Name : Nooriya Ilyas.
ID : 15770.
Semester : 2nd
BS (SE).

Q no.1 Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve the system.

$$\begin{bmatrix} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Solution:-
Writing the equations outside the matrice:-

$$\begin{aligned} x_1 + 7x_2 + 3x_3 &= 5 \\ x_2 &= 7 \\ x_3 &= -6 \\ x_4 &= 7 \end{aligned} \qquad \left[\begin{array}{cccc|c} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$$

★ In-order to remove $3x_3$ from row 1, we will multiply 3 by eq.3 and then subtract it from eq.1.

$3R_2 - R_1$:-

$$x_1 + 7x_2 + \cancel{3x_3} = 5$$

$$\underset{\ominus}{\pm} \cancel{3x_3} = \underset{\oplus}{\mp} 18$$

$$x_1 + 7x_2 = 23.$$

We get:-

$$\begin{aligned} x_1 + 7x_2 &= 23 \\ x_2 &= 7 \\ x_3 &= -6 \\ x_4 &= 7 \end{aligned} \qquad \left[\begin{array}{cccc|c} 1 & 7 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$$

★ Multiply 7 by eq.2 and subtract it from eq.1 to get the value of x_1.

$R_1 - 7R_2$:-

$$x_1 + \cancel{7x_2} = 23$$
$$\overset{+}{\ominus}\ \cancel{7x_2} = \overset{+}{\ominus}49.$$

$$x_1 = -26$$

we get:-

$x_1 = -26$

$x_2 = 7$

$x_3 = -6$

$x_4 = 7$

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & -26 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$$

verification:-

putting the values in eq.1.

$$x_1 + 7x_2 + 3x_3 = 5$$
$$(-26) + 7(7) + 3(-6) = 5$$
$$5 = 5$$

Since the result matches the right side of the original system, $\{-26, 7, -6, 7\}$ is the solution of the system.

Q2 (a) Find elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution:-

In order to transform the first matrix into the second, we will multiply -2 with row 2 and then add the row 2 to row 3.

$-2R_2 + R_3$:-

$-2 \times$ row 2 : $0 \quad 1 \quad -4 \quad 2$

$= 0 \quad -2 \quad 8 \quad -4$

Now adding it to row 3.

$$\begin{array}{rrrrr} & 0 & \cancel{-2} & 8 & -4 \\ + & 0 & \cancel{2} & -5 & -1 \\ \hline & 0 & 0 & 3 & -5. \end{array}$$

Putting the newly formed row 3 into the first matrix :-

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Hence we transformed the first matrix into the second matrix.

Now applying reverse row operation and turning the second matrix into the first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Multiplying 2 with row 2 and adding it to row 3.

$2R_2 + R_3$:-

$$\begin{array}{cccc} 0 & 2 & -8 & 4 \\ 0 & 0 & 3 & -5 \\ \hline 0 & 2 & -5 & -1 \end{array}$$

putting back the new row 3 obtained above, into the second matrix and transforming it to matrix no. first.

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}.$$

The elementary row operation to convert the first matrix,

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

into the second matrix, is

$-2R_2 + R_3$.

And the elementary row operation to convert the second matrix,

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

into the first matrix is

$2R_2 + R_3$

Q2) b) Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a). $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form.

Answer:-

This matrix is not in echelon form because the leading entry is not 1.

b). $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

Answer:- This matrix is in echelon form.

c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Answer:- It is not in reduced row echelon form because the leading entry is not 1.

d). $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Answer:- This matrix is not in reduced row echelon form because the row consisting of all zeros is not at the bottom of the matrix.

Q3) a) The row echelon form is used to solve the system of linear equations. What is the difference b/w the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give example.

Answer:-

Row echelon form:-

A matrix is said to be in echelon form when it satisfies the following conditions:-

1. The first non-zero element in each row (column), called the leading entry is 1.

2. Each leading entry is in a column (row) to the right of the leading entry in the previous row.

3. Rows with all zero elements, if any, are below the rows having a non-zero element.

Reduced row echelon form:-

A matrix is said to be in reduced row echelon form when it satisfies the following conditions.

1. The matrix satisfies conditions for a row echelon form.

2. The leading entry in each row is the only non-zero entry in it's column.

For example:-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced row echelon form is a type of matrix used to solve systems of linear equations.

Q3) b) Find an echelon form for the below matrix using row operations.

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\_First\_last \end{bmatrix}$$

Solution:-

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$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & -4 & 10 \end{bmatrix}$$

$$-2R_1 + R_2 = R_2$$

$$= \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ -7 & 0 & 0 \\ 1 & -4 & 10 \end{bmatrix}$$

$7R_1 + R_3 = R_3$:-

$$= \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 1 & -4 & 10 \end{bmatrix}$$

$-R_1 + R_4$:-

$$= \begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 0 & -9 & 2 \end{bmatrix}$$

$-1/2\, R_2$:-

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 35 & 56 \\ 0 & -9 & 2 \end{bmatrix}$$

$-35 R_2 + R_3$:-

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & -\frac{483}{2} \\ 0 & -9 & 2 \end{bmatrix}$$

$\frac{-2}{483} R_3$:-

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & -9 & 2 \end{bmatrix}$$

$9R_2 + R_4$:-

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 157/2 \end{bmatrix}$$

$\frac{-157}{2} R_3 + R_4$:-

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

this is the required echelon form.