

Sample Problem

Polarization and intensity with three polarizing sheets

Figure 33-15a, drawn in perspective, shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y axis, that of the second sheet is at an angle of 60° counterclockwise from the y axis, and that of the third sheet is parallel to the x axis. What fraction of the initial intensity I_0 of the light emerges from the three-sheet system, and in which direction is that emerging light polarized?

KEY IDEAS

1. We work through the system sheet by sheet, from the first one encountered by the light to the last one.
2. To find the intensity transmitted by any sheet, we apply either the one-half rule or the cosine-squared rule, depending on whether the light reaching the sheet is unpolarized or already polarized.
3. The light that is transmitted by a polarizing sheet is always polarized parallel to the polarizing direction of the sheet.

First sheet: The original light wave is represented in Fig. 33-15b, using the head-on, double-arrow representation of Fig. 33-10b. Because the light is initially unpolarized, the intensity I_1 of the light transmitted by the first sheet is given by the one-half rule (Eq. 33-36):

$$I_1 = \frac{1}{2}I_0.$$

Because the polarizing direction of the first sheet is parallel to the y axis, the polarization of the light transmitted by it is also, as shown in the head-on view of Fig. 33-15c.

Second sheet: Because the light reaching the second sheet is polarized, the intensity I_2 of the light transmitted by that sheet is given by the cosine-squared rule (Eq. 33-38). The angle

θ in the rule is the angle between the polarization direction of the entering light (parallel to the y axis) and the polarizing direction of the second sheet (60° counterclockwise from the y axis), and so θ is 60° . (The larger angle between the two directions, namely 120° , can also be used.) We have

$$I_2 = I_1 \cos^2 60^\circ.$$

The polarization of this transmitted light is parallel to the polarizing direction of the sheet transmitting it—that is, 60° counterclockwise from the y axis, as shown in the head-on view of Fig. 33-15d.

Third sheet: Because the light reaching the third sheet is polarized, the intensity I_3 of the light transmitted by that sheet is given by the cosine-squared rule. The angle θ is now the angle between the polarization direction of the entering light (Fig. 33-15d) and the polarizing direction of the third sheet (parallel to the x axis), and so $\theta = 30^\circ$. Thus,

$$I_3 = I_2 \cos^2 30^\circ.$$

This final transmitted light is polarized parallel to the x axis (Fig. 33-15e). We find its intensity by substituting first for I_2 and then for I_1 in the equation above:

$$\begin{aligned} I_3 &= I_2 \cos^2 30^\circ = (I_1 \cos^2 60^\circ) \cos^2 30^\circ \\ &= \left(\frac{1}{2}I_0\right) \cos^2 60^\circ \cos^2 30^\circ = 0.094I_0. \end{aligned}$$

Thus, $\frac{I_3}{I_0} = 0.094$. (Answer)

That is to say, 9.4% of the initial intensity emerges from the three-sheet system. (If we now remove the second sheet, what fraction of the initial intensity emerges from the system?)



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33-8 Reflection and Refraction

Although a light wave spreads as it moves away from its source, we can often approximate its travel as being in a straight line; we did so for the light wave in Fig. 33-5a. The study of the properties of light waves under that approximation is called *geometrical optics*. For the rest of this chapter and all of Chapter 34, we shall discuss the geometrical optics of visible light.

