University of Engineering and Technology Peshawar, Pakistan

CE-409: Introduction to Structural Dynamics and Earthquake Engineering

MODULE 3:

FUNDAMENTALS OF DYNAMIC ANALYSIS FOR S.D.O.F SYSTEMS

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Structural Degrees of Freedom

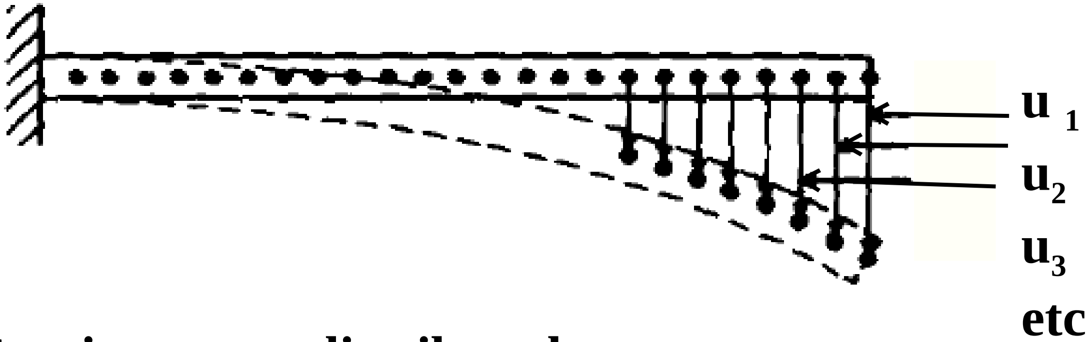
Degrees of freedom (DOF) of a system is defined as the number of ***independent*** variables required to completely determine the positions of all parts of a system at any instant of time.

Discrete vs. Continuous systems

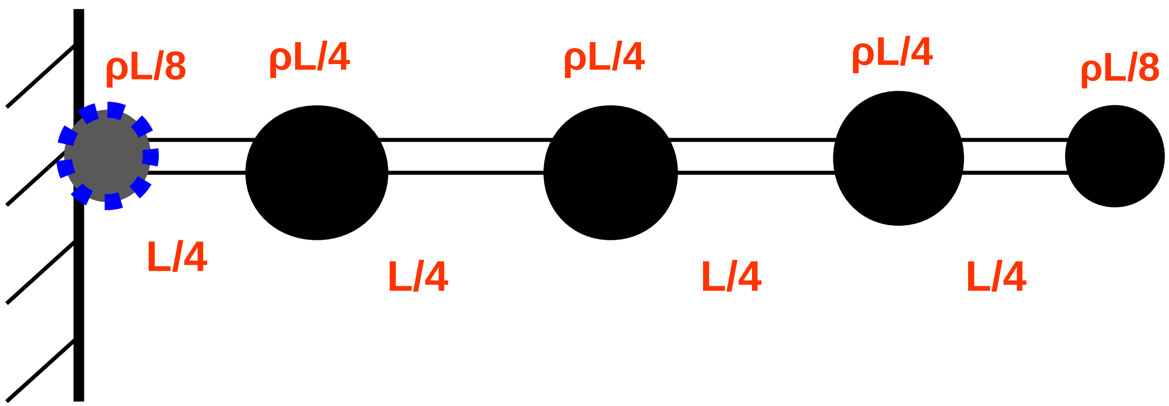
- Some systems, specially those involving continuous elastic members, have an infinite number of DOF. As an example of this is a cantilever beam with self weight only (*see next slide*). This beam has infinites mass points and need infinites number of displacements to draw its deflected shape and thus has an infinite DOF. Systems with infinite DOF are called ***Continuous or Distributed systems***.

- Systems with a finite number of degree of freedom are called ***Discrete or Lumped mass parameter systems***.

Discrete vs. Continuous systems



Continuous or distributed system



Corresponding lumped mass system of the above given cantilever beam with DOF= 4 (*How? there are 5 lumped masses.*)

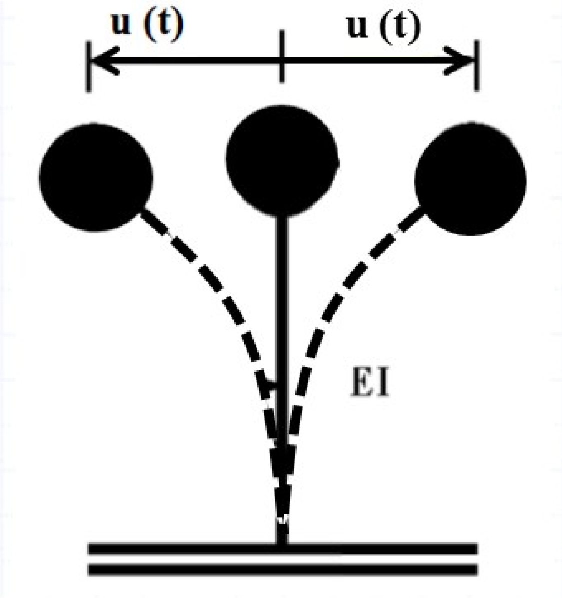
ρ = Mass per unit length

Single Degree-of-Freedom (SDOF) System

- In a single degree of freedom system, the deformation of the entire structure can be described by a single number equal to the displacement of a point from an **at-rest position**.
- Single Degree of freedom systems do not normally exist in real life. We live in a three-dimensional world and all mass is distributed resulting in systems that have an infinite number of degrees of freedom. There are, however, instances where a structure may be approximated as a single degree of freedom system.
- The study of SDOF systems is an integral step in understanding the responses of more complicated and realistic systems.

Idealization of a structural system as SDOF system

This 3-dimensional water tower may be considered as a single degree of freedom system when one considers vibration in **one horizontal direction only**.



