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Section

B

Subject

Hydraulic Engineering

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Solution:- The pressure drop Δp is expected to depend upon the gate opening h , the overall depth d , the velocity V , density ρ and viscosity μ

List the relevant variables:

$\Delta p, h, d, V, \rho, \mu$

Write down dimensions

$$\Delta p \quad ML^{-1}T^{-2}$$

$$h \quad L$$

$$d \quad L$$

$$V \quad LT^{-1}$$

$$\rho \quad ML^{-3}$$

$$\mu \quad ML^{-1}P^{-1}$$

Number of variable $n=6$

Number of independent dimension $m=3$

Number of non dimensional groups $n-m=3$

Choose $m (=3)$ scaling variables:

geometric (l); kinematic time-dependent (V)

dynamical mass dependent (ρ)

Form dimensionless groups by non dimensionalising the remaining variables, Δp , h and μ .

$$\Pi_1 = \Delta p l^a V^b \rho^c$$

$$\begin{aligned} M^0 L^0 T^0 &= (ML^{-1}T^{-2})(L)^a (LT^{-1})^b (ML^{-3})^c \\ &= M^{1+c} L^{-1+a+b-3c} T^{-2-b} \end{aligned}$$

$$M: 0 = 1+c \quad \Rightarrow c = -1$$

$$T: 0 = -2-b \quad \Rightarrow b = -2$$

$$L: 0 = -1+a+b-3c \quad \Rightarrow a = 1+3c-b=8$$

$$\Rightarrow \Pi = \Delta p l^8 V^{-2} \rho^{-1} = \frac{\Delta p}{\rho V^2}$$

$\Pi_2 = \frac{h}{d}$ (by inspection since h is a length)

$$\Pi_3 = \mu d^a V^b \rho^c$$

$$\begin{aligned} M^0 L^0 T^0 &= (ML^{-1}T^{-1}) (L)^a (LT^{-1})^b (ML^{-3})^c \\ &= M^{1+c} L^{-1+a+b-3c} T^{-1-b} \end{aligned}$$

$$M: 0 = 1 + c \quad \Rightarrow c = -1$$

$$T: 0 = -1 - b + 0 \quad \Rightarrow b = -1$$

$$L: 0 = -1 + a + b - 3c \quad \Rightarrow a = 1 + 3c - b = -1$$

$$\Rightarrow \Pi_3 = \mu d^{-1} V^{-1} \rho^{-1} = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace Π_3 by

$$\Pi_3' = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence, dimensional analysis yields

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

i.e

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{b}{d}, \frac{\rho V d}{\mu}\right)$$

(a) Dynamic similarity require that all non-dimensional groups be the same in model and prototype i.e

$$\Pi_1 = \left[\frac{\Delta P}{\rho V^2}\right]_p = \left[\frac{\Delta P}{\rho V^2}\right]_m$$

$$\Pi_2 = \left[\frac{b}{d}\right]_n = \left[\frac{b}{d}\right]_m \quad \text{(Automatic of similar shape i.e geometric similarity)}$$

$$\Pi_3 = \left[\frac{\rho V d}{\mu}\right]_p = \left[\frac{\rho V d}{\mu}\right]_m$$

From the last, we have a velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p d_m}{(\mu/\rho)_m d_p}$$

$$= \frac{0.002}{800} \times \frac{1}{5}$$

$$= 0.5$$

Hence,

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ m s}^{-1}$$

(b) The ratio of quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{Velocity} \times \text{area})_m}$$

$$= \frac{V_p}{V_m} \left[\frac{d_p}{d_m} \right]^2$$

$$= 0.5 \times 5^2 = 12.5$$

(c) Finally, for the drop pressure,

$$\Pi_1 = \left[\frac{\Delta P}{\rho V^2} \right]_p = \left[\frac{\Delta P}{\rho V^2} \right]_m$$

$$\Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left[\frac{V_p}{V_m} \right]^2$$

$$= \frac{800}{1000} \times 0.5^2 = 0.2$$

Hence

$$\Delta P_p = 0.2 \times \Delta P_m = 0.2 \times 60 = 12.0 \text{ kPa}$$

Q No 2

Design a Practical Profile of gravity dam with following data

1. Maximum Depth of water in the reservoir

$$\text{is } (78) \quad H = 78 \text{ m}$$

2. Specific gravity of dam material is

$$G = 2.4$$

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(8) (5)

3. Allowable compressive strength for the dam masonry is 785 T/m^2

4. Height of wave is $H_w = 1.2 \text{ m}$

5. G and H_w is $\mu = 0.7$

$$C_u = 0$$

$$\begin{aligned} \text{Sol: } \textcircled{1} H_{\text{limiting}} &= \frac{G_{all}}{\gamma_w (G - C_u + 1)} \\ &= \frac{785 \times 1000}{1000 (2.4 - 0 + 1)} \end{aligned}$$

$$H_{\text{limiting}} = 230.88 > \text{Hw } 78 \text{ m}$$

So it is low Gravity Dam.