The photograph in Fig. 33-16a shows an example of light waves traveling in approximately straight lines. A narrow beam of light (the *incident beam*), angled downward from the left and traveling through air, encounters a *plane* (flat) water surface. Part of the light is **reflected** by the surface, forming a beam directed upward toward the right, traveling as if the original beam had bounced from the surface. The rest of the light travels through the surface and into the water, form-

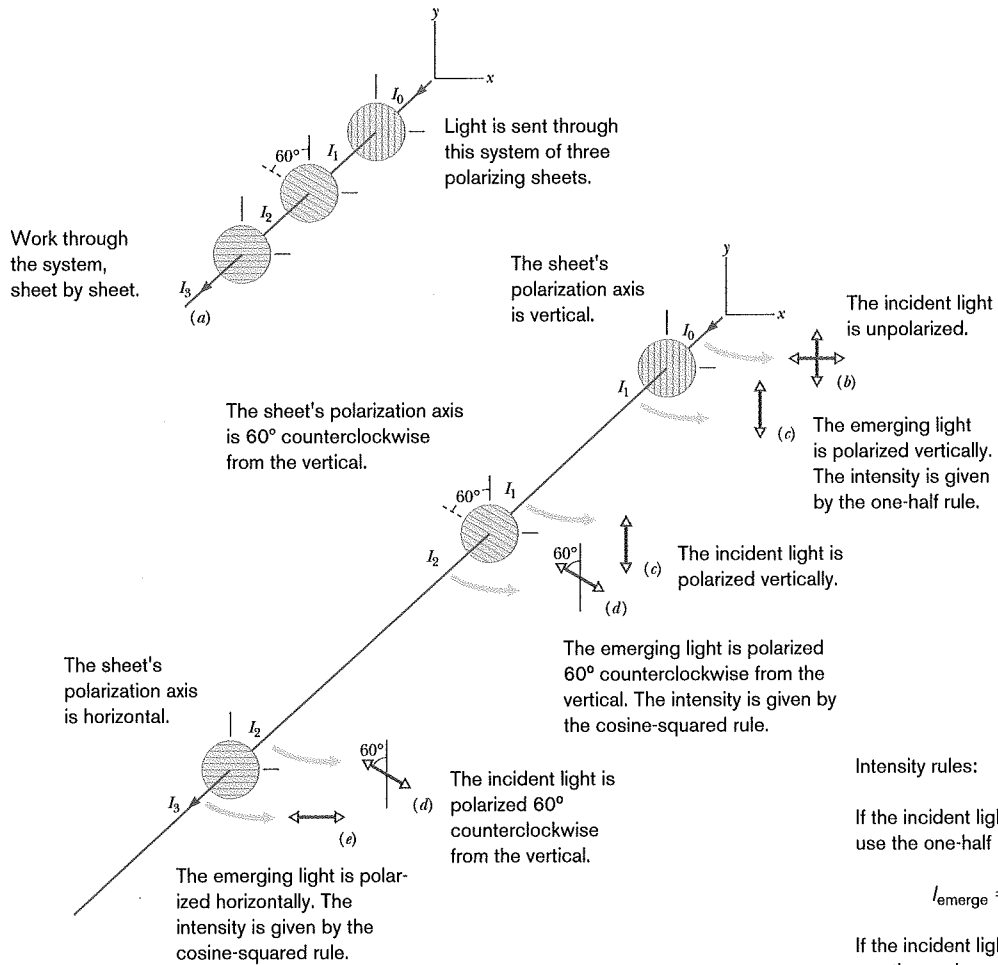


Fig. 33-15 (a) Initially unpolarized light of intensity I_0 is sent into a system of three polarizing sheets. The intensities I_1 , I_2 , and I_3 of the light transmitted by the sheets are labeled. Shown also are the polarizations, from head-on views, of (b) the initial light and the light transmitted by (c) the first sheet, (d) the second sheet, and (e) the third sheet.

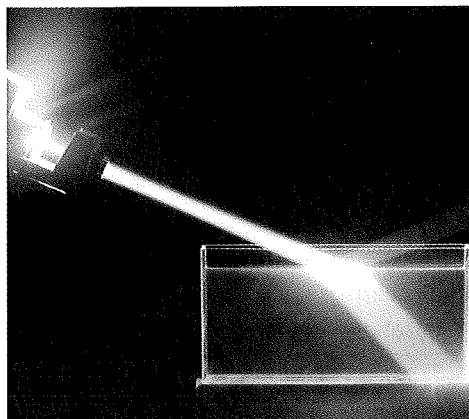


Fig. 33-16 (a) A photograph showing an incident beam of light reflected and refracted by a horizontal water surface. (©1974 FP/Fundamentals Photography) (Fig. 33-16 continues on next page.)

