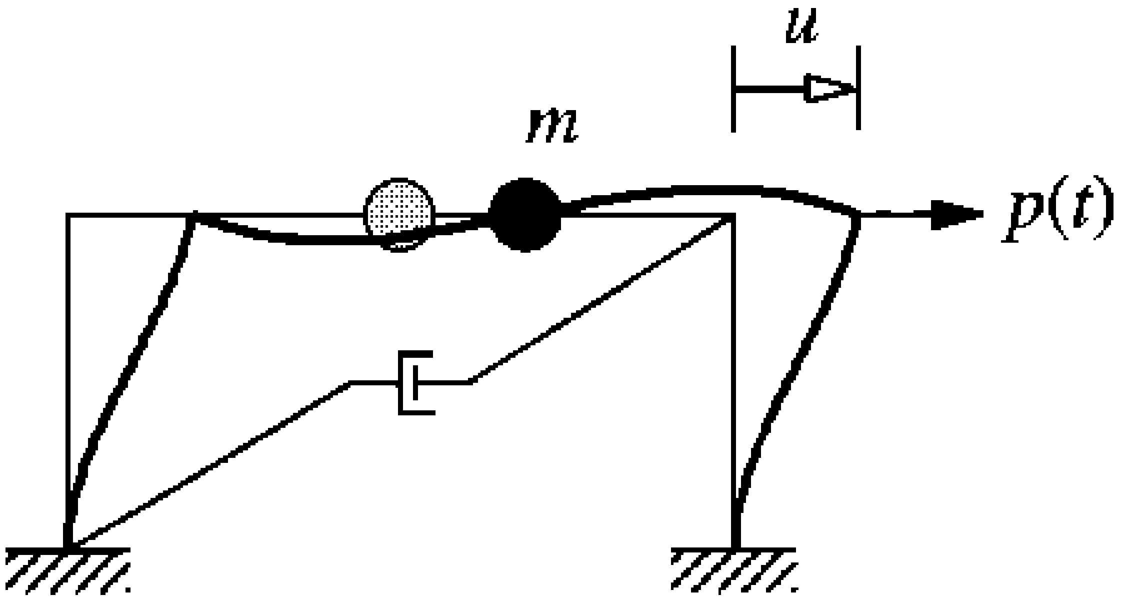
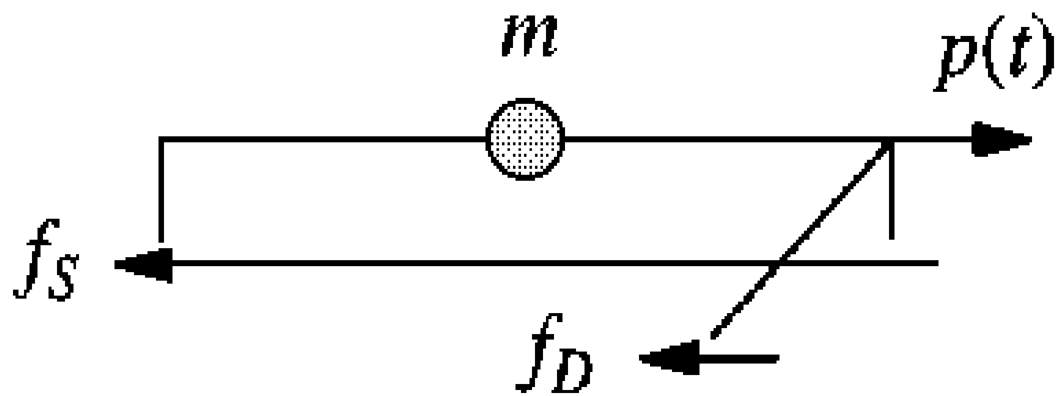
EQUATION OF MOTION (E.O.M) OF A SINGLE STORY FRAME UNDER EXTERNAL DYNAMIC FORCE



Two commonly used vector mechanics based approaches are:

1. **NEWTON'S SECOND LAW OF MOTION**
2. **D'ALEMBERT PRINCIPLE OF DYNAMIC EQUILIBRIUM**

E.O.M USING NEWTON'S SECOND LAW OF MOTION



- The Resultant force along x-axis $= p(t) - f_S - f_D$

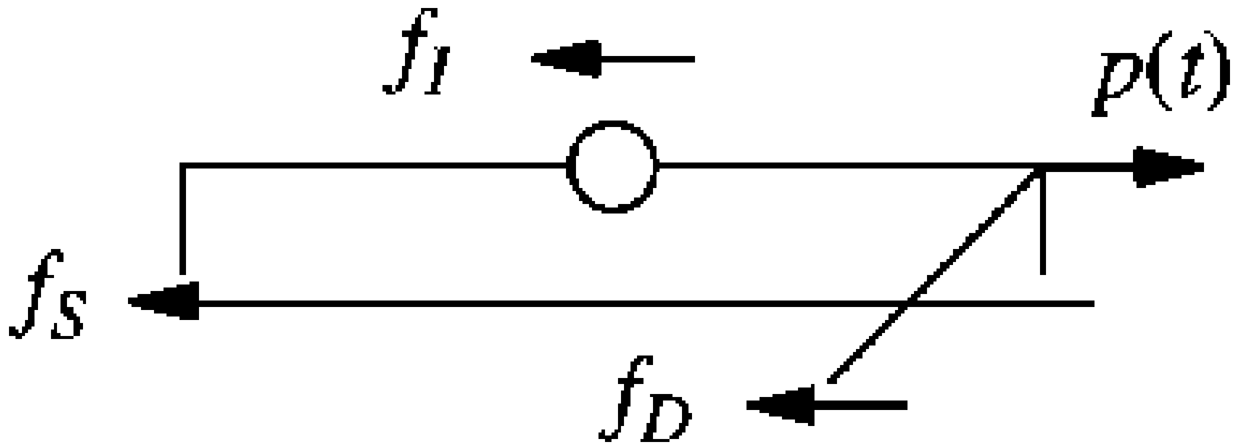
Where f_S = Elastic resisting force; (also known elastic restoring force), f_D = Damping resisting force

- According to Newton's second law, the resulting force causing acceleration $= p(t) - f_S - f_D = m\ddot{u}$ or;

$f_S + f_D + m\ddot{u} = p(t)$; or $$\mathbf{ku + c\dot{u} + m\ddot{u} = p(t)}$$

E.O.M USING DYNAMIC EQUILIBRIUM

Using D'Alembert's Principle, a state of *dynamic equilibrium* can be defined by assuming that a fictitious ***inertial force*** f_I acts on the mass during motion.



