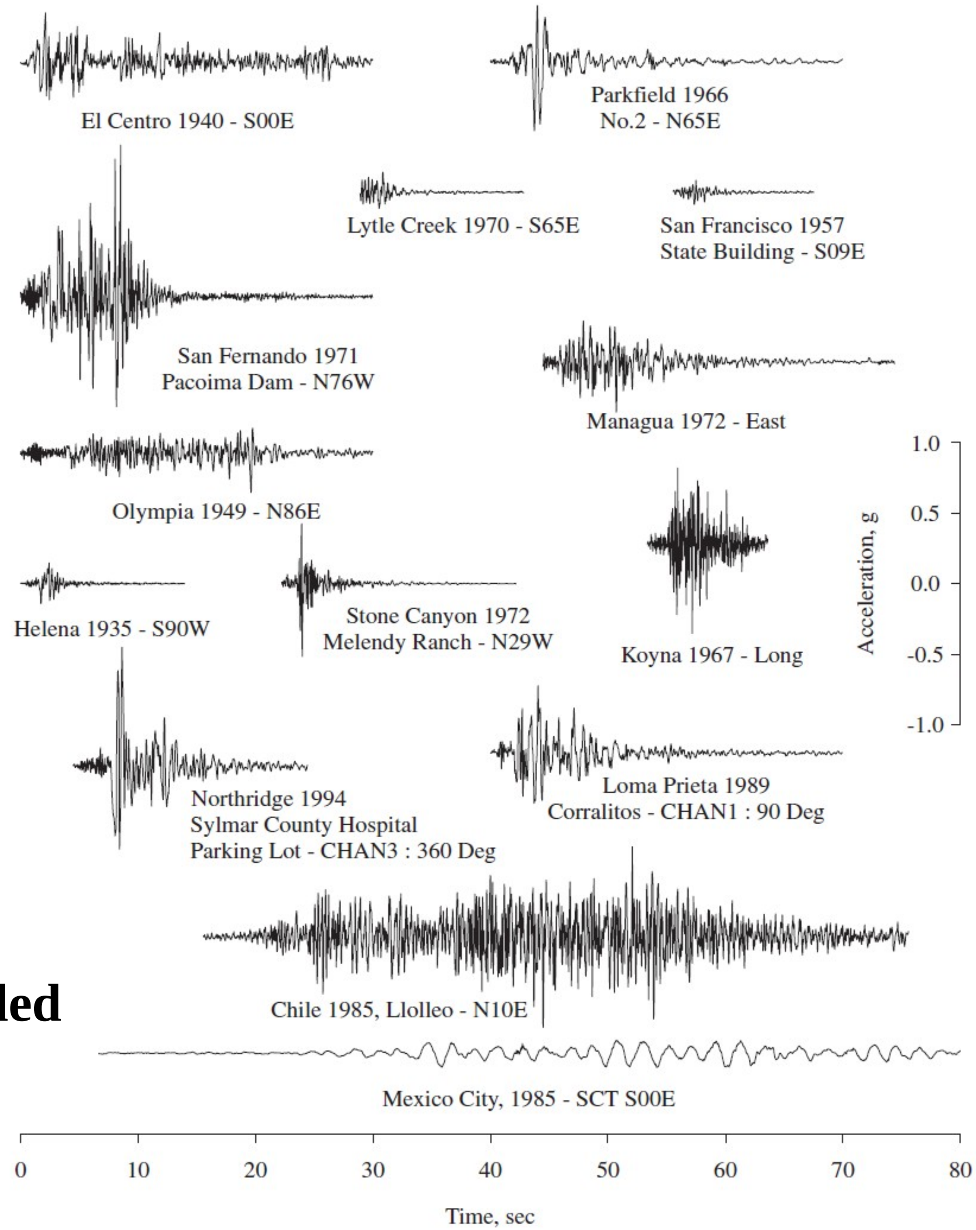


Figure : Ground motions recorded during several earthquakes.

