

24-10 Calculating the Field from the Potential

In Section 24-5, you saw how to find the potential at a point f if you know the electric field along a path from a reference point to point f . In this section, we propose to go the other way—that is, to find the electric field when we know the potential. As Fig. 24-3 shows, solving this problem graphically is easy: If we know the potential V at all points near an assembly of charges, we can draw in a family of equipotential surfaces. The electric field lines, sketched perpendicular to those surfaces, reveal the variation of \vec{E} . What we are seeking here is the mathematical equivalent of this graphical procedure.

Figure 24-14 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV . As the figure suggests, the field \vec{E} at any point P is perpendicular to the equipotential surface through P .

Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface. From Eq. 24-7, we see that the work the electric field does on the test charge during the move is $-q_0 dV$. From Eq. 24-16 and Fig. 24-14, we see that the work done by the electric field may also be written as the scalar product $(q_0\vec{E}) \cdot d\vec{s}$, or $q_0 E(\cos \theta) ds$. Equating these two expressions for the work yields

$$-q_0 dV = q_0 E(\cos \theta) ds, \quad (24-38)$$

or

$$E \cos \theta = -\frac{dV}{ds}. \quad (24-39)$$

Since $E \cos \theta$ is the component of \vec{E} in the direction of $d\vec{s}$, Eq. 24-39 becomes

$$E_s = -\frac{\partial V}{\partial s}. \quad (24-40)$$

We have added a subscript to E and switched to the partial derivative symbols to emphasize that Eq. 24-40 involves only the variation of V along a specified axis (here called the s axis) and only the component of \vec{E} along that axis. In words, Eq. 24-40 (which is essentially the reverse operation of Eq. 24-18) states:

The component of \vec{E} in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

If we take the s axis to be, in turn, the x , y , and z axes, we find that the x , y , and z components of \vec{E} at any point are

$$E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z}. \quad (24-41)$$

Thus, if we know V for all points in the region around a charge distribution—that is, if we know the function $V(x, y, z)$ —we can find the components of \vec{E} , and thus \vec{E} itself, at any point by taking partial derivatives.

For the simple situation in which the electric field \vec{E} is uniform, Eq. 24-40 becomes

$$E = -\frac{\Delta V}{\Delta s}, \quad (24-42)$$

where s is perpendicular to the equipotential surfaces. The component of the electric field is zero in any direction parallel to the equipotential surfaces because there is no change in potential along the surfaces.

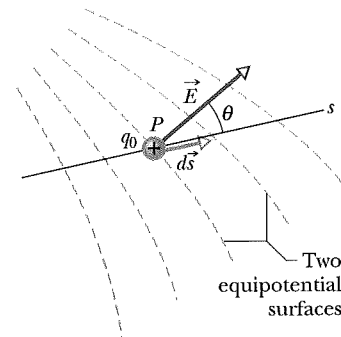


Fig. 24-14 A test charge q_0 moves a distance $d\vec{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\vec{s}$ makes an angle θ with the direction of the electric field \vec{E} .

CHECKPOINT 6

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?