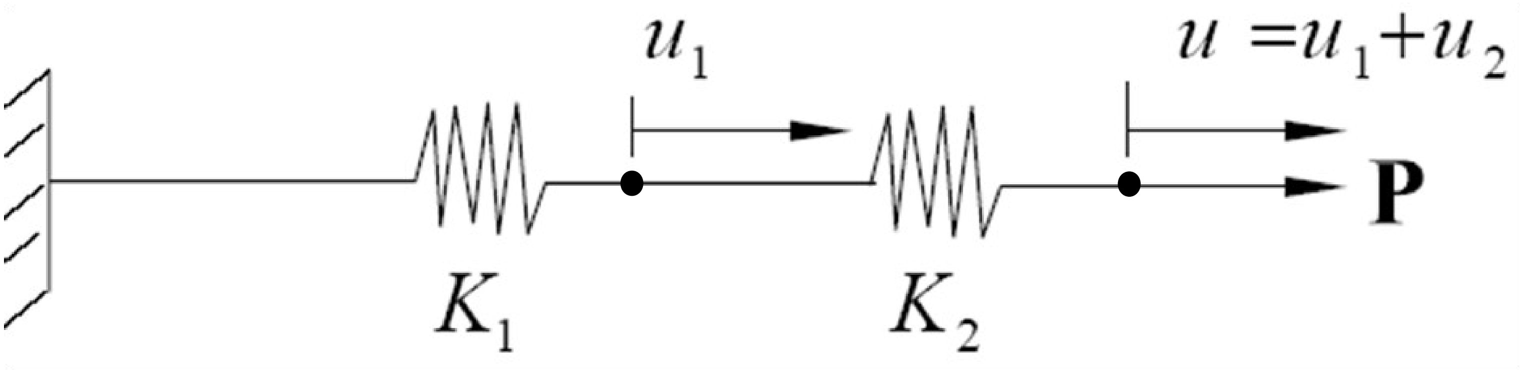
Equilibrium along x-axis requires that:

$-f_S - f_D - f_I + p(t) = 0$ or;

$f_S + f_D + f_I = p(t)$ or;

$$\mathbf{ku + c\dot{u} + m\ddot{u} = p(t)}$$

STIFNESS OF SPRINGS IN SERIES



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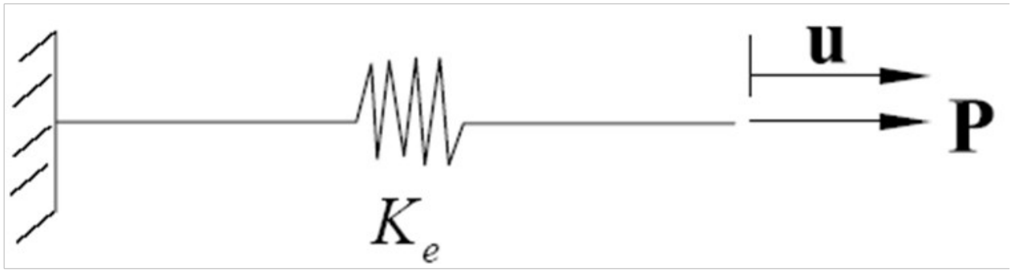

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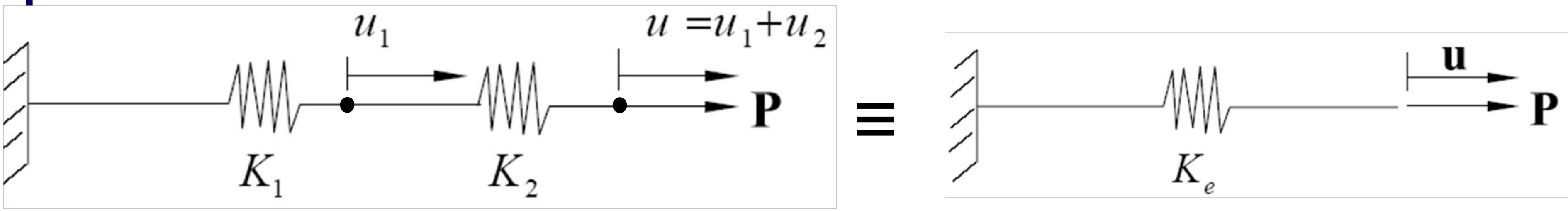
STIFNESS OF SPRINGS IN SERIES



$$u = u_1 + u_2 \qquad \Rightarrow \frac{p}{k_e} = \frac{p_1}{k_1} + \frac{p_2}{k_2}$$

$$\text{Since } p_1 = p_2 = p \quad \Rightarrow \frac{p}{k_e} = \frac{p}{k_1} + \frac{p}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

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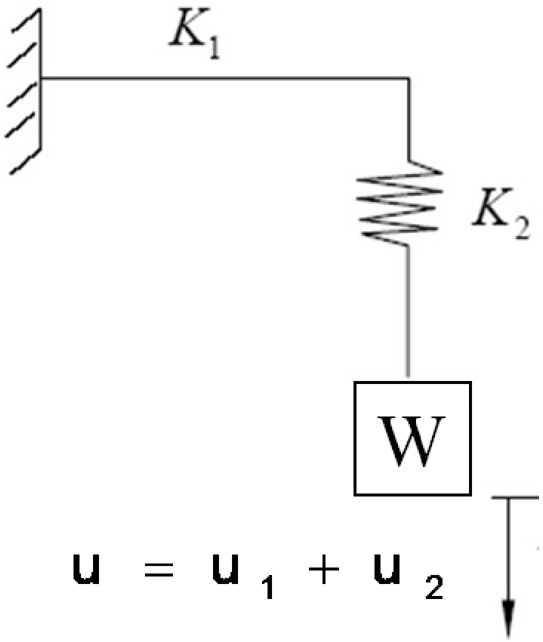
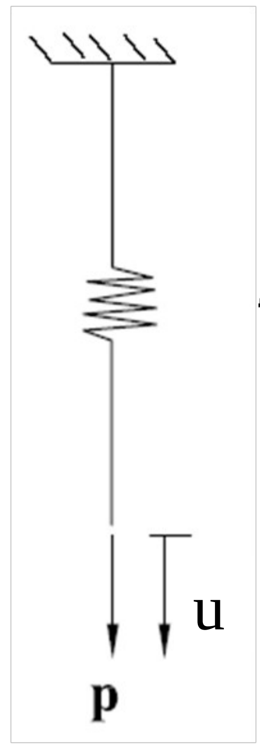
EAXMPLE