Strong ground motion: Near source effect

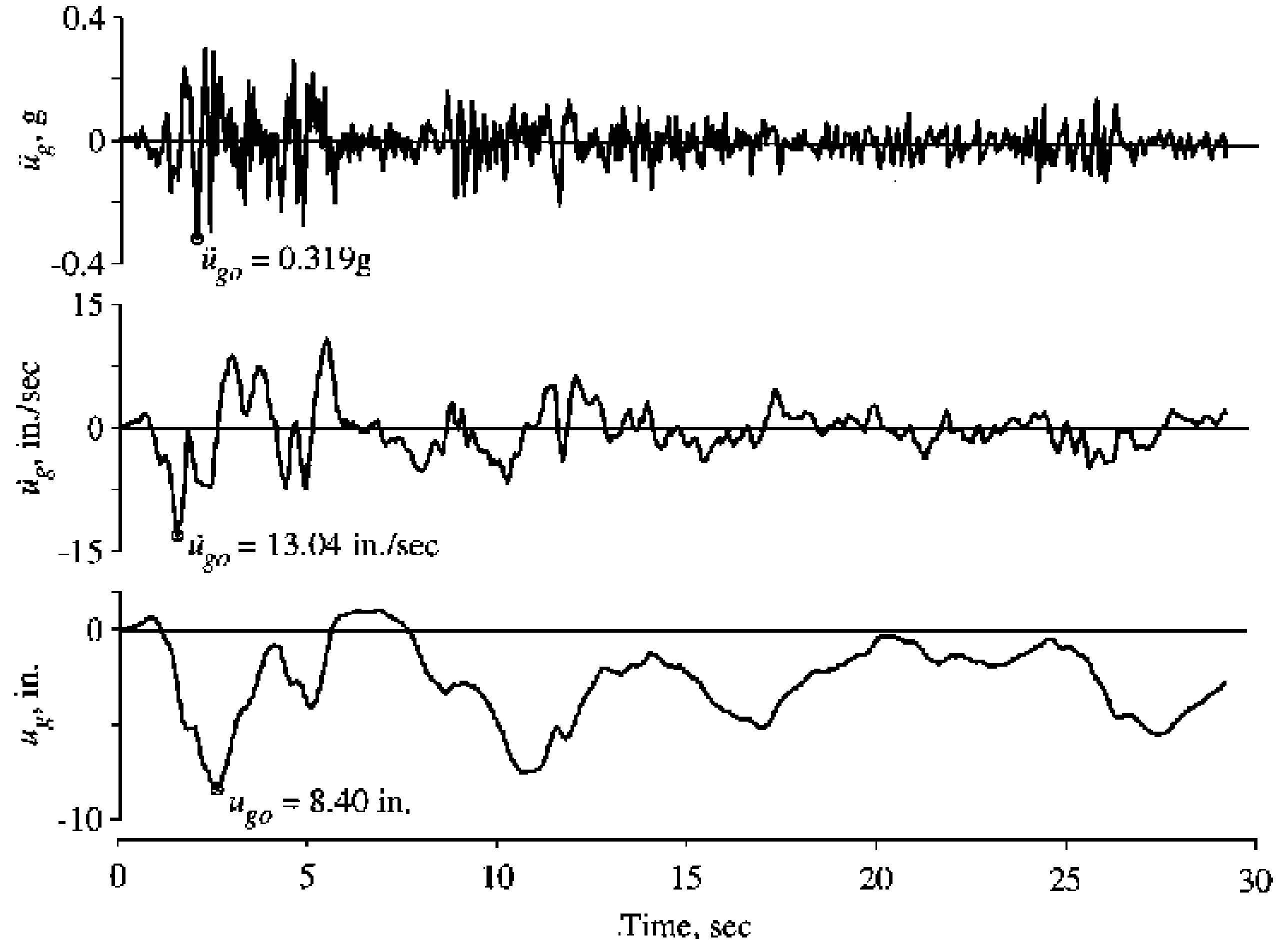
Richter Magnitude (M_L)	Mercalli Intensity (Approx.) at Epicenter	Effect on People and Buildings	Max. Acceleration (Approx. % of g)[a]	Maximum Velocity of Back-and-Forth Shaking (cm/sec.)	Time of Shaking[a] Near Source (sec.)	Displacement or Offset	Surface Rupture Length (km)
<2	I–II	Not felt by most people.	0.1–0.2				
~3	III	Felt indoors by some people.	<1.4	<1	0–2		
~4	IV–V	Felt by most people; dishes rattle, some break.	1.4–9	1–8	0–2		
~5	VI–VII[b]	Felt by all; many windows and some masonry cracks or falls.	9–34	8–31	2–5	~1 cm	1
~6	VIII–IX[c]	People frightened; most chimneys fall; major damage to poorly built structures.	34–124	31–116	10–15	60–140? cm	~8
~7	X–XI	People panic; most masonry structures and bridges destroyed.	>124	<116	20–30	~2 m	50–80
~8	XII	Nearly total damage to masonry structures; major damage to bridges, dams; rails bent.	>124	>150	>30	4.1 m	200–400
~9+	>XII	Nearly total destruction; people see ground surface move in waves; objects thrown into air.	>124		>80	13 m	>1,200

