

26-4 Resistance and Resistivity

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical **resistance**. We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i} \quad (\text{definition of } R). \quad (26-8)$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This combination occurs so often that we give it a special name, the **ohm** (symbol Ω); that is,

$$\begin{aligned} 1 \text{ ohm} &= 1 \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A}. \end{aligned} \quad (26-9)$$

A conductor whose function in a circuit is to provide a specified resistance is called a **resistor** (see Fig. 26-7). In a circuit diagram, we represent a resistor and a resistance with the symbol $\sim\sim\sim$. If we write Eq. 26-8 as

$$i = \frac{V}{R},$$

we see that, for a given V , the greater the resistance, the smaller the current.

The resistance of a conductor depends on the manner in which the potential difference is applied to it. Figure 26-8, for example, shows a given potential difference applied in two different ways to the same conductor. As the current density streamlines suggest, the currents in the two cases—hence the measured resistances—will be different. Unless otherwise stated, we shall assume that any given potential difference is applied as in Fig. 26-8b.



Fig. 26-8 Two ways of applying a potential difference to a conducting rod. The gray connectors are assumed to have negligible resistance. When they are arranged as in (a) in a small region at each rod end, the measured resistance is larger than when they are arranged as in (b) to cover the entire rod end.

As we have done several times in other connections, we often wish to take a general view and deal not with particular objects but with materials. Here we do so by focusing not on the potential difference V across a particular resistor but on the electric field \vec{E} at a point in a resistive material. Instead of dealing with the current i through the resistor, we deal with the current density \vec{J} at the point in question. Instead of the resistance R of an object, we deal with the **resistivity** ρ of the *material*:

$$\rho = \frac{E}{J} \quad (\text{definition of } \rho). \quad (26-10)$$

(Compare this equation with Eq. 26-8.)

If we combine the SI units of E and J according to Eq. 26-10, we get, for the unit of ρ , the ohm-meter ($\Omega \cdot \text{m}$):

$$\frac{\text{unit}(E)}{\text{unit}(J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}} \text{ m} = \Omega \cdot \text{m}.$$

(Do not confuse the *ohm-meter*, the unit of resistivity, with the *ohmmeter*, which is an instrument that measures resistance.) Table 26-1 lists the resistivities of some materials.

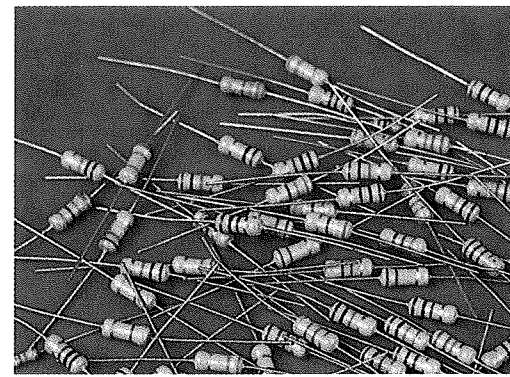


Fig. 26-7 An assortment of resistors. The circular bands are color-coding marks that identify the value of the resistance. (*The Image Works*)

Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot \text{m}$)	Temperature Coefficient of Resistivity, α (K^{-1})
<i>Typical Metals</i>		
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin ^a	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
<i>Typical Semiconductors</i>		
Silicon, pure	2.5×10^3	-70×10^{-3}
Silicon, <i>n</i> -type ^b	8.7×10^{-4}	
Silicon, <i>p</i> -type ^c	2.8×10^{-3}	
<i>Typical Insulators</i>		
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

^aAn alloy specifically designed to have a small value of α .

^bPure silicon doped with phosphorus impurities to a charge carrier density of 10^{23} m^{-3} .

^cPure silicon doped with aluminum impurities to a charge carrier density of 10^{23} m^{-3} .

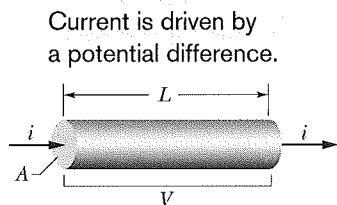


Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A , establishing a current i .

We can write Eq. 26-10 in vector form as

$$\vec{E} = \rho \vec{J}. \quad (26-11)$$

Equations 26-10 and 26-11 hold only for *isotropic* materials—materials whose electrical properties are the same in all directions.

