Department of Electrical Engineering
Assignment
Date: 25/06/2020

Course Details

Course Title: Signals & Systems **Module:** 04
Instructor: ______ **Total Marks:** 50

Student Details

Name: ______ **Student ID:** ______

Q1.	(a)	**Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.	Marks 06+08
			CLO 3
	(b)	If $\mathbf{x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]}$ $\mathbf{h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]}$ **Produce** Y(z) and y[n]	
Q2.		$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$ **Retrieve** the Fourier series for the given function.	Marks 10
			CLO 3
Q3.		If $X(z) = 2z^2 + 2z \big/ (z^2 + 2z - 3)$ **Retrieve** x[n] using inverse Z-transform method.	Marks 10
			CLO 3
Q4.		**Express** the transfer function using the given data. $\mathbf{A} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\mathbf{C} = [1 \quad 2]$ $\mathbf{D} = [0]$	Marks 09
			CLO 3
Q5.		**Apply** Fourier transform on the signal, **x (t) = e** $^{-a\,\|t\|}$ **u (t)** where u (t) is a unit step function.	Marks 07
			CLO 3

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Course Title: Signals & Systems **Module:** 4th

Instructor: eng Mujtaba Ihsan sir **Date:** 25/06/2020

Department: Electrical engineering.

QNo1 :- Part B

if $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$
$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Produce $Y(Z)$ & $y(n)$

Solution:- $Y(z) = H(z) \cdot X(z)$

Find $Y[n]$

$$X(z) = 2 - 4z^{-2} + 2z^{-3}$$

$$H(z) = 3 + 1z^{-1} + 2z^{-2}$$

Now

$$Y(z) = H(z) * X(z)$$

$$Y(z) = (2-4z^{-2}+2z^{-3}).(3+z^{-1}+2z^{-2})$$

$$= 6+2z^{-1}+4z^{-2}-12z^{-2}-4z^{-3}-8z^{-4}+8z^{-3}$$
$$+2z^{-4}+4z^{-5}$$

$$= 6+2z^{-1}-8z^{-2}-2z^{-3}+6z^{-4}+4z^{-5}$$

To find $y[n]$ use the delay property

$$y[n] = 6\delta[n]+2\delta[n-1]-8\delta[n-2]-2\delta[n-3]$$
$$+6\delta[n-4]+4\delta[n-5]$$

QNo1 Part A

Show with a help of an equation that the differentiation of a function in time domain results in the multiplication by $j\omega$ in frequency domain.

Ans:-

Fourier transform of differention integration of continuos - Time

Let $x(t)$ be a continuos - time

Signal with a fourier transform of $x(j\omega)$.

i.e

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Differentiating both side with respect to (t)

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, \frac{d}{dt}\{e^{j\omega t}\}\, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\{e^{j\omega t} \cdot j\omega\}\, d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{j\omega X(j\omega)\}\, e^{j\omega t}\, d\omega$$

$\Rightarrow$

$$\mathcal{F}\left\{\frac{d}{dt}x(t)\right\} = j\omega X(j\omega).$$

Result:

we concluded that if a function is differentiated in time domain. it is multiplied by $j\omega$ in frequency domain.

Q No2 :-

$$f(x) = \begin{cases} -\pi/2 & -\pi \leq x \leq 0 \\ \pi/2 & 0 \leq x \leq \pi \end{cases}$$

Retrive the Fourier series for the given function.

Sol:-

$$f(x) = \begin{cases} -\pi/2 & -\pi \leq x \leq 0 \\ \pi/2 & 0 \leq x \leq \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_0^{-\pi} f(x)\,dx$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} f(x)\,dx + \int_0^{\pi} f(x)\,dx \right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\pi/2\,dx + \int_0^{\pi} \pi/2\,dx$$

$$= \frac{1}{2\pi} \left[-\pi/2 \int_{-\pi}^{0} 1\,dx + \pi/2 \int_0^{\pi} 1\,dx \right.$$

$$= \frac{1}{2\pi} \left[-\pi/2\, x \Big|_{-\pi}^{0} + \pi/2\, x \Big|_0^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\pi/2 \left[0 - (-\pi) \right) + \pi/2 (\pi - 0) \right]$$

$$= \frac{1}{2\pi} \left[-\pi/2\, \pi + \pi/2 (\pi) \right]$$

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$$= \frac{1}{2\pi}\left\{-\frac{\pi^2}{2} + \frac{\pi^2}{2}\right\}$$

$$= \frac{1}{2\pi}\left[\frac{0}{2}\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \int_{0}^{\pi} \pi/2 \cos nx\,dx$$

$$= \frac{1}{\pi}\left[+\pi/2\,\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \pi/2\,\frac{\sin nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\pi/2\{\sin n(0) - \sin n(-\pi)\} + \pi/2\{\sin n(\pi)\right.$$

$$\left. - \sin n(0)\}\right\}$$

$$= \frac{1}{n\pi}\left[-\pi/2\,(0) + \pi/2\,(0)\right]$$

$$= \frac{1}{n\pi}(0)$$

$$\boxed{a_n = 0}$$

Now

$$b_n = \frac{1}{\pi}\int_{\pi}^{\pi} \sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi/2\,\sin nx\,dx + \int_{0}^{\pi} \pi/2\,\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0}\sin nx\,dx + \pi/2\int_{0}^{\pi}\sin nx\,dx.\right.$$