Accelerogram used in these lectures

Ground acceleration, $\ddot{u}_g$

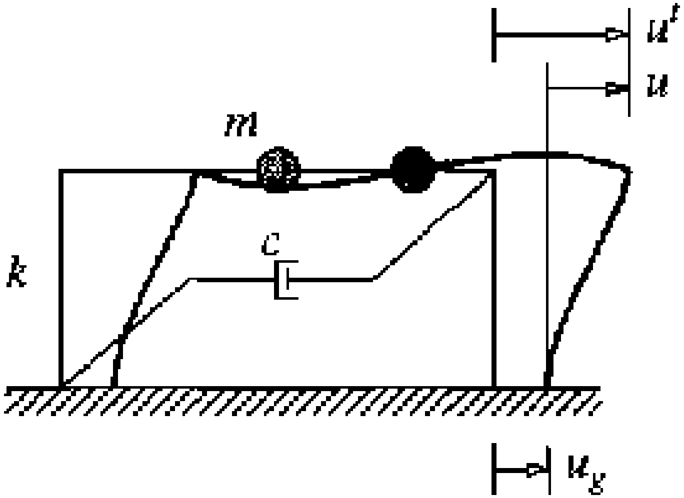
Ground velocity, $\dot{u}_g$

Ground displacement, u_g



N-S component of horizontal ground acceleration recoded at El Centro, California during the Imperial Valley earthquake of 1940

Equation of motion for SDOF system subjected to EQ excitations



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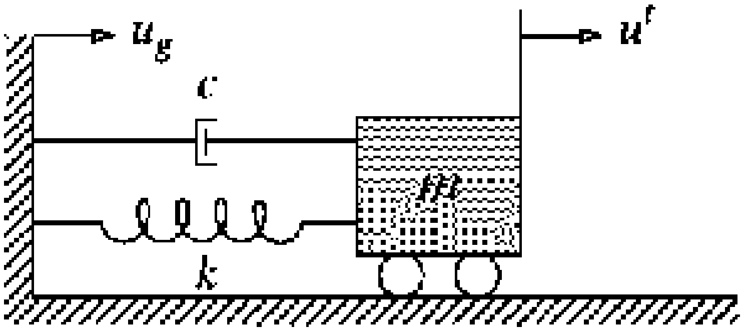
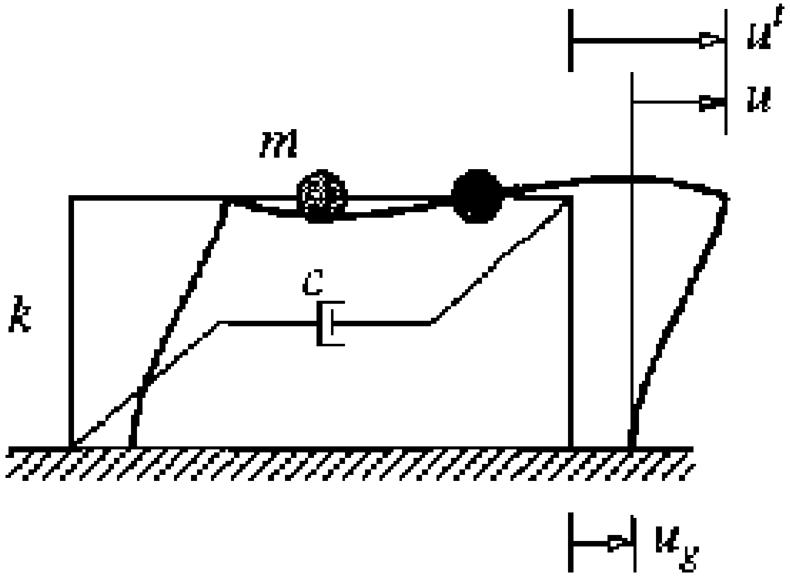


$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t) \Rightarrow \ddot{u} + \frac{c}{m}\dot{u} + \frac{k}{m}u = -\ddot{u}_g(t)$$

$$\text{Since } c = \zeta c_{cr} = \zeta\left(2m\omega_n\right) \text{ and } \sqrt{\frac{k}{m}} = \omega_n$$

$$\Rightarrow \ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^{2}u = -\ddot{u}_g(t)$$

Response quantities



Response is the structural system reaction to a demand coming from ground acceleration record

- Thus a response quantity may be structural displacement, velocity, acceleration, internal shear, bending moment, axial force etc.

- Sometime, the total acceleration, $\ddot{u}_o^t$, of the mass would be needed if the structure is supporting sensitive equipment and the motion imparted to the equipment is to be determined.