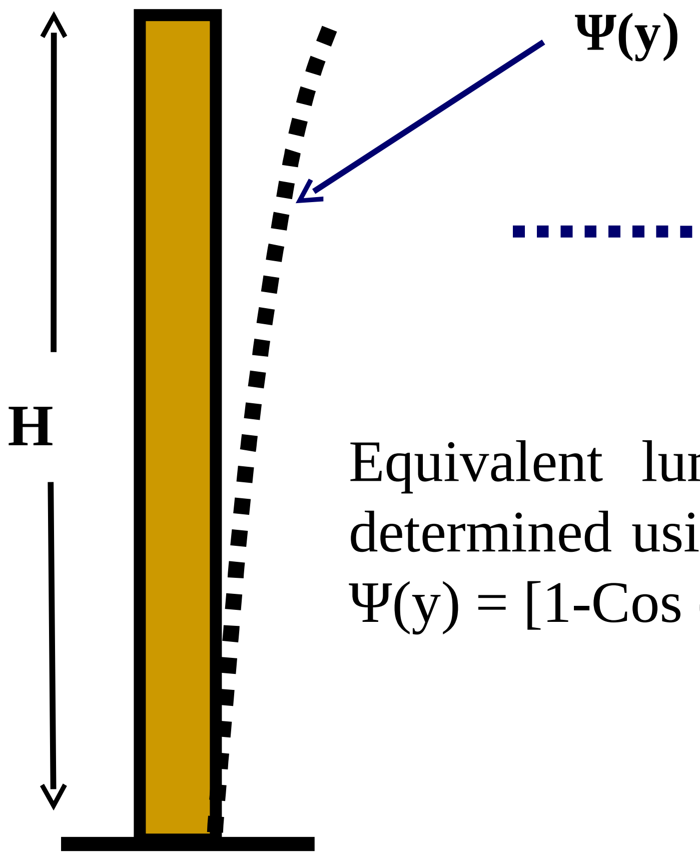
SDOF model of water tank

Idealization of a structural system as SDOF system

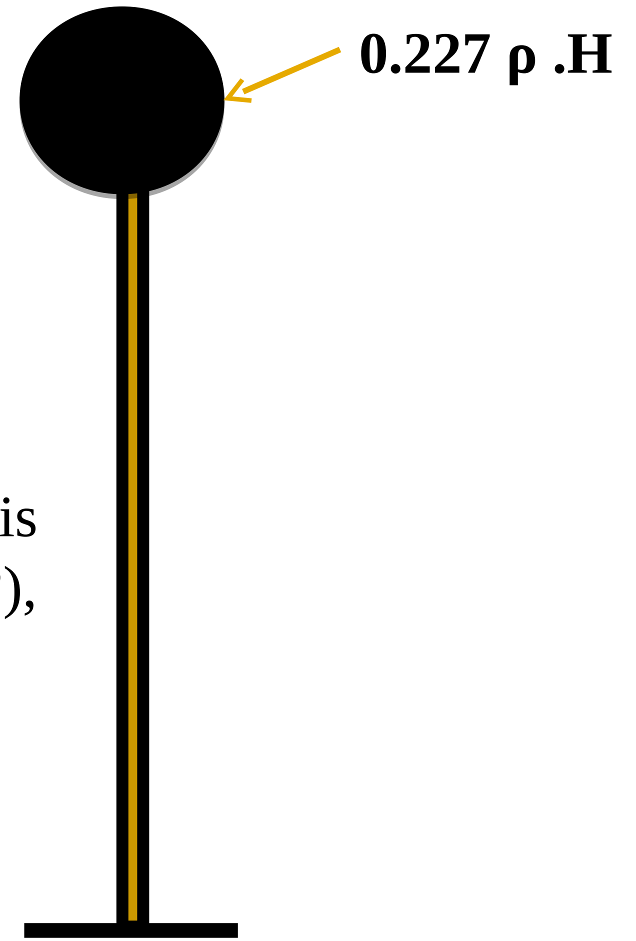
- The structural system of water tank may be simplified by assuming that the column has negligible mass along its length. This is reasonable, assuming that the tube is hollow and that the mass of the tube is insignificant when compared with the mass of the water tank and water at the top.

- This means that we can consider that the tank is a point mass

Equivalent lumped mass SDOF system of a cantilever wall with uniform x-sectional area



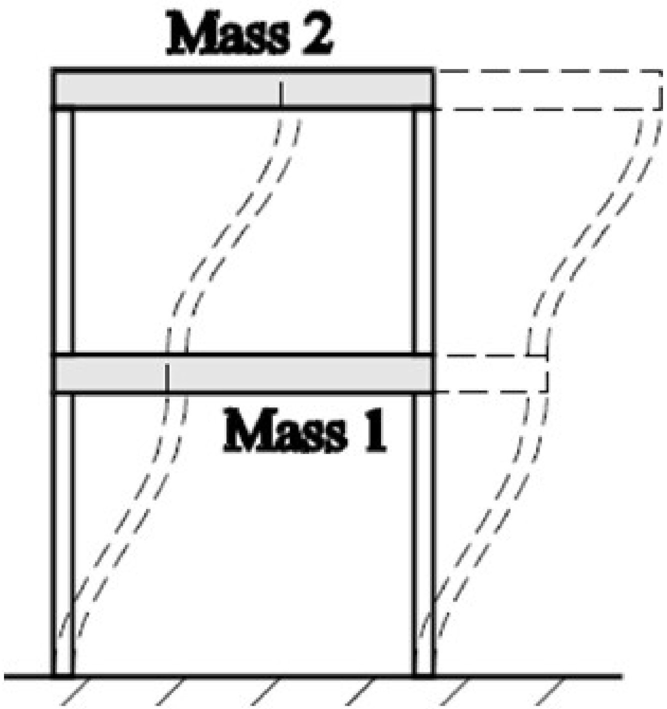
Equivalent lumped mass model is determined using a shape function (?), $\Psi(y) = [1 - \text{Cos}\,(\pi y/2H)]$.

