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Section B

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QNO # 01 :->

we genrally visit beach and if we stand on an ocean shore and take a snapshots of the wave, the Picture shows a Reguler pattern of Peaks and valleys in an instant of time. we see periodic vertical motion in space, with Respect to distance. If we stand in the water. we can feel the rise and fall of the water as the waves go by. we see periodic vertical motion in time. This beauty symmetry is expressed by the one-dimensional wave equation.

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

where w is the wave height x is the distance variable, t is the time variable. and c is the velocity with which the wave are propagated.

(2)

Show that the following function are all solution the wave equation by determining relevant partial derivatives.

(i) $w = \sin(x+ct) + \cos(2x+2ct)$

(ii) $w = \tan(2x+2ct)$

Solution:→

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

(i) $w = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \quad (1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

③

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= [-\sin(x+ct) - 4\cos(x+2ct)]$$

$$\frac{\partial^2 w}{\partial t^2} = c^2[-\sin(x+ct) - 4(\cos(2x+2ct))]$$

$$c^2 \cdot \partial w / \partial x^2$$

(ii) $w = \tan(2x - 2ct)$

$$\frac{\partial w}{\partial x} = \frac{1+2}{[1+(2x-2ct)^2]} = \frac{2}{[1+4x^2+4c^2t^2]}$$

$$\frac{\partial^2 w}{\partial x^2} = -2(1+4x^2-8xct+4c^2t^2)^{-2} \cdot (8x-8ct)$$

$$\frac{\partial w}{\partial t} = \frac{1+(-2c)}{1+(2x-2ct)^2} = \frac{-2c}{[1+4x^2+4c^2t^2+8xct]}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{-2c(-1)(4c^2+2t-8xc)}{[1+4x^2+4c^2t^2-8xct]}$$

(4)

$$\Rightarrow \frac{2c^2[4 + 8ct - 8x]}{[1 + 4x^2 - 8xct + 4c^2t^2]^2}$$

$$\frac{\partial^2 w}{\partial t^2} \Rightarrow \frac{-2(8x - 8ct)}{[1 + 4x^2 - 8xct + 4c^2t^2]^2}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2} \cdot c^2$$

Q No #02

Expand the following function in a fourier series.

$$F(x) = x \qquad -\pi < x \leq 0$$

$$= 2x \qquad 0 \leq x \leq \pi$$

Fourier series.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Now we will find a_0, a_n, b_n.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left|\frac{x^2}{2}\right|_{-\pi}^{0} + \frac{2}{\pi}\left|\frac{x^2}{2}\right|_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\cdot\frac{\pi^2}{2} - 0$$

$$a_0 = \frac{-\pi}{2} + \pi \Rightarrow \boxed{\frac{\pi}{2} = a_0}$$

1

2

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos x \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x \cos nx) \, dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right|_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$\Rightarrow \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{2}\right]$$

$$a_n \Rightarrow \frac{(-1)^n - 1}{\pi n^2}$$

(3)

So

$$a_n = \begin{cases} \dfrac{-2}{\pi n^2} & ; \text{ If } n \text{ is odd} \\ 0 & ; \text{ if } n \text{ is even} \end{cases} \quad -(2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x) \sin nx \, dx$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi}^{0} x \sin nx \, dx + \frac{2}{\pi}\int_{0}^{\pi} x \sin nx \, dx$$

$$\Rightarrow \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right|_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right|_{0}^{\pi}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi \cos n\pi}{n}\right] =$$

$$\frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

(4)

Now put These value of a_0, a_n, b_n in the fourier series.

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

Q No #03

$$y'' - 4y' + 13y = 8\sin 3x$$

$y(0) = 1 \quad y'(0) = 2$

Solve the initial value problem

Solution:→

$$y'' - 4y' + 13y = 8\sin 3x$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8\sin 3x$$

$$\left(\frac{d^2}{dx^2} - 4\frac{d}{dx} + 13\right)y = 8\sin 3x$$

Put $\frac{d}{dx} = D$

$$(D^2 - 4D + 13)y = 8\sin 3x.$$

The characteristic equation as

$$D^2 - 4D + 13 = 0$$

Now we use quadratic formula to find it Roots.