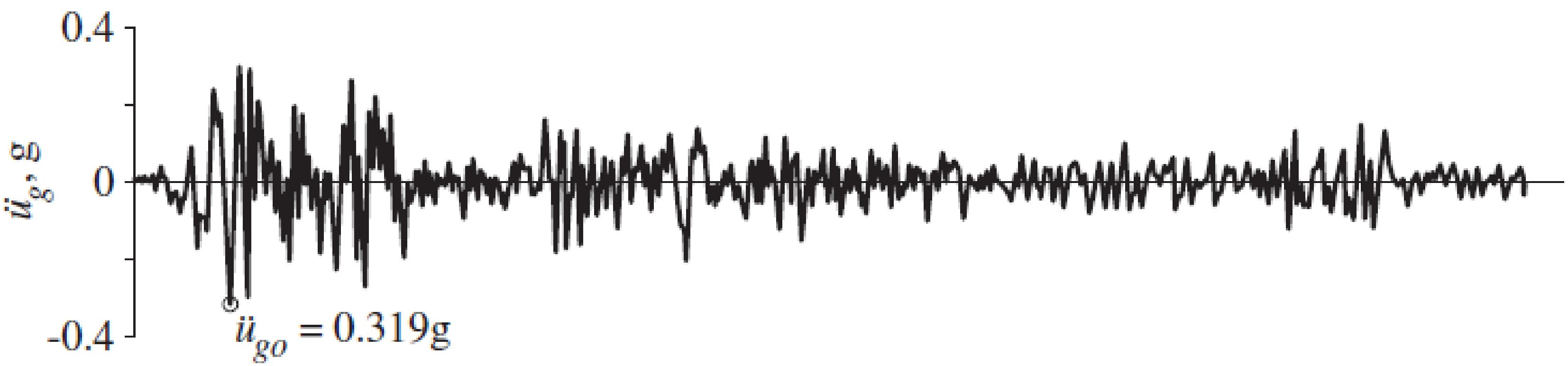
Response quantities

One of the important response quantity is total lateral displacement at the top end of structural system, u_o^t, required to provide enough separation between adjacent buildings to prevent their pounding against each other during an earthquake

Pounding damage, Hotel de carlo, Mexico city, 1985 earthquake

Solution to equation of motion for SDOF system subjected to EQ excitation

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$



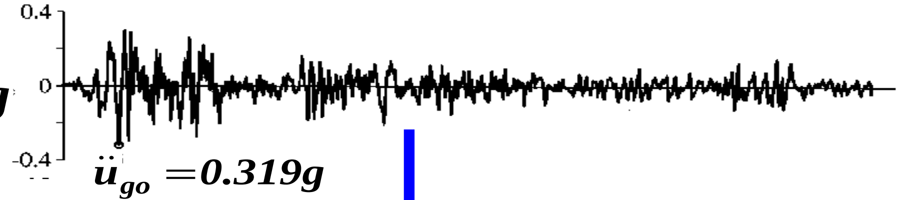
- The time variation of ground displacement, from the given time variation of ground acceleration, can be determined by using any appropriate time stepping numerical method.

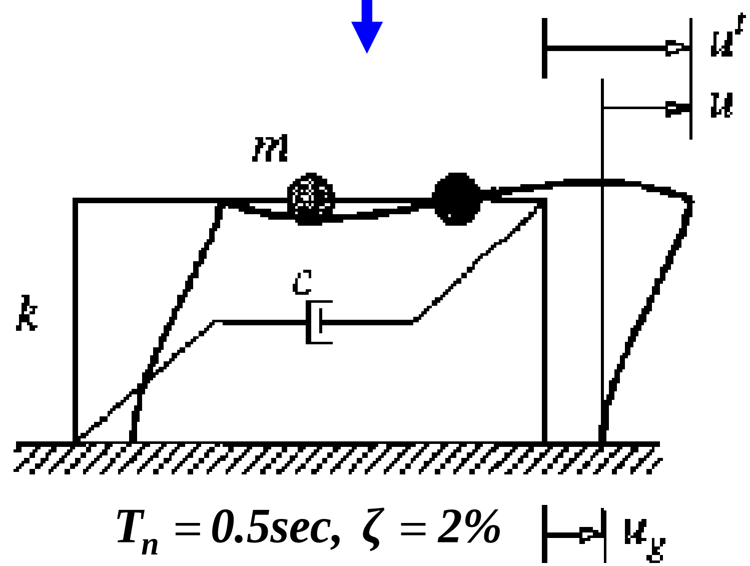
- Closer the time interval, more accurate will be solution. Typically, the time interval is chosen to be 1/100 to 1/50 of a second, requiring 1500 to 3000 ordinates to describe the round motion of above given El- Centro , 1940, ground acceleration record having a duration of 30 sec.