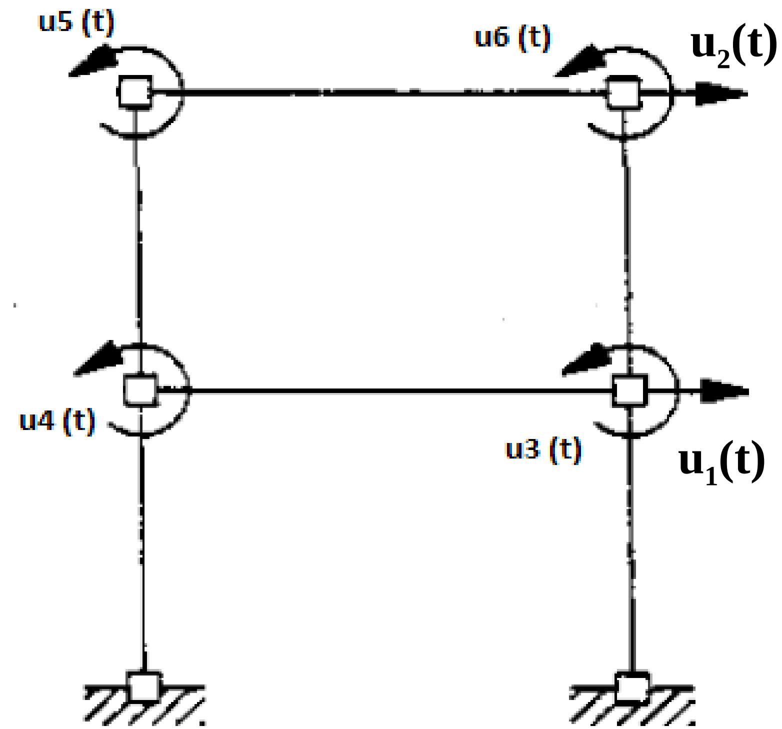


ρ = Mass per unit height, H= total height, y= Any distance along height and k = lateral stiffness of cantilever member = EI/H^3

Multiple Degree-of-Freedom (MDOF) System

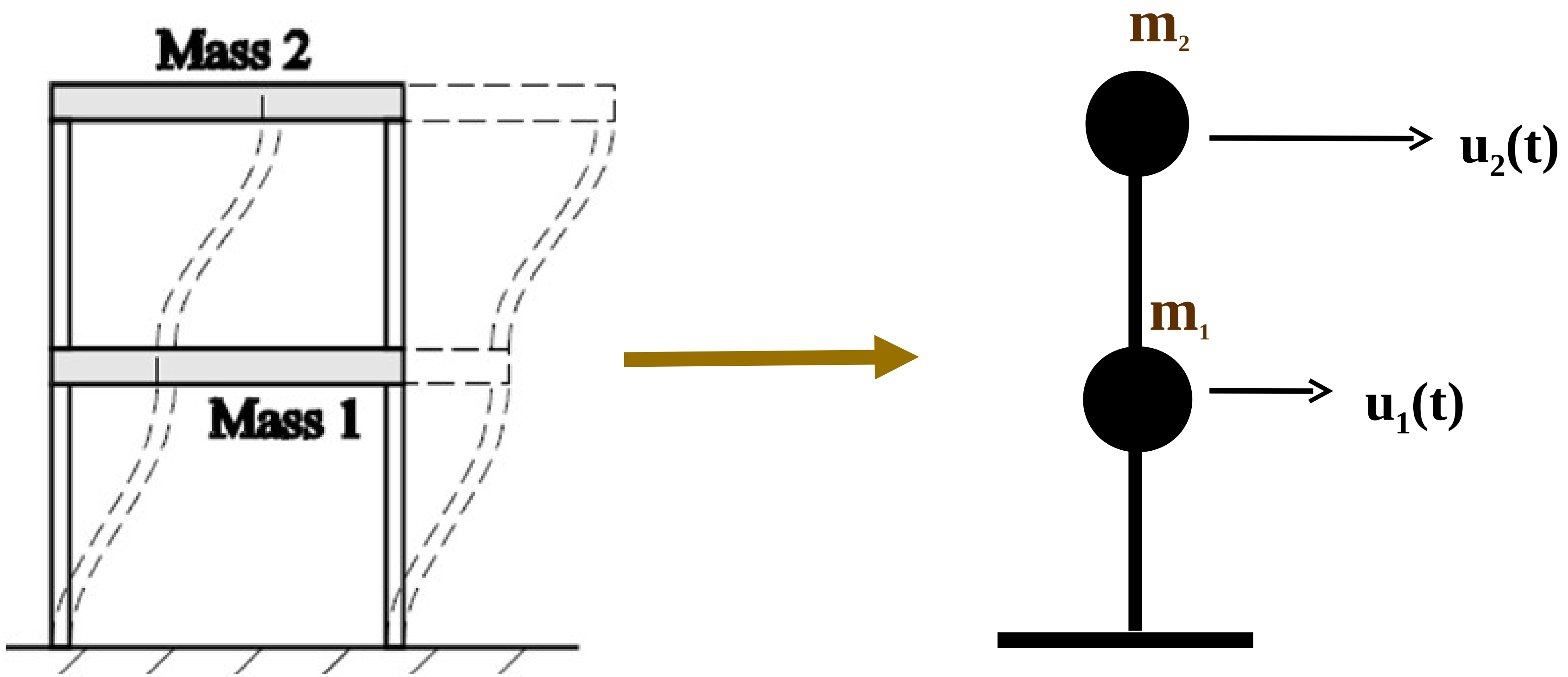
In a Multi degree of freedom system, the deformation of the entire structure cannot be described by a single displacement. More than one displacement coordinates are required to completely specify the displaced shape.





Considering all DOFs

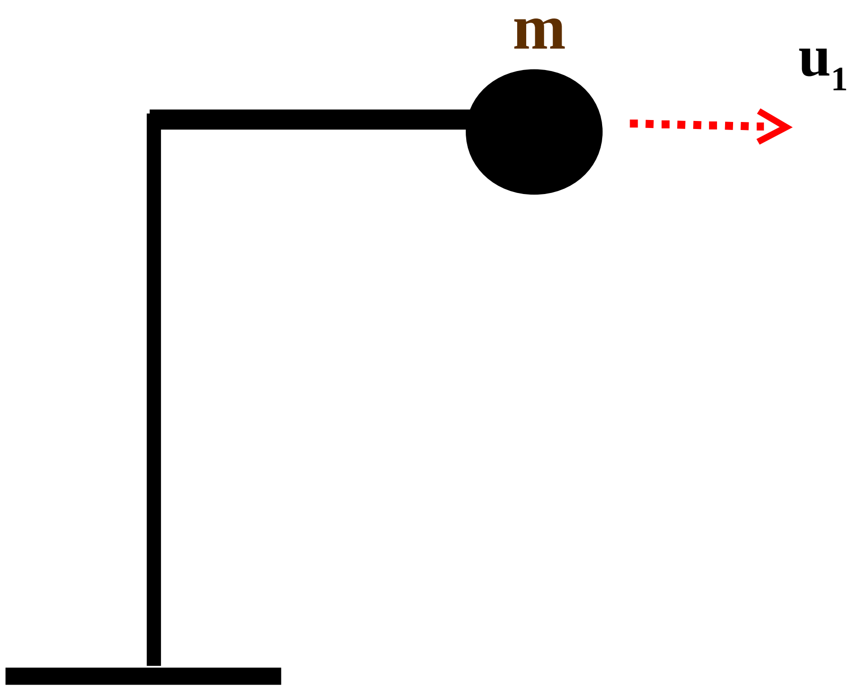
Multiple Degree-of-Freedom (MDOF) System