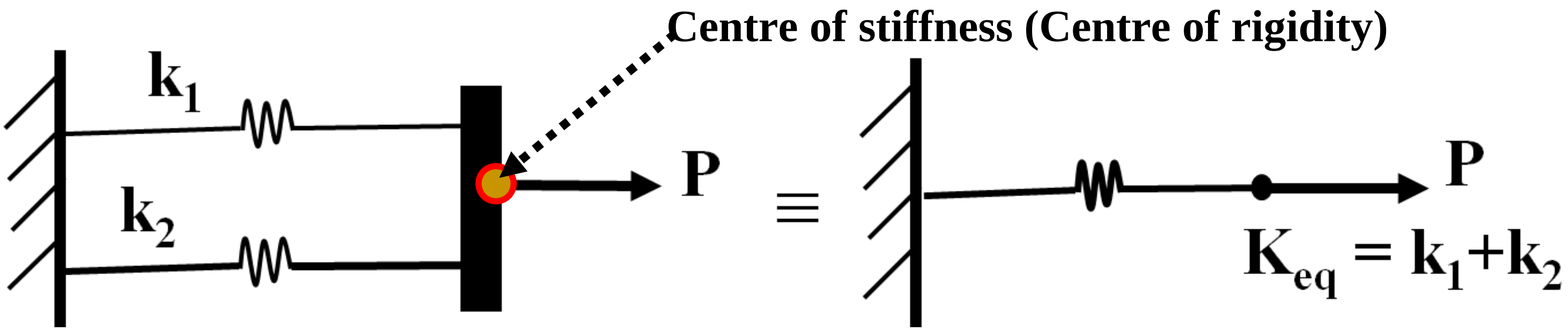
$$u = u_1 + u_2$$

=



$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

STIFNESS OF SPRINGS IN PARALLEL



$$p = p_1 + p_2$$

$$\Rightarrow \; k_e u = k_1 u_1 + k_2 u_2$$

$$\text{Since } u_1 = u_2 = u$$

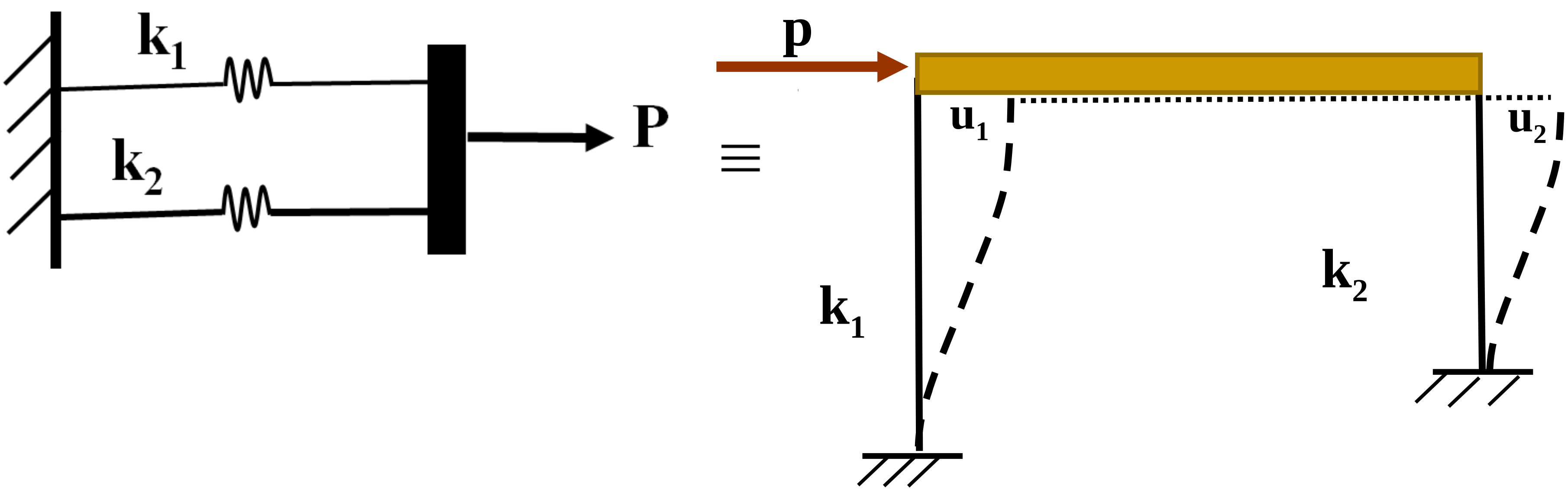
$$\Rightarrow \; k_e u = k_1 u + k_2 u$$

$$\Rightarrow \; k_e = k_1 + k_2$$

STIFNESS OF SPRINGS IN PARALLEL

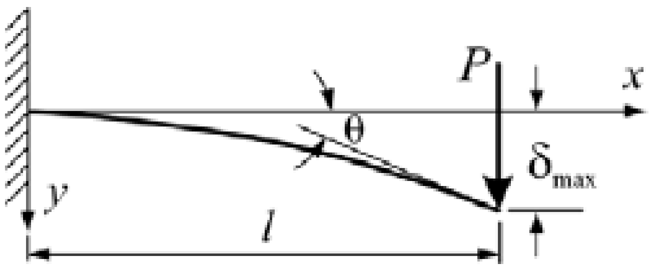
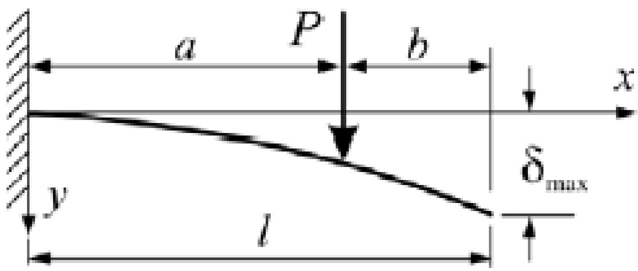
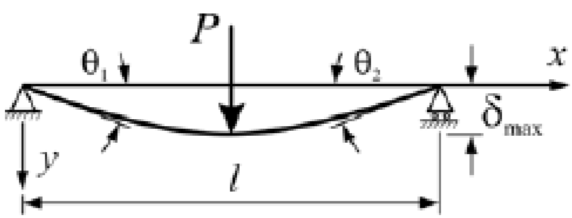
$$k_e = k_1 + k_2$$

EAXMPLE

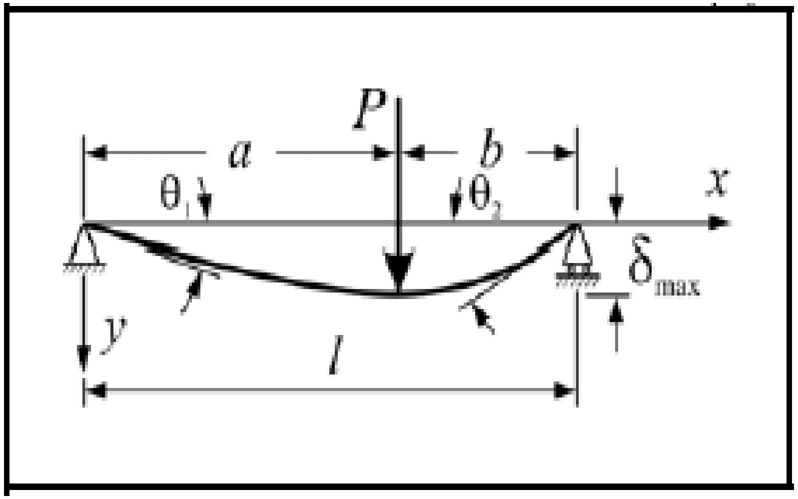


$u_1 = u_2$ provided the change in axial length of beam is neglected (a reasonable assumption).

Deflection in beam and their stiffness

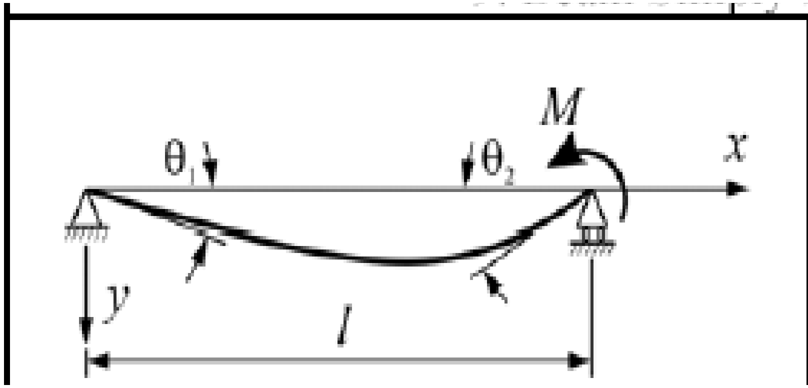
BEAM TYPE	MAXIMUM DEFLECTION	
1. Cantilever Bea		
	$\delta_{max} = \dfrac{Pl^3}{3EI}$	$k = P/\delta_{max} = 3EI/l^3$
2. Cantilever Bea		
	$\delta_{max} = \dfrac{Pa^2}{6EI}(3l - a)$	$k = 6EI/\left\{a^2(3l - a)\right\}$
	$\delta_{max} = \dfrac{Pl^3}{48EI}$	$k = P/\delta_{max} = 48EI/l^3$

Deflection in beam and their stiffness



$$\delta_{max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI} \text{ at } x = \sqrt{\left(l^2 - b^2\right)/3}$$

