Linear Algebra
BS-CS
Time: 9:00am – 3:00pm
Date: 25/06/2020

Instructions:

- Allowed time is 6 hours (9:00am to 3:00pm)
- Mark all the answers sheets with page numbers and ID on every sheet number
- Answers copied will both be marked zero
- Late Submission will not be accepted
- Submit in PDF format

1. Consider the following vectors R^3:

   $V_1 = \begin{pmatrix} ID1 \\ ID2 \\ ID3 \end{pmatrix}$, $V_2 = \begin{pmatrix} ID2 \\ ID3 \\ ID4 \end{pmatrix}$, $V_3 = \begin{pmatrix} ID3 \\ ID4 \\ ID5 \end{pmatrix}$

   Solve the system and find if these vectors are linearly independent?

2. Suppose that a company produces two products X and Y. For each unit of each product produced, money must be spent on materials, labor and overhead.

Cost per unit	Product X	Product Y
Materials	Rs. 450	Rs. 400
Labor	Rs. 250	Rs. 350
Overhead	Rs. 150	Rs. 150

   The production vector x = $\begin{bmatrix} x1 \\ x2 \end{bmatrix}$ whereas, x1 and x2 are total number of units for product A and B i.e. 1000 and 500 respectively.

   a. Find the total cost.

   b. Explain the linear transformation properties with the help of above problem as an example.

      - $T(u + v) = T(u) + T(v)$
      - $T(cu) = cT(u)$

3. What are the four main things we need to define for a vector space? Which of the following is a vector space over R? For those that are not vector spaces, modify one part of the definition to make it into a vector space.
    a. V = { 2 x 2 matrices with entries in R}, usual matrix addition, and

$$k.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix} for\ k \in R$$

    b. V = {Polynomials with complex coefficients of degrees ≤3}, with usual addition and scalar multiplication of polynomials.
4. Determinants: Let M = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2x2 matrix.
    a. For which values of det M does M have an inverse?
    b. Write down all 2x2 bit matrices with determinant 1. (Remember bits are either 0 or 1 and 1+1 = 0 in bits)
    c. Write down 2x2 bit matrices with determinant 0
    d. Compute det A for below 3x3 matrix.

$$A = \begin{pmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{pmatrix}$$