

## 35-7 Interference from Thin Films

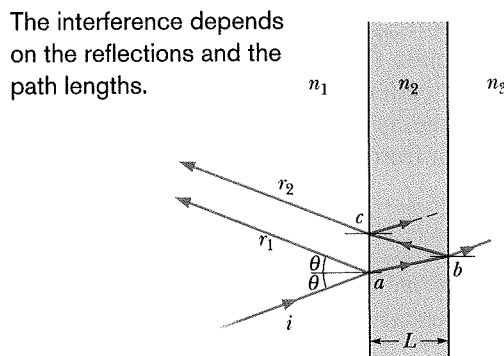
The colors we see when sunlight illuminates a soap bubble or an oil slick are caused by the interference of light waves reflected from the front and back surfaces of a thin transparent film. The thickness of the soap or oil film is typically of the order of magnitude of the wavelength of the (visible) light involved. (Greater thicknesses spoil the coherence of the light needed to produce the colors due to interference.)

Figure 35-15 shows a thin transparent film of uniform thickness  $L$  and index of refraction  $n_2$ , illuminated by bright light of wavelength  $\lambda$  from a distant point source. For now, we assume that air lies on both sides of the film and thus that  $n_1 = n_3$  in Fig. 35-15. For simplicity, we also assume that the light rays are almost perpendicular to the film ( $\theta \approx 0$ ). We are interested in whether the film is bright or dark to an observer viewing it almost perpendicularly. (Since the film is brightly illuminated, how could it possibly be dark? You will see.)

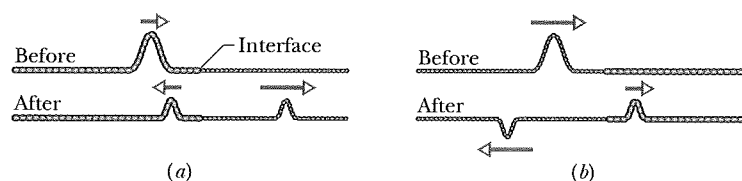
The incident light, represented by ray  $i$ , intercepts the front (left) surface of the film at point  $a$  and undergoes both reflection and refraction there. The reflected ray  $r_1$  is intercepted by the observer's eye. The refracted light crosses the film to point  $b$  on the back surface, where it undergoes both reflection and refraction. The light reflected at  $b$  crosses back through the film to point  $c$ , where it undergoes both reflection and refraction. The light refracted at  $c$ , represented by ray  $r_2$ , is intercepted by the observer's eye.

If the light waves of rays  $r_1$  and  $r_2$  are exactly in phase at the eye, they produce an interference maximum and region  $ac$  on the film is bright to the observer. If they are exactly out of phase, they produce an interference minimum and region  $ac$  is dark to the observer, *even though it is illuminated*. If there is some intermediate phase difference, there are intermediate interference and brightness.

Thus, the key to what the observer sees is the phase difference between the waves of rays  $r_1$  and  $r_2$ . Both rays are derived from the same ray  $i$ , but the path involved in producing  $r_2$  involves light traveling twice across the film ( $a$  to  $b$ , and then  $b$  to  $c$ ), whereas the path involved in producing  $r_1$  involves no travel through the film. Because  $\theta$  is about zero, we approximate the path length difference between the waves of  $r_1$  and  $r_2$  as  $2L$ . However, to find the phase difference between the waves, we cannot just find the number of wavelengths  $\lambda$  that is equivalent to a path length difference of  $2L$ . This simple approach is impossible




**Fig. 35-15** Light waves, represented with ray  $i$ , are incident on a thin film of thickness  $L$  and index of refraction  $n_2$ . Rays  $r_1$  and  $r_2$  represent light waves that have been reflected by the front and back surfaces of the film, respectively. (All three rays are actually nearly perpendicular to the film.) The interference of the waves of  $r_1$  and  $r_2$  with each other depends on their phase difference. The index of refraction  $n_1$  of the medium at the left can differ from the index of refraction  $n_3$  of the medium at the right, but for now we assume that both media are air, with  $n_1 = n_3 = 1.0$ , which is less than  $n_2$ .



**Fig. 35-16** Phase changes when a pulse is reflected at the interface between two stretched strings of different linear densities. The wave speed is greater in the lighter string. (a) The incident pulse is in the denser string. (b) The incident pulse is in the lighter string. Only here is there a phase change, and only in the reflected wave.

for two reasons: (1) the path length difference occurs in a medium other than air, and (2) reflections are involved, which can change the phase.

 The phase difference between two waves can change if one or both are reflected.

Before we continue our discussion of interference from thin films, we must discuss changes in phase that are caused by reflections.

### Reflection Phase Shifts

Refraction at an interface never causes a phase change—but reflection can, depending on the indexes of refraction on the two sides of the interface. Figure 35-16 shows what happens when reflection causes a phase change, using as an example pulses on a denser string (along which pulse travel is relatively slow) and a lighter string (along which pulse travel is relatively fast).

When a pulse traveling relatively slowly along the denser string in Fig. 35-16a reaches the interface with the lighter string, the pulse is partially transmitted and partially reflected, with no change in orientation. For light, this situation corresponds to the incident wave traveling in the medium of greater index of refraction  $n$  (recall that greater  $n$  means slower speed). In that case, the wave that is reflected at the interface does not undergo a change in phase; that is, its *reflection phase shift* is zero.

