

24-9 Potential Due to a Continuous Charge Distribution

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation of Eq. 24-27 to find the potential V at a point P . Instead, we must choose a differential element of charge dq , determine the potential dV at P due to dq , and then integrate over the entire charge distribution.

Let us again take the zero of potential to be at infinity. If we treat the element of charge dq as a point charge, then we can use Eq. 24-26 to express the potential dV at point P due to dq :

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} \quad (\text{positive or negative } dq). \quad (24-31)$$

Here r is the distance between P and dq . To find the total potential V at P , we integrate to sum the potentials due to all the charge elements:

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}. \quad (24-32)$$

The integral must be taken over the entire charge distribution. Note that because the electric potential is a scalar, there are *no vector components* to consider in Eq. 24-32.

We now examine two continuous charge distributions, a line and a disk.

Line of Charge

In Fig. 24-12a, a thin nonconducting rod of length L has a positive charge of uniform linear density λ . Let us determine the electric potential V due to the rod at point P , a perpendicular distance d from the left end of the rod.

We consider a differential element dx of the rod as shown in Fig. 24-12b. This (or any other) element of the rod has a differential charge of

$$dq = \lambda dx. \quad (24-33)$$

This element produces an electric potential dV at point P , which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. 24-12c). Treating the element as a point

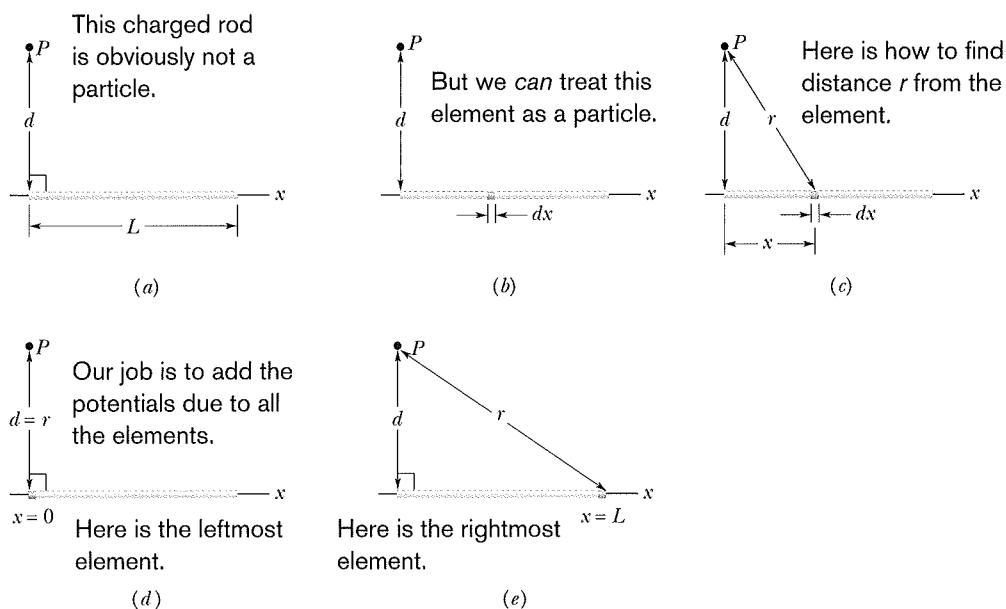


Fig. 24-12 (a) A thin, uniformly charged rod produces an electric potential V at point P . (b) An element can be treated as a particle. (c) The potential at P due to the element depends on the distance r . We need to sum the potentials due to all the elements, from the left side (d) to the right side (e).

charge, we can use Eq. 24-31 to write the potential dV as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}. \quad (24-34)$$

Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know from Section 24-6 that dV in Eq. 24-34 must be positive.

We now find the total potential V produced by the rod at point P by integrating Eq. 24-34 along the length of the rod, from $x = 0$ to $x = L$ (Figs. 24-12*d* and *e*), using integral 17 in Appendix E. We find

$$\begin{aligned} V &= \int dV = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx \\ &= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(x + (x^2 + d^2)^{1/2} \right) \right]_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(L + (L^2 + d^2)^{1/2} \right) - \ln d \right]. \end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$. We then find

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]. \quad (24-35)$$

Because V is the sum of positive values of dV , it too is positive, consistent with the logarithm being positive for an argument greater than 1.

Charged Disk

In Section 22-7, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density σ on one surface. Here we derive an expression for $V(z)$, the electric potential at any point on the central axis.

In Fig. 24-13, consider a differential element consisting of a flat ring of radius R' and radial width dR' . Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

in which $(2\pi R')(dR')$ is the upper surface area of the ring. All parts of this charged element are the same distance r from point P on the disk's axis. With the aid of Fig. 24-13, we can use Eq. 24-31 to write the contribution of this ring to the electric potential at P as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}. \quad (24-36)$$

We find the net potential at P by adding (via integration) the contributions of all the rings from $R' = 0$ to $R' = R$:

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z). \quad (24-37)$$

Note that the variable in the second integral of Eq. 24-37 is R' and not z , which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that $z \geq 0$.)

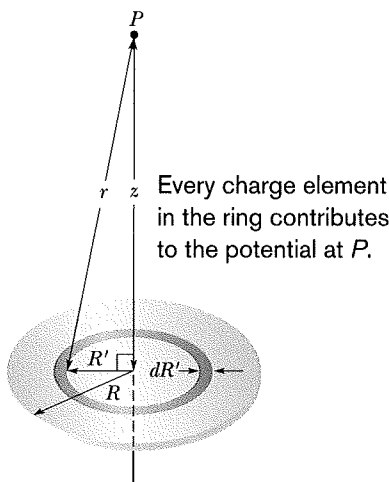


Fig. 24-13 A plastic disk of radius R , charged on its top surface to a uniform surface charge density σ . We wish to find the potential V at point P on the central axis of the disk.