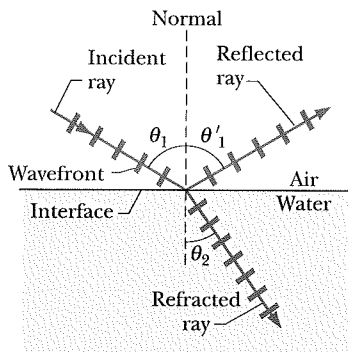


Fig. 33-16 (Continued) (b) A ray representation of (a). The angles of incidence (θ_1), reflection (θ'_1), and refraction (θ_2) are marked.

ing a beam directed downward to the right. Because light can travel through it, the water is said to be *transparent*; that is, we can see through it. (In this chapter we shall consider only transparent materials and not opaque materials, through which light cannot travel.)

The travel of light through a surface (or *interface*) that separates two media is called **refraction**, and the light is said to be *refracted*. Unless an incident beam of light is perpendicular to the surface, refraction changes the light's direction of travel. For this reason, the beam is said to be "bent" by the refraction. Note in Fig. 33-16a that the bending occurs only at the surface; within the water, the light travels in a straight line.

In Figure 33-16b, the beams of light in the photograph are represented with an *incident ray*, a *reflected ray*, and a *refracted ray* (and wavefronts). Each ray is oriented with respect to a line, called the *normal*, that is perpendicular to the surface at the point of reflection and refraction. In Fig. 33-16b, the **angle of incidence** is θ_1 , the **angle of reflection** is θ'_1 , and the **angle of refraction** is θ_2 , all measured relative to the normal. The plane containing the incident ray and the normal is the *plane of incidence*, which is in the plane of the page in Fig. 33-16b.

Experiment shows that reflection and refraction are governed by two laws:

Law of reflection: A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence (both relative to the normal). In Fig. 33-16b, this means that

$$\theta'_1 = \theta_1 \quad (\text{reflection}). \quad (33-39)$$

(We shall now usually drop the prime on the angle of reflection.)

Law of refraction: A refracted ray lies in the plane of incidence and has an angle of refraction θ_2 that is related to the angle of incidence θ_1 by

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \quad (\text{refraction}). \quad (33-40)$$

Here each of the symbols n_1 and n_2 is a dimensionless constant, called the **index of refraction**, that is associated with a medium involved in the refraction. We derive this equation, called **Snell's law**, in Chapter 35. As we shall discuss there, the index of refraction of a medium is equal to c/v , where v is the speed of light in that medium and c is its speed in vacuum.

Table 33-1 gives the indexes of refraction of vacuum and some common substances. For vacuum, n is defined to be exactly 1; for air, n is very close to 1.0 (an approximation we shall often make). Nothing has an index of refraction below 1.

Table 33-1

Some Indexes of Refraction^a

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP) ^b	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

^aFor a wavelength of 589 nm (yellow sodium light).

^bSTP means "standard temperature (0°C) and pressure (1 atm)."