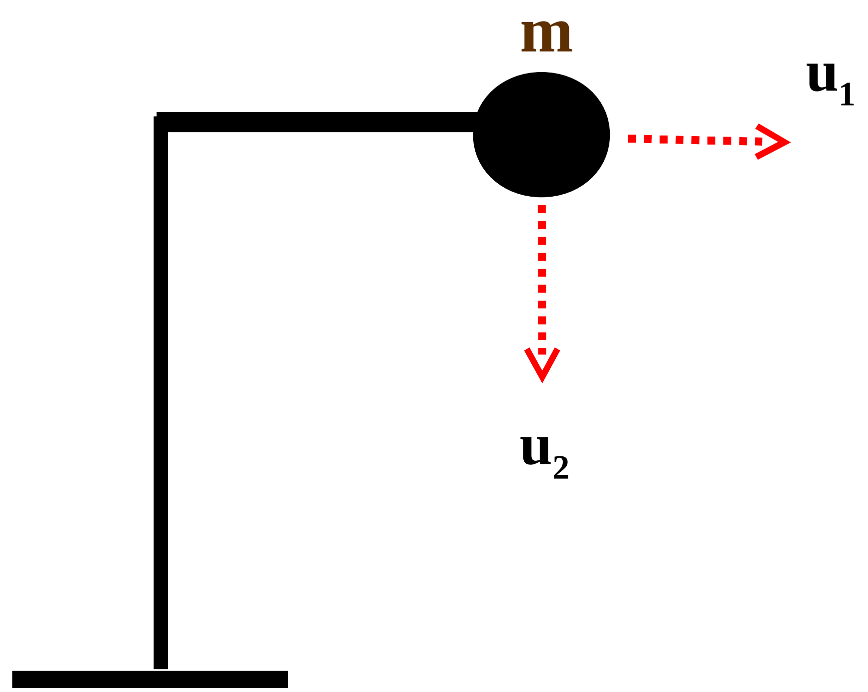
Lumped mass model of building (DOF=2). $u_3(t)$ to $u_6(t)$ (shown on previous slide) got eliminated by lumping the masses at mid length of beam.

Multiple Degree-of-Freedom (MDOF) System

What is the DOF for this system…?



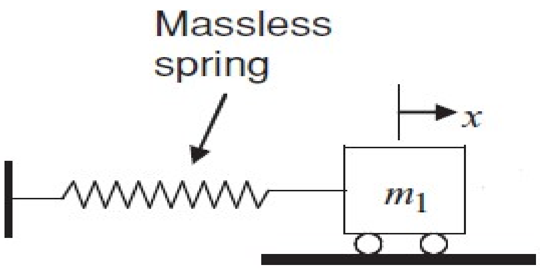
DOF may be taken 1 when flexural stiffness of beam is taken infinite/ too high



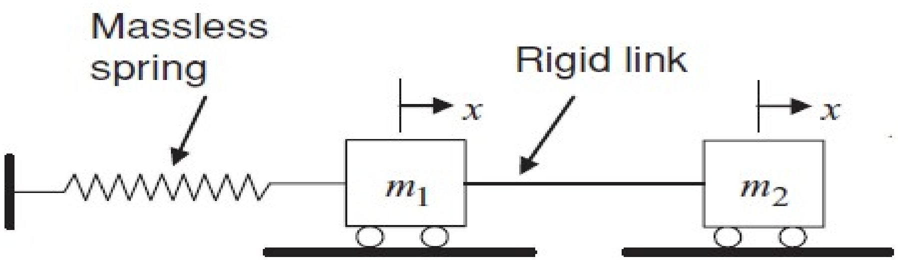
DOF is 2 when we have a flexible beam

Home Assignment No. M3H1

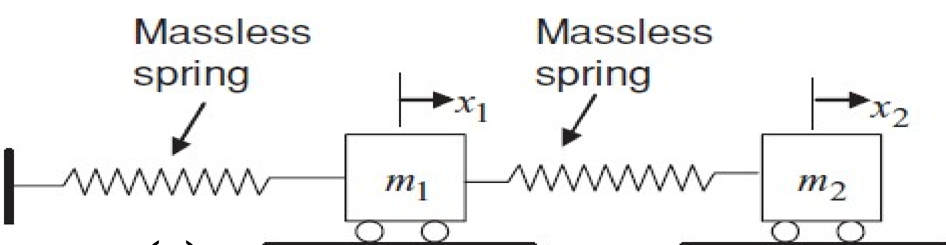
Determine the DOF of systems shown in given figures. Support you answer with argument(s)



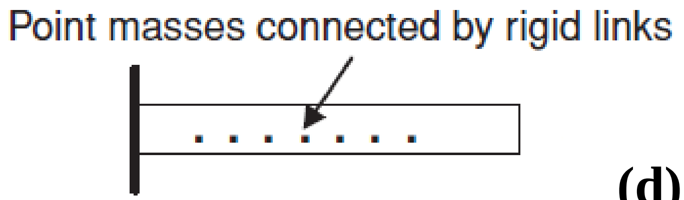
(a)



(b)

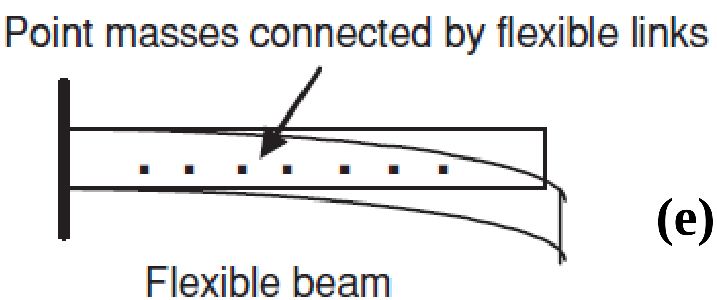


(c)



