$= x^3$

Integration by Method of Substitution

Q8:- Evaluate $\int \frac{a\,dt}{2\sqrt{at+b}} \rightarrow (1)$

Let $at+b = u \Rightarrow a\,dt = du$

Putting values in (1), we get

$$\int \frac{a\,dt}{2\sqrt{at+b}} = \int \frac{du}{2\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{1}{2}\left[\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]+c$$

$$= \frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]+c = u^{\frac{1}{2}}+c = \sqrt{at+b}+c$$

Q9:- Evaluate $\int \frac{x}{\sqrt{4+x^2}}dx$

Put $4+x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{1}{2}dt$

$$\int \frac{x}{\sqrt{4+x^2}}dx = \int \frac{1}{\sqrt{t}}\left(\frac{1}{2}\right)dt = \frac{1}{2}\int t^{-\frac{1}{2}}dt = \frac{1}{2}\frac{t^{1/2}}{1/2}+c$$

$$= \sqrt{t}+c = \sqrt{4+x^2}+c$$

Q10: $\int x\sqrt{x-a}\,dx$, let $x-a = t \Rightarrow x = a+t$, $dx = dt$

$$\int x\sqrt{x-a}\,dx = \int (a+t)\sqrt{t}\,dt = \int \left(at^{\frac{1}{2}} + t^{\frac{3}{2}}\right)dt$$

$$= a\int t^{\frac{1}{2}}dt + \int t^{\frac{3}{2}}dt = a\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{5}{2}}}{\frac{5}{2}}+c$$

$$= \frac{2a}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}}+c = 2t^{\frac{3}{2}}\left(\frac{a}{3}+\frac{1}{5}t\right)+c$$

$$= 2(x-a)^{\frac{3}{2}}\left(\frac{a}{3}+\frac{1}{5}(x-a)\right)+c = 2(x-a)^{\frac{3}{2}}\left(\frac{5a+3(x-a)}{15}\right)+c$$