Name :- Hamza khan Yousafzai

ID :- 7487

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Subject :- Differential Equation

Section :- B

Submitted to :- Mam Shomaila Mazhar

Q.no 1

(i)

The order of matrix Ab is $\underline{m \times n}$

(ii) The number of non-zero rows in an Echelon form is $\underline{\text{one}}$

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix than $a = \underline{8}$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1) \qquad \therefore i^2 = -1$$

$$= 2+1$$

$$= 3$$

(v) The matrix $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is Scaler Matrix

The given matrix A is a scaler matrix because the diagonal are zero

(vi) Solution of $\frac{dy}{dx} + 2xy = y = ?$

Solution:-

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x) \quad \text{Taking } y \text{ common}$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{dx} = (1-2x)\,dx$$

taking integration

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

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$\ln y = \int 1\,dx - \int 2x\,dx$

$\ln y = x - \frac{2x^2}{2} + C$

$\ln y = x - x^2 + C$

~~$\ln y = x$~~

$e^{\ln y} = x - x^2 + C$

$e^{\ln y} = e^{x - x^2 + C}$

$y = e^{x(1-x)+C}$

(Vii)

The order and degree of differential Equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is

=> Solution :-

Order = 1

Degree = 3

(viii) The order and degree of differential Equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is

Solution :-

Order = 2

Degree 1

(ix) The differential Equation $2\,dy/dx + x^2 y = 2x+3,\ y=(0)=5$ is ______ ?

$\Rightarrow$ Solution:- $2y' + x^2 y = x^2+3,\ y(0) = 5$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2}dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6} \left(\frac{x^2}{2}\right) y = \frac{1}{2} e^{x^3/6} (x^2+3)$$

$$Y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + C}{2e^{x^3/6}}$$

$$Y(0) = \frac{0+3}{2} = 3/2$$

$$Y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$

Ans

(X)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by C_1

$$1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1 \begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1 \begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$

Ans

Qno2

(A) Express the determiant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c

Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 \\ a^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3 - b^2)$$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^2cb^3 - a^3b^2c$

common abc

$\Rightarrow abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$

$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)]$

Ans

Q no 2

(B)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$\Rightarrow$ Solution :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\rightarrow |A-\lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determient

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \text{ —— } B$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda)$$

$$-(1)(-1)\big) - 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (2+\lambda-1) -$$
$$(+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda + 4 + \lambda$$
$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \text{ --- } (A)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \text{ —(b)}$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$- \left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$\Rightarrow \left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1) \right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\cancel{\lambda}\lambda^2 + 6\lambda - 8} \rightarrow c$$

Put a, b, c in B

$$(2-\lambda)\left[-\lambda^3 + 8\lambda^2 - 18\lambda + 8\right] - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8$$

$$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$$

$$= \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda + 6\lambda + 16\cancel{3} - 16$$

$$= \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

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⇒ By Synthatic Diversion we get

$$\lambda(-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization Method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda=4, \lambda=4$$

$$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4, \lambda_4=4}$$

Ans

Q no 3

$$(x^2+3y^2)dx-2xy\,dy=0$$
$$x=2,\ y=6$$

=> Solution:-

$$(x^2+3y^2)dx-2xy\,dy=0$$
$$\Rightarrow (x^2+3y^2)dx=2xy\,dy$$

Dividing both sides by $2xy\,dx$ we get

$$\frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx}=\frac{x^2}{2xy}+\frac{3y^2}{2xy}$$

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{x}{y}+\frac{3y}{x}\right]\longrightarrow A$$

Let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (b)$$

Put (b) in (a)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{xv} + 3\frac{vx}{x}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

multiplying both sides by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

multiplying bothsides by $\frac{v}{x(1+v^2)}$

We get

$$\frac{v}{1+v^2}dv = \frac{1}{x}dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}dv = \int \frac{1}{x}dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1 + v^2 = xe$$

$I + v^2 = xc$

Put $v = y/x$

$$1 + \left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2 + y^2 = x^3 c \longrightarrow ©$$

Put $x=2,\ y=6$ in ©

$$(4) + (36) = 8c$$

$$c = \frac{40}{8}$$

$\boxed{c = 5} \longrightarrow$ put in Eq ©

So

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking root on both sides

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}$$

Ans