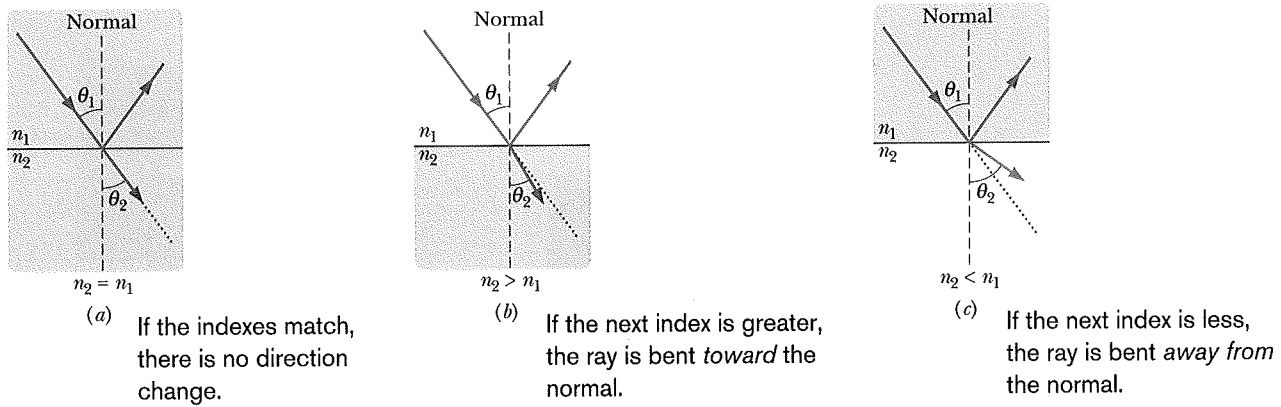


Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2 . (a) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (b) toward the normal when $n_2 > n_1$ and (c) away from the normal when $n_2 < n_1$.

We can rearrange Eq. 33-40 as

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 \quad (33-41)$$

to compare the angle of refraction θ_2 with the angle of incidence θ_1 . We can then see that the relative value of θ_2 depends on the relative values of n_2 and n_1 :

1. If n_2 is equal to n_1 , then θ_2 is equal to θ_1 and refraction does not bend the light beam, which continues in the *undeflected direction*, as in Fig. 33-17a.
2. If n_2 is greater than n_1 , then θ_2 is less than θ_1 . In this case, refraction bends the light beam away from the undeflected direction and toward the normal, as in Fig. 33-17b.
3. If n_2 is less than n_1 , then θ_2 is greater than θ_1 . In this case, refraction bends the light beam away from the undeflected direction and away from the normal, as in Fig. 33-17c.

Refraction *cannot* bend a beam so much that the refracted ray is on the same side of the normal as the incident ray.

Chromatic Dispersion

The index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light. The dependence of n on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction. This spreading of light is called **chromatic dispersion**, in which “chromatic” refers to the colors associated with the individual wavelengths and “dispersion” refers to the spreading of the light according to its wavelengths or colors. The refractions of Figs. 33-16 and 33-17 do not show chromatic dispersion because the beams are *monochromatic* (of a single wavelength or color).

Generally, the index of refraction of a given medium is *greater* for a shorter wavelength (corresponding to, say, blue light) than for a longer wavelength (say, red light). As an example, Fig. 33-18 shows how the index of refraction of fused quartz depends on the wavelength of light. Such dependence means that when a beam made up of waves of both blue and red light is refracted through a surface, such as from air into quartz or vice versa, the blue *component* (the ray corresponding to the wave of blue light) bends more than the red component.