(d)

A rigid beam fixed at one end



(e)

A flexible beam fixed at one end

Vibrations vs. Oscillations

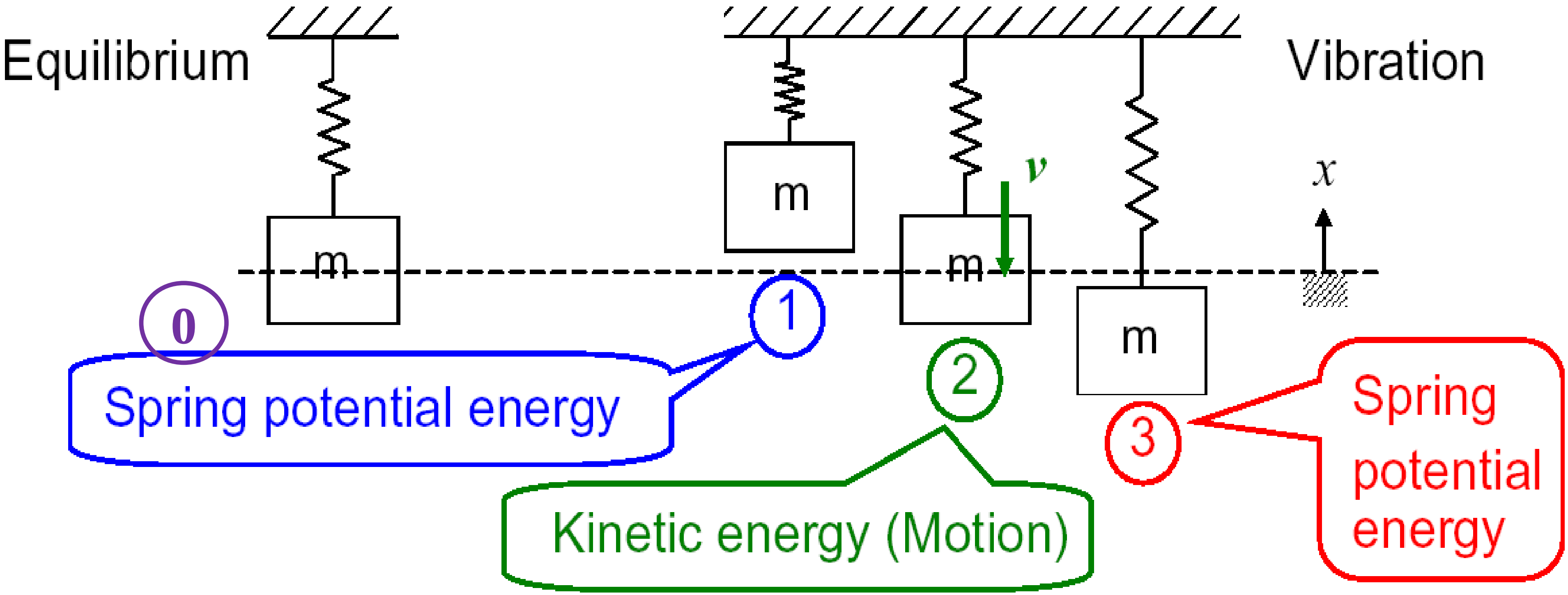
- **Vibration is "the rapid to and fro motion of an elastic /inelastic system whose equilibrium is disturbed"**

- *Vibrations are oscillations due to an elastic restoring force.*

- A flexible beam or string ***vibrates*** while a pendulum ***oscillates***.

- In most of the text books written on the subject, vibration and oscillations are interchangeably used

