

We know that Avogadro's number N_A is 6.02×10^{23} molecules/mol, or $6.02 \times 10^{23} \text{ mol}^{-1}$, and from Table 15-1 we know that the density of water ρ_{mass} under normal conditions is 1000 kg/m^3 . We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining $18 \text{ g/mol} = 0.018 \text{ kg/mol}$. So, the current of negative charge due to the electrons in the water is

$$\begin{aligned} i &= (10)(1.6 \times 10^{-19} \text{ C})(6.02 \times 10^{23} \text{ mol}^{-1}) \\ &\quad \times (0.018 \text{ kg/mol})^{-1}(1000 \text{ kg/m}^3)(450 \times 10^{-6} \text{ m}^3/\text{s}) \\ &= 2.41 \times 10^7 \text{ C/s} = 2.41 \times 10^7 \text{ A} \\ &= 24.1 \text{ MA.} \end{aligned} \quad (\text{Answer})$$

This current of negative charge is exactly compensated by a current of positive charge associated with the nuclei of the three atoms that make up the water molecule. Thus, there is no net flow of charge through the hose.



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26-3 Current Density

Sometimes we are interested in the current i in a particular conductor. At other times we take a localized view and study the flow of charge through a cross section of the conductor at a particular point. To describe this flow, we can use the **current density** \vec{J} , which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the element as $\vec{J} \cdot d\vec{A}$, where $d\vec{A}$ is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}. \quad (26-4)$$

If the current is uniform across the surface and parallel to $d\vec{A}$, then \vec{J} is also uniform and parallel to $d\vec{A}$. Then Eq. 26-4 becomes

$$i = \int J dA = J \int dA = JA,$$

so

$$J = \frac{i}{A}, \quad (26-5)$$

where A is the total area of the surface. From Eq. 26-4 or 26-5 we see that the SI unit for current density is the ampere per square meter (A/m^2).

In Chapter 22 we saw that we can represent an electric field with electric field lines. Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call *streamlines*. The current, which is toward the right in Fig. 26-4, makes a transition from the wider conductor at the left to the narrower conductor at the right. Because charge is conserved during the transition, the amount of charge and thus the amount of current cannot change. However, the current density does change—it is greater in the narrower conductor. The spacing of the streamlines suggests this increase in current density; streamlines that are closer together imply greater current density.

Drift Speed

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now

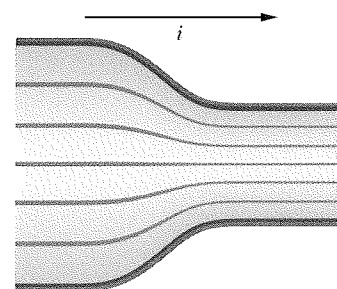
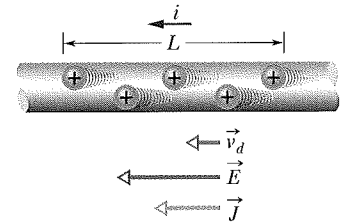


Fig. 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

Current is said to be due to positive charges that are propelled by the electric field.

Fig. 26-5 Positive charge carriers drift at speed v_d in the direction of the applied electric field \vec{E} . By convention, the direction of the current density \vec{J} and the sense of the current arrow are drawn in that same direction.



they tend to *drift* with a **drift speed** v_d in the direction opposite that of the applied electric field that causes the current. The drift speed is tiny compared with the speeds in the random motion. For example, in the copper conductors of household wiring, electron drift speeds are perhaps 10^{-5} or 10^{-4} m/s, whereas the random-motion speeds are around 10^6 m/s.

We can use Fig. 26-5 to relate the drift speed v_d of the conduction electrons in a current through a wire to the magnitude J of the current density in the wire. For convenience, Fig. 26-5 shows the equivalent drift of *positive* charge carriers in the direction of the applied electric field \vec{E} . Let us assume that these charge carriers all move with the same drift speed v_d and that the current density J is uniform across the wire's cross-sectional area A . The number of charge carriers in a length L of the wire is nAL , where n is the number of carriers per unit volume. The total charge of the carriers in the length L , each with charge e , is then

$$q = (nAL)e.$$

Because the carriers all move along the wire with speed v_d , this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Equation 26-1 tells us that the current i is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d. \tag{26-6}$$

Solving for v_d and recalling Eq. 26-5 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

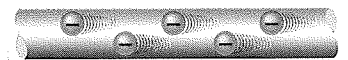
or, extended to vector form,

$$\vec{J} = (ne)\vec{v}_d. \tag{26-7}$$

Here the product ne , whose SI unit is the coulomb per cubic meter (C/m^3), is the *carrier charge density*. For positive carriers, ne is positive and Eq. 26-7 predicts that \vec{J} and \vec{v}_d have the same direction. For negative carriers, ne is negative and \vec{J} and \vec{v}_d have opposite directions.

CHECKPOINT 2

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current i , (b) the current density \vec{J} , (c) the electric field \vec{E} in the wire?



Sample Problem

Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m². What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

KEY IDEA

Because the current density is uniform across the cross section, the current density J , the current i , and the cross-sectional area A are related by Eq. 26-5 ($J = i/A$).

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$\begin{aligned} A' &= \pi R^2 - \pi \left(\frac{R}{2}\right)^2 = \pi \left(\frac{3R^2}{4}\right) \\ &= \frac{3\pi}{4} (0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2. \end{aligned}$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

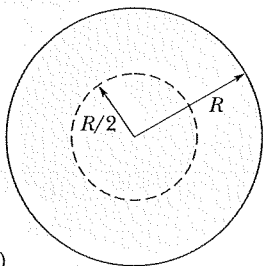
$$\begin{aligned} i &= (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2) \\ &= 1.9 \text{ A.} \end{aligned} \quad (\text{Answer})$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m⁴ and r is in meters. What now is the current through the same outer portion of the wire?