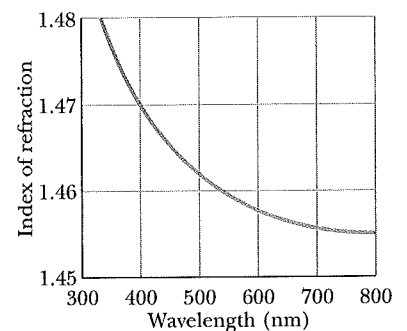


Fig. 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

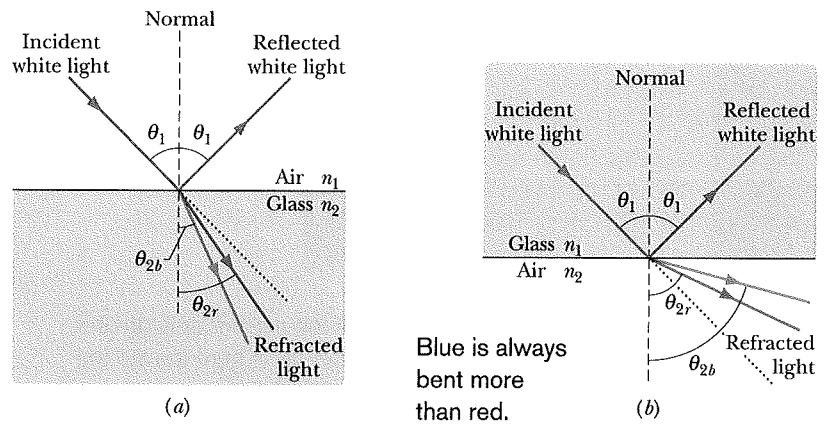


Fig. 33-19 Chromatic dispersion of white light. The blue component is bent more than the red component. (a) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (b) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.

A beam of *white light* consists of components of all (or nearly all) the colors in the visible spectrum with approximately uniform intensities. When you see such a beam, you perceive white rather than the individual colors. In Fig. 33-19a, a beam of white light in air is incident on a glass surface. (Because the pages of this book are white, a beam of white light is represented with a gray ray here. Also, a beam of monochromatic light is generally represented with a red ray.) Of the refracted light in Fig. 33-19a, only the red and blue components are shown. Because the blue component is bent more than the red component, the angle of refraction θ_{2b} for the blue component is *smaller* than the angle of refraction θ_{2r} for the red component. (Remember, angles are measured relative to the normal.) In Fig. 33-19b, a ray of white light in glass is incident on a glass–air interface. Again, the blue component is bent more than the red component, but now θ_{2b} is greater than θ_{2r} .

To increase the color separation, we can use a solid glass prism with a triangular cross section, as in Fig. 33-20a. The dispersion at the first surface (on the left in Figs. 33-20a, b) is then enhanced by the dispersion at the second surface.

Rainbows

The most charming example of chromatic dispersion is a rainbow. When sunlight (which consists of all visible colors) is intercepted by a falling raindrop,

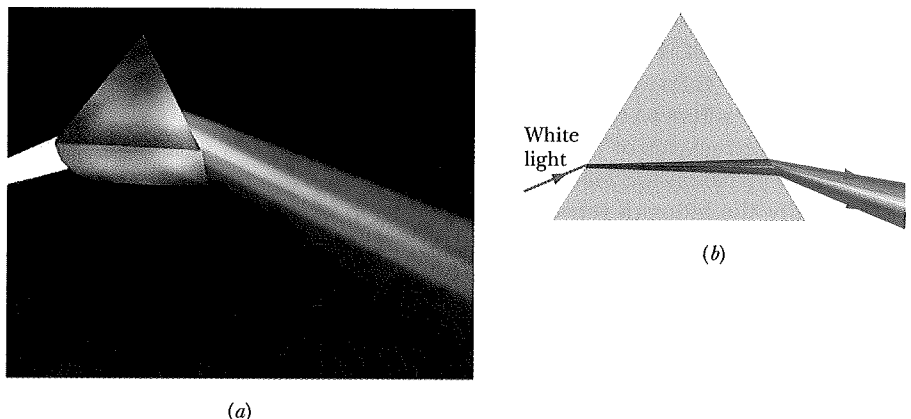


Fig. 33-20 (a) A triangular prism separating white light into its component colors. (b) Chromatic dispersion occurs at the first surface and is increased at the second surface. (Courtesy Bausch & Lomb)