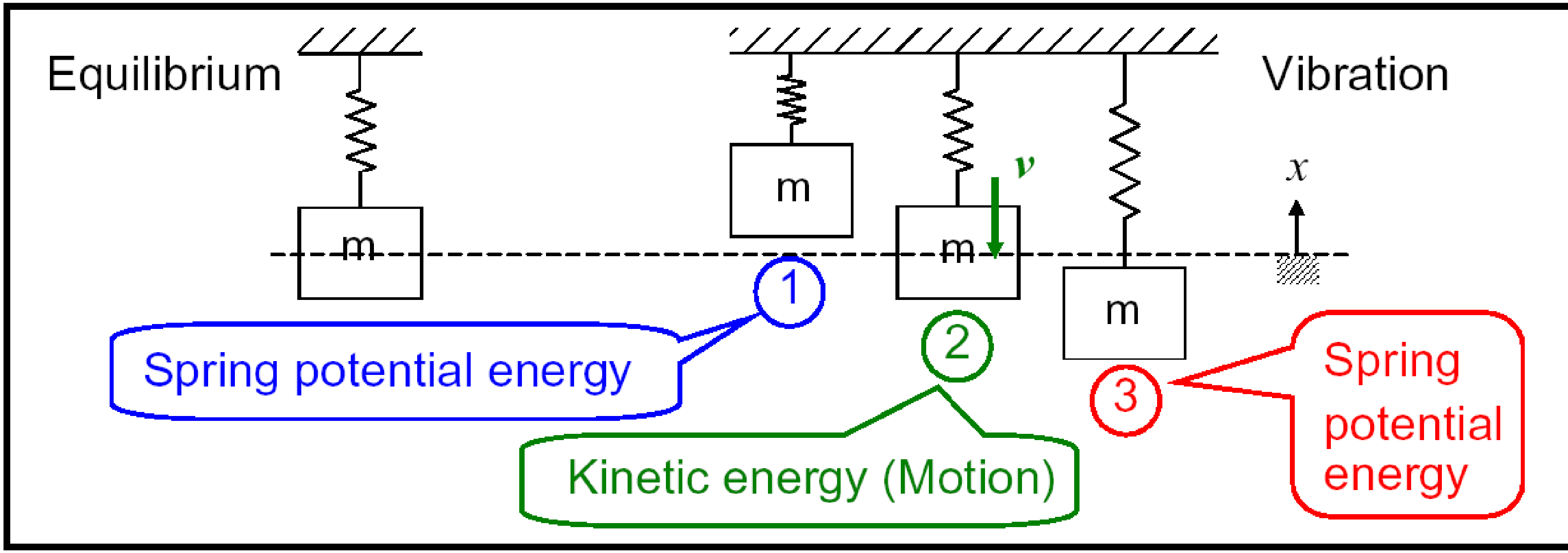
Physical Explanation of Vibration

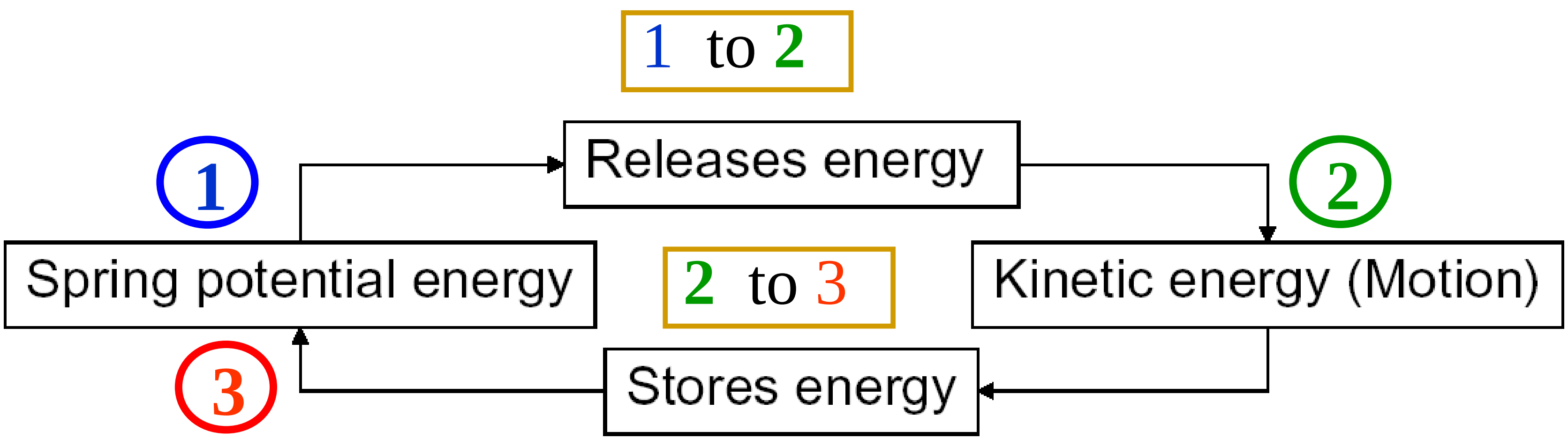


Activity….

Graphically represent the position of mass for positions 0,1,2, 3

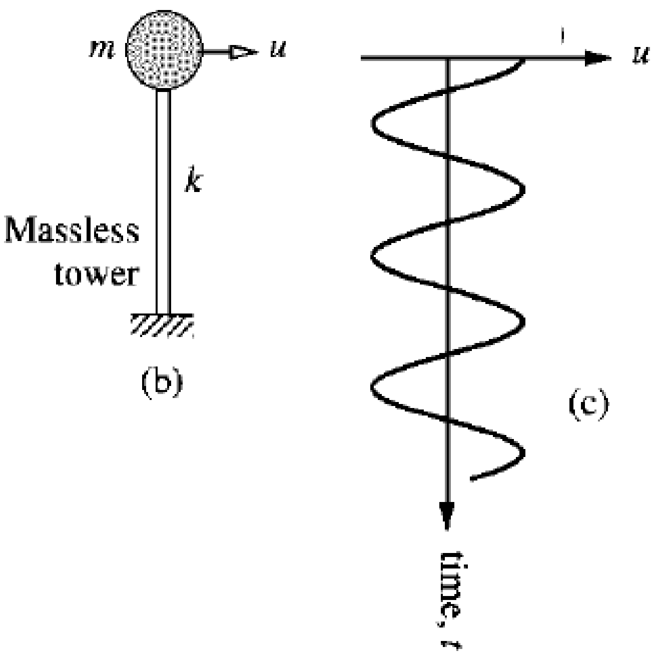
Physical Explanation of Vibration



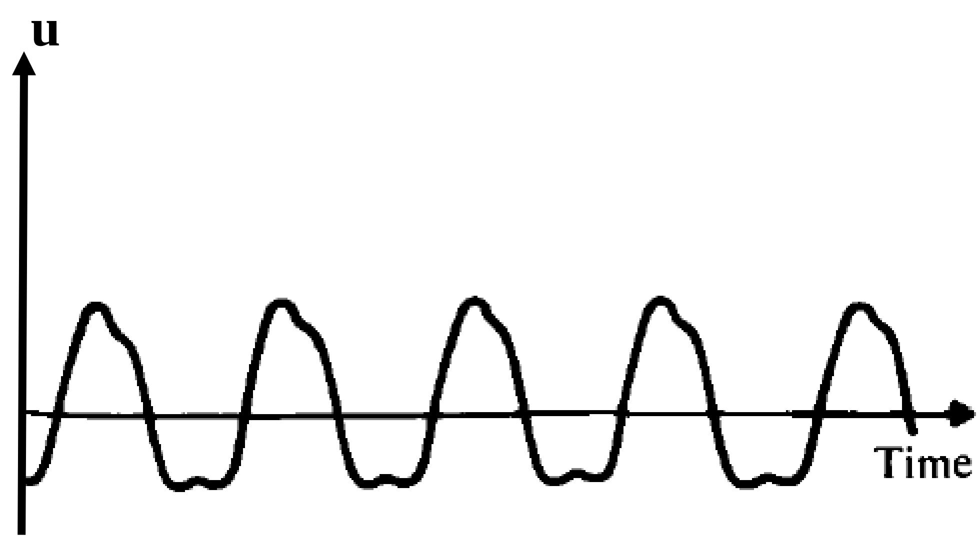


Periodic and Random vibrations

- The vibration can be Periodic (cyclic) or Random (arbitrary).
- If the motion is repeated after equal intervals of time, it is called ***Periodic motion***.
- The simplest type of periodic motion is Harmonic motion.