Structural disp., u , due to ground acceleration, $\ddot{u}_g$

El Centro,1940, ground acceleration

$\ddot{u}_g$, g



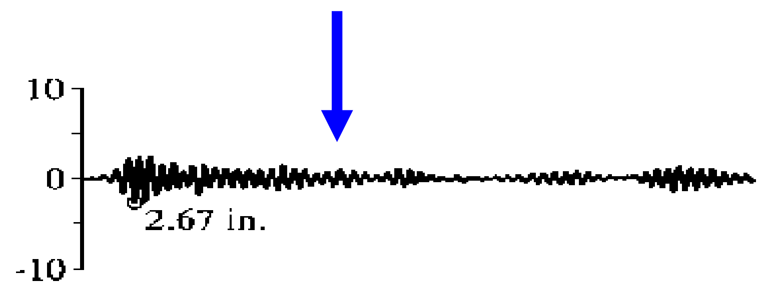


SDOF system with

$$T_n = 0.5\text{sec}, \zeta = 2\%$$

Corresponding relative displacement at the top end of the SDOF frame

u, in



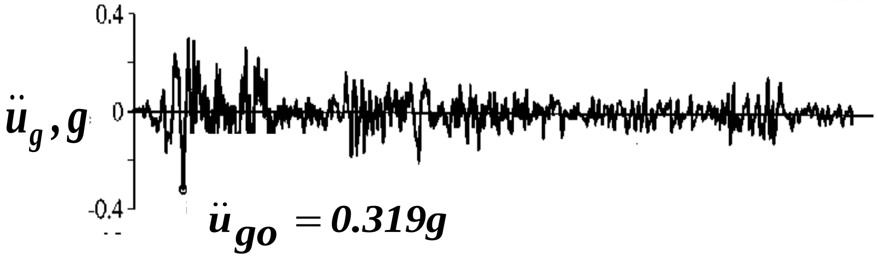
Influence of T_n and ζ on Peak displacement, u_o , in a liner elastic SDOF system

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$

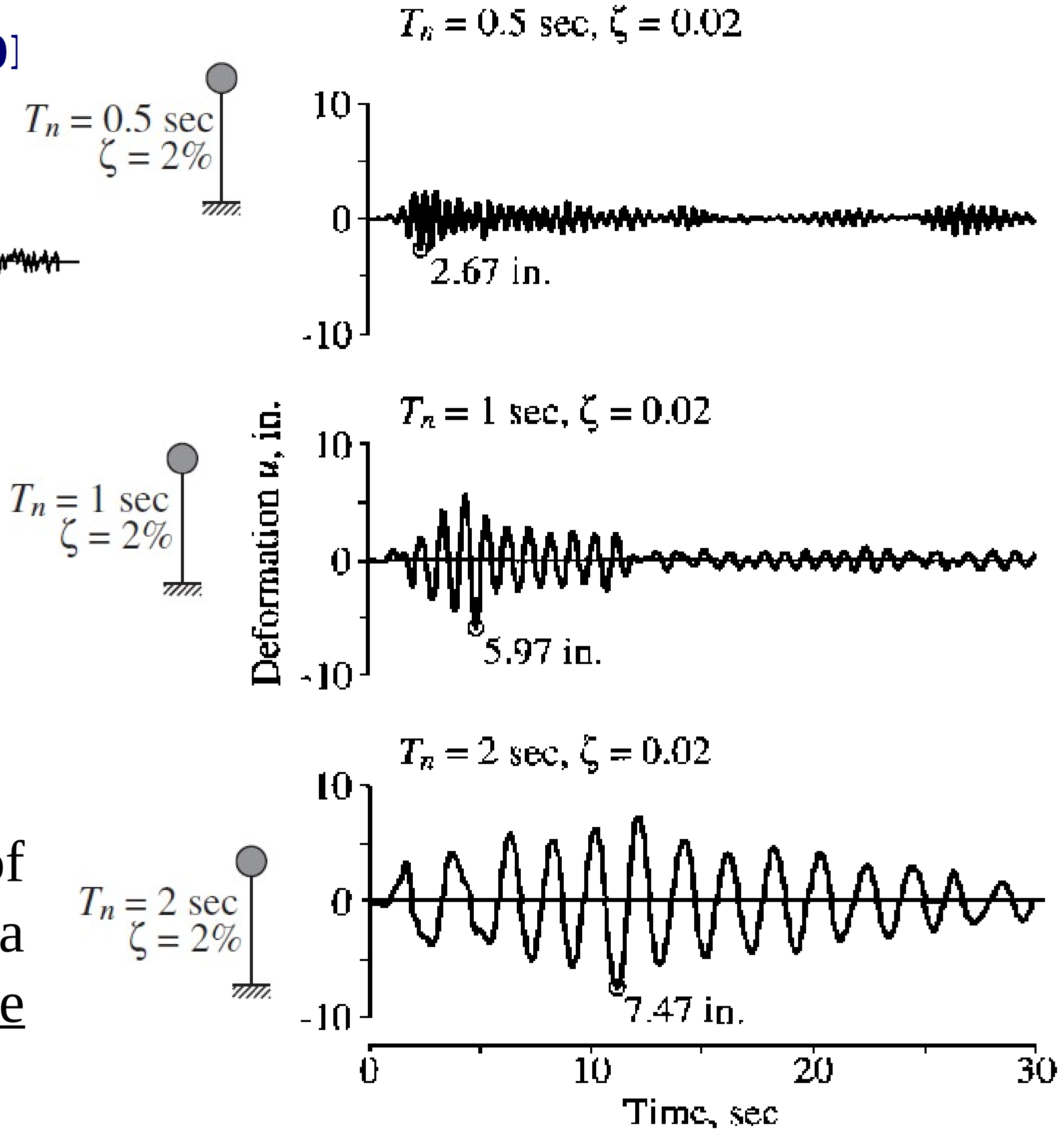
The above given equation indicates that $u = f(T_n, \zeta)$