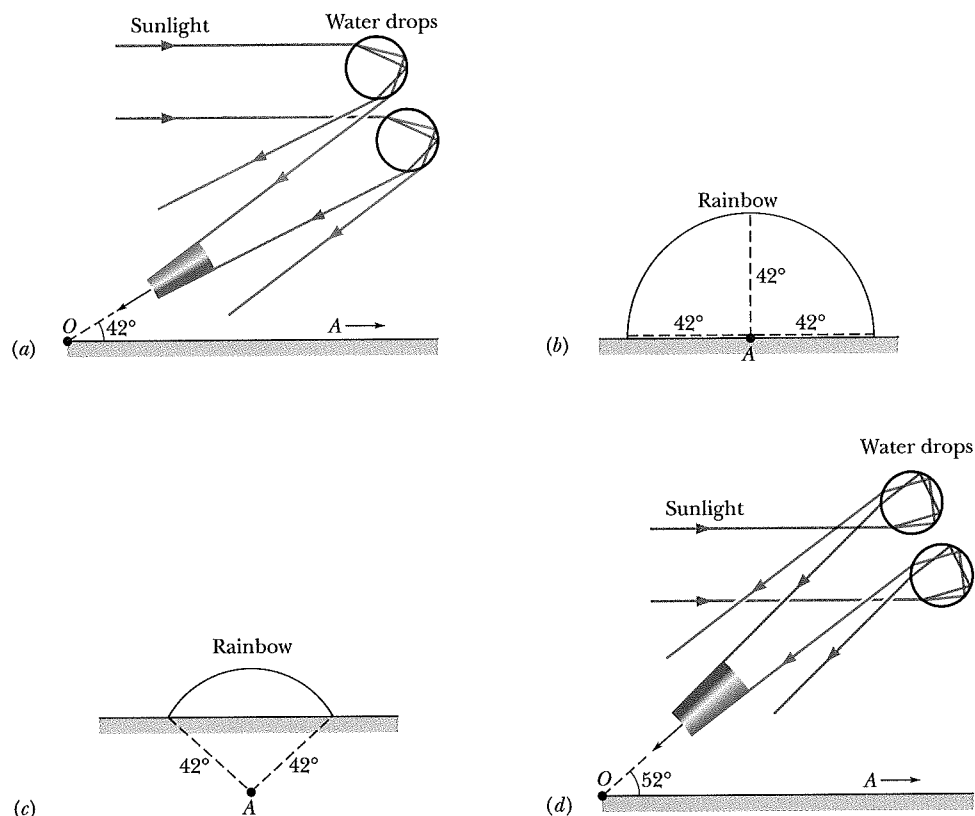


Fig. 33-21 (a) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point A is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of A . (b) Drops at 42° from A in any direction can contribute to the rainbow. (c) The rainbow arc when the Sun is higher (and thus A is lower). (d) The separation of colors leading to a secondary rainbow.

some of the light refracts into the drop, reflects once from the drop's inner surface, and then refracts out of the drop. Figure 33-21a shows the situation when the Sun is on the horizon at the left (and thus when the rays of sunlight are horizontal). The first refraction separates the sunlight into its component colors, and the second refraction increases the separation. (Only the red and blue rays are shown in the figure.) If many falling drops are brightly illuminated, you can see the separated colors they produce when the drops are at an angle of 42° from the direction of the *antisolar point* A , the point directly opposite the Sun in your view.

To locate the drops, face away from the Sun and point both arms directly away from the Sun, toward the shadow of your head. Then move your right arm directly up, directly rightward, or in any intermediate direction until the angle between your arms is 42° . If illuminated drops happen to be in the direction of your right arm, you see color in that direction.