$\textcircled{2}$ Top width 'a'

$$\text{Free board} = 1.5 \text{ hwave} = 1.5 \times 1.2$$

$$\boxed{F.B = 1.8 \text{ m}}$$

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(7) (6)

$$\begin{aligned} \text{height of Dam} = HD &= HW + F.B \\ &= 78 + 1.8 \end{aligned}$$

$$HD = 79.8 \text{ m}$$

$$a = 14\% \text{ of } HD$$

$$a = 0.14 \times 79.8$$

$$a = ~~11.172~~ 11.172 \text{ m}$$

③ Base width (with out offset)

(i) - For No sliding criteria

$$b' = \frac{HW}{\mu C} = \frac{78}{0.7 \times 2.4}$$

$$b' = 46.42$$

~~46.42~~

(i) - For No tension criteria

$$b' = \frac{HW}{\sqrt{C}} = \frac{78}{\sqrt{2.4}}$$

$$b' = 50.34 \text{ m}$$

x Depth of Vertical Portion on U/S Side

$$h' = 2a \sqrt{G - Cu}$$

$$h' = 2 \times 11.172 \sqrt{2.4 - 0}$$

$$h' = 34.40 \text{ m}$$

$$\begin{aligned} \textcircled{5} \text{ Upstream offset} &= \frac{a}{16} \\ &= \frac{11.172}{16} \\ &= 0.69 \text{ m} \end{aligned}$$

⑥ Depth below the water level to the end of inclined portion in U/S

$$= 3.14 a \sqrt{G}$$

$$= 3.14 \times 11.172 \sqrt{2.4}$$

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$$= 54.02 \text{ m}$$

⑦ total width of the base of dam

$$b = b' + \frac{a}{16}$$

$$= 50.34 + 0.69$$

$$b = 51.03 \text{ m}$$

$$\textcircled{8} \tan Q = \frac{b'}{H} = \frac{50.34}{78} =$$

$$\textcircled{9} = \tan^{-1} 0.64$$

$$Q = 32.61^\circ$$

⑨ Depth of vertical position on Dis (from WL on U/S side)

$$\tan Q = \frac{a}{d'} =$$

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Depth of vertical position D/s