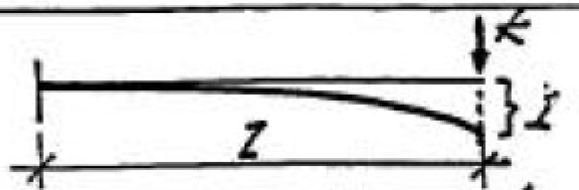
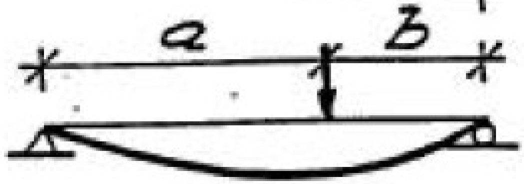
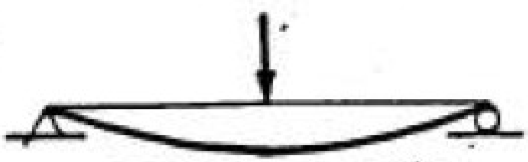
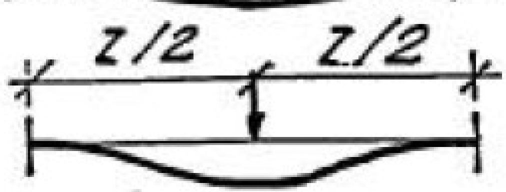
$$\delta = \frac{Pb}{48EI}\left(3l^2 - 4b^2\right) \text{ at the center, if } a > b$$



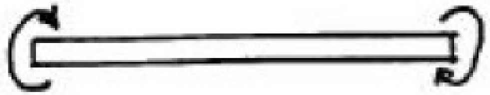
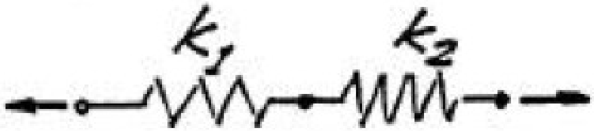
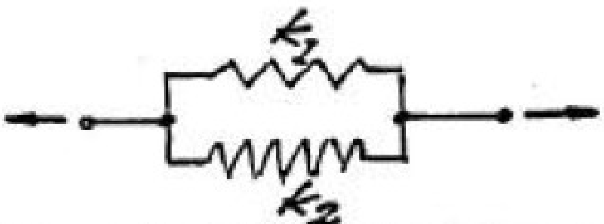
$$\delta_{max} = \frac{Ml^2}{9\sqrt{3}\,EI} \text{ at } x = \frac{l}{\sqrt{3}}$$

$$\delta = \frac{Ml^2}{16EI} \text{ at the center}$$

STIFFNESS CONSTANTS OF SOME STRUCTURAL ELEMENTS

	Structure	k =	Comment
1.		$\frac{3EI}{\ell^3}$	Fixed-Pinned
2.		$\frac{12EI}{\ell^3}$	Fixed-Fixed
3.		$\frac{3EI\ell}{a^2b^2}$	Pinned-Pinned
4.		$\frac{48EI}{\ell^3}$	Pinned-Pinned
5.		$\frac{192EI}{\ell^3}$	Fixed-Fixed

STIFFNESS CONSTANTS OF SOME STRUCTURAL ELEMENTS

7.		$\frac{EA}{\ell}$	Axial
8.		$\frac{GJ}{\ell}$	Torsion
9.		$\frac{1}{1/k_1+1/k_2}$	Springs in Series
10.		k_1+k_2	Parallel Springs

I = moment of inertia of cross-sectional area

A = cross-sectional area

J = torsional constant of cross section

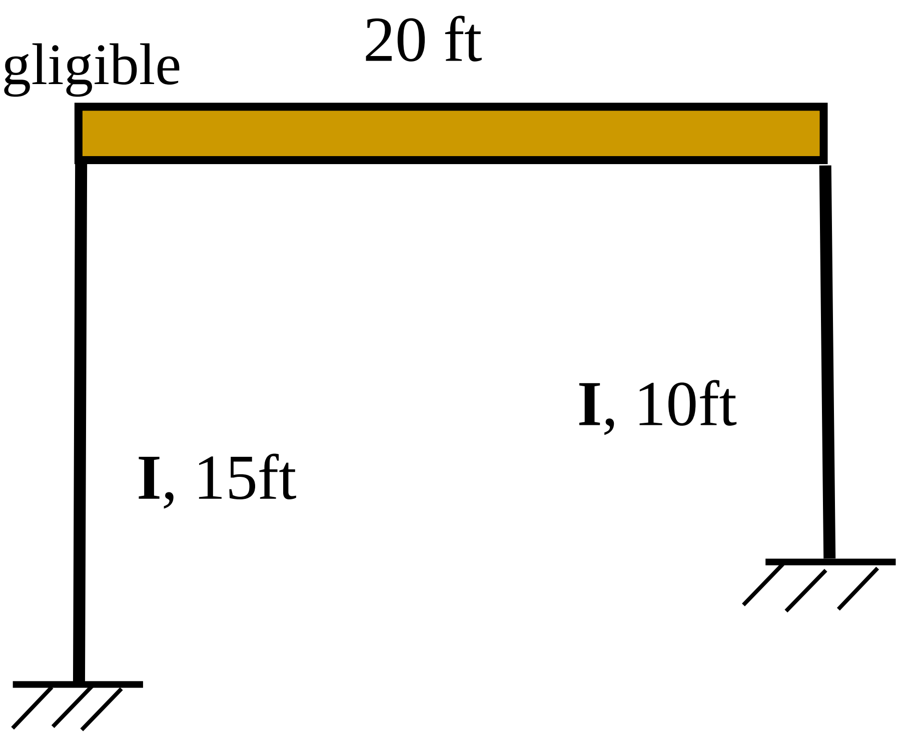
ℓ = length of element

Problem M 3.1

Determine lateral stiffness of the frame if a lateral load is applied at beam level. Assume:

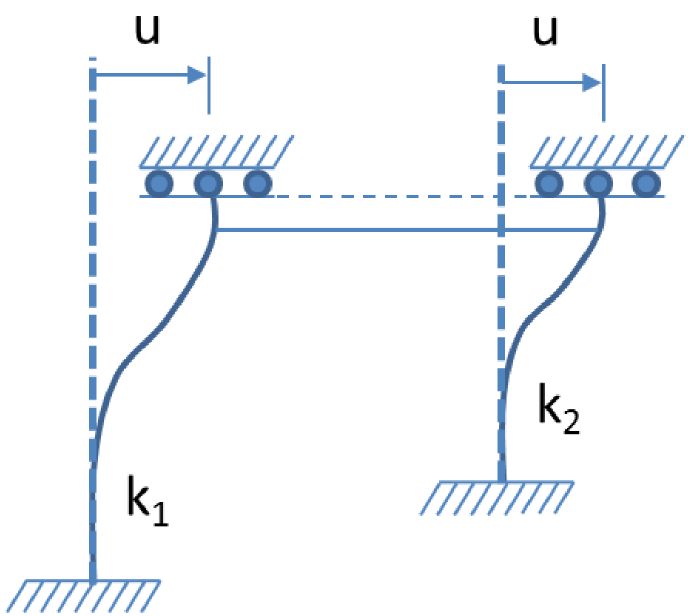
1. The flexural stiffness of beam is too high as compared to that of connected columns.
2. Axial deformations in beam is negligible

Take **E** = 29,000 ksi, **I** = 1200 in^4



Solution M3.1

$$k_{eq} = k_1 + k_2$$



$$k = \frac{12EI}{h_1^3} + \frac{12EI}{h_2^3}$$

$$= 12EI\left[\frac{1}{h_1^3} + \frac{1}{h_2^3}\right]$$

$$= 12 \times (29000\ k/in^2) \times (1200in^4)\left[\frac{1}{(15 \times 12\ in)^3} + \frac{1}{(10 \times 12\ in)^3}\right]$$

