Iqra National University, Peshawar
Department of Electrical Engineering

Mid – Term Examinationsummer2020
Date:20/8/2020

Course Code:	MTH 101		**Course Title:**	Linear Algebra	
Prerequisite:	NA		**Instructor:**	HIMAYTULLAH	
Module:	1	**Program:** BEE	**Total Marks:**	30	**Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive **Name: Fawad Ahmad (13204)**

Q1.	(a)	. Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$. **Identify** the (3,2) entry of AB.	Marks 5 PLO1 C1
	(b)	**Label** the quadratic polynomial that interpolate the points (1,3), (2,4), (3,4)	Marks 5 PLO1 C1
Q2	(a)	. If A and B are $n \times n$ matrices where $\lvert A \rvert = 2$ and $\lvert B \rvert = -3$,calculate $\lvert A^{-1}\mathrm{B}^{\mathrm{T}} \rvert$.	Marks 5 PLO2 C2
	(b)	**Estimate** the linear system of equation $x + y + 2z = 1$ $x - 2y + z = -5$ $3x + y + z = 3$	Marks 5 PLO2 C2
Q3		Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$.	Marks 10 PLO2 C2

Name: FAWAD AHMAD – ID: 13204

Ques # 1 (A) Fawad Ahmad (13204)

Sol:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

Row 3 (A) and Column 2 (B)

$$\begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0 \times 4) + (1 \times -1) + (-2 \times 2)$$

$$= 0 + (-1) + (-4)$$

$$= 0 - 1 - 4$$

$$\boxed{= -5}$$ Answer

Question: 1 (B)

Que# 1 (B)

Sol :-

Points = (1, 3) (2, 4) (3, 4)

As $a_2 x_1^2 + a_1 x_1 + a_0 = y_1$

$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$

$a_3 x_3^2 + a_1 x_3 + a_0 = y_3$

Now $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (2, 4)$

$(x_3, y_3) = (3, 4)$ put in Above Eq

$a_2 (1)^2 + a_1 (1) + a_0 = 3$

$a_2 (2)^2 + a_1 (2) + a_0 = 4$

$a_2 (3)^2 + a_1 (3) + a_0 = 4$

$a_2 + a_1 + a_0 = 3$

$4a_2 + 2a_1 + a_0 = 4$

$9a_2 + 3a_1 + a_0 = 4$

$$A_6 = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 4 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -23 \end{bmatrix} \begin{matrix} R_2 - 4R_1 \\ R_3 - 9R_1 \end{matrix}$$

$R_2 - 4R_1$

$$\begin{array}{rrrr} 4 & 2 & 1 & 4 \\ -4 & 4 & 4 & 12 \\ \hline 0 & -2 & -3 & -8 \end{array}$$

$R_3 - 9R_1$

$$\begin{array}{rrrr} 9 & 3 & 1 & 4 \\ -9 & 9 & 9 & 27 \\ \hline 0 & -6 & -8 & -23 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_3 - 3R_2$$

$R_3 - 3R_2$

$$\begin{array}{rrrr} 0 & -6 & -8 & -23 \\ 0 & -6 & -9 & -24 \\ \hline 0 & 0 & 1 & 1 \end{array}$$

As $a_0 = 1$

So $-2a_1 + a_0 = -8$

Put $a_0 = 1$

$-2a_1 + 1 = -8$

$-2a_1 = -8 - 1$

$-2a_1 = -9$

$$\boxed{a_1 = -9/2}$$

Now $a_2 + a_1 + a_0 = 3$

$a_2 + (-9/2) + 1 = 3$

$a_2 - \frac{9}{2} - 1 = 3$

$a_2 = \frac{-9-2}{2} = 3$

$a_2 = \frac{11}{2} = 3$

$a_2 = 3 + \frac{11}{2}$

$a_2 = \frac{6+11}{2}$

$$\boxed{a_2 = \frac{17}{2}}$$

Question: 2 (A)

Q2 (A) Fawad Ahmad (13204)

Solution :-

$$|A^{-1}B^t| = |A^{-1}|\,|B^t|$$

Since $= \frac{1}{|A|}|B|$ Because $|B^t| = |B|$

So $|A^{-1}B^t| = \frac{1}{|A|}|B|$

$= \frac{1}{2} \cdot 3 = \boxed{\frac{3}{2}}$ Ans

Question: 2 (B)

Que # 2 (B)

Sol:- $x+y+2z=1$

$x-2y+z=-5$

$3x+y+z=3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right] R_3-3R_1$$

$$\begin{array}{r} 3\ 1\ 1\ /3 \\ -\ 3\ 3\ 6\ /3 \\ \hline 0\ -2\ -5\ /0 \end{array}$$

$$\underset{R}{\sim}\left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] R_2-R_1$$

$$\begin{array}{r} 1\ -2\ 1\ /-5 \\ -\ 1\ 1\ 2\ /-7 \\ \hline 0\ -3\ -1\ /-6 \end{array}$$

From Above matrix

$-3y-2=-6 \longrightarrow$ Eq (1)

$-2y-5z=0 \longrightarrow$ Eq (2)

multiply Eq (1) by Eq (2) and Eq (2) by (3)

we get $-6y-2z=-12 \longrightarrow$ Eq (3)

$-6y-15z=0 \longrightarrow$ Eq (4)

Now Subtract Eq (4) from Eq (3)

$$-6y - 2z = -12$$
$$-8y + 15z = 0$$
$$0 - 13z = -12$$

$$z = \frac{-12}{13}$$

Now from Eq (2)

$$-2y - 5z = 0$$
$$-2y - 5\left(\frac{-12}{13}\right) = 0$$
$$-2y + \frac{60}{13} = 0$$
$$+2y = +\frac{60}{13}$$
$$y = \frac{60}{13 \times 2}$$
$$y = \frac{60}{26} = \frac{30}{13} \qquad y = \frac{30}{13}$$

As $x + y + z = 1$

$$x + \left(\frac{-12}{13}\right) + \frac{30}{13} = 0$$
$$x = \frac{18}{13} = 0$$

$$\boxed{x = \frac{18}{13}}$$ Answer

Question: 3

Q(3)

Solution :-

$$|A| = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$= 3\begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6) + 2(-15-2) + (0-6)$$

$$|A| = -94$$

Now $A_{11} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ 17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$

END of Paper

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