Thus any two systems having the same values of T_n and ζ will have the same deformation response u(t) even though one system may be more massive than the other or one may be stiffer than the other

Effect of T_n on Deformation response histo



El Centro ground acceleration



In general, peak value of displacement at the top end of a SDOF increases with the increase in the time period of the system.

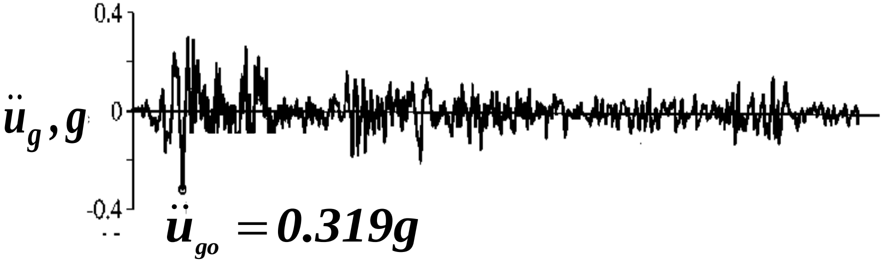
Response of SDOF systems with different values of T_n to El Centro ground acceleration

Effect of Time Period

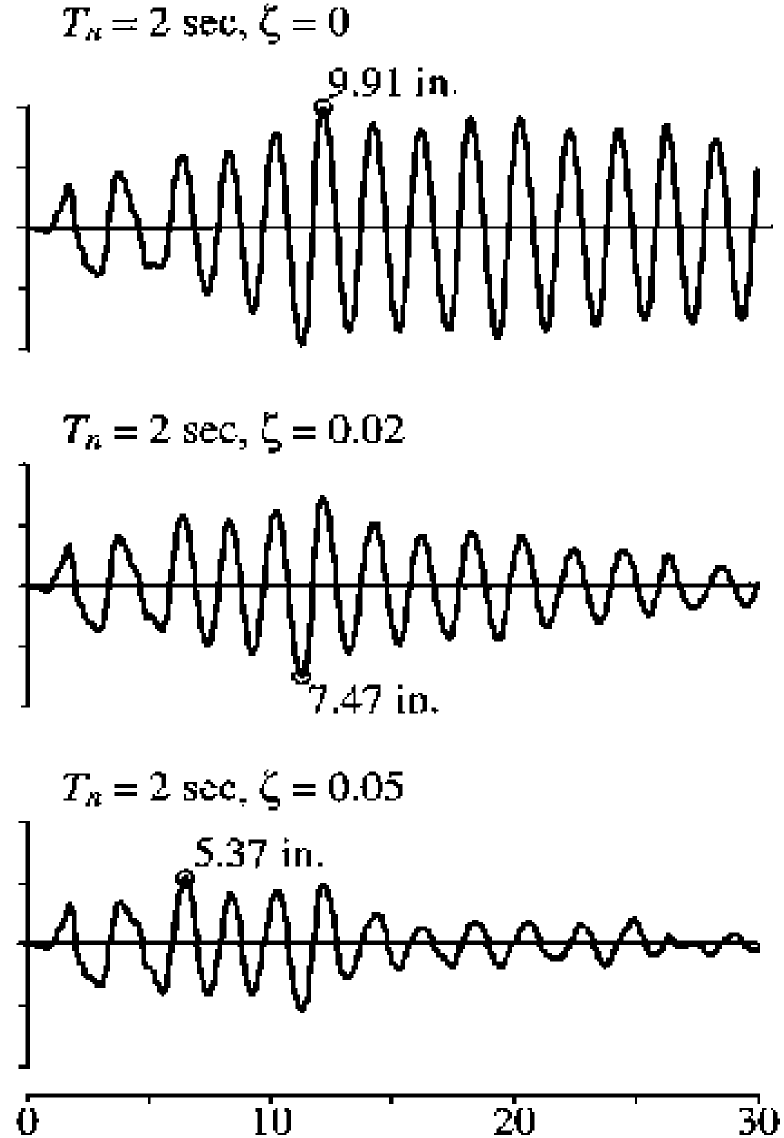
In general, for ground motions recorded on rock or firm soil sites an increase in period of vibration produces an *increase* in maximum relative displacement.