We often speak of the **conductivity** σ of a material. This is simply the reciprocal of its resistivity, so

$$\sigma = \frac{1}{\rho} \quad (\text{definition of } \sigma). \quad (26-12)$$

The SI unit of conductivity is the reciprocal ohm-meter, $(\Omega \cdot \text{m})^{-1}$. The unit name mhos per meter is sometimes used (mho is ohm backwards). The definition of σ allows us to write Eq. 26-11 in the alternative form

$$\vec{J} = \sigma \vec{E}. \quad (26-13)$$

Calculating Resistance from Resistivity

We have just made an important distinction:



Resistance is a property of an object. Resistivity is a property of a material.

If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance. Let A be the cross-sectional area of the wire, let L be its length, and let a potential difference V exist between its ends (Fig. 26-9). If the streamlines representing the current density are uniform throughout the wire, the electric field and the current density will be constant for all points within the wire and, from Eqs. 24-42 and 26-5, will have the values

$$E = V/L \quad \text{and} \quad J = i/A. \quad (26-14)$$

We can then combine Eqs. 26-10 and 26-14 to write

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}. \quad (26-15)$$

However, V/i is the resistance R , which allows us to recast Eq. 26-15 as

$$R = \rho \frac{L}{A}. \quad (26-16)$$

Equation 26-16 can be applied only to a homogeneous isotropic conductor of uniform cross section, with the potential difference applied as in Fig. 26-8b.

The macroscopic quantities V , i , and R are of greatest interest when we are making electrical measurements on specific conductors. They are the quantities that we read directly on meters. We turn to the microscopic quantities E , J , and ρ when we are interested in the fundamental electrical properties of materials.



CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.

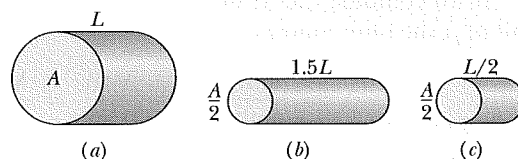
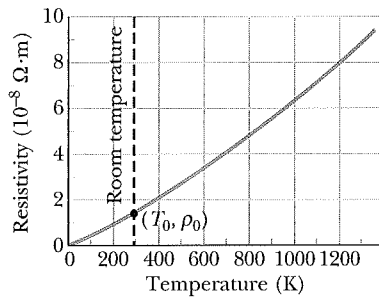


Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8} \Omega \cdot \text{m}$.



Resistivity can depend on temperature.

Variation with Temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 26-10, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0). \quad (26-17)$$

Here T_0 is a selected reference temperature and ρ_0 is the resistivity at that temperature. Usually $T_0 = 293$ K (room temperature), for which $\rho_0 = 1.69 \times 10^{-8} \Omega \cdot \text{m}$ for copper.

Because temperature enters Eq. 26-17 only as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity α in Eq. 26-17, called the *temperature coefficient of resistivity*, is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range. Some values of α for metals are listed in Table 26-1.

Sample Problem

A material has resistivity, a block of the material has resistance

A rectangular block of iron has dimensions $1.2 \text{ cm} \times 1.2 \text{ cm} \times 15 \text{ cm}$. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8b). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions $1.2 \text{ cm} \times 1.2 \text{ cm}$) and (2) two rectangular sides (with dimensions $1.2 \text{ cm} \times 15 \text{ cm}$)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A , according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15 \text{ cm} = 0.15 \text{ m}$ and

$$A = (1.2 \text{ cm})^2 = 1.44 \times 10^{-4} \text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8} \Omega \cdot \text{m})(0.15 \text{ m})}{1.44 \times 10^{-4} \text{ m}^2} \\ &= 1.0 \times 10^{-4} \Omega = 100 \mu\Omega. \end{aligned} \quad (\text{Answer})$$

Similarly, for arrangement 2, with distance $L = 1.2 \text{ cm}$ and area $A = (1.2 \text{ cm})(15 \text{ cm})$, we obtain

$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8} \Omega \cdot \text{m})(1.2 \times 10^{-2} \text{ m})}{1.80 \times 10^{-3} \text{ m}^2} \\ &= 6.5 \times 10^{-7} \Omega = 0.65 \mu\Omega. \end{aligned} \quad (\text{Answer})$$



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