(2)

$\Rightarrow D^2 - 4D + 13 = 0$

here $a = 1,\ b = -4,\ c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{36 i^2}}{2}$$

$$m = \frac{4 \pm 6i}{2} = \frac{\cancel{2}(2 + 3i)}{\cancel{2}}$$

$$\boxed{m = 2 + 3i}$$

Hance Root are Complex

3

$$y_c = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

$$y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x)$$

Let $y_p = A\cos 3x + B\sin 3x$ — *

Differentiate w.r.t x.

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again defferentiate w.r.t x

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

Put in 1

$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x +$
$3B\cos^2 x) + B(A\cos 3x + B\sin 3x) = 8\sin 3x$

$\Rightarrow -9A\cos 3x - 12B\cos 3x + BA\cos 3x - 9B\sin 3x$
$+12A\sin 3x + 13B\sin 3x = 8\sin 3x$

(4)

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)$

$\sin 3x = 8\sin 3x$

Comparing Co-efficients.

$\sin 3x \Rightarrow 4B + 12A + 8$ — (a)

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$\boxed{A = 3B}$ — (b)

Put (b) in eq (a)

$\Rightarrow 4B + 12(3B) = 8$

$\Rightarrow 4B + 36B = 8$

$\Rightarrow 40B = 8$

$\Rightarrow \boxed{B = 1/5}$ — (c)

Put c in eq (b)

$\Rightarrow \boxed{A = 3/5}$

put c and d in (*)-eq.

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \quad \text{—— (B)}$$

The genral solution is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1\cos 3x + c_2 \sin 3x) + 3/5 \cos x$$
$$+ 1/5 \sin 3x \quad \text{—— (C)}$$

Now we Need to find the value of c_1 & c_2 for this

put $y(0) = 1$, $x = 0$, $y = 1$ — in (C)

$$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin 3(0) + 3/5 \cos(0)$$
$$+ 1/5 \sin 3(0)$$

$$1 = c_1(1) + c_2(0) + 3/5(1) + 1/5(0)$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - 3/5 \Rightarrow \boxed{c_1 = \frac{2}{5}}$$

(6)

Differentiate (C) w.r.t x

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \quad \text{—— (D)}$$

Put $y' = 2$, $x = 0$ $\qquad y'(0) = 2$

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$$

Put $y' = 2$, $x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$$

$$2 = c_1(2) + c_2(3) + 0 + \frac{3}{5}$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

Put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = 3/5 \qquad \boxed{c_2 = 3/15}$$

Put the value of c_1 and c_2 in General solution.

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + 3/15 \sin 3x\right) + 3/5 \cos 3x + 1/5 \sin 3x$$

$$\boxed{y = \frac{2}{5} e^{2x} \cos 3x + \frac{3}{15} e^{2x} \sin 3x + \frac{3}{5} \cos 3x + 1/5 \sin 3x}$$

This is the Required General solution.

Q No # 04

Solve :→ $(D^2 - DD')Z = \cos x \cdot \cos 2y$

$\Rightarrow (D^2 - DD')Z = \cos x \cdot \cos 2y$ — (a)

it is already in symbolic form.

Put A.E $\quad D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m \quad$ i-e $D = m, D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

There fore $C.F = f_1(y) + f_2(y+x)$

From eq (a)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cdot \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cdot \cos 2y$$

(2)

As $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$C.F = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2 - 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

By Genral Method.

$m = -1$; $y - x = c$

$$\Rightarrow \frac{1}{D+D'}\int [(2c + \sin(-c)] \, dx$$

$$\Rightarrow \frac{1}{D+D'}[2(x - \sin c)x]$$

Replacing $y - x = c$

$$\Rightarrow \int (2xc - x\sin c)\,dx \Rightarrow cx^2 - \frac{x^2}{2}\sin c$$

$$= x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^2$$

$$+ \frac{x^2}{2}\sin(x-y)$$

(3)

Hance The Required solution is.

$$Z = C.F + P.I = f_1(y-x) + x f_2(y-x) + x^2 y - x^2 \sin(x-y)$$