When a pulse traveling more quickly along the lighter string in Fig. 35-16b reaches the interface with the denser string, the pulse is again partially transmitted and partially reflected. The transmitted pulse again has the same orientation as the incident pulse, but now the reflected pulse is inverted. For a sinusoidal wave, such an inversion involves a phase change of  $\pi$  rad, or half a wavelength. For light, this situation corresponds to the incident wave traveling in the medium of lesser index of refraction (with greater speed). In that case, the wave that is reflected at the interface undergoes a phase shift of  $\pi$  rad, or half a wavelength.

We can summarize these results for light in terms of the index of refraction of the medium off which (or from which) the light reflects:



Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength

This might be remembered as “higher means half.”

### Equations for Thin-Film Interference

In this chapter we have now seen three ways in which the phase difference between two waves can change:

1. by reflection
2. by the waves traveling along paths of different lengths
3. by the waves traveling through media of different indexes of refraction

When light reflects from a thin film, producing the waves of rays  $r_1$  and  $r_2$  shown in Fig. 35-15, all three ways are involved. Let us consider them one by one.

We first reexamine the two reflections in Fig. 35-15. At point  $a$  on the front interface, the incident wave (in air) reflects from the medium having the higher of the two indexes of refraction; so the wave of reflected ray  $r_1$  has its phase shifted by 0.5 wavelength. At point  $b$  on the back interface, the incident wave reflects from the medium (air) having the lower of the two indexes of refraction; so the wave reflected there is not shifted in phase by the reflection, and thus neither is the portion of it that exits the film as ray  $r_2$ . We can organize this information with the first line in Table 35-1, which refers to the simplified drawing in Fig. 35-17 for a thin film in air. So far, as a result of the reflection phase shifts, the waves of  $r_1$  and  $r_2$  have a phase difference of 0.5 wavelength and thus are exactly out of phase.

Now we must consider the path length difference  $2L$  that occurs because the wave of ray  $r_2$  crosses the film twice. (This difference  $2L$  is shown on the second line in Table 35-1.) If the waves of  $r_1$  and  $r_2$  are to be exactly in phase so that they produce fully constructive interference, the path length  $2L$  must cause an additional phase difference of 0.5, 1.5, 2.5, . . . wavelengths. Only then will the net phase difference be an integer number of wavelengths. Thus, for a bright film, we must have

$$2L = \frac{\text{odd number}}{2} \times \text{wavelength} \quad (\text{in-phase waves}). \quad (35-31)$$

The wavelength we need here is the wavelength  $\lambda_{n_2}$  of the light in the medium containing path length  $2L$ —that is, in the medium with index of refraction  $n_2$ . Thus, we can rewrite Eq. 35-31 as

$$2L = \frac{\text{odd number}}{2} \times \lambda_{n_2} \quad (\text{in-phase waves}). \quad (35-32)$$

If, instead, the waves are to be exactly out of phase so that there is fully destructive interference, the path length  $2L$  must cause either no additional phase difference or a phase difference of 1, 2, 3, . . . wavelengths. Only then will the net phase difference be an odd number of half-wavelengths. For a dark film, we must have

$$2L = \text{integer} \times \text{wavelength} \quad (\text{out-of-phase waves}). \quad (35-33)$$

where, again, the wavelength is the wavelength  $\lambda_{n_2}$  in the medium containing  $2L$ . Thus, this time we have

$$2L = \text{integer} \times \lambda_{n_2} \quad (\text{out-of-phase waves}). \quad (35-34)$$

Now we can use Eq. 35-8 ( $\lambda_n = \lambda/n$ ) to write the wavelength of the wave of ray  $r_2$  inside the film as

$$\lambda_{n_2} = \frac{\lambda}{n_2}, \quad (35-35)$$

where  $\lambda$  is the wavelength of the incident light in vacuum (and approximately also in air). Substituting Eq. 35-35 into Eq. 35-32 and replacing “odd number/2” with  $(m + \frac{1}{2})$  give us

$$2L = (m + \frac{1}{2}) \frac{\lambda}{n_2}, \quad \text{for } m = 0, 1, 2, \dots \quad (\text{maxima—bright film in air}). \quad (35-36)$$

Table 35-1

An Organizing Table for Thin-Film Interference in Air (Fig. 35-17)<sup>a</sup>

Reflection	$r_1$	$r_2$
phase shifts	0.5 wavelength	0
Path length difference	$2L$	
Index in which path length difference occurs	$n_2$	
In phase <sup>a</sup> :	$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2}$	
Out of phase <sup>a</sup> :	$2L = \text{integer} \times \frac{\lambda}{n_2}$	

<sup>a</sup>Valid for  $n_2 > n_1$  and  $n_2 > n_3$ .

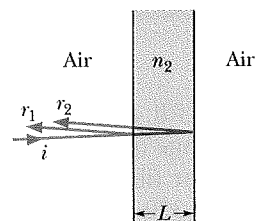


Fig. 35-17 Reflections from a thin film in air.