$$= \frac{1}{\pi}\left\{-\pi/2\;\frac{-\cos nx}{n}\Big|_{-\pi}^{0} + \pi/2\;\frac{-\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\pi/2\left[-\cos n(0) + \cos n(-\pi)\right] + \pi/2\left[-\cos n\pi + \cos n(0)\right]\right]$$

$$= \frac{1}{n\pi}\left\{-\pi/2\left[-1 + \cos n(\pi) \cancel{+ \frac{\pi}{2}\left[-\cos n\pi + 1\right]}\right] + \pi/2\left[-\cos n\pi + \cos n(0)\right]\right]$$

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$$\pi/2 = \frac{1}{n\pi}\left\{-1\left[-1+\cos n(-\pi)\right]+1\left[-\cos n\pi+1\right]\right\}$$

$$= \frac{1}{2n}\left\{1-\cos n\pi - \cos n\pi + 1\right\}$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd.} \end{cases}$$

$$\left\{b_n = 4/2n\right\}.$$

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \cdots\cdots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \cdots\cdots$$

$$f(x) = (0) + (0)\cos x + (0)\cos 2x + (0)\cos 3x + \cdots\cdots$$
$$= \frac{4}{2}\sin x + (0)\sin 2x + \frac{4}{3(2)}\sin 3x + \cdots\cdots$$

$$\left\{\frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \cdots\cdots\right\}$$

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QNo 4:- Express the transfer functions using given data.

Sol:-

$$A=\begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B=\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 2 \end{bmatrix} \quad D=\begin{bmatrix} 0 \end{bmatrix}.$$

we know that

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H(s) = C(sI-A)^{-1}B + D$$

putting values.

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix}\left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\left\{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right\}^{-1}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = (s+2)s+1 = s^2+2s+2$$

$$\Rightarrow s^2+3s+2$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2+3s+2}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

so

$$H(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+3s+2}\begin{bmatrix} s & +0 \\ 1 & \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+3s+2}$$

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(5)

Q Nu5:- Apply Fourier transform on the signal, $x(t) = e^{-a|t|}\, u(t)$ where $u(t)$ is a unit step function. $a > 0$

Ans:-

~~CTFT (continuous-Time Fourier Transform)~~

Sol:-

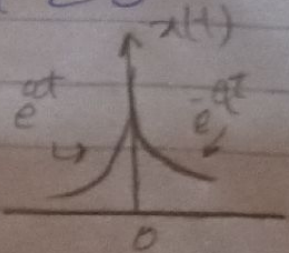
Fourier transform of the given function $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt.$$

Note:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$\therefore$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

(10) (?)

$$\Rightarrow \frac{e^{(a-jw)t}}{a-jw}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+jw)t}}{-(a+jw)}\Bigg|_{0}^{\infty}$$

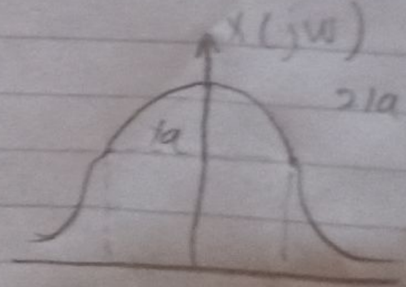
$$\Rightarrow \frac{1}{(a-jw)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+jw)}\left[e^{-\infty} - e^{0}\right]$$

$$\Rightarrow \frac{1}{(a-jw)}\left[1-0\right] - \frac{1}{a+jw}\left[0-1\right]$$

$$\Rightarrow \frac{1}{(a-jw)} + \frac{1}{(a+jw)}$$

$$\Rightarrow \frac{a+jw+a-jw}{a^2-(jw)^2}$$

$$\Rightarrow \frac{2a}{a^2+(jw)^2}$$



Q No 3 If $X(z) = \dfrac{2z^2+2z}{(z^2+2z-3)}$

Retrieve $x(n)$

Ans:

$$X(z) = \frac{2z^2+2z}{(z^2+2z-3)}$$

$$X(z) = \frac{2z(z+1)}{(z^2+2z-3)}$$

$$\frac{X(z)}{z} = \frac{2z+2}{z^2+2z-3}$$

$$\frac{2z+2}{z^2+2z-3} = \frac{A}{(z+1)} + \frac{B}{(z-3)} \quad \text{—— (1)}$$

or

$$2z+2 = A(z-3) + B(z+1)$$

Put $z = 3$ in (1)

$$2(3)+2 = A(3-3) + B(3+1)$$

$$8 = 4B$$

$$\boxed{B = 2}$$

(12)

put $z = -1$

$$2(-1) + 2 = A(-1-3) + B(\cancel{-1}+1)$$

$$0 = -A$$

$$\boxed{A = 0}$$

Now put A, B in (1)

$$\frac{2z+2}{z^2+2z-3} = \frac{0}{z+1} + \frac{2}{z-3}$$

$$X(z) \quad \frac{2z}{z-3}$$

inverse z-transform.

$$\boxed{x(n) = 2(-3)^n}$$