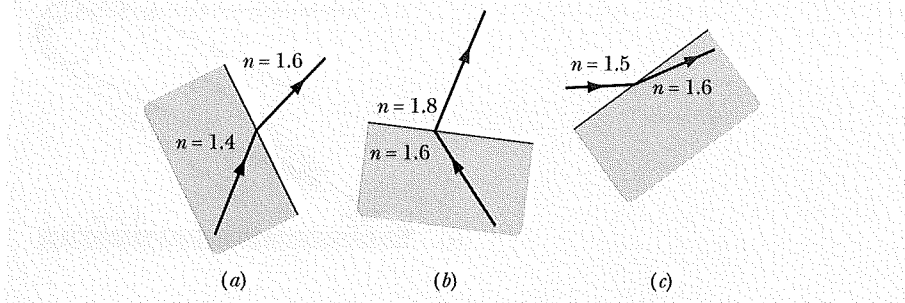
Because any drop at an angle of 42° in any direction from A can contribute to the rainbow, the rainbow is always a 42° circular arc around A (Fig. 33-21b) and the top of a rainbow is never more than 42° above the horizon. When the Sun is above the horizon, the direction of A is below the horizon, and only a shorter, lower rainbow arc is possible (Fig. 33-21c).

Because rainbows formed in this way involve one reflection of light inside each drop, they are often called *primary rainbows*. A *secondary rainbow* involves two reflections inside a drop, as shown in Fig. 33-21d. Colors appear in the secondary rainbow at an angle of 52° from the direction of A . A secondary rainbow is wider and dimmer than a primary rainbow and thus is more difficult to see. Also, the order of colors in a secondary rainbow is reversed from the order in a primary rainbow, as you can see by comparing parts a and d of Fig. 33-21.

Rainbows involving three or four reflections occur in the direction of the Sun and cannot be seen against the glare of sunshine in that part of the sky. Rainbows involving even more reflections inside the drops are too dim to see.

CHECKPOINT 5

Which of the three drawings here (if any) show physically possible refraction?



Sample Problem

Reflection and refraction of a monochromatic beam

(a) In Fig. 33-22a, a beam of monochromatic light reflects and refracts at point A on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point A? What is the angle of refraction there?

KEY IDEAS

(1) The angle of reflection is equal to the angle of incidence, and both angles are measured relative to the normal to the surface at the point of reflection. (2) When light reaches the interface between two materials with different indexes of refraction (call them n_1 and n_2), part of the light can be refracted by the interface according to Snell's law, Eq. 33-40:

$$n_2 \sin \theta_2 = n_1 \sin \theta_1, \quad (33-42)$$

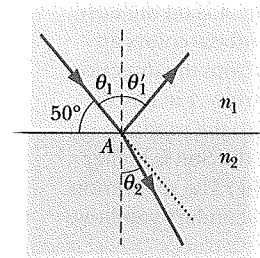
where both angles are measured relative to the normal at the point of refraction.

Calculations: In Fig. 33-22a, the normal at point A is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given 50° but is $90^\circ - 50^\circ = 40^\circ$. Thus, the angle of reflection is

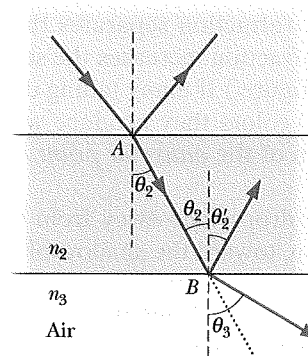
$$\theta'_1 = \theta_1 = 40^\circ. \quad (\text{Answer})$$

The light that passes from material 1 into material 2 undergoes refraction at point A on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22a, the angle of refraction is the angle marked θ_2 . Solving Eq. 33-42 for θ_2 gives us

$$\begin{aligned} \theta_2 &= \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right) = \sin^{-1} \left(\frac{1.33}{1.77} \sin 40^\circ \right) \\ &= 28.88^\circ \approx 29^\circ. \end{aligned} \quad (\text{Answer})$$



(a)



(b)

Fig. 33-22 (a) Light reflects and refracts at point A on the interface between materials 1 and 2. (b) The light that passes through material 2 reflects and refracts at point B on the interface between materials 2 and 3 (air). Each dashed line is a normal. Each dotted line gives the incident direction of travel.

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22a.