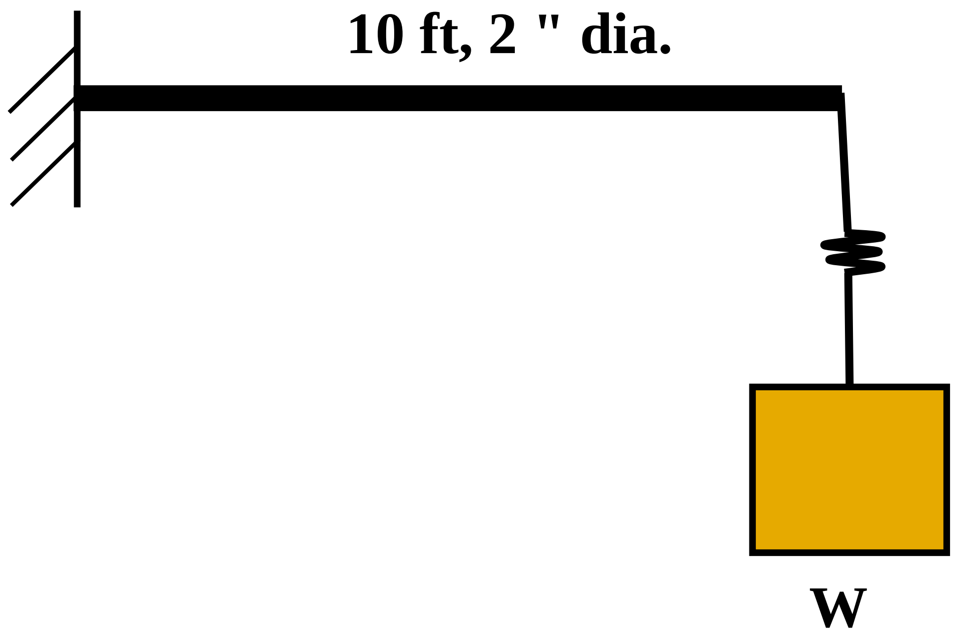
$$= 313.29\, k/in$$

$$= 3759\, k/ft$$

LATERAL STIFFNESS OF A CANTILEVER BEAM CONNECTED TO A SPRING SYSTEM

Problem M 3.2

Determine the stiffness of cantilever beam by assuming that the self weight of beam is negligible Take **E** = 29,000 ksi, $\mathbf{k}_{spring}$= 200 lb/ft.



Solution M3.2

$$k_1 = 200\, lb/ft$$

$$k_2 = \frac{3EI}{l^3} = \frac{3 \times \left(29000\, {}^{k}/_{in^2}\right) \times \left(\frac{\pi}{64} \times (2\ in)^2\right)}{(10 \times 12\ in)^3}$$

$$= 0.0396\, k/in^2 = 474.7\, lb/ft$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{200 \times 474.7}{200 + 474.7}$$

$$k_{eq} = 140.7\, lb/ft$$

Home Assignment No. M3H2

1. Determine the equivalent stiffness of system shown in Figure **a** **(Answer** = 2.6k)

2. Determine the equivalent stiffness of system shown in Figure **b** **(Answer=** 7.03 * 10^7 N/m)

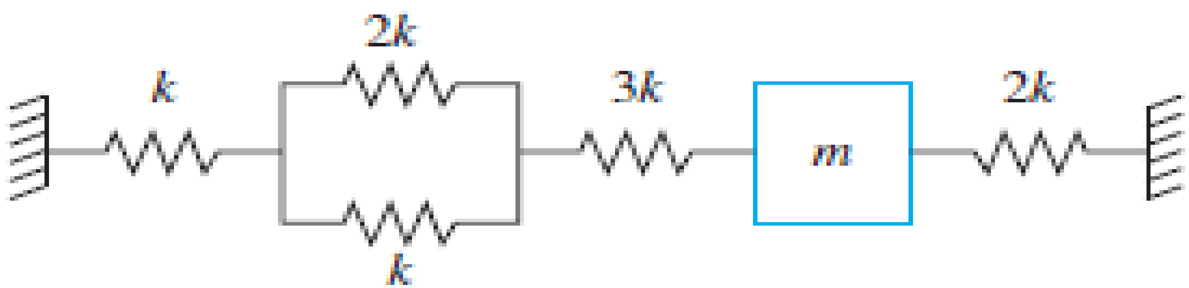


Figure **a**

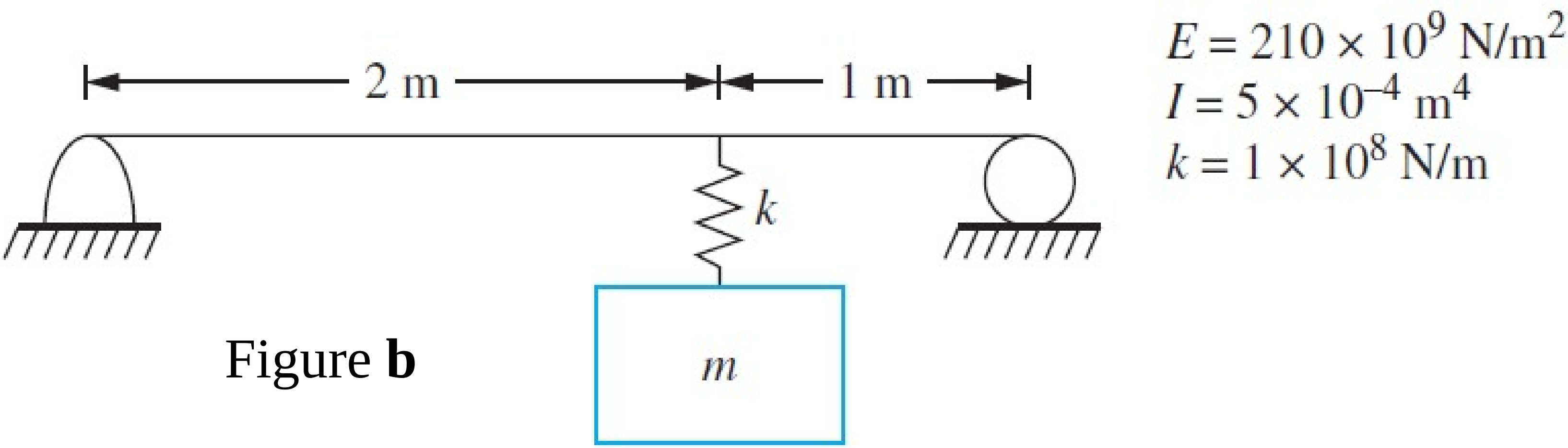


Figure **b**

Home Assignment No. M3H2

3. Determine the equivalent stiffness of the systems (in vertical direction when vertical force is applied at point A) shown in Figures **c & d**. Take $\boldsymbol{k_1}$= 1 kN/m and $\boldsymbol{k_{beam}}$ = 5 kN/m