$$\tan \alpha = \frac{\alpha}{d'} = \frac{11.172}{d'}$$

$$\tan \alpha = \frac{11.172}{d'}$$

$$0.64d' = 11.172$$

$$d' = \frac{11.172}{0.64}$$

$$d' = 17.45 \text{ m}$$

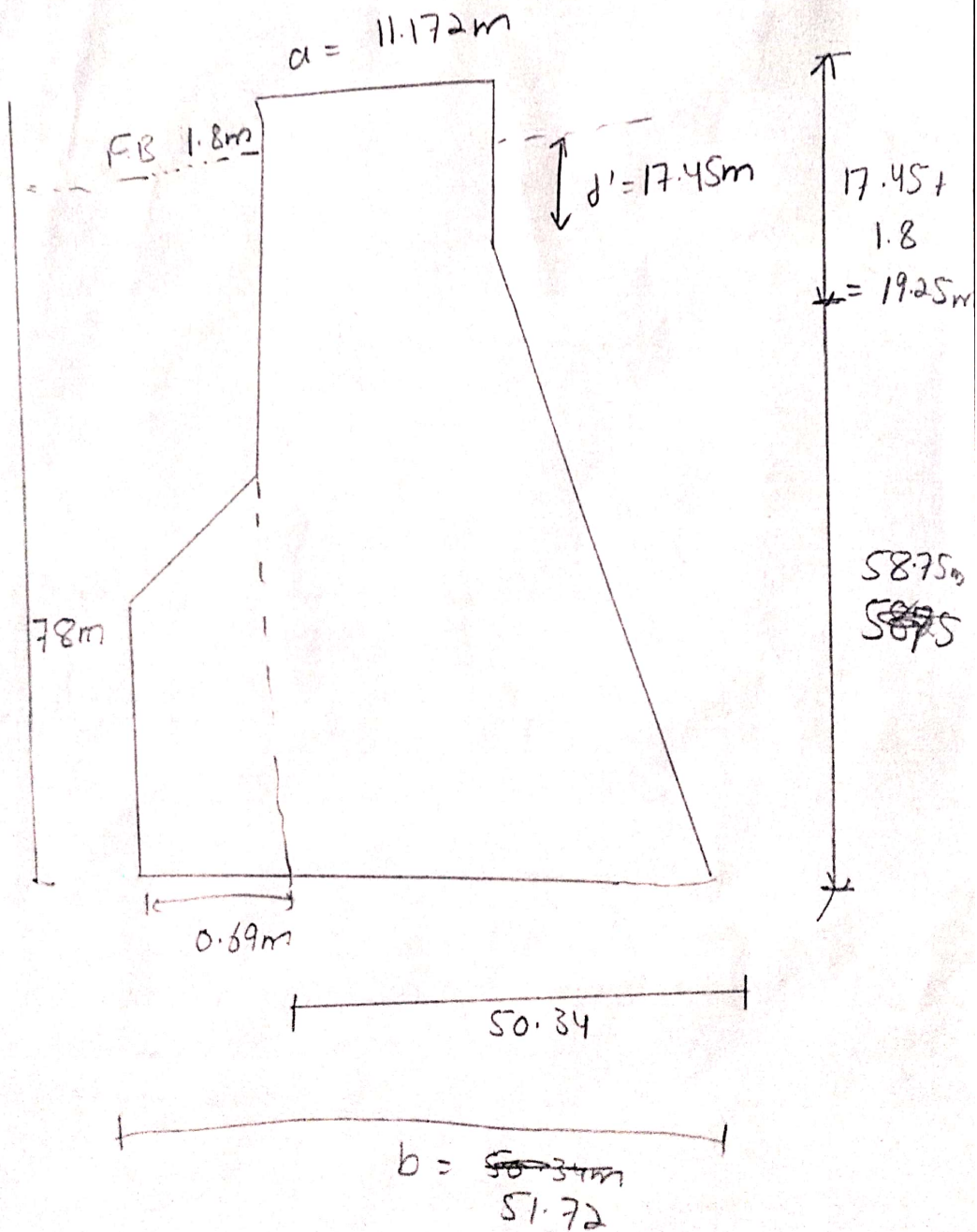
depth of vertical position

$$\begin{aligned} d &= d' + FB \\ &= 17.45 + 1.8 \end{aligned}$$

$$d = 19.25 \text{ m}$$

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Q No 4:

Ans The dynamic problems of liquid-solid interaction are greatly influenced by the sediment properties. The description of the latter, however is exceedingly complex and one is forced to make many simplifying assumptions. The first of which is its subdivision into cohesive and non-cohesive sediments.

Particle Size, shape and density

Particle Size. The most important physical property of a sediment particles is its size.

It has a direct effect on the mobility of particles and range from great boulders, which are rolled only by mountain torrents, to fine clays,

Which once stirred uptake days to settle. The size of particles can be determined in a number of ways. The nominal diameter refers to the diameter of a sphere of same volume as the particles, usually measured by the displaced volume of submerged particle. The sieve diameter is minimum length of the square sieve opening through which a particle will fall. The fall diameter is the diameter of an equivalent sphere of specific gravity = 2.65 having the same terminal setting velocity in water at 24°C.

Shape: - A poor form size, shape effects the transport of sediment but there is no direct quantitative way to measure shape and its effects.

McNown (1951) suggested a shape factor $S.F = c/\sqrt{ab}$ where c is the shortest of three perpendicular axes (a, b, c) of the particle. The shape factor is always less than unity, and values of 0.7 are typical for naturally worn particles.

Density:- Density of the particles is important and must be known. A large variation in density affects sediment transport by segregation, eg the armoring effects of heavy minerals on dune crests. The mass density of solid particles describes the solid mass per unit volume. The particles specific weight corresponds.

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to the solid weight per unit volume of solid. The specific weight of solid also equals the product of mass density of a solid particle ρ_s

times the gravitational accelerating, thus

$$\gamma_s = \rho_s \cdot g$$

Submerged specific weight of particles γ_s owing to Archimedes principle, the specific weight of solid particles γ_s submerged in a fluid of specific weight γ , equals to the difference between two specific weights thus.

$$\gamma_s = \gamma_s - \gamma = (\rho_s - \rho)g$$

where ρ is mass density of fluid.

The relation of specific weight of

fluid at a standard reference temperature defines the specific gravity δ with common reference to water at 4°C , the specific gravity of equal particles is

$$\delta = \frac{\rho_s}{\rho} = \frac{P_s}{P} = 2.65$$

The specific gravity is a dimensionless ratio of specific weights, and thus its value remains independent of the system of units.

