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Question No 1

Part : A

Q: Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect x

Solution:: $\frac{d}{dx}\left(\frac{3x^4-2x^3+5}{x^3+1}\right)$

Use the quotient rule

$$= \frac{d}{dx}\left(\frac{3x^4-2x^3+5}{x^3+1}\right)$$

$$= \frac{(x^3+1)\left(\frac{d}{dx}(3x^4-2x^3+5)\right)-(3x^4-2x^3+5)\left(\frac{d}{dx}(x^3+1)\right)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

Simplify:

$$= \frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

$$= \frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$$

$$= \frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$ Ans:

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Question 1: part B:

Solution: $\frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$

Using the quotient rule;

$$= \frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$$

$$= \frac{(x^3-1)\left(\frac{d}{dx}\left((x^3+1)^2\right)\right) - (x^3+1)^2\left(\frac{d}{dx}(x^3-1)\right)}{(x^3-1)^2}$$

$$= \frac{(x^3-1)(2(x^3+1)(3x^2)) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$$

Simplify;

$$= \frac{(x^3-1)(2(x^3+1)(3x^2)) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$$

$$= \frac{3x^8 - 6x^5 - 9x^2}{(x^3-1)^2}$$

$$= \frac{3x^2(x^6-2x^3-3)}{(x^3-1)^2}$$

Answer: $\frac{3x^2(x^6-2x^3-3)}{(x^3-1)^2}$

Question No: 2

part: A

Solution: Find $\int \frac{1}{\sqrt{x^3}} \, dx =$

$$= \int x^{-3/2} \, dx = \frac{x^{-3/2+1}}{-3/2+1} = \frac{x^{-1/2}}{-\frac{1}{2}} =$$

$$= \frac{-2}{\sqrt{x}} + C.$$

Ans

Question No 2

Part B.

Solution.
$$\int \frac{1}{(8x+7)^8}\,dx$$

Substitute $u = 8x+7 \rightarrow \frac{du}{dx} = 8$ steps

$\rightarrow$ $dx = \frac{1}{8}\,du$:

$$= \frac{1}{8}\int \frac{1}{u^8}\,du$$

Now Solving

$$\int \frac{1}{u^8}\,du$$

Apply power rule

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \quad \text{with } n = -8$$

$$= -\frac{1}{7u^7}$$

Plug in Solved integrals

$$= \frac{1}{8}\int \frac{1}{u^8}\,d\overset{u}{\cancel{x}}$$

$$= \cancel{\frac{1}{8}}\; -\frac{1}{56u^7}$$

Undo Substitution $\Rightarrow u = 8x+7$;

$$= -\frac{1}{56(8x+7)^7}$$

= this problem is Solved

$$\int \frac{1}{(8x+7)^8}\,dx$$

$$= -\frac{1}{56(8x+7)^7} + C$$

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Question 3 part A:

Solution:

$$2x^2 - 8x + 6$$
$$2x^2 - 6x - 2x + 6$$
$$2x(x-3) - 2(x-3)$$
$$(x-3)(2x-2)$$

let $\dfrac{-x+9}{(x-3)(2x-2)} = \dfrac{A}{(x-3)} + \dfrac{B}{(2x-2)}$

xing by $(x-3)(2x-2)$ on Both Sides

$-x+9 = A(2x-2) + B(x-3)$, putting $x=3$

$-3+9 = A(2(3)-2) + B(3-3)$

$6 = A(6-2) + B(0)$

$6 = A(4) + 0$

$\cancel{A = \frac{9}{6}}$ $\quad A = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} = \boxed{A = \frac{3}{2}}$

Now putting $2x - 2 = 0$

$\boxed{x = 1}$

$-1+9 = A(2(1)-2 + B(1-3)$

$8 = A(2-2) + B(-2)$

$B = A(0) + B(-2)$

$B = \frac{-2}{8}$, $\boxed{B = -\frac{1}{4}}$

Thus $\int \frac{-x+9}{(x-3)(2x-2)} dx = \int \left(\frac{A}{(x-3)} + \frac{B}{(2x-2)} \right) dx$

$= \int \frac{-x+9}{(x-3)(2x-3)} dx = \int \left(\frac{2/3}{(x-3)} + \frac{\frac{-1}{4}}{(2x-2)} \right) dx$

$= \frac{2}{3} \int \frac{1}{(x-3)} dx - \frac{1}{4} \int \frac{1}{2x-2} dx$

$\boxed{\Rightarrow \text{Ans} = \frac{2}{3} \ln(x-3) - \frac{1}{4} \ln(2x-2) + C \quad \text{Ans}}$

Question No 4:

Solve each of the following matrix equations

a) $x + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

Solution

$$x = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-3 & 1-(-1) \\ -3-2 & 1-2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} \text{ Ans}$$

Part B : $x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

Solution.

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+(-4) & 6+(-8) \\ 1+(-2) & 5+0 \end{bmatrix}$$

$$x + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} -2-(-1) & -2-0 \\ -1-0 & 5-2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

Ans

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Q4:

Part 3

$$x + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Solution

$$x + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$x + \cancel{2I} - \cancel{2I} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} (2\times1) & 2\times0 \\ (2\times0) & (2\times1 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} (3-2) & (-1-0) \\ (1-0) & (2-2) \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

Ans

Question 5

Solution: if $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

find $= A^2 + BC$

Solution. Find $= A^2$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1) + 4(2) & 1(4) + 4(1) \\ 2(1) + 1(2) & 2(4) + 1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 8 & 4 + 4 \\ 2 + 2 & 8 + 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} \rightarrow (1)$$

Find BC

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3(1) + 2(0) & -3(0) + 2(2) \\ 4(1) + 4(0) & 4(0) + 4(2) \end{bmatrix} = \begin{bmatrix} -3 + 0 & 0 + 4 \\ 4 + 0 & 0 + 8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 \\ 4 & 8 \end{bmatrix} \text{———} (2)$$

Now Find. $A^2 + BC$

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 9+(-3) & 8+4 \\ 9+4 & 9+8 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 17 \end{bmatrix}$$