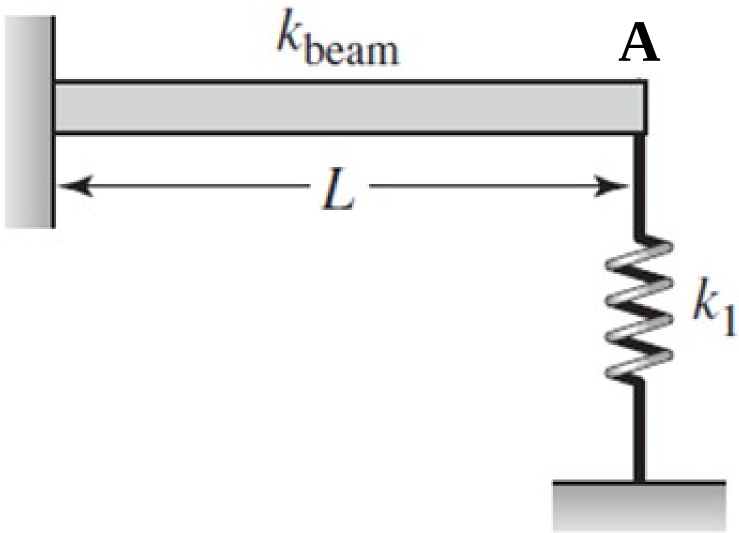


Fig. c

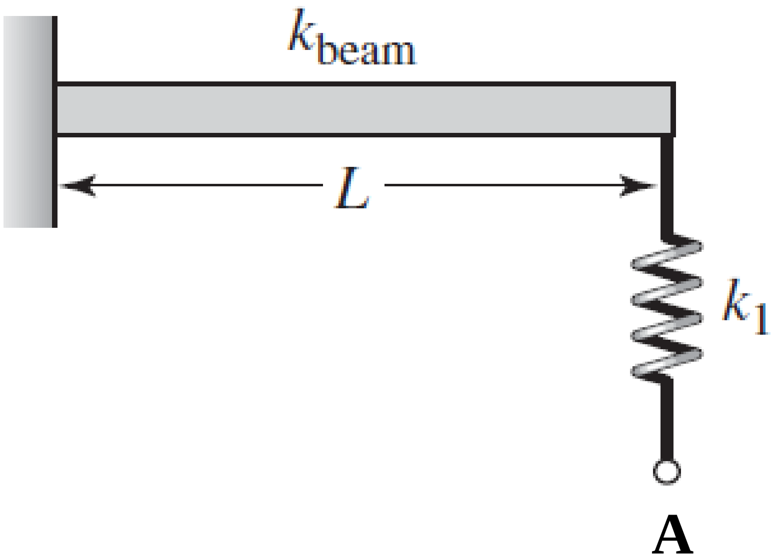


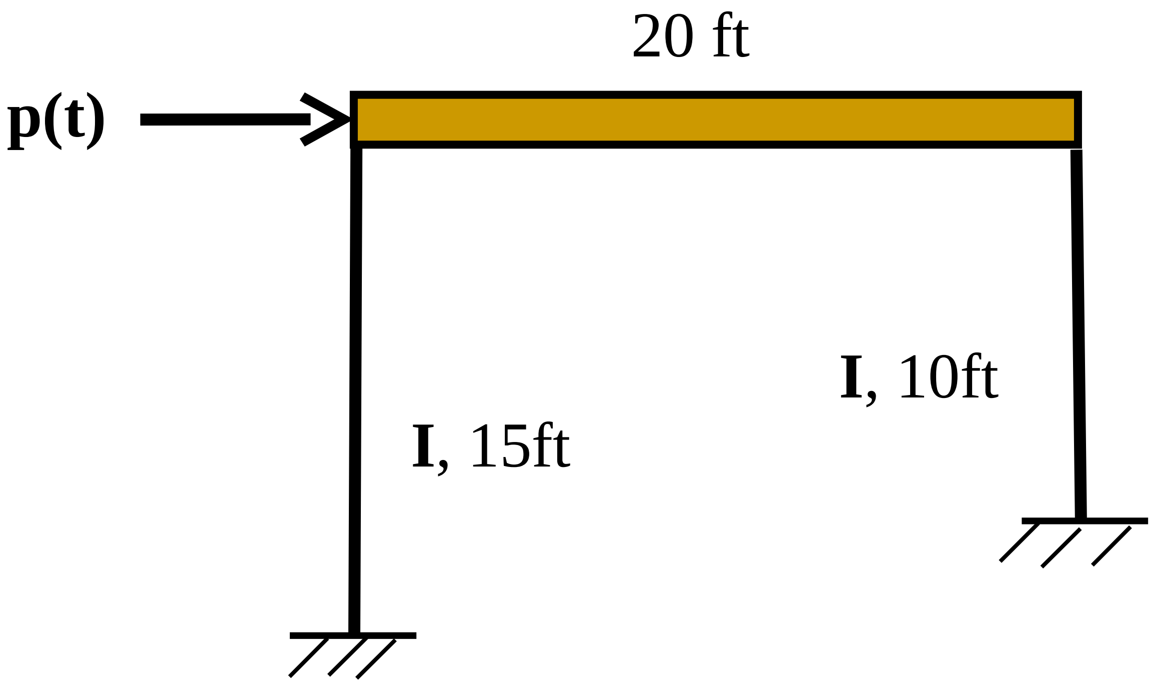
Fig. d

E.O.M FOR A SINGLE STORY FRAME UNDER LATERAL DYNAMIC FORCE

Problem M 3.3

Develop the equation of motion of the frame shown in problem **M 3.1** under the action of a lateral dynamic force **p(t)**. Consider a uniformly Distributed gravity load of 5 k/ft acting on the beam.