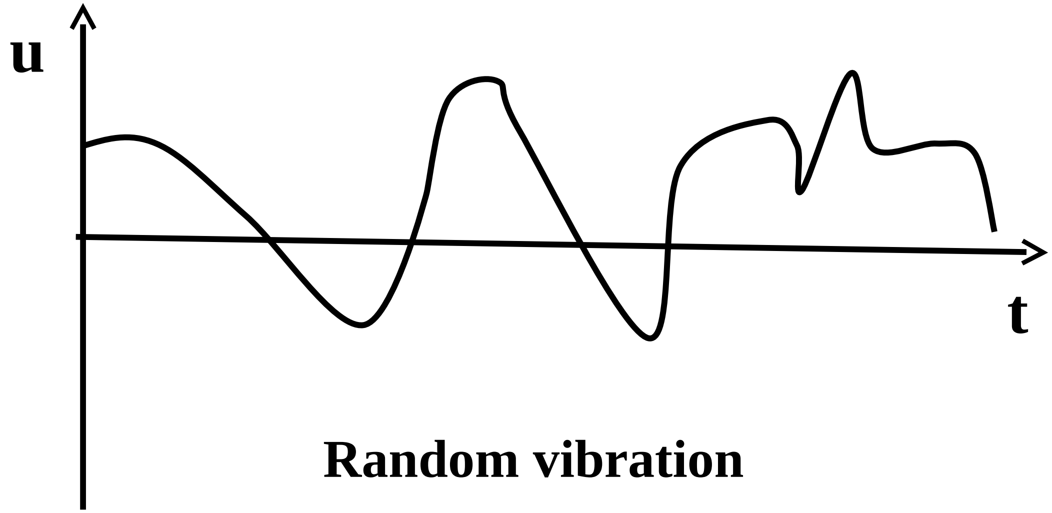


Periodic vibration (Harmonic vibration)



Periodic vibration (Non-harmonic vibration)

Periodic and Random vibrations

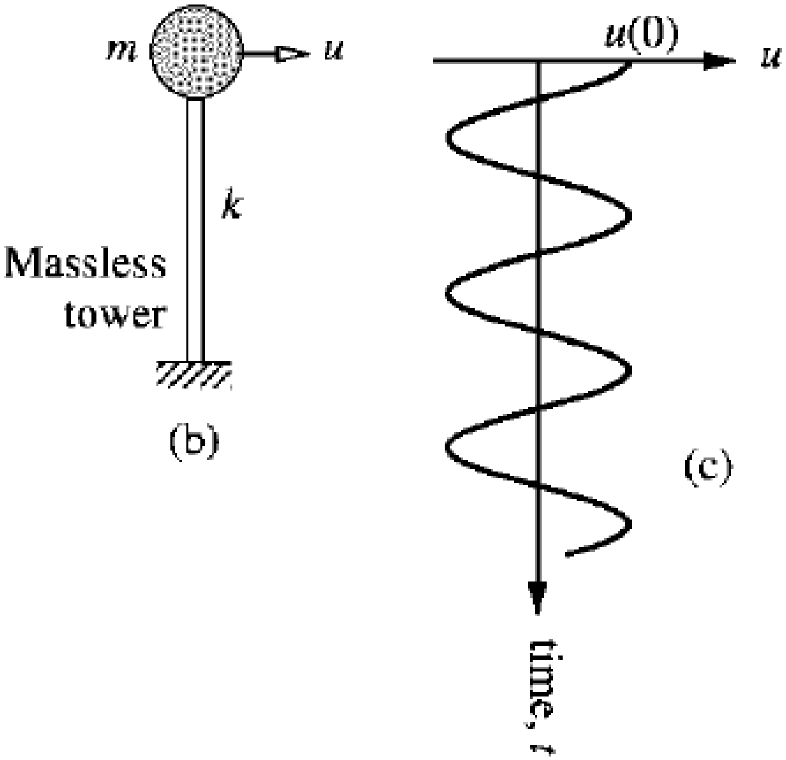


Random vibration

- Such type of vibrations are produced in a system due to wind, earthquake, traffic etc

Free vibrations vs. Forced vibrations

- When a structure vibrates without any externally applied forces, such as when it is pulled out of position, and then released.
- The vibration of strings on a musical instrument after they are struck is a common example of free vibration.



Free vibration of a SDOF lumped mass system when released after being stretched by a displacement u(0) at the top end .

Free vibrations vs. Forced vibrations

- Vibration of a system subjected to an external force is known is known as Forced vibration.
- The vibration that arises in machine such as diesel engines is an example of Forced vibration.
- As stated above vibration of a system in the absence of external force is known is known as Free vibration. Free vibration continues to occur after forced vibration. e.g., vibration of rotating machines continues to occur for some time after power supply is switched off. Similarly, a structure subjected to earthquake continues to vibrate for some time after there are no seismic waves to impart en

Undamped free vibration

If no energy is lost or dissipated in friction or other resistance during vibration, the vibration is known as ***Undamped vibration***

Undamped vibration is a hypothetical phenomena which help in providing an understanding of the Damped vibration.

Damped free vibration

In actual system the energy is always lost due to a number of mechanisms. Such type of vibration is known as ***Damped vibrations***

Damping

- Any energy that is dissipated during motion will reduce the kinetic and potential (or strain) energy available in the system and eventually bring the system to rest unless additional energy is supplied by external sources.

- The term ***Damping*** is used to described all types of energy dissipating mechanisms.

Damping

- In structures many mechanism contributes to the damping. In a vibrating building these include friction at steel connections, opening and closing of microcracks in concrete, and friction between the structures itself and nonstructural elements such as partition walls.

- Since there is considerable uncertainty regarding the exact nature and magnitude of energy dissipating mechanisms in most structural systems, the simple model of a ***dashpot*** is often used to quantify damping.