NOTE: The above observation is valid only on average. For a specific record it may not be valid over the entire period range. It also does not apply to very long periods (e.g., with periods longer than 3 or 4 s)

Effect of ζ on Deformation response history



El Centro ground acceleration



In general, peak value of displacement at the top end of a SDOF <u>increases with the decrease in the damping ratio of the system</u>

Response of SDOF systems with different values of ζ to El Centro ground acceleration

Approximate Periods of Vibration (ASCE 7-05)

$$T_a = C_t h_n^x$$

C_t = 0.028, x = 0.8 for steel moment frames
C_t = 0.016, x = 0.9 for concrete moment frames
C_t = 0.030, x = 0.75 for eccentrically braced frames
C_t = 0.020, x = 0.75 for all other systems

Note: This applies ONLY to building structures!

Where h is the height of building in ft.

$$T_a = 0.1N$$

For moment frames < 12 stories in height, minimum story height of 10 feet. N = number of stories.

Approximate Periods of Vibration

- Thus, structural systems with T_n=0.5sec, 1 and 2 sec may be considered as 5, 10 and 20 story height buildings, respectively.
- A building with 3 story height can be considered as Multi DOF system with at least 3 DOFs.
- To keep the discussion simple at this stage, it will be a reasonable assumption to state that (out of 3 natural time periods of the 3 story building) we consider only fundamental natural time period (T_n=0.3 sec) to determine the response quantities for the building.
- Later on we will discuss how all 3 vibration modes (and the corresponding natural time periods) are calculated and are taken into account to find the total response of a building with DOF =3
- Because the empirical period formula is based on measured response of buildings, it should not be used to estimate the period for other types of structure (bridges, dams, towers).

Response spectrum concept

- A plot of the peak value of a response quantity as a function of the natural vibration period T_n of the system, or a related parameter such as circular frequency ω_n or cyclic frequency f_n, is called the *response spectrum* for that quantity.

- **Response** is the structural system reaction to a demand coming from ground acceleration record (i.e. Accelerogram) and when the peak response commodities such as structural system displacement (u_o), velocity $(\dot{u}_o)$ and acceleration $(\ddot{u}^t_o)$ are plotted against the structural system natural time period (or frequencies) will be called **spectrum**

Response spectrum concept

- Peak values of **response quantities** and shape of **response spectrum** depends on the **accelerogram**

- Each such plot is for SDOF system having a fixed damping ratio ζ, and several such plots for different values of ζ are included to cover the range of damping values encountered in actual structures.

- The *deformation response spectrum* is a plot of u_o against T_n for fixed ζ. A similar plot for $\dot{u}_o$ is the *relative velocity response spectrum*, and for $\ddot{u}_o^t$ is the *total acceleration response spectrum*.