(b) The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22*b*. The interface through B is parallel to that through A . At B , some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

Calculations: We first need to relate one of the angles at point B with a known angle at point A . Because the interface through point B is parallel to that through point A , the incident angle at B must be equal to the angle of refraction θ_2 , as shown in Fig. 33-22*b*. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at B is

$$\theta_2' = \theta_2 = 28.88^\circ \approx 29^\circ. \quad (\text{Answer})$$

Next, the light that passes from material 2 into the air undergoes refraction at point B , with refraction angle θ_3 . Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \quad (33-43)$$

Solving for θ_3 then leads to

$$\begin{aligned} \theta_3 &= \sin^{-1} \left(\frac{n_2}{n_3} \sin \theta_2 \right) = \sin^{-1} \left(\frac{1.77}{1.00} \sin 28.88^\circ \right) \\ &= 58.75^\circ \approx 59^\circ. \quad (\text{Answer}) \end{aligned}$$

This result means that the beam swings away from the normal (it was at 29° to the normal and is now at 59°). The reason is that when the light travels across the interface, it moves into a material (air) with a lower index of refraction.



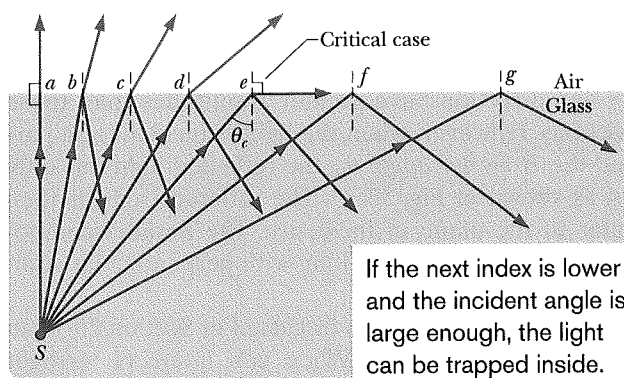
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33-9 Total Internal Reflection

Figure 33-23*a* shows rays of monochromatic light from a point source S in glass incident on the interface between the glass and air. For ray a , which is perpendicular to the interface, part of the light reflects at the interface and the rest travels through it with no change in direction.

For rays b through e , which have progressively larger angles of incidence at the interface, there are also both reflection and refraction at the interface. As the angle of incidence increases, the angle of refraction increases; for ray e it is 90° , which means that the refracted ray points directly along the interface. The angle of incidence giving this situation is called the **critical angle** θ_c . For angles of incidence larger than θ_c , such as for rays f and g , there is no refracted ray and *all* the light is reflected; this effect is called **total internal reflection**.

To find θ_c , we use Eq. 33-40; we arbitrarily associate subscript 1 with the glass and subscript 2 with the air, and then we substitute θ_c for θ_1 and 90° for θ_2 ,



If the next index is lower and the incident angle is large enough, the light can be trapped inside.

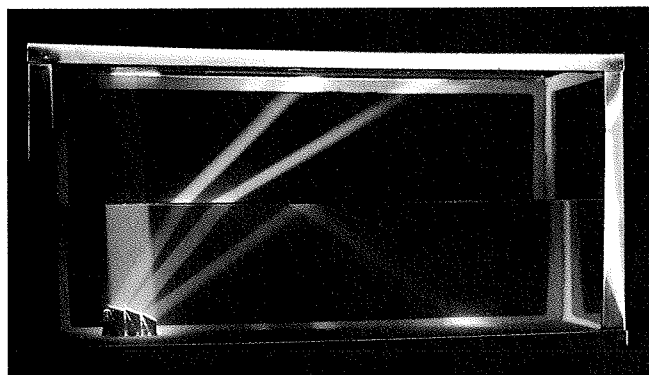


Fig. 33-23 (a) Total internal reflection of light from a point source S in glass occurs for all angles of incidence greater than the critical angle θ_c . At the critical angle, the refracted ray points along the air–glass interface. (b) A source in a tank of water. (Ken Kay/Fundamental Photographs)