Neglect damping effect



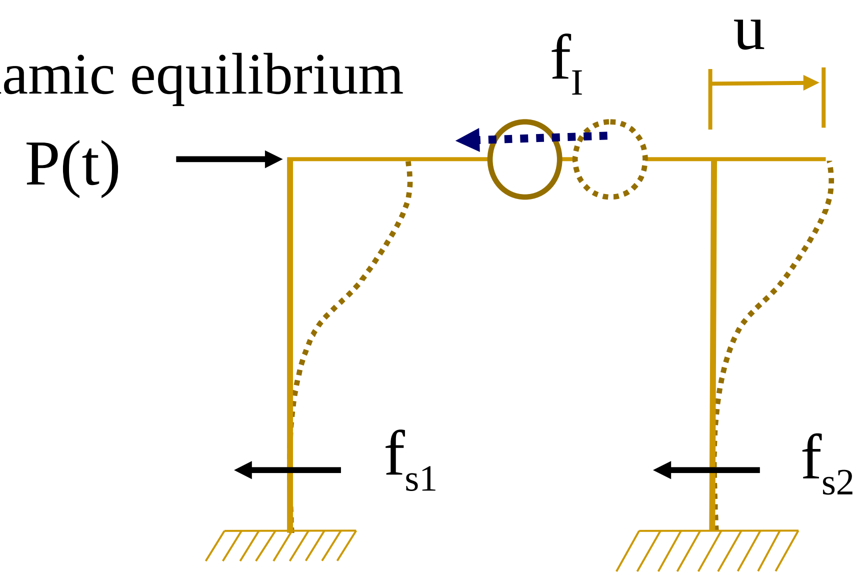
Solution M3.3

$$m = \frac{w}{g} = \frac{5 \times 20k}{32.2\, ft/sec^2}$$

$$m = 3.106\, k.sec^2/ft = 3106\ lb.sec^2/ft$$

$$m = 3.106\, slug$$

Using D-Alembert's Principle of dynamic equilibrium



$$P(t) - f_I - f_{s_1} - f_{s_2} = 0$$

$$P(t) - m\ddot{u} - \left(f_{s_1} + f_{s_2}\right) = 0$$

$$(k_1 u + k_2 u) + m\ddot{u} = P(t)$$

$$(ku) + m\ddot{u} = P(t)$$

As , k= 3759k/ft

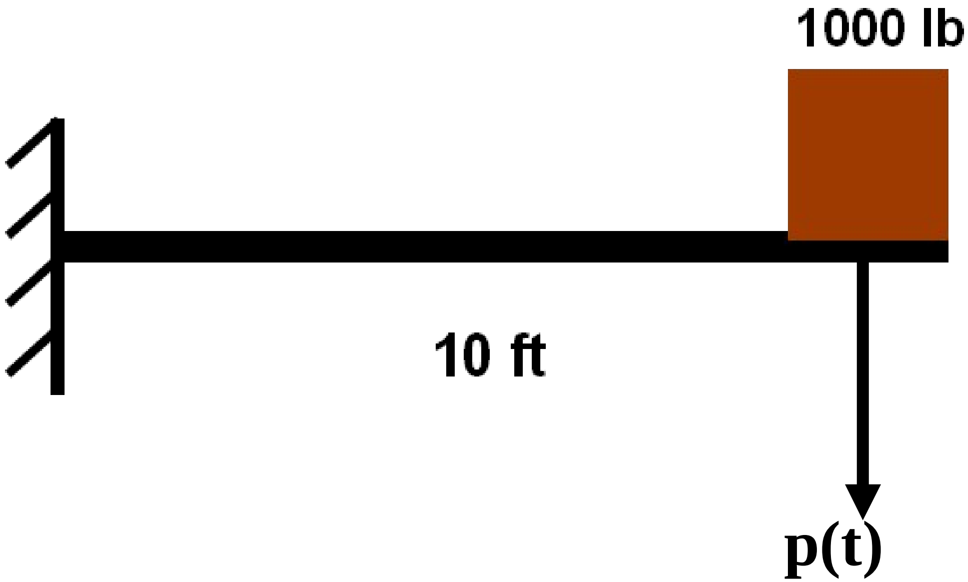
$$3106\ddot{u} + 3.76*10^6 u = p(t)$$

Where u & p(t) are in ft and lb, respectively

E.O.M FOR A CANTILVER BEAM UNDER LATERAL DYNAMIC FORCE

Problem M 3.4

Develop the equation of motion for the cantilever beam under the action of **p(t)**. Neglect the self weight of beam as well as damping effect. Take **E**= 29000 ksi & **I** = 150 in^4



M-3.4

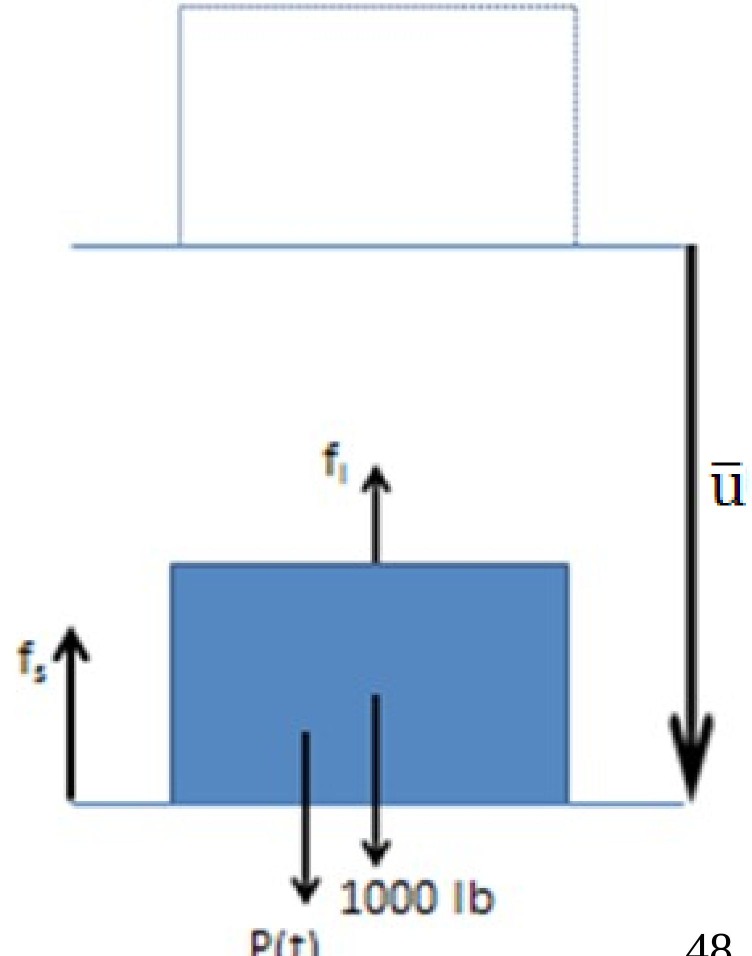
The displacement comprise of two parts.

1.Constant deflection at free end, δ_{st}, due to static load of 100lb.

2.Time dependent displacement [u(t)] at free end due to P(t).

Total Displacement, $\overline{u} = \delta_{st} + u$......... eq (i)

$\sum Fy = 0 \quad +\uparrow \quad -\downarrow$



$f_s + f_I - W - P(t) = 0$

$K.\overline{u} + m\,\overline{\ddot{u}} - W = P(t)$ ....eq (ii)

$\overline{u} = \delta_{st} + u$

$\overline{\dot{u}} = \dot{u}$

$\overline{\ddot{u}} = \ddot{u}$

Inserting values in eq (ii)

$K(\delta_{st} + u) + m\ddot{u} - W = P(t)$

$K\delta_{st} + Ku + m\ddot{u} - K\delta_{st} = P(t)$

$Ku + m\ddot{u} = P(t)$

the above equation indicate that EOM is unaffected by static displacements. As considering dynamic displacements from static equilibrium position result in same EOM.

Now

However, the gravity loads must be considered if they act as either restoring forces (e.g., Pendulum) or as destabilizing forces (inverted pendulum).

$$K = \frac{3EI}{L^3} = \frac{3*(29000\ ^{k}/_{in^2})*(150\,in^4)}{(10*12in)^3}$$

K = 7.552 K/in = 90625 lb/ft

$$m = w/g = \frac{1000\,lb}{32.2\frac{ft}{sec^2}} = 31.06\ \text{slug}$$

substituting in eq (iii)

$90625u + 31.06\,\ddot{u} = P(t)$

Where u & p(t) are in ft and lb, respectively

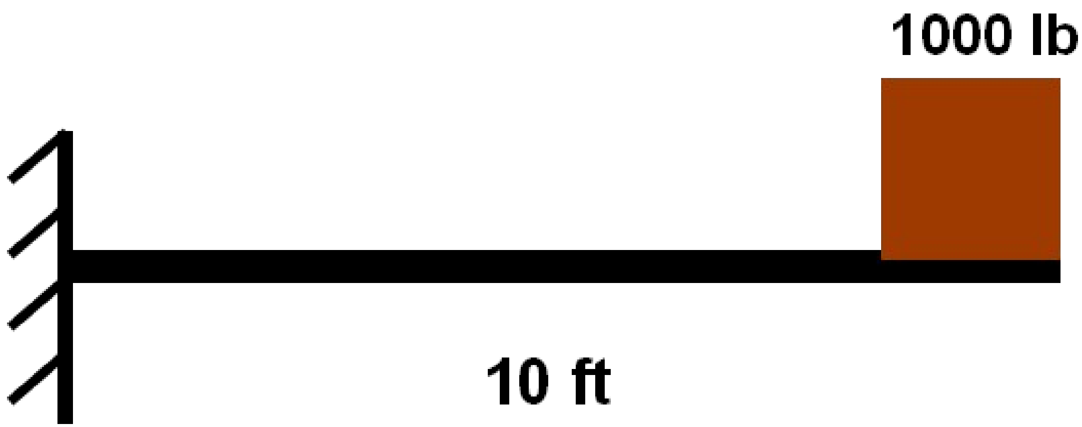
E.O.M FOR A CANTILVER BEAM UNDER THE GRAVITY LOAD (SUDDENLY PLACED)

Problem M 3.5

Develop the equation of motion for the cantilever beam under the action 1000 lb weight. Assume that time required to place the weight on beam is Insignificant as compared to natural time period of beam (?).