KEY IDEA

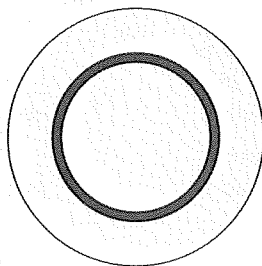
Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

We want the current in the area between these two radii.



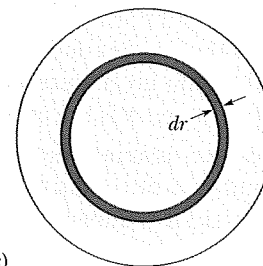
(a)

If the current is nonuniform, we start with a ring that is so thin that we can approximate the current density as being uniform within it.



(b)

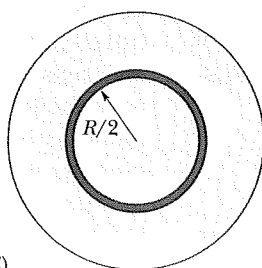
Its area is the product of the circumference and the width.



(c)

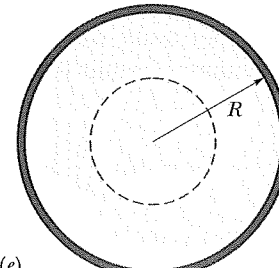
The current within the ring is the product of the current density and the ring's area.

Our job is to sum the current in all rings from this smallest one ...



(d)

... to this largest one.



(e)

Fig. 26-6 (a) Cross section of a wire of radius R . If the current density is uniform, the current is just the product of the current density and the area. (b)–(e) If the current is nonuniform, we must first find the current through a thin ring and then sum (via integration) the currents in all such rings in the given area.



Calculations: The current density vector \vec{J} (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

$$\vec{J} \cdot d\vec{A} = J dA \cos 0 = J dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate with r as the variable of integration. Equation 26-4 then

gives us

$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J dA \\ &= \int_{R/2}^R ar^2 2\pi r dr = 2\pi a \int_{R/2}^R r^3 dr \\ &= 2\pi a \left[\frac{r^4}{4} \right]_{R/2}^R = \frac{\pi a}{2} \left[R^4 - \frac{R^4}{16} \right] = \frac{15}{32} \pi a R^4 \\ &= \frac{15}{32} \pi (3.0 \times 10^{11} \text{ A/m}^4)(0.0020 \text{ m})^4 = 7.1 \text{ A.} \end{aligned}$$

(Answer)

Sample Problem

In a current, the conduction electrons move very slowly

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900 \mu\text{m}$ when it has a uniform current $i = 17 \text{ mA}$? Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

KEY IDEAS

1. The drift speed v_d is related to the current density \vec{J} and the number n of conduction electrons per unit volume according to Eq. 26-7, which we can write as $J = nev_d$.
2. Because the current density is uniform, its magnitude J is related to the given current i and wire size by Eq. 26-5 ($J = i/A$, where A is the cross-sectional area of the wire).
3. Because we assume one conduction electron per atom, the number n of conduction electrons per unit volume is the same as the number of atoms per unit volume.

Calculations: Let us start with the third idea by writing

$$n = \left(\frac{\text{atoms}}{\text{per unit volume}} \right) = \left(\frac{\text{atoms}}{\text{per mole}} \right) \left(\frac{\text{moles}}{\text{per unit mass}} \right) \left(\frac{\text{mass}}{\text{per unit volume}} \right).$$

The number of atoms per mole is just Avogadro's number $N_A (= 6.02 \times 10^{23} \text{ mol}^{-1})$. Moles per unit mass is the inverse of the mass per mole, which here is the molar mass M of copper. The mass per unit volume is the (mass) density ρ_{mass} of copper. Thus,

$$n = N_A \left(\frac{1}{M} \right) \rho_{\text{mass}} = \frac{N_A \rho_{\text{mass}}}{M}.$$

Taking copper's molar mass M and density ρ_{mass} from Appendix F, we then have (with some conversions of units)

$$\begin{aligned} n &= \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(8.96 \times 10^3 \text{ kg/m}^3)}{63.54 \times 10^{-3} \text{ kg/mol}} \\ &= 8.49 \times 10^{28} \text{ electrons/m}^3 \end{aligned}$$

or $n = 8.49 \times 10^{28} \text{ m}^{-3}$.

Next let us combine the first two key ideas by writing

$$\frac{i}{A} = nev_d.$$

Substituting for A with $\pi r^2 (= 2.54 \times 10^{-6} \text{ m}^2)$ and solving for v_d , we then find

$$\begin{aligned} v_d &= \frac{i}{ne(\pi r^2)} \\ &= \frac{17 \times 10^{-3} \text{ A}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(2.54 \times 10^{-6} \text{ m}^2)} \\ &= 4.9 \times 10^{-7} \text{ m/s,} \end{aligned} \quad (\text{Answer})$$

which is only 1.8 mm/h, slower than a sluggish snail.

Lights are fast: You may well ask: "If the electrons drift so slowly, why do the room lights turn on so quickly when I throw the switch?" Confusion on this point results from not distinguishing between the drift speed of the electrons and the speed at which *changes* in the electric field configuration travel along wires. This latter speed is nearly that of light; electrons everywhere in the wire begin drifting almost at once, including into the lightbulbs. Similarly, when you open the valve on your garden hose with the hose full of water, a pressure wave travels along the hose at the speed of sound in water. The speed at which the water itself moves through the hose—measured perhaps with a dye marker—is much slower.



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