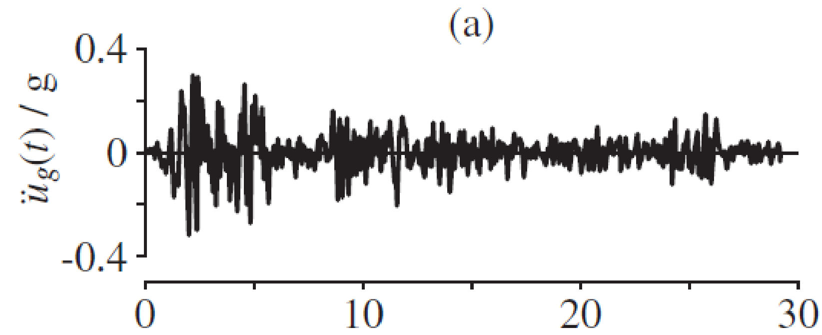
Deformation response spectrum

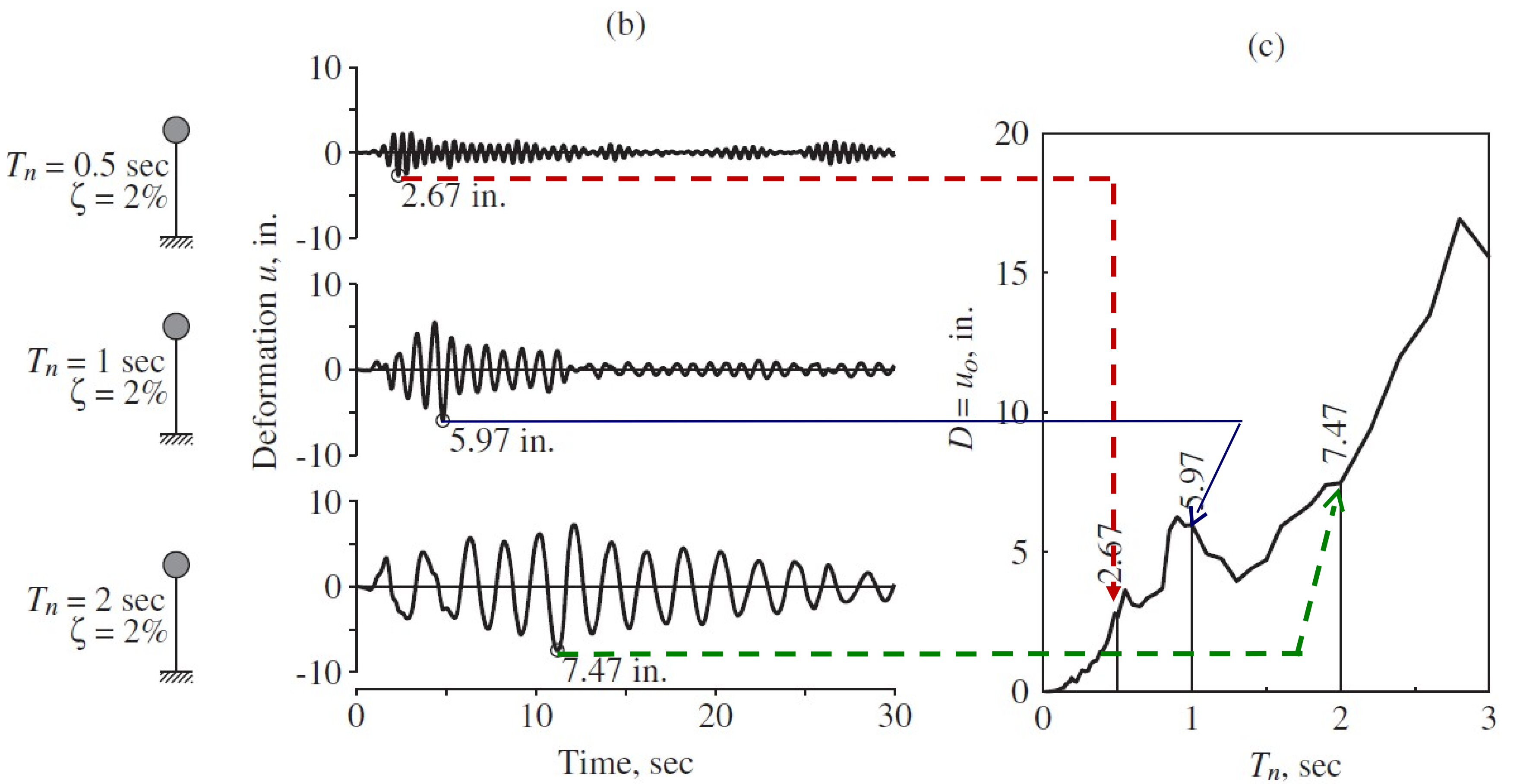
- Figure on next slide shows the procedure to determine the deformation response spectrum. The spectrum is developed for El Centro ground motions, as shown in part (a) of the figure.

- The time variation of deformation induced by this ground motion in three SDF systems is presented in part (b) of the figure

- The peak value of deformation $D \equiv u_o$, determined for SDF system with different T_n is determined and shown in part (c) of the Figure

Construction of deformation response spectrum





(a) El-centro ground acceleration; (b) Deformation response of three SDF systems with ζ=2% and T_n=0.5,1, and 2 sec; (c) Deformation response spectrum for ζ=2%