Fall Velocity

The fall or settling velocity of a particle is assumed to be steady state motion. It is also a function of size, shape, density and viscosity

of fluid.

In addition it depends on extent of fluid in which it falls, on the number of particles falling and on the level of turbulence intensity. Turbulent condition occur when settling takes place in flowing fluid and can also occur when a dust of particles is settling.

For grain diameter d greater than 2mm , the fall velocity w can be approximated by the following equation

$$w = 3.32 \sqrt{d \text{ (mm)}}$$

Falling under the influences of gravity the particles will reach a constant velocity named the terminal when the

drag equal the terminal velocity, i.e. difference of solid and fluid

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$v_s = v = w$, we obtain following equation

$$C_D \frac{\pi d^3}{4} \frac{\rho w^2}{2} = \frac{\pi d^3}{6} \rho g (p_s - p)$$

$$w^2 = \frac{4}{3} \frac{1}{C_D} g d \left(\frac{p_s - p}{\rho} \right)$$

Thus the problem reduces to finding the value of drag coefficient C_D , for the particles in question for spherical particles of diameter d in viscous fluid of dynamic viscosity μ the drag coefficient is fairly well defined in laminar flow region for $Re < 0.5$ and approximately up to 1.0, where $Re = \frac{w d}{\nu}$.

$$F_D = 3 \mu d w, \text{ and } C_D = \frac{24}{Re}$$

$$C_D = \frac{24}{Re} \left(1 + \frac{3}{16} Re \right)$$

Czadd stein provides a more complete solution for Oseen approximation and gives the drag coefficient

$$C_D = \frac{24}{Re} \left(1 + \frac{3}{16} Re - \frac{19}{1280} Re^2 + \frac{71}{20480} Re^3 \right)$$

for $Re \leq 2$

The value of drag coefficient C_D , depends strongly on level of free stream turbulence,

Schiller and Nauman (1933) suggest a formula which give good results for $Re < 800$

$$C_D = \frac{24}{Re} (1 + 0.150 Re^{0.687})$$

Oseen (1961) suggests that the drag coefficient can be well represented by following

for $Re < 100$

$$C_D = \frac{24}{Re} \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}}$$

Q No 3 Using any hydraulic model and explain the concept of Dimensional analysis and similitude.

Ans In fluid dynamics, Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in static pressure or a decrease in fluid's potential energy. The principle is named after Daniel Bernoulli who published it in his book *Hydrodynamica* in ~~1738~~ 1738. Although Bernoulli deduced that pressure decreases when the flow speed increases, it was Leonhard Euler who derived Bernoulli's equation in its usual form in 1752. The principle is only applicable for

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isentropic flows. When the effects of irreversible processes (like turbulence and non-adiabatic processes (eg heat radiation)) are small and can be neglected

Bernoulli's principle can be applied to various types of fluid flow, resulting in various forms of Bernoulli's equation. There are different forms of Bernoulli's equation for different types of flow.

The simple form of Bernoulli's equation is valid for incompressible flows.

More advanced forms may be applied to compressible flows at higher Mach numbers.

Bernoulli's principle can be derived from the principle of conservation of energy.

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Derivation of Bernoulli equation

Derivation through integrating
Newton second law of motion

The simplest derivation is to first ignore gravity and consider constrictions and expansion in pipes that are otherwise straight, as seen in Venturi effect. Let the x axis be directed down to axis of pipe.

Flow velocity $v = \frac{dx}{dt}$. Define a

parcel of moving through a pipe with cross sectional area A , length of parcel is dx and the volume of parcel $A \cdot dx$. If mass density is ρ , mass of parcel is density multiplied by its volume $m = \rho A dx$. The change

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Pressure over distance dx is dp and

$$\text{flow velocity } v = \frac{dx}{dt}$$

Apply Newton second Law of motion

$$m \frac{dv}{dt} = F$$

$$PA dx \frac{dv}{dt} = -A dp$$

$$\rho \frac{dv}{dt} = -\frac{dp}{dx}$$

In steady flow the velocity field is constant

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt}$$

$$= \frac{dv}{dx} v$$

$$= \frac{d}{dx} \left(\frac{v^2}{2} \right)$$

with density ρ constant

$$\frac{d}{dx} \left(\rho \frac{v^2}{2} + p \right) = 0$$

by integrating with respect to x

$$\frac{V^2}{2} + \frac{P}{\rho} = C$$

Where C is constant

In the above derivation, no external work energy is involved. Rather Bernoulli's principle was derived by a simple manipulation of Secor Law.

Application

- An injector on a steam locomotive
- The pitot tube and static port on an aircraft are used to determine the air speed of aircraft.