Damping

- The Dashpot or viscous damper is a ***'device'*** that limit or retard vibrations.
- Dashpot can be imagined as a cylinder fi ous fluid ar

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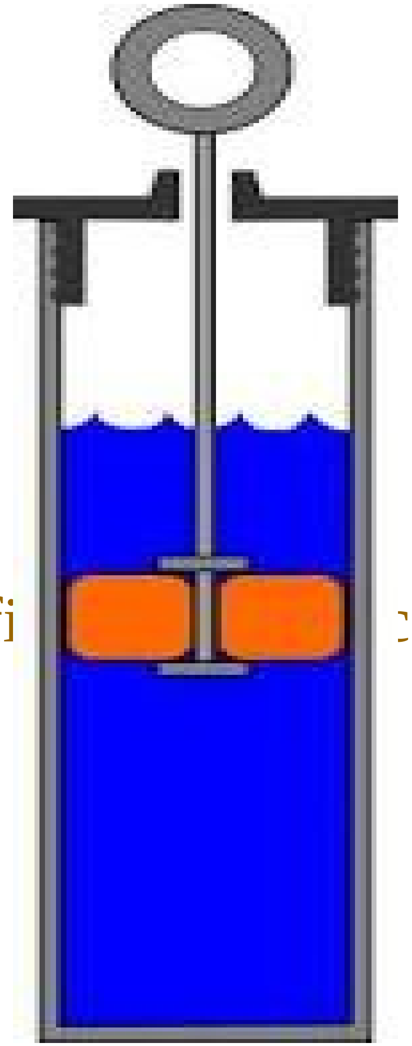
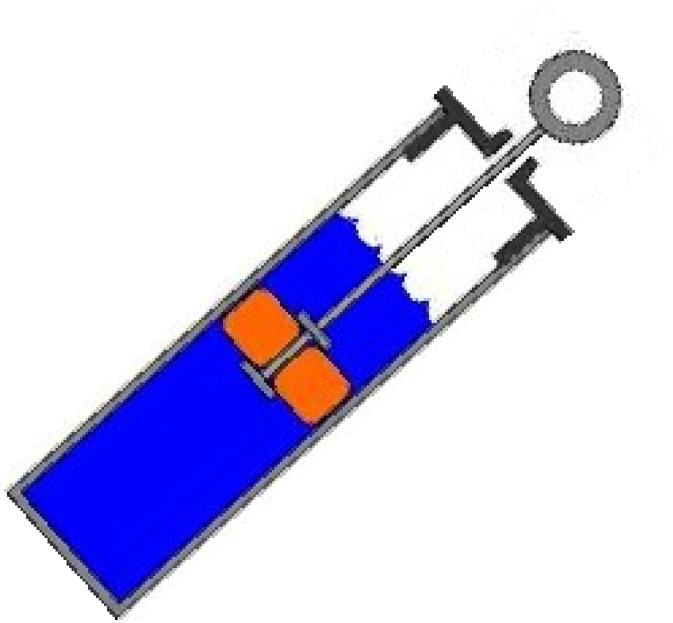
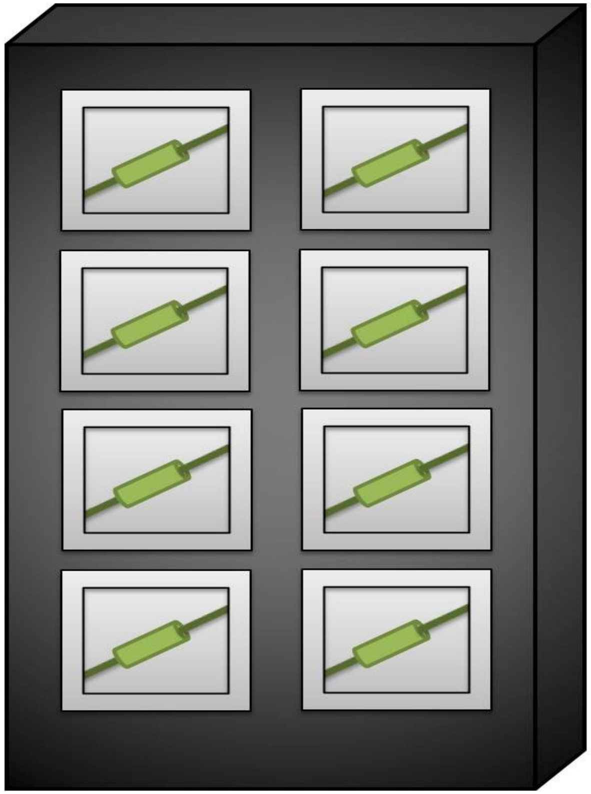


Simplified diagram of linear dashpot

Damping





Dash pots are ***imagined*** to exist diagonally in a building for dissipating energy .

Damping

- Simple dashpots as shown schematically in below given figure exert a force $\mathbf{f_D}$ whose magnitude is proportional to the velocity of the vibrating mass

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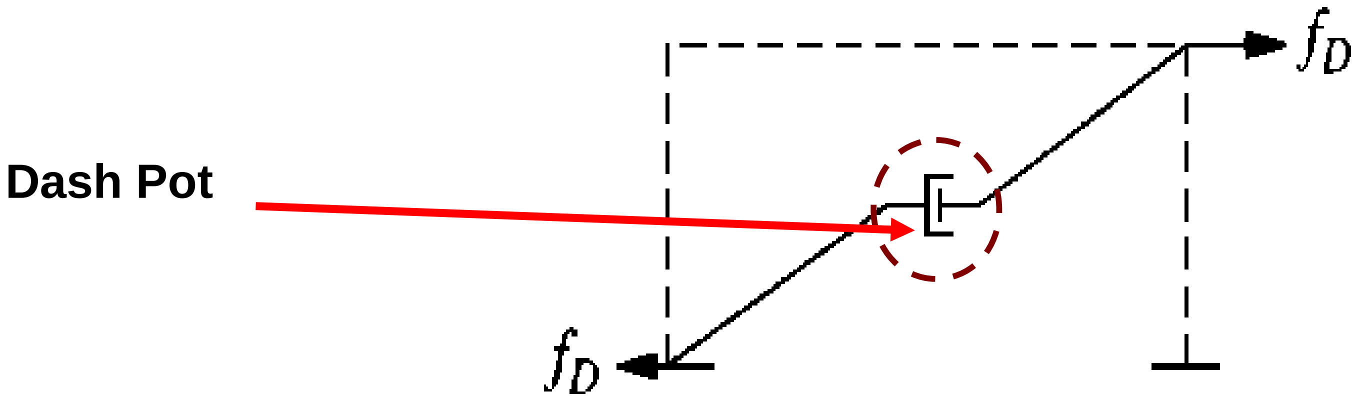

Damping

Figure **a** shows a linear viscous damper subjected to a force $\mathbf{f_D}$ along the DOF **u**. The internal force in the damper is equal and apposite to the external force $\mathbf{f_D}$ (Figure **b**). The damping force $\mathbf{f_D}$ is related to the velocity $\dot{u}$ across the linear viscous damper by:

$$f_D = c\dot{u}$$

Where the constant **c** is the *viscous damping coefficient*