Neglect the self weight of beam as well as damping effect.

Take **E** = 29000 ksi & **I** = 150 in^4



M-3.5

The gravity load in given case acts as dynamic load (as per statement of problem)

$\sum Fy = 0 \quad + \uparrow \quad - \downarrow$

$f_s + f_I - 1000 = 0$

$K.u + m\ddot{u} - 1000 = 0$

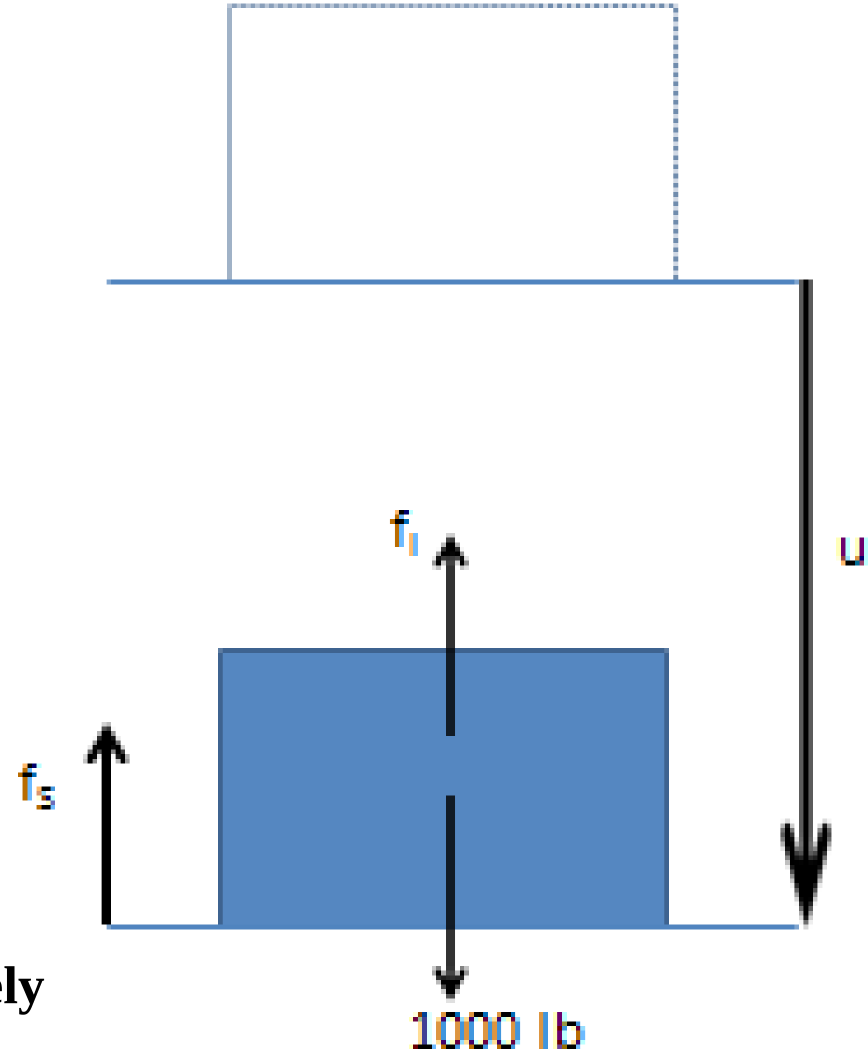
From problem 3.4

K=90625 lb/ft

And m=31.06 slug

So,

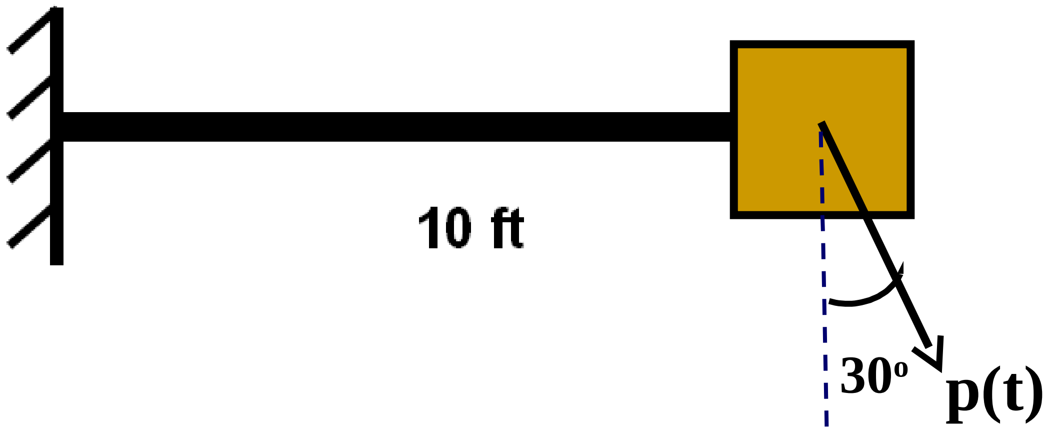
$90625u + 31.06\ \ddot{u} = 1000$



Where u & p(t) are in ft and lb, respectively

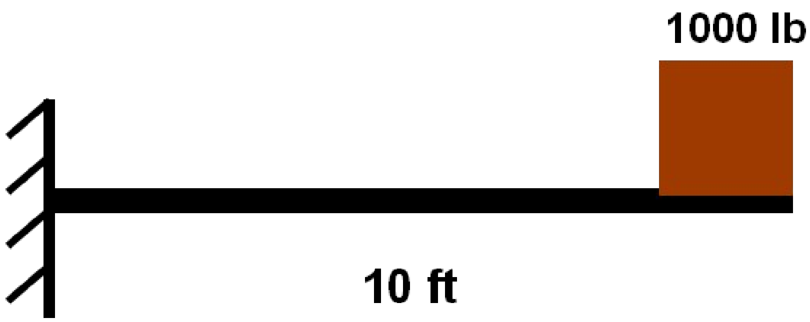
Home Assignment No. M3H3

Q1: Develop the EOMs for the cantilever beam along axes which are parallel and perpendicular to the beam, under the action of **p(t)**. Neglect the self weight of beam as well as damping effect. Take **E**= 29000 ksi and **I** = 200 in^4 , **A**= 50 in^2. Ignore the dimensions of 1000 lb weight



Home Assignment No. M3H3

Q 2. Develop the equation of motion for the cantilever beam under the action 1000 lb weight. A weight of 500 lb is placed on already acting weight of 1000 lb. Assume that the time required to place the 500 lb weight on beam is insignificant as compared to natural time period of beam. Neglect the self weight of beam as well as damping effect. Take **E** = 29000 ksi & **I** = 150 in^4



Q 3. Develop the EOMs for the beams mentioned in Figures **c** and **d** (Q 3, M3H2) for a vertical dynamic load **p(t)** acting at **A**. Consider a

METHODS OF SOLUTIONS FOR DIFFRENTIAL EQUATIONS

The differential equation of the type $ku + c\dot{u} + m\ddot{u} = p(t)$

can be solved by any one of the following four methods

1. **Classical mathematical solutions**

2. **Duhamel's Integral**

3. **Frequency- Domain method**

4. **Numerical methods**