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Section = A

Subject = Differential Equations

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Exam

Date

QNo1 Solve the Initial value problem

$$\frac{dy}{dx} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

Sol $\Rightarrow$

$$\frac{dy}{dx} = \frac{e^y e^t}{\cos(y)}(1+t^2)$$

$$e^{-y}\cos(y)\,dy = e^{-t}(1+t^2)\,dt$$

Integration on both side.

$$\int e^{-y}\cos(y)\,dy = \int e^{-t}(1+t^2)\,dt$$

$$\frac{e^{-y}}{2}\sin(y) - \cos(y) = -e^{-t}(t^2+2t+3)$$

Applying the Initial Condition gives.

$$\tfrac{1}{2}(-1) = -(3) + c \quad c = 5/2$$

Therefore the Implicit Solution is.

$$\frac{e^{-y}}{2}\sin(y) - \cos(y) = -e^{-t}(t^2+2t+3)$$

It is not possible to find an explicit solution for this problem.

QNo2

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right.\,dy=0$$

Sol $\Rightarrow$ $\left(\sqrt{x+y}+(\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$

$$\left((\sqrt{x+y})+\sqrt{x-y}\right)dx=\left((\sqrt{x+y})-\sqrt{x-y}\right)dy=0$$

$$\Rightarrow \left((\sqrt{x+y})+\sqrt{x-y}\right)dx=\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

$$\frac{dy}{dx}=\frac{(\sqrt{x+y})+(\sqrt{x-y})}{(\sqrt{x+y})-(\sqrt{x-y})}$$

$$\frac{dy}{dx}\Rightarrow\frac{\cancel{\sqrt{x}}\left(1+\frac{y}{\sqrt{x}}\right)+\cancel{\sqrt{x}}\left(1-\frac{y}{\sqrt{x}}\right)}{\cancel{\sqrt{x}}\left(1+\frac{y}{\sqrt{x}}\right)-\cancel{\sqrt{x}}\left(1-\frac{y}{\sqrt{x}}\right)}$$

$$\frac{dy}{dx}\Rightarrow\frac{\left(1+\frac{y}{\sqrt{x}}\right)+\left(1-\frac{y}{\sqrt{x}}\right)}{\left(1+\frac{y}{\sqrt{x}}\right)-\left(1-\frac{y}{\sqrt{x}}\right)}$$

x

Taking Sequar roots. $\delta 7$ y

$$\frac{dy}{dx} = \frac{\left(1+\frac{\sqrt{y}}{\sqrt{x}}\right)+\left(1-\frac{\sqrt{y}}{\sqrt{x}}\right)}{\left(1+\frac{\sqrt{y}}{\sqrt{x}}\right)-\left(1-\frac{\sqrt{y}}{\sqrt{x}}\right)}$$

$g(x) = \frac{\sqrt{y}}{\sqrt{x}}$ form

$v = \frac{y}{x} \Rightarrow y = vx$

W.r.t. x

$$\frac{dy}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ put equation.

$$v + x\frac{dv}{dx} = \frac{(1+\sqrt{v})+(1-\sqrt{v})}{(1+\sqrt{v})-(1-\sqrt{v})}$$

$$\frac{dy}{dx} = \frac{(1-1\sqrt{v})^2}{1-(\sqrt{v})^2} \Rightarrow \frac{1-v}{1+v}$$

$$v + x\frac{dv}{dx} = \frac{1-v}{1+v} - v$$

$$x\frac{dv}{dx} = \frac{1-v-(1+v)}{1+v} \Rightarrow \frac{1-v-v+v^2}{1+v}$$

$$x\frac{dv}{dx} \Rightarrow \frac{1-2v+v^2}{1+v}$$

$$x\frac{dv}{dx} = \frac{v^2-2v+1}{1+v}$$

$$\frac{dv}{dx} = \frac{v^2-2v+1}{1+v}\,x$$

$$\int\frac{dx}{x} = \int\frac{1+v}{v^2-2v+1}$$ Multipling by 2 Nomavator or Right Si

$$= \int\frac{dx}{x} = \int\frac{2v+2}{v^2+2v+1}$$

$$\ln x = \ln(v^2+2v+1)$$

$x = v^2+2v+1$ Put value of v

(b)

$$x = \left(\frac{y}{x}\right)^2 + 2\left(y/x\right) + 1$$

$$\boxed{x = \frac{y^2}{x^2} + \frac{2y}{x} + 1}$$

Q No 3 $(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

Sol $(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

$\Rightarrow \quad f(D)y) = f(x)$

As it is non-homogenous linear equation.

So solution will be

$y = y_c + y_p \rightarrow (1)$

Complementry solution y_c

$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D = 0}$

$D^2 + 1 = 0 \rightarrow D^2 = -1$

$D = \sqrt{-1} \Rightarrow \boxed{D = i}$ or $\boxed{D = 0 + i}$

Roots are real and complex

$y_c = C_1 e^{0x} \, e^{0x}(C_2 \cos x + C_3 \sin x)$

$y_c = C_1 + C_2 \cos x + C_3 \sin x$

$y_p = \frac{1}{f(D)} f(x)$

$y_p = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

So $f'(D) = 4D^3 + D2$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again differentiating

$f''(D) = 12D + 2$

So for $D = 0$

$f''(0) \quad 12(0) + 2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow \quad y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2}\, 2\cos x$$

Putting $D = 0$ in all

$$y_p = \frac{x^2\, 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So

putting in Equation (i)

$$y = c_1 + c_2 \cos x + c_3 \cos x + \frac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \tfrac{3}{2} x^4}$$