

$$Q9:- \int \frac{2x^2 + 6x}{(x^2+1)(x^2+2x+3)} dx$$

Let

$$\frac{2x^2 + 6x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$$

$$2x^2 + 6x = (Ax+B)(x^2+2x+3) + (Cx+D)(x^2+1)$$

$$2x^2 + 6x = Ax^3 + 2Ax^2 + 3Ax + Bx^2 + 2Bx + 3B + Cx^3 + Cx + Dx^2 + D$$

Equating coefficient of $x^3, x^2, x, 1$

$$x^3 \Rightarrow 0 = A + C \Rightarrow C = -A \rightarrow (1)$$

$$x^2 \Rightarrow 2 = 2A + B + D \rightarrow (2)$$

$$x \Rightarrow 6 = 3A + 2B + C \rightarrow (3)$$

$$1 \Rightarrow 0 = 3B + D \rightarrow (4)$$

Subtracting (2) and (4)

$$\begin{array}{r} 2 = 2A + B + D \\ -0 = - 3B + D \\ \hline 2 = 2A - 2B \end{array}$$

$$1 = A - B \rightarrow (5)$$

Now putting $C = -A$ in (3), we get

$$6 = 2A + 2B \Rightarrow 3 = A + B \rightarrow (6)$$

Adding (5) and (6), we get

$$4 = 2A \Rightarrow \boxed{A = 2}$$

$$\text{Putting in (5) } B = A - 1 \Rightarrow \boxed{B = 1}$$

$$\text{Putting } B = 1 \text{ in (4) } 0 = 3 + D \Rightarrow \boxed{D = -3}$$

$$\text{and } \boxed{C = -2}$$